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Modeling Credit Risk for Small and Medium-Sized Enterprises

Evidence from Norway

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Abstract

Primarily, this paper investigates the determining factors of default in the Norwegian small and medium-sized enterprises (SMEs). Using logit regressions on a database comprises over 280,000 Norwegian firms (with sales less than 500 million kroner, and employees less than 250 persons), three default prediction models are developed. These three models designed to predict default event in one, three, and five year from now based on the today's available information. These models have out of sample prediction powers which are approximately 15% (on average) higher than the models which are available for Norwegian SMEs. A secondary objective of this paper is to examine the proposed models' ability to decrease bank capital requirements based on the latest Basel Capital Accord's guidelines for SMEs. Throughout breakeven analyses, for any combination of SMEs (as retail customers and corporates) in banks' portfolios, all models show lower capital requirements than the one suggested by the Basel III.

Furthermore, the Basel III suggested a one-year default probability model as the basis for capital requirements calculation under the Internal Rating Based (IRB) approach. By a simulation over a sample of randomly selected SMEs, capital requirements are calculated using probabilities resulted from the one-year model and mixture of the one, three, and five-year models (corresponding to maturities of intended loans). This simulation confirms that using one-year probabilities of default for longer maturities slightly underestimates the calculated capital requirements under IRB approach.

Keywords: SME finance; Modeling credit risk; Basel III; Bank capital requirements; IRB approach

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Introduction

Risk of loss results from a debtor's inability to repay a loan or any other credit line has been defined as credit risk[1]. When a debtor has failed to fulfil her contractual obligations, or has violated a condition of loan contract, she defaults. Today's highly competitive financial market requires up-to-date financial knowledge supported by quantitative skill to apply mathematical modelling techniques for successful risk management.

Small and medium-sized enterprises (SMEs) are believed as the main block of the economy for many countries worldwide. SMEs form more than 97 percent of the total number of enterprises for OECD[2] members (Altman and Sabato, 2005). According to a report[3] published by OECD, between 60 to 70 percent of employments for most of the OECD's member countries is being handled by SMEs. However, they often confront problems for financing; for instance, higher interest rates due to their relatively higher profitability and growth variances; and credit rationing as a consequence of collateral scarcity[4].

SMEs are appeared to be dissimilar from large corporations for the credit risk measurement matter. They are more risky than large firms, however, large corporations have higher asset correlation between them than small and medium-sized enterprises (Dietsch and Petey, 2004). Moreover, small enterprises often have less transparent information compare to large corporations, for two main reasons; small businesses often have no credible financial information resulted from lack of certified audited financial statements, also there is no market prices or publicly available rating for them as they usually do not have publicly traded equity or debt (Berger and Frame, 2007). Altman and Sabato (2007) suggest that having separate default prediction models for SMEs and large corporates, instead of using a model established

[1] Basel Committee on Banking Supervision, & Bank for International Settlements, (2000).

[2] OECD stands for "The Organization for Economic Co-operation and Development". Norway is a member of OECD since 4 July 1961.

[3] "Small Businesses, Job Creation and Growth: Facts, Obstacles and Best Practices" available at: https://www.oecd.org/cfe/smes/2090740.pdf.

[4] According to a research carried out by Ghimire and Abo (2013) SMEs' inadequate collateral is one of the two major constraints that restrain the flow of credit from banks. The other issue is information asymmetry.

for large corporates default prediction on SMEs data, will consequence in more prediction power and therefore a better performance of the corporate portfolio.

The main focus of this research is to analyze a comprehensive group of financial ratios of Norwegian SMEs along with number of qualitative indicators for finding the most efficient predictors to estimate SMEs' credit worthiness. One of the first studies that exclusively focused on SMEs' credit risk modelling was carried by Edmister (1972). He used multivariate discriminant analysis (MDA) for analyzing 19 financial ratios on a sample of SMEs obtained from Small Business Administration (SBA)[5] data over the period of 1954-1969. Altman and Sabato (2005, 2007) expanded and improved Edmister's research using the Basel II SME definition (enterprises with sales less than 50 million Euros) for the first time. They applied logit regression analyses in their both studies to predict one-year[6] probability of defaults (PD) over samples of American, Italian, and Australian SMEs. However, they had no opportunity to employ qualitative indicators due to their data limitation. Lehmann (2003) and Grunet, Norden and Weber (2005) found out that using qualitative variables as predictors can improve default prediction models. Behr and Guttler (2007) on their study on German SMEs showed that qualitative predictors are statistically good predictors of SMEs failures using logistic regression.

One motivation of this research is to figure out if using one-year PD model, which is required clearly by the latest Basel to calculate capital requirements[7] under Internal Rating Based (IRB) approach, will result differently using more specific PD models for credit risk assessment. That is, using three-year, and five-year PD models for calculating capital requirements for loans with maturities of three-year and five-year, respectively. A five-year PD model is already available for Norwegian SMEs by a study carried out by Moody[8].

This paper's analyses are carried out on relevant subsamples of 255,063 small and medium-sized enterprises (all with sales less than 500 million Norwegian Korner, and less than

[5] SBA is a United States government organization that provides support to small businesses and entrepreneurs.

[6] Using one-year PD was suggested by "Basel Committee on Banking Supervision", June 2004.

[7] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 67 and 121.

[8] Moody's study for small and medium-sized firms in Norway, "MOODY'S KMV RISKCALC V3.1", July 2006.

250 employees[9]), including 22,898 defaulted SMEs over period of 1995 to 2015[10]. In the next chapter, a summary of the most relevant literature about default prediction is represented. First, methodologies are discussed, and the choice of employing logistic regressions to construct models exclusively for small and medium-sized enterprises is vindicated. Then, a survey of the recent SMEs' studies and their finding is provided. In the third chapter, research questions and methodology applied in this research are explained. In Chapter 4, data that is used in this research, and sampling steps are described. Then, explanation of variable selection steps is given, and finally empirical results are represented in Chapter 5. Limitations are discussed in Chapter 6. Three different default probability prediction models are developed in this study. I found that using specific model for different PD applied on specific sample (that only includes the SMEs that can receive specific loans) results in higher capital requirements compared by using a general one-year PD models on samples include all SMEs. However, when Internal Rating Based (IRB) approach is implemented all models demonstrate lower capital requirements for Norwegian SMEs compares to the Basel III minimum capital requirements. This finding can potentially result in lower cost of debts for SME customers. Another finding is that using a one-year default probability model to predict defaults over a longer period (suggested by the Basel III) results in a spuriously lower default probabilities, and consequently, lower capital requirements for loans with maturities longer than one year. Using specific models for different maturities on a sample of only relevant SMEs[11] will give a more accurate default probability, and therefore, a better estimate of minimum capital requirements. More detailed conclusion is available in Chapter 7.

[9] OECD definition of SME available at: https://stats.oecd.org/glossary/detail.asp?ID=3123; Exchange rate is approximated at 10 NOK for each Euro.

[10] The data that is used in this research is the Norwegian Corporate Accounts (Working Paper No. 11/16) collected by SNF and NHH (Berner, Mjøs, & Olving, 2016). However, all the sampling processes, data cleaning, data restrictions, and analyses have been done independently from SNF.

[11] Given that loans with longer maturities than 1 year may not be available for small-sized businesses in the market, including them in the capital requirement calculation for medium-sized enterprises will introduce some error. First, the default frequency for relatively smaller enterprises is higher. On the other hand, enterprises with relatively bigger sizes which can have access to longer maturity and higher value loans will have higher exposures in default events. This higher exposure requires higher capital requirements to cover the loss at default. However, these two variations cannot accurately neutralize each other in the final capital requirements calculation, and as a result there would be always some unwanted error in the estimates using a one-year default probability model to predict relatively longer maturities.

2. Literature Review

In this chapter, some of the most important studies about the default prediction methodologies are reviewed. In the first part, the most employed statistical techniques that are used for developing credit risk assessment models in previous studies is summarized. Then studies that focused on the credit risk modelling for small and medium-sized enterprises are discussed.

2.1 Default prediction literatures

There is a tremendous literature available that investigating default prediction methodologies. During the last 50 years, many researchers have studied various practical statistical techniques for predicting default probabilities for businesses and individuals. Univariate and multivariate models using a group of financial ratios for predicting business failure were developed by Beaver (1966, 1968) and Altman (1968). A dichotomous classification test was used by Beaver to discover the error rates a potential creditor would face whether the creditor classified firms as failed or non-failed based on individual financial ratios. Using 14 financial ratios, his model accurately classified 78% of sample (consisting 158 firms: 79 failed and 79 non-failed firms) five year prior to failure. For solving inconsistency problem related to the Beaver's univariate analysis, and testing a more comprehensive financial profile of enterprises, Altman (1968) used multiple discriminant analysis (MDA). He started with 22 financial ratios, and ended up using a weighted combination of five financial ratios. His results were 95% effective in detecting future default one year ahead of the default event on a sample of 66 firms (consisting 33 failed and 33 non-failed coaptations). However, by increasing the number of year prior to bankruptcy, the predictive power of Altman model was decreasing such that Beaver's dichotomous classification test using only one financial ratio (Cash Flow/Total Debt) had better predictive power (less misclassification) through the second to fifth year prior to the default event. Although Altman's empirical results show that his method has less predictive ability than Beaver's method, the method used by Altman is more intuitive (Deakin, 1972).

Multiple discriminant analysis technique (MDA) had been the most popular statistical technique for many years to predict defaults. Many researchers applied MDA in their studies;

Deakin (1972) concluded that MDA can be used to predict business failure relatively accurately from accounting data up to three years prior to actual bankruptcy event. Edmister (1972) applied MDA for small business failures predictions using financial ratios. Example of other authors that used MDA in their studies of failure prediction can be listed as: Blum (1974), Eisenbeis (1977), Taffler and Tisshaw (1977), Altman, Haldeman, and Narayanan (1977), Bilderbeek (1979), Micha (1984), Lussier (1995), Altman, Hartzell, and Peck (1998), and Altman and Sabato (2007). In most of these works, however, the two primary assumptions of MDA, that predictors used in the model are multivariate normally distributed, and variance-covariance matrices are equal across the failing and non-failing group, are usually violated when applied to default prediction problems. Additionally, it is not possible to interpret the standardized coefficient resulted from applying MDA the same way as the slopes of a regression, and therefore, there is no possibility to denote the relative importance of the variables (Altman and Sabato, 2007).

Taking inherent problems of MDA into account, conditional logit model was applied to the default prediction studies by Ohlson (1980) for the first time. Using logit model for default prediction has some practical advantages; two restrictive assumptions of MDA is not required to apply logit methodology. Moreover, it makes it possible to work with disproportional samples. Ohlson (1980) analyzed 9 predictors, included two binary variables and seven financial ratios, over a sample of 2,058 non-defaulted and 105 defaulted firms. Ohlson’s models represented lower classification accuracy, as model performance measure, in comparison with MDA models applied in previous studies by Altman (1968) and Altman et al. (1977). However, some reasons were mentioned by Ohlson to prefer logistic regression. Altman and Sabato (2007) studied credit risk modeling specifically for SMEs and applied both MDA and logistic regression on a sample of 2010 (120 defaults and 1890 non-defaults) small and medium-sized firms. They concluded that, using the same variables as predictors in both models, logistic regression default prediction models are expected to have higher power to separate defaulted and non-defaulted enterprises than MDA models.

Logit analysis appears to be an appropriate fit for the characteristics of default prediction studies, statistically, such that the dependent variable is binary (default or non-default) and with the discreate, identifiable, and non-overlapping groups. It produces a score between zero and one which is easily transformable into the probability of default (PD) for individuals or enterprises. Moreover, contrary to MDA coefficient estimates, logistic regression coefficients are distinctly interpretable as the importance or significance of each of the predictors toward the estimated probability of default explanation. Most of studies, after Ohlson (1980), applied logistic regression in their default predictions (Sabato 2010); for example, Zavgren (1983), Gentry, Newbold, and Whitford (1985), Keasy and Watson (1987), Aziz, Emanuel, and Lawson (1988), Platt and Platt (1990), Mossman, Bell, and Turtle (1998), Becchetti and Sierra (2003), and Altman and Sabato (2005, 2007).

According to Sabato (2010), no significant benefits has been observed over the prediction accuracy of credit scoring models using other statistical techniques[12] that have attempted to improve the logit prediction accuracy. For instance, Coats and Fant (1993), Altman, Marco, and Varetto (1994), Wilson and Sharada (1994), Lee, Han, and Kwon (1996), and West (2000) have used Artificial Intelligence (AI), more specifically neural network to construct credit scoring and failure prediction models. However, this machine learning approach typically produces very complicated models. Furthermore, obtained models are also extremely contingent on the samples and experimental data (Chen, Wang, and Wu 2010). Fantazzini and Figini (2009) used Random Survival Forest (RSF) method (introduced by Ishwaran, Kogalur, and Blackstone (2008)) in their study of SME credit risk management. They developed a comparison between a non-parametric procedure (Random Survival Forest) and a parametric procedure (logit) to predict the SMEs' probability of default. They found that Random Survival Forest (RSF) model provides a better in-sample description of SMEs default data. However, they reported that using a simple logit model, in the term of out-of-sample forecast accuracy, performed better than RSF model. Their conclusion confirms findings of a study by Fuertes and Kalotychou (2006) that the logit model is equally or even more preferred to other more sophisticated computing models.

After West (2000) used individual machine learning (IML) method on corporate credit risk prediction problem, Huang et al. (2004), Tsai and Wu (2008), and Nanni and Lumini (2009) used ensemble machine learning (EML) method and their models resulted in higher accuracy ratios than IML method, particularly on those cases that different structure of machine learning approaches result into independent errors. Furthermore, Wang and Ma (2011) applied

[12] Linear regression, probit analysis, Bayesian methods, and neural network are mentioned as examples in his article.

integrated ensemble machine learning (IEML), more specifically random subspace (RS) boosting, and concluded that IEML can be applied on corporate credit risk prediction problem as an alternative. Zhu et al. (2017) compared various machine learning approaches on SMEs credit risk prediction and concluded that RS-boosting performed better compare to other methods[13]. However, they did not compare their results with a possible logistic regression result on their sample. Hence, there is no evidence that their proposed methodology is more beneficial than the logit method.

Finally, after considering the characteristics of the problem in hand and the purpose of this study, I have decided to use the logistic regression (logit) as an appropriate statistical technique throughout this research.

2.2 SME literature

After the Basel II publication in June 2004, many analysts started to study SME segment. Governments and SME associations have started to criticize the high capital charges for SMEs, arguing that it could result into credit rationing of small firms, and therefore, taking the importance of the small firms in the economy into the account, decrease in economic growth (Altman and Sabato, 2007). Number of studies have investigated the potential impact of the Basel II on bank capital requirements for SMEs such as Schwaiger (2002), Saurina and Tracharte (2004), Dietsch and Petey (2004), Repullo and Suarez (2004), Udell (2004), Jacobson, Lindé, and Roszbach (2005), Berger (2006), Altman and Sabato (2005, 2007), and Scellato and Ughetto (2010). However, the above-mentioned studies, except Altman and Sabato (2007), have not investigated or just slightly got into the problem of modeling credit risk specially for SMEs.

Berger and Udell (2006) studied the lending strategies and structures for SME finance. They discussed that the lending infrastructure[14] may directly impact SME credit availability through its effect on the choice of different lending technologies. Through a restrictive

[13] They compared RS-boosting, multi-boosting, decision tree (DT), bagging, boosting, and random subspace (RS).

[14] According to Berger and Udell (2006), lending infrastructure refers to the tax and regulatory environments, the information environment, and the legal, judicial and bankruptcy environment.

regulatory environment, lending infrastructure also may indirectly impact SME credit availability by constraining the potential financial institution structure, and consequently limit SME credit availability. They concluded that better lending infrastructures may facilitate the use of the various lending technologies, and therefore significantly impact SME credit availability. Moreover, investigating U.S. data, they reported relatively little relationship between SME credit availability and the local market shares of large and small banks.

Analyzing the U.S. data over the period of 1994-2001, Kolari and Shin (2003), investigated the profitability and riskiness of SMEs in the banking industry. They concluded that lending to small business normally does not have a negative consequence on bank profitability. Moreover, although it is generally believed that small business lending is risky, they found that it has a tendency to decrease the banks' probability of failure (regardless of their asset size). Therefore, banks are expected to continue to play a fundamental role in providing credits to small enterprises.

Using survey data and focusing on the specific problem of innovation activities of Italian SMEs, Scellato and Ughetto (2010) examined the relationship between traditional credit suppliers and SMEs. They performed an analysis of the expected effect of the Basel II Accord guidelines on banks' capital requirements, which in turn might distress lending strategies for dissimilar kinds of borrowers. Scellato and Ughetto concluded that the Basel II may negatively impact young innovative SMEs' cost of loan.

The Basel II opened the way for capital requirements to be closely correlated to the specific underlying risk of each bank's loan portfolio by introducing Internal Rating Based (IRB) approach. Repullo and Suarez (2004) theoretically analyzed IRB approach and found that risky companies encounter higher cost of debt under IRB approach, while low risk enterprises will benefit from lower loan rates. Contrarily, Saurina and Trucharte (2004) studied effects of the Basel II on Spanish SMEs corporate lending and found no significant impact. They have tested a huge database of almost entire loans made by the whole Spanish banking system under both the IRB approach and the Standard Approach (SA), and found that final capital requirements for Spanish enterprises is slightly below the 8% (as required by the Basel II) on average; IRB approach resulted in 7.27% and SA showed 7.28%.

Nevertheless, OECD (2012) quantitively studied ex-post measurable effects of the Basel II on the company side based on a survey of SMEs in 18 countries over the period of 2007 to 2010 and reported that 34 to 54 percent of the surveyed SMEs faced an increase in their interest rates, whereas 10 to 29 percent of the respondents experienced decreasing interest rates. Moreover, increased collateral is reported for 34 to 39 percent of the survey participants. Recently, Schindele and Szczesny (2016) analyzed two groups of German SMEs over the period of 2007 to 2010 for ex-post effects of the Basel II; SMEs that have debt relations with banks that use Revised Standardized Approach (RSA) and those that use Internal Rating Based Approach (IRBA)[15]. Their result showed that SMEs that have debt relation with IRBA banks faced a significant overall increase of the cost of debt. Moreover, they found lower loan costs for low risk firms under IRBA, while riskier businesses confronted relatively higher loan rates after the Basel II implementation. On the other hand, for the SMEs that have debt relation with banks that use RSA, their results indicated less obvious effect although it is observable for companies with high level of risk. Schindele and Szczesny (2016) concluded that credit pricing is more risk-sensitive under IRBA, specifically SMEs with higher level of risk suffer more from regulatory reforms.

According to the large proportion of previous studies, there is enough evidence that small business lending has strong significant beneficial impact on bank profitability (Berger, 2006; Kolari and Shin, 2003). However, small and medium-sized businesses are often riskier than large corporates (Dietsch and Petey, 2004; Saurina and Tracharte, 2004). Regarding Dietsch and Petey (2004), classifying SMEs as retail customers[16] results in less minimum equity capital requirement from banks for given default probabilities. They justified this finding by the assumption of retail credits and loans of small businesses are less sensitive to systematic risk. Altman and Sabato (2007) developed a specific credit risk model for U.S. small and medium-sized firms and confirmed their hypothesis from their previous SME study (Altman and Sabato (2005)) that the SMEs' credit supply can be expanded, and consequently, this may imply a lower cost of credit. Contrary to Altman and Sabato, analyzing the credit portfolio of

[15] IRBA is the same as IRB approach. In this paper, these two abbreviations are used interchangeably.

[16] Considering the Basel II definition (also the Basel III), banks can classify SMEs as retail or corporate clients, based on the SMEs exposures. Later in this paper, following Altman and Sabato (2005, 2007), SMEs with sales less than 50 million kroner (5 Million Euro) will be classified as retail customers, and those with greater sales as corporates.

two Swedish banks, Jacobson et al. (2005) found no significant difference in SMEs capital requirement whether they are classified as corporate or retail customers.

3. Research Questions and Methodology

In this research, two main goals are tracked. First aim is to construct specific models for predicating Norwegian SME's probability of default and calculate minimum capital requirements, under the Basel III guideline, for banks based on constructed models. The second purpose of this paper is to examine whether using a general one-year probability of default (PD) model (as suggested by the Basel III) for more than one-year default prediction is a good choice in compare with employing exclusive duration PD models.

3.1 SME Model Development

Resulting from their simple structures, SMEs can respond rapidly to altering economic conditions and meet needs of their local customers, developing into large and powerful corporations occasionally or defaulting right after starting up (Altman and Sabato, 2007). However, the most important source of external SME financing is borrowing from commercial banks (Altman and Sabato, 2005). Thus, they are extremely sensitive to the banks' credit adjustments. Introduction of the Basel II substantially changed banks' credit risk assessment for SMEs by explicitly differentiating between large corporates and SMEs capital requirements. Furthermore, the Basel III made this differentiation even more noticeable by introducing a risk weight of 85% for SMEs as corporates[17]. Modelling credit risk for SMEs is initiated in an article by Edmister (1972). There is number of studies that exclusively focused on modelling credit risk for SMEs (see literature review), however, the only work that I aware of that studied Norwegian SMEs is an article published by Moody's on 2006[18]. However, their model has relatively lower power comparing to the models that are developed for other countries by other authors (for example, Altman and Sabato (2007) developed a model for US SMEs with more than 20 percentage points higher prediction accuracy ratio)[19]. When the Advance Internal Rating Based (A-IRB) approach is being used, predication accuracy

[17] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, paragraph 43.

[18] KMV Model has been introduced by Moody's study for small and medium-sized firms in Norway, "MOODY'S KMV RISKCALC V3.1", July 2006.

[19] Moody's model for Norwegian SMEs has 66.3% accuracy ratio as predictive power indicator for their one-year PD model. However, Altman and Sabato (2007) reported an accuracy ratio of 89.8% for US SMEs one-year PD model.

improvement is likely to have positive impact on the capital requirements for SMEs, and therefore can potentially provide lower cost of credit for SMEs (Altman and Sabato, 2007). Thus, models with more predictive power for Norwegian SMEs also can potentially result in a better credit allocation. The initial objective of this research is to provide information mainly for banks to determine default probability for Norwegian SMEs more precisely, and consequently, calculating the capital requirement based on their SMEs loan portfolio under the Basel III adjustments. Moreover, instead of only using a one-year probability of default (PD) model (suggested by the both Basel II and III), a three-year, and five-year PD models are attempted to be constructed exclusively for Norwegian SMEs in the case of loans with longer maturities than one year. Thus, the first two research questions can be listed as follow:

1) What are the main factors for default predication on Norwegian SMEs with probability of default in one-year, three-year, and five-year?

2) What are the capital requirements calculated based on the models specifically for Norwegian SMEs?

In order to answer the first question logistic regressions are used to estimate default probabilities in one-year, three-year, and five-year. Practical default prediction is an important feature of those models. More specifically, one-year PD model is to predict default probability in one year based on the most recent recorded information, and three and five-year PD models are to estimate default probability in the next three and five years based on the latest information. One of the advantages of using logit is that the outcome is in the term of probability between zero and one, as such, requires no further adjustment to be transformed into probabilities. Ohlson (1980) applied conditional logit model to a default prediction study for the first time, and it is the most popular method in default prediction studies.

Probability of default (PD) models longer than one year can be constructed based on the relevant information for that time span. For example, to construct a probability model that estimates the likelihoods of default in span of three years a data set that includes defaults distributed on that period is required. That is, default event can occur in any year during the loan’s life. For constructing specific PD models, after finding defaults in the whole database, the distance from the last submitted financial statement and the first registered default following that financial statement is the key information. For instance, we assume, a bank plans to give a loan with maturity of three years to a firm, default may happen in the first, second, or the third year; however, bank only has the financial information which is available today. Thus, the bank may need a model that simulate the probability of default during the life of the intended loan. For constructing a sample which includes default further than one year, there are at least two ways. First, finding default events and keep, for example, financial information from one year, two year, or three year ago[20]. The other way is to find the last financial information before registration of the default event. Many firms stop to officially submit their financial information in the case of financial distress, therefore, there would be always some missing information before default. After constructing the sample based on the intended maturity we can construct a model that estimates default probability during the intended duration (t = 0 to t = T) based on the today's information (t = 0).

Second question is calculated based on the Basel III latest release "Finalising Post-Crisis Reforms" regulations. As it is mentioned, the Basel III certainly suggested that banks need to use a one-year PD model for calculating their customers probability of default under IRBA whether assuming SMEs as retail customers or corporates[21]. There are maturity and size adjustments in the calculation suggested by the latest Basel. First, a minimum capital requirement is calculated based on one-year PD model and sample included all SMEs, using the same approach as Altman and Sabato (2005, 2007) have used in their studies of SMEs.

Additionally, capital requirements based on different maturities are also calculated under the default probabilities resulted from the specific PD models (i.e., three-year, and five-year). The same sales classification as Altman and Sabato (2005, 2007) is used in three-year and five-year capital requirements' calculations. However, for each specific class, only the target customers (firms that are eligible to receive loans with maturity equal *T* years) are kept for calculating the capital requirement for that certain group. For example, SMEs with sales more than 50 million kroner (5 million Euros) and less than 250 million kroner (25 million

[20] The first default is important here, so if a firm defaulted in the first year, it cannot also default in the second year, and similarly the third year.

[21] Considering Altman and Sabato (2005, 2007) classification of loan life time based on SME's sales, firms with sales less than 5 million Euros can receive loans which need to be repaid in one year and they were classified as retail customers. In the same way, SMEs with sales more than 5 million Euros but less than 25 million Euros are classified as small size corporates and they can receive loans with maturity of 3 years. Similarly, SMEs with sale more than 25 million Euros and less than 50 million Euros are classified as medium size corporates and can receive loan with maturity of 5 years.

Euros) are assumed to be eligible to receive loans with maturity of three years (Altman and Sabato, 2005 and 2007). Thus, a subsample of SMEs regarding their sales is created and then probabilities of default are estimated by the relevant probability model within that specific subsample. Finally, capital requirements are calculated and aggregated (detailed calculations are available in sections 5 of this paper).

3.1.1 Methodology

The Logit Model

Logit model[22] is used to generate value for each SMEs based on independent variables. Each PD model used different samples of population. In each model $y_i \in \{0,1\}$, 0 for non-default and 1 for default in the specific timeline (i.e., one, three, or five-year period):

$$y_i = \begin{cases} 1 & if\ the\ i-th\ \ firm\ is\ defaulted \\ 0 & otherwise. \end{cases}$$

Then π_i is defined as the probability which depends on a vector of independent variables $x_{i1}, x_{i2}, \dots , x_{in}$ such that:

$$\pi_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_n x_{in} \equiv \boldsymbol{X_i'\beta},$$

where $\boldsymbol{\beta}$ denotes a vector of linear combination coefficients. Although probability π_i must be between zero and one, the right-hand side equation ($\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_n x_{in}$) can take any real value. Thus, to have the predicted values in the meaningful range ($0 \leq \pi_i \leq 1$) some restrictions are needed. As a solution, probability π_i needs to be transformed and then the transformation be modeled as a linear function of the independent variables ($x_{i1}, x_{i2}, \dots , x_{in}$). This can be done in two steps:

First, probability π_i is replaced by the *odds* which is defined as the ration of defaults to non-defaults in this case:

$$odds_i = \frac{\pi_i}{1-\pi_i}$$

[22] The discussion about the logit model is mainly taken from Rodríguez (2002).

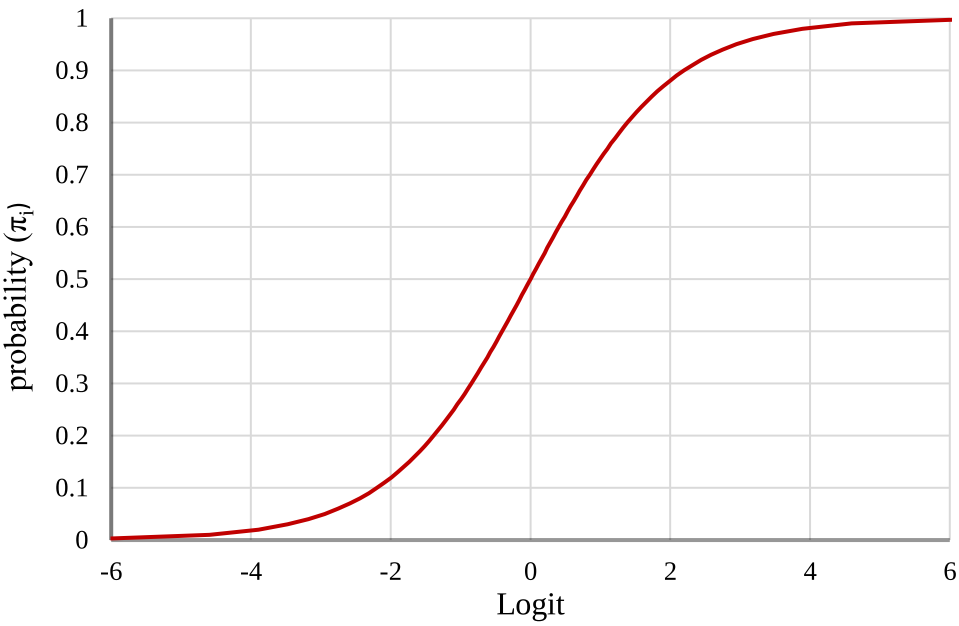
By the above shift from probability π_i to *odds*, the potential problem of having negative values in the probability measure has been solved, as the right-hand-side of the equation is always positive. Nevertheless, it can take any positive values while probability has to be between zero and one.

Second, logarithm is taken from *odds* (so-called *logit* or *log-odds*):

$$\eta_i = \text{logit}(\pi_i) = \log \frac{\pi_i}{1 - \pi_i}$$

Now, the floor restriction is removed. That is, when probability $\pi_i$approaches zero the *odds* also go down to zero and the log-odds (or logit) approaches the negative infinity $(-\infty)$. On the other hand, the logit and odds approach infinity $(+\infty)$ when probability $\pi_i$approaches one. Therefore, probabilities from zero to one are mapped to the entire real values line by the logit. The relationship between probability π_i and the logit is plotted in Figure 1.

Figure 1. The logit transformation



In order to get back from logits to probabilities, an inverse transformation is need (also known as the *antilogit*). It can be solved as follow:

$$\pi_i = \text{logit}^{-1}(\eta_i) = \frac{\exp(\eta_i)}{1 + \exp(\eta_i)}$$

Assuming that the logit of the probability π_i follows a linear model, instead of the probability π_i, it is possible to define the logit model (logistic regression model) as follow:

$$\text{logit}(\pi_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_n x_{in} \equiv \boldsymbol{X}_i'\boldsymbol{\beta},$$

where $\boldsymbol{\beta}$ denotes a vector of regression coefficients, and $\boldsymbol{X_i}$ is a vector of independent variables. The model shown in the above equation is a generalized linear model with link logit and binomial response (y_i). It is possible to interpret the regression coefficients $\boldsymbol{\beta}$ along the same line as in the linear models, however, there is a logit in the left-hand-side of the equation which is not a mean. Therefore, holding all other independent variables constant, β_k signifies the change in the logit of the probability linked to a unit change in the *k*-th independent variable.

Odds for the *i*-th observation can be find by exponentiating the above equation:

$$odds_i = \exp\{\boldsymbol{X}_i'\boldsymbol{\beta}\} = \frac{\pi_i}{1 - \pi_i}$$

The above equation can be seen as a multiplicative model for the odds. For instance, holding all other predictors constant, multiplying the odds by $\exp\{\beta_k\}$ shows a one-unit change in the the *k*-th independent variable (i.e., one-unit increase in x_k will result $\boldsymbol{X}_i'\boldsymbol{\beta}$ increases by β_k, or $\boldsymbol{X}_i'\boldsymbol{\beta} + \beta_k$; exponentiating $\boldsymbol{X}_i'\boldsymbol{\beta} + \beta_k$ gives $\exp\{\boldsymbol{X}_i'\boldsymbol{\beta}\} \times \exp\{\beta_k\}$).

Probability $\pi_i$can be find by solving the following equation:

$$\pi_i = \frac{\exp\{\boldsymbol{X}_i'\boldsymbol{\beta}\}}{1 + \exp\{\boldsymbol{X}_i'\boldsymbol{\beta}\}}$$

Thus, the conditional probability of default linked to the SME *i* can be written as:

$$\Pr(y_i = 1 \mid \boldsymbol{X}_i'\boldsymbol{\beta}) = \frac{\exp\{\boldsymbol{X}_i'\boldsymbol{\beta}\}}{1 + \exp\{\boldsymbol{X}_i'\boldsymbol{\beta}\}} = \frac{1}{1 + \exp\{-\boldsymbol{X}_i'\boldsymbol{\beta}\}}$$

Estimated parameters are shown by $\beta_i (i = 0,1, \ldots \ , n)$ and independent variables by $x_i (i = 1,2, \ldots \ , n)$. In the other PD models $y_i \in \{0,1\}$, 0 for non-default and 1 for default in the relative distance from the information date. For example, a firm that recorded as 1 (defaulted) in a three-year PD has defaulted at some point in three years' timeline after the financial information recording date.

Functional Misspecification Test

Underlying assumption that the logistic regression models are built on is that a linear combination of the independent variables generates the logit of the outcome variable. However, it is difficult to assume all the ratio has linear relationship with default probability. For example, Moody's (2006) on a study of Norwegian SMEs reported a U-shape relationship between growth indicator and probability of default. Therefore, it is necessary to test if the linear combination of independent variables is appropriate. Moreover, functional misspecification may result in omitting relevant predictors. To test for misspecification, the framework suggested by Pregibon (1980) is applied. The predicted value ($\hat{y}$) on each model and the squared term of predicted value ($\hat{y}^2$) as independent variables are regressed on the outcome variable such that:

$$\hat{y} = \hat{\beta}X$$

$$y = \beta_0 + \beta_1 \hat{y} + \beta_2 \hat{y}^2$$

For a model to be correctly specified, $\hat{y}$ should be statically significant as it is predicted value from the fitted model, however, squared term ($\hat{y}^2$) should not be statistically significant.

Hosmer-Lemeshow Goodness-of-Fit Test

For evaluating whether the number of predicted values imitate the number of observed values in the data, Hosmer-Lemeshow (H-L) goodness-of-fit test (Hosmer and Lemeshow, 1989) is used. Based on the value of the predicted probability from the respective models each SMEs is ranked and grouped. A group number of 8 is used for this study models. Assuming the number of firms equal to k, the first group includes $k_1 = k/8$ firms with the lowest predicted default probabilities, and $k_8 = k/8$ SMEs with the highest estimated default probabilities form the last group. If the test statistic is statistically significant the model is considered a poor fit for the data. That is, the H-L test statistic denotes existence of a statistically significant difference between at least one group in the number of predicted values, compared to the observed number of values.

3.2 Minimum Capital Requirements and Models for Longer Maturities

As it was mentioned earlier, the Basel III (in the same way as the Basel II) suggests banks to calculate their minimum capital requirements base on one-year PD model under the Internal Rating Based (IRB) approach. However, there are adjustments suggested by the Basel III for SMEs' size and loans' maturities. This may impose two implicit assumptions for calculating the minimum capital requirements under the IRB approach. First, default rate is similar for all SMEs regardless of their size. Second, one-year default probability model has enough predictive power to estimate defaults that may happen duration longer than one year (i.e., three-year, and five-year). These two implicit assumptions result into another implicit assumption for minimum capital requirement calculation. That is, size and maturity adjustments (suggested by Basel III) can empower the formula to replicate the real default rates, and therefore, true minimum capital requirements for different size of SMEs and loans with longer maturities than one year. For testing these assumptions, after developing models for one, three, and five-year PDs, capital requirements based on conventional approach suggested by the Basel III and the new approach (unconventional) for each specific PDs and sizes are calculated. Then the difference of the outcome using these two approaches is examined. This comparison, as far as I aware of, has never been done by other authors for small and medium-sized enterprises (SMEs). The third research question can be summarized as follow:

3) Does using a specific PD model for a certain maturity result in different capital requirements?

For answering the third question, a bank is assumed which needs to calculate capital requirements for its SME customers, on a new sample of 5,000 Norwegian SMEs which are randomly drawn from the year 2015 data and none of them is already bankrupt (2,500 as retail customers, and 2,500 as corporates). Assuming that one-year PD model is sufficient for capital requirements calculations, both approaches need to result into similar capital requirements. Capital requirement for each firm is calculated separately once based on the one-year PD model and ranking system, and then based on the specific maturity of the intended loans PD model. At the end average calculated capital requirements will be compared.

4. Data and Variables Selection

I developed three specific models to estimate one-year, three-year, and five-year SME's probability of default. In this chapter, first, data cleaning and sampling steps are described. Then variable selection stages, and model construction are explained in detail.

4.1 The data set

The data that is used in this research is obtained from an updated version of the Norwegian Corporate Accounts (Working Paper No. 11/16)[23] data base collected by SNF[24] and NHH[25]. This database has accounting and company information for Norwegian companies from 1992 to 2015. All the data restrictions, data cleaning, sampling, and analyses are done afterwards and independently from the above mentioned working paper.

SME is defined by the OECD statistics portal[26] by two milestones: number of employees, and turnover (Updated on December 2005). According to the OECD, firms that have less than 250 employees, and turnover of less than €50 million (approximately NOK 500 million) are classified as SMEs. Following the KMV[27] data exclusions, small companies with total assets less than NOK 1,590,000 (2015 Norwegian Kroner)[28], and financial institutions are excluded from the data set as they are dissimilar to the typical middle-market firms. Main reason for the first exclusion is that the future prosperity of such firms depends on the key individual finances. Financial institutions' balance sheets often indicate higher financial leverages in comparison to the typical private firms. The other three exclusions suggested by Moody's study are public sector and non-profit institutions, start-up companies, and real estate development companies as all these three groups represent different behaviors than the typical

[23] Berner, Mjøs, & Olving, (2016)

[24] Center for Applied Research at NHH

[25] Norwegian School of Economics (Norges Handelshøyskole)

[26] https://stats.oecd.org/glossary/detail.asp?ID=3123

[27] KMV Model has been introduced by Moody's study for small and medium-sized firms in Norway, "MOODY'S KMV RISKCALC V3.1", July 2006.

[28] NOK 1,250,000 in 2002 Norwegian Kroner has been suggested by the Moody's study.

middle market firms. All the exclusions have been implemented on the data set. For further investigation, sector dummies have been created in order to find a possible difference between various sectors. Observations before 1995 in the data set are excluded from the data due to significant number of missing values.

4.1.1 One-year Default Probability Model Samples

To create a sample for a model to estimate one-year probability of default (PD), 3,072 defaulted SMEs without any missing data has been selected. Then, the same number of non-defaulted firms for each year have randomly been chosen to obtain a balanced probability sample. For example, if there are 82 defaulted firms that have financial information submitted in 2005, 82 non-defaulted firms which have non-missing financial information in year 2005 are randomly selected and added to the sample. Moreover, another sample is randomly selected from non-defaulted firms over same period to replicate average default rate as close as possible to the Norwegian SMEs' expected average default rate (2.1%)[29]. For instance, the expected one-year default rate is 2.1% and there are 82 defaulted firms that have financial information submitted in 2005, ($\frac{82}{0.021} - 82 =$) 3,823 non-defaulted firms which have non-missing financial information in year 2005 are randomly selected and added to the sample. The first sample will be used to construct a one-year default probability model, then the second sample will be employed to find minimum capital requirements. The main reasons of using a balanced sample for constructing default probability models is having a sample size of 146 thousand observations while 143 thousand of them are 0's (non-defaults) often causes that logistic regression never converges[30] in the models that are being used in this research. Following a study of rare events by King and Zeng (2002), using sample size of fewer than 200 observations causes logit coefficients to be biased; on the other hand, having a sample sizes of thousands which are always in the same directions would also cause considerably meaningful biases in probabilities. According to their study, it is possible to collect all the available 1's and randomly sample a small sample of 0's without deterioration of consistency or even obtain a more

[29] This default rate's expected average for Norwegian SMEs has been obtained from a Moody's study for small and medium-sized firms in Norway, "MOODY'S KMV RISKCALC V3.1", July 2006.

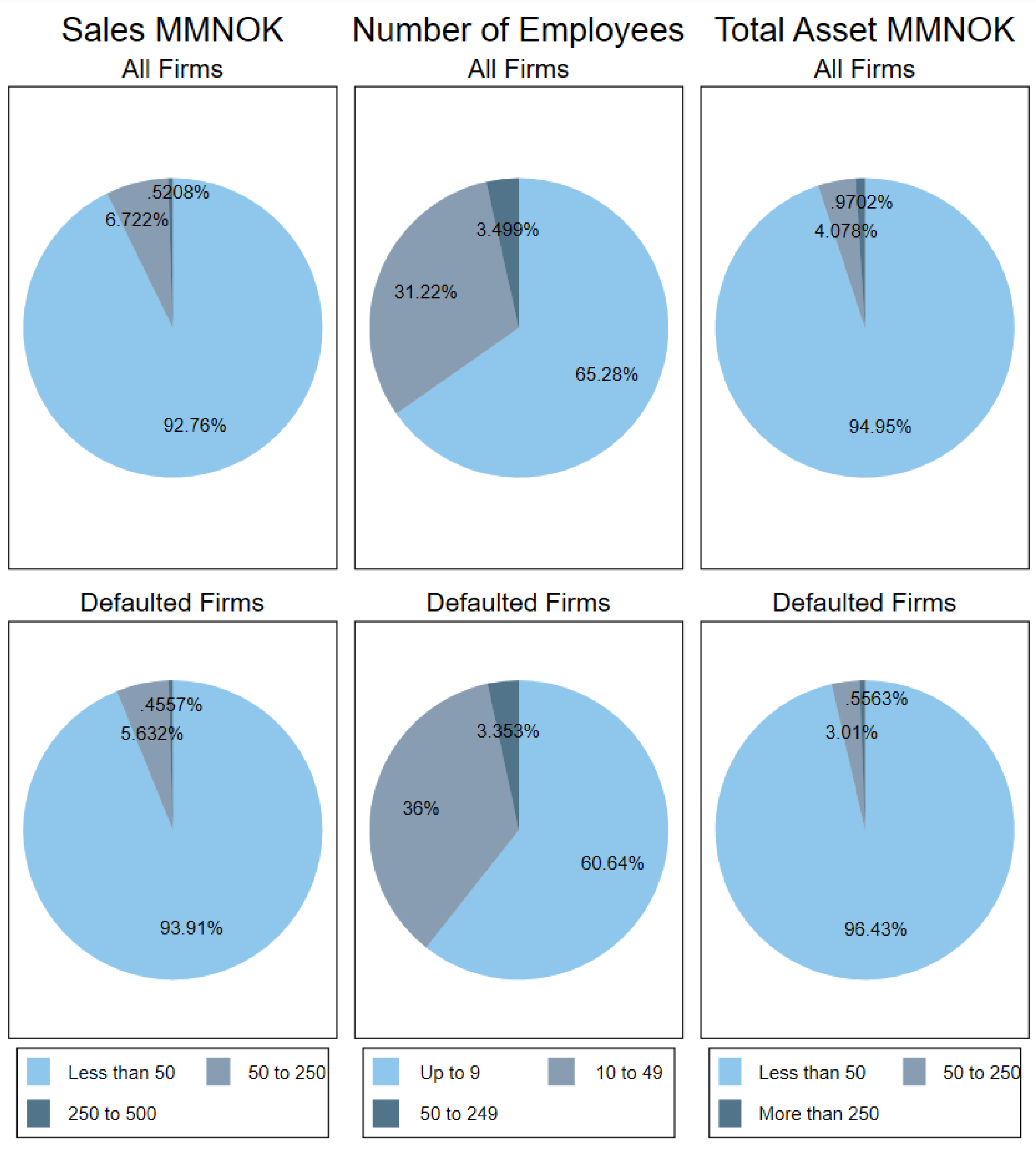
[30] The main reason of convergence failure in logistic regression is resulted from specific pattern in data known as complete or quasi-complete separation (see Allison, 2008).

efficient sample relative to using the full sample. Table 1 shows the number of defaults and non-defaults in each year for both balanced and imbalanced sample. Figure 2 demonstrates the distributions of sales, number of employees, and total asset for both all firms and the defaulted firms in the balanced sample (See appendix for relevant distributions of the imbalanced sample).

Table 1. Data sets for the Norwegian SMEs in the one-year probability of default

Year		**Balanced Sample**		**Non-Balanced Sample**	
	Defaults	Non-defaults	Total sample	Non-defaults	Total sample
1995	91	91	182	4,242	4,333
1996	79	79	158	3,683	3,762
1997	90	90	180	4,196	4,286
1998	154	154	308	7,179	7,333
1999	186	186	372	8,671	8,857
2000	228	228	456	10,629	10,857
2001	293	293	586	13,659	13,952
2002	292	292	584	13,613	13,905
2003	178	178	356	8,298	8,476
2004	157	157	314	7,319	7,476
2005	82	82	164	3,823	3,905
2006	71	71	142	3,310	3,381
2007	155	155	310	7,226	7,381
2008	133	133	266	6,200	6,333
2009	185	185	370	8,625	8,810
2010	170	170	340	7,925	8,095
2011	91	91	182	4,242	4,333
2012	124	124	248	5,781	5,905
2013	100	100	200	4,662	4,762
2014	91	91	182	4,242	4,333
2015	122	122	244	5,688	5,810
Total	3,072	3,072	**6,144**	143,213	**146,285**

Figure 2. Distribution of sales, number of employees, and total asset in the Norwegian SMEs' sample for one-year PD



4.1.2 Three-year Default Probability Model Samples

To form a sample for the three-year PD model, 11,209 defaulted SMEs without any missing data has been obtained. Then, the same number of non-defaulted firms for each year has randomly been selected to make a balanced probability sample. The same method as one-year PD has been used to build a sample with cumulative average default probability for three-year default probability closest to expected average default rate (4.2%)[31]. The sample with cumulative PD for three-year has 266,882 observations. To construct ranking classes exclusively for small-sized firms as corporates (with sales between 50 million to 250 million kroner), a sub-sample with only small-sized firms is drawn from the cumulative PD sample. The specific sample for ranking has 20,397 observations (includes 533 defaulted small-sized firms).

4.1.3 Five-year Default Probability Model Samples

Five-year probability of default balanced sample is formed by selecting 11,449 defaulted SMEs without any missing data and randomly sampling the same number of non-defaulted firms for each year. The imbalanced sample for five-year default probability is constructed in the same way as the previous imbalanced samples with expected average default rate (8.4%)[32]. The sample with cumulative PD for five-year has 136,297 observations. To construct ranking classes exclusively for medium-sized firms as corporates (with sales between 250 million to 500 million kroner), a sub-sample with only medium-sized firms is drawn from the cumulative PD sample. The specific sample for ranking has 1,471 observations (includes 37 defaulted medium-sized firms). Detailed numbers of defaulted and non-defaulted firms for each year, and distribution of sales, number of employees, and total asset are available in appendix for both three and five-year samples.

[31] The three-year central tendencies of default probability is derived from one-year estimate based on the formula available in the study of “Probability of default and default correlations” by Li (2016).

[32] The five-year central tendencies of default probability is derived from one-year estimate based on the formula available in the study of “Probability of default and default correlations” by Li (2016).

4.2 Candidate Variables

Prior studies have shown a large number of potential candidate financial ratios (Altman and Sabato, 2007); Chen and Shimerda (1981) found that approximately 50 percent of more than 100 financial ratios were proved to be useful at least in one empirical research. Regarding Lehman (2003) and Grunet et al. (2004) using qualitative variables such as type of the industry, employees count, geographical region, and the legal form of the business would increase predictive power of models to forecast SME default.

According to prior studies, a company's financial profile can be described by five main accounting ratio categories: leverage, liquidity, profitability, coverage, and activity (Altman and Sabato, 2007). In Moody's KMV study of Norwegian SMEs default prediction, growth and size have also shown to be good predictors. In this study, all those five accounting ratio categories together with growth and size variables have been used as the main candidate variables. In addition to the above variables, age[33], industry type, number of employees, geographical region for the main business, and ownership structure of firm also have been added to the candidate variables. Table 2 lists the candidate variables in this study, following the table there are short explanation of each candidate predictor.

[33] Laitinen (2005) used age as a predictor in his study of "Survival analysis and financial distress prediction: Finnish evidence".

Table 2. Candidate Variables List

Definition	Category
Short Term Debt/Equity Equity/Liabilities Liabilities/Total Assets Public Charges/Total Income Public Charges/Total Assets Total interest-bearing liabilities/Total Assets	Leverage
Cash/Total Assets Working Capital/Total Assets Cash/EBIT Intangible Assets/Total Assets	Liquidity
EBIT/Total Income EBIT/Sales EBITDA/Total Assets Net Income/Total Assets Net Pre-Tax Income/Total Assets Net Income/Total Income Net Income/Sales	Profitability
EBITDA/Interest Expenses EBIT/Interest Expenses	Coverage
Total Income/Total Assets Sales/Total Assets Trade Creditors/Total Income Trade Creditors/Sales Trade Debtors/Liabilities Payroll Expenses/Total Asset Depreciation/Total Asset	Activity
ROA(t)- ROA(t-1)	Growth
Natural Logarithm of Total Assets in 2015 Norwegian Kroner	Size
Age of the Firm (Based of registration date)	Age
Industry Type Geographical Region Number of Employees Ownership Categories	Qualitative

4.2.1 Leverage

There are six leverage ratios in this category which principally indicate the solidity of the firms in the context of financing structure. The solidity of an enterprise shows that how the enterprise would react when the external circumstances change (for example, change in the interest rate, and downturn in the economy). For instance, a firm that is relatively more debt-financed will be impacted more in the case of change in interest rate. Table 3 (and Table IV, and V in appendix) shows means for defaulted and non-defaulted firms, defaulted firms in each sample show a weaker performance in terms of leverage.

Leverage ratio 1 (Altman and Sabato, 2007)

$$Lev1 = \frac{Short\ Term\ Debt}{Equity\ (Book\ Value)}$$

First leverage ratio describes information on a firm total short-term debt to its total book value of equity. Based on the data in this research, defaulted SMEs on average have relatively lower rates of short-term debt to equity. This is inconsistent with the result from the study of SMEs by Altman and Sabato (2007); they found a negative relationship between this predictor and non-defaulted firms. However, based on the univariate analyses, this ratio seems to explain the default events poorly, specially on longer time durations (it is the weakest predictor in term of univariate analysis in three, and five-year samples).

Leverage ratio 2 (Altman and Sabato, 2007)

$$Lev2 = \frac{Equity\ (Book\ Value)}{Total\ Liabilities}$$

Leverage ratio 2 gives information on amount of a firm's equity financing to the amount of debt financing. Total liabilities amount simply signifies sum of the long-term and short-term liabilities. Defaulted SMEs consistently show a lower equity to liabilities ratio in all samples. A lower equity-to-debt ration generally explains that a firm has been less aggressive toward financing itself with equity, and there would be a relatively greater potential for default in the case of financial distress.

Leverage ratio 3 (Altman and Sabato, 2007)

$$Lev3 = \frac{Total\ Liabilities}{Total\ Assets}$$

The third leverage ratio indicates the proportion of a firm's total assets which are financed through debt. Total asset is sum of equity and debt (liabilities). Throughout this research defaulted SMEs show a higher liabilities-to-asset ratios.

Leverage ratio 4

$$Lev4 = \frac{Public\ Charges}{Total\ Income}$$

This ratio shows how much of a firm's income goes to tax withholdings, employees' national insurance owed by the firm, and VAT (Value-added tax). Hypothetically, social welfare and tax system of Norway can potentially have impact on defaults. The purpose of including this ratio and the Leverage ratio 5 is to investigate if this hypothesis is true or not. However, public charges and total income are correlated (i.e., more income results more VAT). Therefore, Leverage ratio 5 is defined in the case of high correlation of Lev4 ratio such there would be no variation.

Leverage ratio 5

$$Lev5 = \frac{Public\ Charges}{Total\ Assets}$$

This ratio is an alternative form of the Leverage ratio 4. As public charges amount is less correlated with total asset in comparison with total income, this leverage can potentially divulge some information that Lev4 may not be able (i.e., public charges amount is less correlated to total asset in compare to total income). Defaulted firms have higher Lev4 and Lev5 ratios in all samples.

Leverage ratio 6

$$Lev6 = \frac{Total\ interest - bearing\ liabilities}{Total\ Assets}$$

Total interest-bearing liabilities amount shows liabilities that lender expects and an implicit interest payment for the given loan. Including this ratio, it is possible to observe if there is any difference between Lev3 and this ratio in order to predict defaults. It has the same relationship with defaults in all samples, however, it seems to be a weaker predictor compares to Lev3, based on univariate analyses.

4.2.2 Liquidity

When a firm is not able to meet its obligations on-time that firm is insolvent. Thus, that is important for a firm to have enough liquidity to avoid insolvency and potentially defaulting. Liquidity ratios generally measure how much liquid assets or cash a firm holds. The first two liquidity ratios are on average lower for devalued SMEs in all samples, and both have good predictive powers, based on univariates analyses. The third liquidity ratio, however, has lowest predictive among other liquidity ratios, but the same relationship (i.e., higher liquidity for non-defaulted). The fourth liquidity ratio is also consistent in all samples. It is important to note that intangible assets are not highly liquid, so it has positive relationship with default.

Liquidity ratio 1(Altman and Sabato, 2007)

$$Liq1 = \frac{Cash}{Total\ Assets}$$

Liquidity ratio 1 is a measure of how much cash a firm has relative to its total asset. This ratio clearly shows how liquid a firm is. Generally, this ratio can be adjusted internally by the firms; that is a firm can change this ratio by changing the amount of cash they keep (if it has enough cash and enough investment opportuinties).

Liquidity ratio 2 (Altman, 1968)(Ohlson, 1980)(Altman and Sabato, 2007)

$$Liq2 = \frac{Working\ Capital}{Total\ Assets}$$

The working capital is equal to current assets minus current liabilities. Current asset is defined as assets associated with the enterprise’s sales of good and services, receivable that must be repaid within a year, and investments which are not planned for permanent use or ownership. This liquidity ratio also measures of how liquid a firm is, and it is combined with the current liabilities.

Liquidity ratio 3

$$Liq3 = \frac{Cash}{EBIT}$$

Cash to earnings before interest and taxes (EBIT) specifies the effectiveness of the firm's credit and receivable collection strategies, and the quantity of cash required as safeguard for unpredicted suspensions in cash collection. EBIT is a measure of a firm’s net profit before interest and income tax expenses.

Liquidity ratio 4 (Altman and Sabato, 2007)

$$Liq4 = \frac{Intangible\ Assets}{Total\ Assets}$$

Intangible assets can potentially contribute to decreases future costs or generating future income in the way of the asset is used to produce or sell goods and services; however, intangible assets have no physical elements. It includes investments in research and development (R&D) to generate future income and acquired goodwill as results of accruing other companies. Thus, this liquidity ratio should not have negative relationship with default events.

4.2.3 Profitability

Being profitable in the long run makes a firm attractive for investors. An attractive business can raise needed capital more easily. Having access to more capital makes that firm to fulfill their obligations, and gives it more opportunity to expand (e.g., through investment in new project). Thus, a leveraged firm, specially, needs to reach a certain level in profitability to be able to meet its obligations. Low profitability can be a reason for future default. There are seven Profitability ratios in this research which all of them measure the financial performance of the SMEs. All the profitability ratios in this research consistently show a lower level for defaulted SMEs compares to non-defaulted ones on average (in all samples).

Profitability ratio 1

$$Pro1 = \frac{EBIT}{Total\ Income}$$

Cash to earnings before interest and taxes (EBIT) on total income shows how a firm is efficient before paying tax and interest expenses. Total income can be from operational and non-operational income.

Profitability ratio 2 (Altman and Sabato, 2007)

$$Pro2 = \frac{EBIT}{Sales}$$

This ratio is an alternative for the first profitability ratio (Pro1). The main difference is that it shows EBIT on operational incomes (sales).

Profitability ratio 3 (Altman and Sabato, 2007)

$$Pro3 = \frac{EBITDA}{Total\ Assets}$$

Earnings before interest, taxes, depreciation and amortization (EBITDA) is a museare of a firm's financial performance. It is an alternative form of net income or earning. By excluding expenses linked to debt, this ratio can measure a firm's performance independet of its capital structure.

Profitability ratio 4 (Altman and Sabato, 2007)

$$Pro4 = \frac{Net\ Income}{Total\ Assets} = \ Return\ on\ assets\ (ROA)$$

This ratio is known as return on assets (ROA) which is a standard performance measure. Compared to the previous profitability ratio (Pro3), ROA includes interest, taxes, depreciation and amortization expenses.

Profitability ratio 5 (Altman and Sabato, 2007)

$$Pro5 = \frac{Net\ Pre\ Tax\ Income}{Total\ Assets}$$

Net pre-tax income quantifies all the incomes resulted from ordinary and extraordinary activities, before the tax expenses. It is relatively close measure to EBIT but does not exclude interest expenses.

Profitability ratio 6

$$Pro6 = \frac{Net\ Income}{Total\ Income}$$

This ratio is an alternative for the first profitability ratio (Pro1).

Profitability ratio 7 (Altman and Sabato, 2007)

$$Pro7 = \frac{Net\ Income}{Sales}$$

This profitability ratio is also known as profit margin ratio, or gross profit ratio. It shows what fraction of sales remains after all expenses are paid by the company.

4.2.4 Coverage

A coverage ratio is a degree of a firm's ability to repay its debt and fulfil its financial obligations. Having a higher coverage ratio for an enterprise is an indication of that enterprise is able to pay its interest payment on its debt easier. There are two coverage ratios in this research. In all the samples defaulted SMEs have significantly lower coverage ratios than non-defaulted SMEs.

Coverage ratio 1 (Altman and Sabato, 2007)

$$Cov1 = \frac{EBITDA}{Interest\ Expenses}$$

Coverage ratio 2 (Altman and Sabato, 2007)

$$Cov2 = \frac{EBIT}{Interest\ Expenses}$$

4.2.5 Activity

Activity ratios measure a firm's ability to translate diverse accounts within its balance sheets into sales, liabilities, total assets, total income and etcetera. It is a measure of the relative efficiency of how a firm use its assets, leverage, or other similar balance sheet items. It can also be seen as a measure of a firm’s management efficiency of generating cash and income from the firm available resources.

Activity ratio 1

$$Act1 = \frac{Total\ Income}{Total\ Assets}$$

The first activity ratio shows how much total income a firm earns to its total asset. Total income includes all income that a firm receive. It represents the sum of sales and other operating income.

Activity ratio 2 (Altman and Sabato, 2007)

$$Act2 = \frac{Sales}{Total\ Assets}$$

This ratio is an alternative form of the first ratio (Act1) with this difference that it measures sales on total asset. Both the first and second activity ratios represent slightly higher values among all samples in this research. This potentially has some reasons apart from the activity of SMEs solely, such as higher interest payments (resulted from a firm being highly leveraged with interest-bearing liabilities) or higher tax rate (resulted from a firm being classified in a higher income level).

Activity ratio 3

$$Act3 = \frac{Trade\ Creditors}{Total\ Income}$$

The third activity ratio measures how much credit a firm received extended credit from sellers (as result of purchasing goods and services) to its total income. Trade creditors typically have 30 to 90 days extended credit period. Therefore, the higher this ratio is a firm must to repay a higher portion of its income to the seller in a short period. In all samples, this ratio is significantly higher for defaulted SMEs on average.

Activity ratio 4

$$Act4 = \frac{Trade\ Creditors}{Sales}$$

This ratio is an alternative form of the first ratio (Act3). There is no significant difference between defaulted and non-defaulted SMEs in the one-year sample. However, in both three, and five-year sample, this ratio is significantly higher for defaulted SMEs.

Activity ratio 5

$$Act5 = \frac{Trade\ Debtors}{Liabilities}$$

Trade debtors includes money that is owed to the firm by its customers, advance payment to suppliers, and short-term receivables such as fixed assets held for sale. This ratio represents how much a firm suppose to receive under trade debtors to its liabilities. It is one of the resources that a firm can fulfil its obligation from it. This ratio is lower for defaulted firm, on average, consistently in all samples of this research.

Activity ratio 6

$$Act6 = \frac{Payroll\ Expenses}{Total\ Asset}$$

Payroll expenses is a firm’s remunerations to its current and former employees and executives. It comprises all form of payment from a company to it employees, such as salary, holiday pays, gifts, bonuses, company car, and etcetera. This ratio simply shows how big are these payments relatively to the firm’s total assets. In all samples, on average, this ratio is approximately 30 percentage points higher for defaulted firms.

Activity ratio 7

$$Act7 = \frac{Depreciation}{Total\ Asset}$$

The depreciation is categorized as operating expenses, and it appears on the income statement of a firm. Depreciation has been defined as an expense resulted from the limited expected useful life of the fixed assets[34]. For example, if a fixed asset has an expected useful life of *Y* years, and its initial cost is *P* dollars, it must to be depreciated at the rate of $\frac{P}{Y}$ dollars per year. While it is a non-cash expense, depreciation has positive impact on a firm’s cash flow

[34] If depreciation is negative for an asset it means that the asset gaining value over time.

through the tax shield it provides[35]. However, this ratio (Act7) is higher for defaulted firms in all samples.

4.2.6 Growth

$$Gro = ROA_t - ROA_{t-1}$$

Change in ROA (return on assets) is used as the growth predictor in this research. Moody's KMV (2004) for Norway used this indicator as one of its model predictors. According to Moody's (2004), this indicator represents a non-linear behaviour such that both sharp negative growth and rapid growth have a tendency to increase the probability of default of a firm.

4.2.7 Size

Size of the firm may have an impact on the probability of default (PD). There are two typical ways to include the size in the analyses; first to create dummy variables for different size classes; second, taking the natural logarithm. In this research, total assets assumed to be a size indicator. First all the total assets from different year calculated in 2015 value, and the natural logarithms are taken. The reason to make all the size in a certain year value is to increase the accuracy for size comparison. However, all the other financial indicators are in ratios, and there is no need to discount ratios (both nominator and denominator have to be discounted with the same rate). Size indicator shows a slightly lower value for defaulted firms in all samples of this study.

4.2.8 Age

It is common to consider younger companies more prone to defaults. Thus, an age indicator is included in this research analyses. This indicator has been calculated from registration date. That is, date of the financial information (in year) minus date of registration (in year).

[35] That is, unlevered net income is calculated as (revenues – costs – depreciation) multiply by (1 – tax-rate). However, in calculation of the free-cash-flow, depreciation is added back to the unlevered net income. Thus, it provides tax-shield equal to the tax-rate multiply by depreciation.

4.2.9 Qualitative

Lehman (2003) and Grunet et al. (2004) suggested that using qualitative variables like type of the industry, employees count, geographical region, and the legal form of the business would increase the predictive power of models to forecast SME default. Thus, all these four indicators are included in this research.

Industry Type

The industry a firm operates in is influencing many aspects of its activities. Political and economic changes may affect each industry in a different way as industries are market related. Moreover, specific regulations for a certain industry will mainly affect those firms that are active in that industry or partially dependant on it. The industry type indicator in this research has 9 different sectors as follow: Agriculture, Offshore/Shipping, Transport, Manufacturing, Telecom/IT/Tech, Electricity, Construction, Wholesale/Retail, and Other services. SMEs belonged to Finance/Insurance and Real Estate sectors are excluded from this research’s sample for their different characteristics (Explained in section 4.1 of this paper). Dummy variables are defined for each industry type.

Geographical Region

Different geographical regions may have some advantages for some firms, and others may have disadvantages. For example, a firm may benefit from higher demand in a certain geographical region, while it would not be such demand in other region. Moreover, some regulation which exclusively issued for a certain region affect businesses on that region not on the other ones. This can impact default probability of firms that are active in different region. For investigating that, seven dummy variables are defined based on seven regions in Norway. Table III in appendix describes the geographical region dummies.

Number of Employees

Having different number of employees may impact the probability of default in some ways. For example, different regulations may apply for different employees count in a certain country. For examining employee count on probability of default (PD) there is a variable that indicates the number of full-time employees in each SMEs.

Ownership Categories

Ownership structure of a firm may have effect on default probability either directly or indirectly through its impact on the firm performance. There are nine ownership categories recorded in data base that is used in this research. These nine categories are as follow:

1. Unknown ownership structure
2. Publicly listed or part of such concern
3. Company-owned, or Norw. Co.s have majority
4. Owned by individuals, one or more
5. Combined ownership (individuals/company)
6. Public sector ownership (>50%)
7. ASA, not publicly listed
8. Cooperative
9. Owned by foreigners

A dummy variable is defined for each of the above-mentioned category.

4.3 Variable Selection

After defining and calculating candidate predictors, a six steps variable selection is used for each of the three different samples. First, a univariate test for each variable which is not classified as a qualitative variable in Table 2 is estimated; the first 20 variables for each sample are selected from the F-test result of the univariate regressions (Table 3 exhibits the result for the balanced sample of one-year probability of default model, tables for three-year and five-year PDs are available in appendix). In the next stage, an exhaustive list of all possible combinations based on their categories of the candidate variables is programmed. That is, one variable from each category is drawn and estimated using logistic regression in each regression[36]. Then the best 3 combinations regarding their AIC, and BIC are selected.

[36] For one-year PD balanced samples 360 non-redundant logistic regressions with 8 independent variables were listed and ran. 576 non-redundant regressions with 8 independent variables were tested for each of the three and five-year PD samples.

Table 3. Variables list based on one period prior to default data (one-year PD balanced sample).

Variable			Population means		Univariate
No.	Category*	Name	Defaulted	Non-Defaulted	*F*-test
1	3	Net Pre-Tax Income/Total Assets	-0.2601	0.0919	1235.0
2	3	EBITDA/Total Assets	-0.1441	0.1417	1092.3
3	3	Net Income/Total Assets	-0.2567	0.0628	1047.7
4	2	Cash/Total Assets	0.0789	0.1895	631.1
5	2	Working Capital/Total Assets	-0.1850	0.1345	588.3
6	1	Public Charges/Total Assets	0.1243	0.0647	383.6
7	5	Payroll Expenses/Total Asset	0.7331	0.4643	326.6
8	1	Equity/Liabilities	-0.0176	0.6885	231.0
9	8	Age of the Firm (Based of registration date)	9.8545	14.4310	227.3
10	1	Liabilities/Total Assets	1.2291	0.7581	217.6
11	6	ROA(t)- ROA(t-1)	-0.1397	0.0047	129.8
12	5	Sales/Total Assets	2.6319	1.9576	129.4
13	7	Natural Logarithm of Total Assets in 2015	8.6066	8.9322	128.6
14	5	Total Income/Total Assets	2.6852	2.0116	127.4
15	2	Intangible Assets/Total Assets	0.0440	0.0218	81.1
16	5	Depreciation/Total Asset	0.0535	0.0396	69.8
17	4	EBIT/Interest Expenses	-24.3017	91.9985	53.5
18	4	EBITDA/Interest Expenses	-20.2921	118.1007	53.5
19	1	Total interest-bearing liabilities/Total Assets	0.5449	0.3263	47.6
20	5	Trade Debtors/Liabilities	0.2178	0.2511	23.7
21	3	EBIT/Total Income	-0.5344	0.0191	19.5
22	5	Trade Creditors/Total Income	0.3277	0.1252	10.8
23	1	Public Charges/Total Income	0.0638	0.0464	10.4
24	1	Short Term Debt/Equity	-1.0235	5.1844	8.1
25	3	EBIT/Sales	-0.8567	1.0016	4.5
26	3	Net Income/Sales	-1.1036	1.5164	4.3
27	3	Net Income/Total Income	-0.6658	-0.1751	3.3
28	2	Cash/EBIT	-0.1073	3.7218	0.4
29	5	Trade Creditors/Sales	0.4522	0.4884	0.0

*Categories:

1 = Leverage

2 = Liquidity

3 = Profitability

4 = Coverage

5 = Activity

6 = Growth

7 = Size

8 = Age

In the third stage, remaining uncorrelated variables from each category are added to the selected models from the second stage; in order to avoid severe multicollinearity, the added variable not only should not be strongly correlated with the variable which is already in the model, but also must not to be correlated with other variable if there is potentially more than one variable remains to add. The fourth stage is to remove insignificant variables from each constructed model, as far as that variable does not add any predictive value to the model. The best model based on AIC and BIC was selected in this stage. Table 4 shows the best models for each sample without any qualitative variables. For testing multicollinearity, variance inflation factor is used; the test result (see appendix) does not denote any multicollinearity problem (see Wooldridge (2000)), none of the variables that represented in Table 5 has a VIF[37] of more than 2.

Table 4. Best model for each sample before adding qualitative variables.

Category	**One-year PD**	**Three-year PD**	**Five-year PD**
Leverage	Liabilities/Assets Public Charges/Assets	Public Charges/Assets interest-bearing liabilities/Assets	Public Charges/Assets interest-bearing liabilities/Assets
Liquidity	Cash/Assets Intangible Assets/Assets	Cash/Assets Working Capital/Assets	Cash/Assets Working Capital/Assets
Profitability	Net Pre-Tax Income/Assets	Net Pre-Tax Income/Assets	Net Pre-Tax Income/Assets
Activity	Payroll Expenses/Asset Depreciation/Asset	Sales/Assets	Income/Asset Depreciation/Asset
Age	Age	Age	Age
Size	-	Log (Assets in 2015 NOK)	Log (Assets in 2015 NOK)

[37] Regarding Wooldridge (2000) variance inflation factor (VIF) has an inverse relationship with the tolerance value (1-R^2); i.e., a tolerance value of 0.10 corresponds to a VIF of 10. There is no predetermined threshold for VIF, it depends on sample size and is rather arbitrary. However, VIF of greater than 4 is a sign for possibility of existence of multicollinearity, and VIF greater than 10 is a sign for serious multicollinearity problem.

Following Lehman (2003) and Grunet et al. (2004), I add qualitative variables in next stage (5^{th} stage); results of adding one qualitative variable to the models from the previous stage is represented in Table 5 and 6. Misclassifications rates are illustrated in two ways: the type I error which shows the percentage of defaulted SMEs classified as non-defaulted; type II error that illustrates the percentage of non-defaulted SMEs classified as defaulted. The average of correctly classified defaulted and non-defaulted SMEs is shown as "correctly classified[38]" in the tables. The average power of the test corresponding to all cut-off rates on default to non-default is denoted as area under curve (AUC). Accuracy ratio (AR) which is defined as the ratio of the area between the rating model cumulative accuracy profile (CAP) being validated and the random model CAP. In fact, the model's power to maximize the distance between defaulted and non-defaulted firms can be measured by AR (Altman and Sabato, 2007). Accuracy ratio (AR) can be calculated as follow (see Engelman et al. (2003) for proof):

$$AR = 2AUC - 1$$

According to the results, adding qualitative variables has overall positive impact on accuracy ratio (AR) of the model; however, this impact is not dramatic at all (Table 5 and 6). The best qualitative variable among the four qualitative variables those are tested appeared to be industry type indicator with positive impact of approximately 0.4 percentage point in each model. Therefore, I decided to keep industry type indicator in all models. Among the three remaining qualitative indicators, ownership category indicator has negative impact on one-year PD model and positive impact in both three-year and five-year PD models; this impact is greater in five-year PD model. However, ownership category indicator data is missing for many firms, and therefore, using it in final models results into remove at least 40 percent observations. Moreover, keeping ownership category in a model contributes only 0.22 percentage point to accuracy ratio at its highest level in the five-year PD model. Thus, keeping the 40 percent of the observations is preferable here.

[38] Correctly classified = 1 – average of error type I and error type II.

Table 5. Models' comparison with and without qualitative variables.

Model	Error Type I[39]	Error Type II[40]	Correctly classified	AUC	AR
One-year PD	19.17%	15.43%	82.70%	89.76%	79.52%
+ Industry	19.01% (-0.16%)	15.04% (-0.39%)	82.98% (0.28%)	89.99% (0.23%)	79.98% (0.46%)
+ Region	19.08% (-0.10%)	15.27% (-0.16%)	82.83% (0.13%)	89.78% (0.02%)	79.56% (0.04%)
+ #Employees	19.04% (-0.13%)	15.27% (-0.16%)	82.85% (0.15%)	89.80% (0.04%)	79.61% (0.08%)
Three-year PD	23.18%	22.09%	77.37%	84.72%	69.44%
+ Industry	22.67% (-0.51%	21.88% (-0.21%)	77.72% (0.36%)	84.90% (0.18%)	69.80% (0.36%)
+ Region	23.12% (-0.06%	22.17% (0.08%)	77.36% (-0.01%)	84.79% (0.06%)	69.57% (0.13%)
+ #Employees	23.15% (-0.03%	22.21% (0.12%)	77.32% (-0.05%)	84.72% (0.00%)	69.45% (0.01%)
Five-year PD	22.78%	22.75%	77.23%	84.64%	69.29%
+ Industry	22.41% (-0.37%	22.28% (-0.47%)	77.65% (0.42%)	84.83% (0.18%)	69.65% (0.37%)
+ Region	22.72% (-0.06%	22.51% (-0.24%)	77.39% (0.15%)	84.70% (0.06%)	69.41% (0.12%)
+ #Employees	22.49% (-0.29%	22.81% (0.06%)	77.35% (0.11%)	84.66% (0.02%)	69.32% (0.04%)

Note: Differences are in parentheses (model with a qualitative variable - model without any qualitative variable)

[39] Error type I is also known as false positive finding, which in here denotes the percentage of non-defaulted firms that classified as defaulted firms under the constructed model prediction.

[40] Error type II is also known as false negative finding, which in here denotes the percentage of defaulted firms that classified as non-defaulted firms under the constructed model prediction.

Table 6. Models' comparison with and without ownership categories[41].

Model	Error Type I	Error Type II	Correctly classified	AUC	AR
One-year PD	18.68%	16.01%	82.65%	89.76%	79.52%
+ Ownership Categories	18.68% (0.00%)	16.44% (0.43%)	82.44% (-0.22%)	89.74% (-0.02%)	79.48% (-0.04%)
Three-year PD	23.01%	22.04%	77.47%	85.01%	70.02%
+ Ownership Categories	22.60% (-0.41%	21.82% (-0.22%)	77.79% (0.32%)	85.09% (0.08%)	70.19% (0.16%)
Five-year PD	22.84%	22.76%	77.20%	84.75%	69.50%
+ Ownership Categories	22.21% (-0.63%	22.86% (0.11%)	77.46% (0.26%)	84.86% (0.11%)	69.73% (0.22%)

Note: Differences are in parentheses (model with a qualitative variable - model without any qualitative variable)

[41] Models represented in this table have smaller sample sizes compare to models in Table 5. Thus, accuracy ratios for models without ownership categories indicator have different accuracy ratios with the ones that are shown in 5. The main point that need to be observed is by how much this qualitative variable will add accuracy to each model.

5. Empirical Results

All the three models show relatively good predictive power for Norwegian SMEs. In this section, first, variables' behavior in each model is discussed. Then out of sample performance and model's power is described. Finally, minimum capital recruitments are calculated using different PD models.

5.1 Logistic regressions results

After selecting potentially best predictors in variable selection steps, I run a Wald test for each predictor, and for all variables together in each category. Moreover, I use log-likelihood test for testing models with industry type variable and without it. The Wald test for each variables and variables in each category together is statistically significant ($p < 0.001$ for all tests). Also, the log-likelihood test between nested (without industry type indicator) and non-nested is statistically significant ($p < 0.001$). Therefore, it is possible to argue a significantly strong relationship between the default event and the selected predictors.

Using original form of predictors (accounting ratios), I found unexpected behavior of the first activity predictors (Payroll expenses/Total Assets, Sales/Total Assets, and Income/Total Assets in one-year, three-year, and five-year PD models respectively) in the constructed models; all these three predictors have positive signs. Other predictors have expected slopes.

I tested both log-transformation of total asset, and original form of total assets in variable selection steps; result suggested to use log-transformed term for total asset (calculated in 2015 Norwegian Kroner). To capture non-linearity, I tried different forms of each independent variables (quadratic term, cubic term, and also log-transformation[42]). Result suggests using the original forms along with quadratic terms for the first activity predictor (Payroll expenses/Total Assets) in one-year PD model, and original term for first activity independent variables ((Sales/Total Assets, and Income/Total assets) together with quadratic

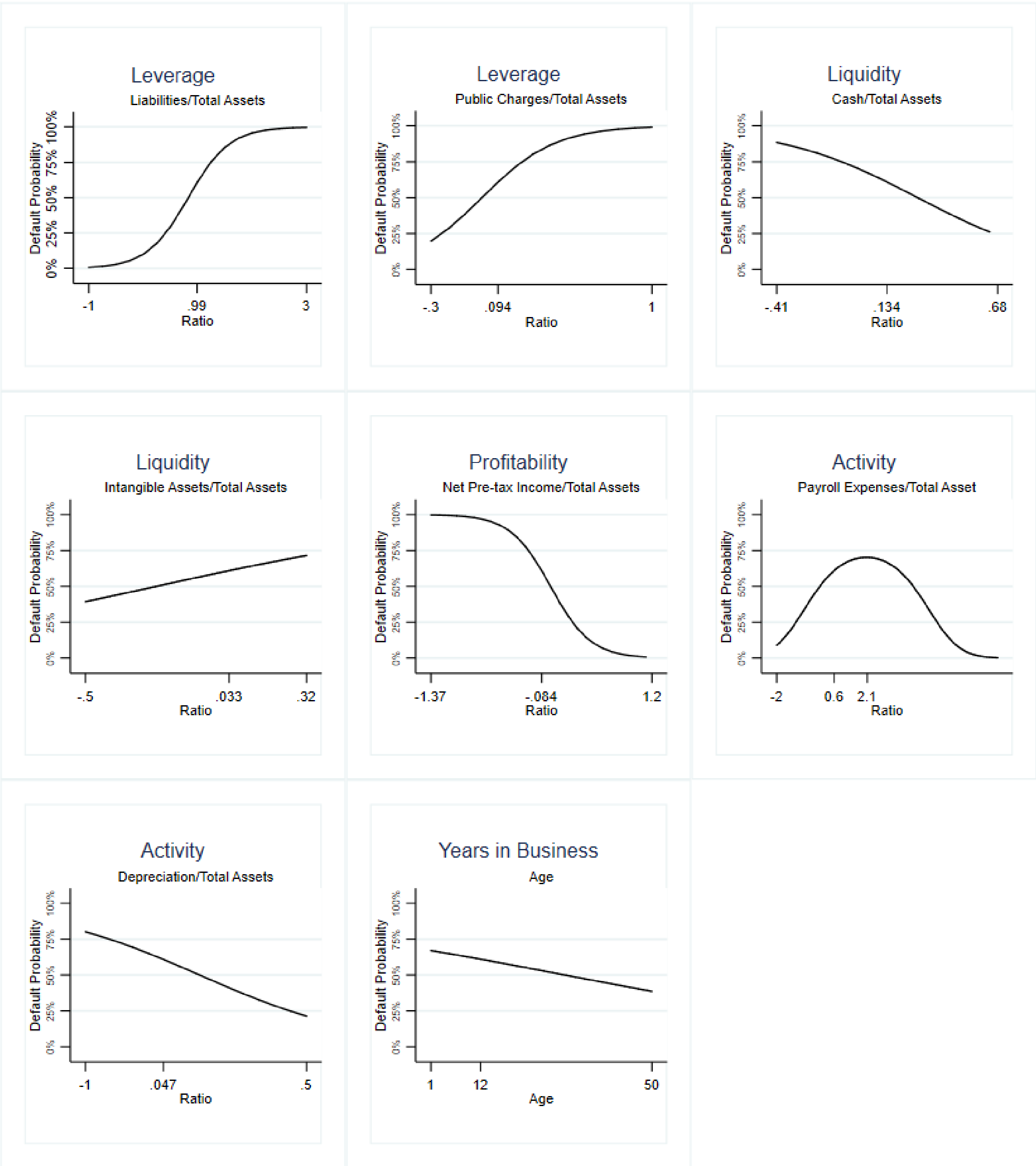
[42] Edward I.Altman and Gabriele Sabato (2007) used log-transformed accounting ratios in their study of "Modeling Credit Risk for SMEs: Evidence from the US Market" and found relatively better result than using original non-transformed predictors.

and cubic terms for both three-year and five-year PD models; original form of all other predictors need to be presented in each model. I run Wald test for added terms, results show significant ($p < 0.001$) model improvement using new terms for all the three models. Moreover, I tested log-likelihood for model with and without (nested) square and cubic terms, results confirm the Wald results. Additionally, I jointly tested original predictor along with added term in each model; results indicate significantly ($p < 0.001$) positive improvement of each model using both forms of the predictors. For testing the statistical technique, logistic regression in this study, Hosmer-Lemeshow (H-L) test (Hosmer and Lemeshow, 1989)[43] is used; following a suggestion by Hosmer, Lemeshow, and Sturdivant (2013), I regrouped the data by predicted probabilities and then formed 8 equal-sized group; I cannot reject any of the constructed model based on the test results (P values equal to 0.5833, 0.6235, and 0.9879 for one-year, three-year, and five-year PD respectively). For all the models, H-L tests show highly nonsignificant p-values. Thus, it is not possible to reject the null hypothesis of that the models are not good fits. Moreover, testing for functional misspecification, the framework suggested by Pregibon (1980) is used for each model. Results for all the three models attested no functional misspecification.

Figure 3 illustrates margin plots for independent variables used in one-year probability of default model (Margin plots for three-year and five-year PDs are available in appendix); the middle values represent population mean for the variable in each plot. Each plot represents that specific predictor marginal change at different level while other predictors holding at their means. Specific margin plot for the first activity predictors (Payroll expenses/Total Assets) is combination of all terms; this ratio shows a bell-shape behavior, that is, first it has positive impact on default probability, and then after a certain point (2.1) it has negative relationship with default probability.

[43] A random subsample of each original sample is used for tests. According to a study by Bentler and Bonett (1980), any model is prone to get rejected as inadequate when sample size is large.

Figure 3. Margin plots for One-year PD



5.2 Out of sample performance of the models

In order to check the out of sample performance of the constructed models, ten sub-samples were randomly sampled in a two-step random sampling method as follow: first 10 random samples for each model were created in the same way as the main samples had been constructed but with different seeds[44]; then, new sub-samples randomly sampled again with 10 percent of their original sample sizes. Applying that method has two attributes: the sub-samples include different non-defaulted SMEs rather than the main samples; the ratio of the defaulted to non-defaulted SMEs will not be the same as the main samples. In addition of the ten subsamples, the relative cumulative default probability samples are also added to the tables. Table 7, 8, and 9 show the results for out of sample tests.

Table 7. One-year PD out of sample test results.

	Error Type I	Error Type II	Correctly classified	AR
Subsample 1	18.33%	14.52%	83.55%	82.05%
Subsample 2	19.22%	15.96%	82.41%	79.44%
Subsample 3	15.29%	11.00%	86.81%	84.81%
Subsample 4	19.35%	17.11%	81.76%	79.10%
Subsample 5	20.36%	11.79%	83.55%	81.72%
Subsample 6	20.54%	17.35%	81.11%	77.27%
Subsample 7	15.19%	16.44%	84.20%	79.20%
Subsample 8	12.74%	16.33%	85.50%	82.93%
Subsample 9	16.09%	19.19%	82.41%	79.71%
Subsample 10	18.60%	10.64%	85.67%	84.77%
Out of sample (mean)	17.57%	15.03%	83.70%	81.10%
Cumulative PD Sample	18.59%	16.75%	83.21%	78.47%
In-Sample	18.59%	15.49%	82.96%	80.09%

According to the sub-samples' performance of one-year PD model, the minimum accuracy ratio for a subsample is 77.27%, and maximum is 84.81%; overall out of sample accuracy ratio represents a mean of 81.10% which attest a good out of sample performance of the constructed model. Predicting defaults on the cumulative default probability sample (with only 2.1% of the firms defaulted) with the model that is derived from the balanced sample also shows a relevantly close accuracy ratio (78.47%) to the balanced sample. Comparing accuracy

[44] Seed specifies the initial value of the random-number used by the random-number functions to create the random samples (see https://www.stata.com/manuals13/rsetseed.pdf for further details).

ratio of balanced sample and imbalanced one (with approximately more than 140 thousand more 0’s observations than balanced sample), a performance difference of only 1.6 percentage point is observable. Consistent with a study of rare events by King and Zeng (2001), as it is mentioned in the previous chapter, using all 1’s (defaulted firms) and a random sample of 0’s (non-defaulted firms), there is not a dramatic drawback in the model performance. Checking the type II error (non-defaulted classified as defaulted), it denotes 1.25 percentage point more misclassification which is relatively a small decrease of performance of the model. Nevertheless, using a relatively huge sample size with more than 100 thousand observations would make a researcher to drop many potentially relevant variables for avoiding complete or quasi-complete separation (see Allison, 2008). Moreover, I have attempted to construct a model based on a cumulative PD sample and use the same variables and then less variables, using approximately 50 different sample seeds for each model, the logistic regression has never converged.

Figure 4 shows the cumulative accuracy profile of the one-year PD model for both balanced and imbalanced sample. Two curved lines represent the model’s performance over balanced and imbalanced samples being evaluated in describing the rate of defaults captured by the model (True-positive rate) over rate of non-defaulted classified as defaulted (False-positive rate). The straight line shows the zero-information case. Clearly, the model shows a very similar power over the both samples.

Figure 4. Receiver operating characteristic (ROC) curves comparison, applying the one-year PD model on balanced and imbalanced samples.

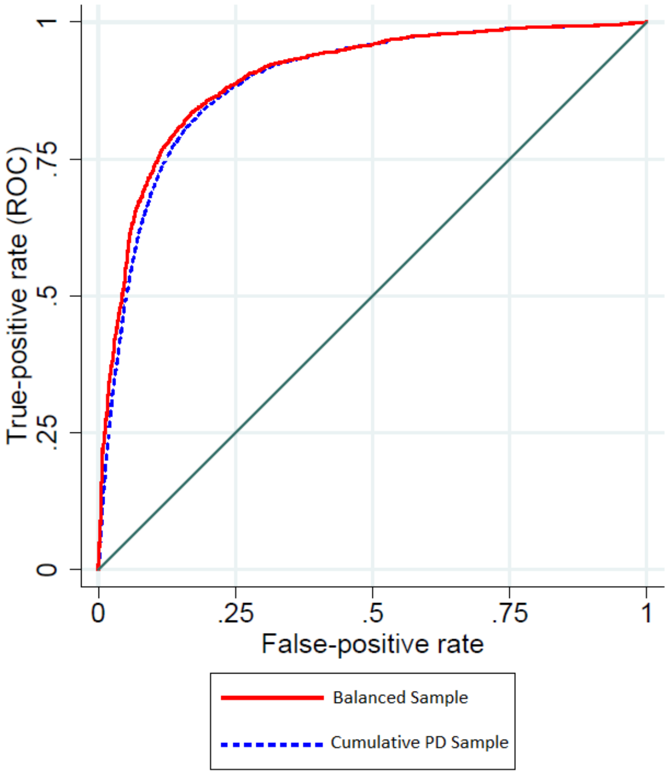


Table 8. Three-year PD out of sample test results.

	Error Type I	Error Type II	Correctly classified	AR
Subsample 1	23.27%	20.05%	78.41%	70.74%
Subsample 2	20.86%	21.67%	78.72%	70.94%
Subsample 3	21.36%	21.81%	78.41%	72.21%
Subsample 4	21.79%	23.35%	77.43%	69.94%
Subsample 5	23.09%	24.62%	76.14%	68.34%
Subsample 6	22.42%	22.09%	77.74%	72.32%
Subsample 7	22.31%	21.67%	78.01%	71.00%
Subsample 8	21.68%	20.53%	78.90%	71.22%
Subsample 9	20.28%	21.21%	79.26%	70.58%
Subsample 10	23.56%	22.09%	77.16%	67.80%
Out of sample (mean)	22.06%	21.91%	78.02%	70.51%
Cumulative PD Sample	18.59%	16.75%	83.21%	69.16%
In-Sample	18.59%	15.49%	82.96%	69.95%

Similar to one-year PD model, three-year PD model also shows a relatively good out of sample performance; smallest accuracy ratio is 67.80% and the greatest is 72.32%. Applying the three-year PD model on balanced and imbalanced denotes a considerably small deterioration of AR, only 0.79 percentage point. Power curve comparison for three-year PD is available in appendix.

Table 9. Five-year PD out of sample test results.

	Error Type I	Error Type II	Correctly classified	AR
Subsample 1	21.18%	21.43%	78.69%	71.15%
Subsample 2	20.24%	22.78%	78.52%	71.00%
Subsample 3	23.18%	20.82%	78.03%	68.93%
Subsample 4	22.65%	21.29%	78.03%	69.99%
Subsample 5	21.50%	23.02%	77.73%	70.43%
Subsample 6	20.50%	22.79%	78.38%	68.97%
Subsample 7	21.86%	25.06%	76.51%	69.90%
Subsample 8	20.39%	22.47%	78.56%	70.23%
Subsample 9	21.52%	22.32%	78.08%	71.42%
Subsample 10	23.72%	21.93%	77.16%	69.70%
Out of sample (mean)	21.67%	22.39%	77.97%	70.17%
Cumulative PD Sample	18.59%	16.75%	83.21%	69.50%
In-Sample	18.59%	15.49%	82.96%	70.06%

Five-year PD model also represents a relatively efficient out of sample predicting power. The lowest AR is 68.93%, and the highest is 71.42%. The difference between balanced and imbalanced sample became smaller in five-year PD model. Power curve comparison for five-year PD over these two samples is available in appendix.

All the three models show good out of sample prediction power. Comparing to existed models[45] for Norwegian SMEs one-year and five-year PD, all models significantly perform better (Table 10).

[45] Moody's study for small and medium-sized firms in Norway, "MOODY'S KMV RISKCALC V3.1", July 2006.

Table 10. Comparison between the existed models and models introduced by the current study.

	One-year Model		Five-year Model	
	Best Accuracy Ratio	Improvement	Best Accuracy Ratio	Improvement
Moody's KMV	66.3%		53.9%	
This study's model	81.1%	**14.8%**	70.1%	**16.2%**

5.3 Basel III capital requirements for SMEs

Similar to the Basel II, SMEs can be classified as retail customers or corporates based on the Basel III. Nevertheless, the main change in the new Basel release, for SMEs capital requirements using the internal rating based (IRB), is imperative. On the Basel III, capital requirements for SMEs as retail customers remained unchanged at 75% risk weight (risk-weighted asset, also known as RWA, is a bank's off-balance-sheet exposure adjusted for risk); however, a risk weight of 85% has been introduced for SMEs as corporates[46]; this reduced the capital requirements for SMEs as corporate to 6.8%.[47]. According to the data in hand for Norway, approximately 90% present of the SMEs that can be classified as corporates have sales between 50 to 250 million NOK, and 10% have sales between 250 to 500 million NOK. Given SMEs distribution in Norway, and the size adjustment in the formula provided by the latest Basel for calculating capital requirements for SMEs as corporates[48], there is a possibility for even lower capital requirements for Norwegian SMEs as corporates. Following the classification from studies by Altman and Sabato (2005, 2007) and the Basel III final release (which clearly states that sales between 5 million Euro and less than 50 million Euro is needed for a SME to be classified as corporate[49]- approximately, between 50 million and 500 million NOK), for calculating capital requirements for SMEs as corporates a three-year and five-year

[46] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, paragraph 41 and 43.

[47] That was 8% on Basel Committee on banking Supervision, June 2004.

[48] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 53, 54 and 70.

[49] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 54.

maturities are used for small-sized (more than 50 to 250) and medium-sized (more than 250 to 500) corporates, respectively.

The Basel II and III[50] suggest using a one-year PD models to calculate capital requirement for different maturities. Following the Basel II, Altman and Sabato (2005, 2007) used same rating classifications in order to calculate capital requirements for SMEs whether as retail customers or corporates, or for longer maturities than one year. That is, a one-year PD model is used in their all calculations. However, I developed three different models for different maturities, given availability of high-quality data for Norway. First, I calculated the capital requirements in the same way as Altman and Sabato, and then applied different rating classes based on specific models for different maturities. In all models the impact on capital requirements have been analyzed whether the whole SMEs' portfolio is regarded as corporates or retail customers. Twelve rating classes are created with the one-year PD model. Each rating classes has been constructed based on specific range of default probability resulted from the model, then the probability of default (PD) of each class is determined by dividing the number of actual defaulted SMEs by the total SMEs in that class. Equivalent bond probability of default distribution is considered as the milestone to create rating classes (starting PD is 0.05% instead of 0.03%; this is clearly mentioned in the latest Basel that PD in the risk weight formula cannot be less than 0.05%[51]). According to the Basel III suggestion under the foundation approach for senior unsecured loan exposure, loss given default (LGD) is set to 40%[52] (It was 45% under the previous Basel Accord). Despite the high-quality of the data, there is no information on total amount of loan by a specific lender; this prevent me to calculate exposure at default (EAD) or LGD for each counterparty. Following the Altman and Sabato (2005), I assumed the loan exposure amount equal to EAD for on-balance sheet items[53]. Nonetheless,

[50] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 67 and 121.

[51] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 68.

[52] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 70.

[53] It explained in paragraph 98 of the Basel III "Internal rating-based approach for credit risk", December 2017.

there is no fixed LGD, and standard EAD calculation approach for retail customers in the Basel III[54].

I assumed a fixed LGD of 40% for both retail customers and corporates. Moreover, instead of using the actual monetary value of the loan exposure as the weight for the capital requirement, the percentage of SMEs in each ranking category is used. These two assumptions eventually do not have substantial impact on the results according to Altman and Sabato (2005).

The formula for risk-weighted asset is:

$$RWA = K \cdot 12.5 \cdot EAD$$

However, as final capital requirement is 8% of this amount (regardless of risk-weights specific for SMEs as retail customers or corporates), we can factor out 8% and 12.5 from the formula. The latest formula for the SMEs classified as retail customers[55] is represented below; for a normal random variable X, $N(X)$ stands for the cumulative distribution function and $G(X)$ denotes the inverse cumulative distribution function.

$$Capital\ requirement\ (K) = \left\{ LGD \cdot N\left[\frac{G(PD)}{\sqrt{(1-R)}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot LGD \right\}$$

$$Correlation\ (R) = 0.03 \cdot \frac{1-e^{-35 \cdot PD}}{1-e^{-35}} + 0.16 \cdot \left(1 - \frac{1-e^{-35 \cdot PD}}{1-e^{-35}}\right)$$

Applying the one-year PD model on the specific cumulative probability of default (2.1%) sample with 146,285 total SMEs (included 6,144 defaulted SMEs), and solving the capital requirements using above formula, results in a capital requirement equal to 2.52% if and only if we assume the whole portfolio as retail customers. The result is represented in table 11; in the fifth column capital requirements (K_{sme}) for each rating class is listed, in the seventh

[54] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 41.

[55] Basel Committee on banking Supervision, "Basel III: Finalising post-crisis reforms", December 2017, Internal rating-based approach for credit risk paragraph 54.

column cumulative weighted capital requirements (Cum. Weighted K_{sme}) are shown which are cumulated sum of products of each rating class's capital requirement and its weight.

Table 11. Capital requirements for all SMEs as retail customers.

Rating	PD	LGD	R_{sme}	K_{sme}	Weight	Cum. Weighted K_{sme}
AAA	0.05%	40%	0.15774	0.0047140	0.00260	0.0012%
AA	0.06%	40%	0.15730	0.0054196	0.01080	0.0071%
A	0.11%	40%	0.15509	0.0085094	0.15065	0.1353%
BBB+	0.15%	40%	0.15335	0.0106227	0.13964	0.2836%
BBB	0.30%	40%	0.14704	0.0169234	0.26109	0.7255%
BBB-	0.50%	40%	0.13913	0.0230124	0.08730	0.9264%
BB	1.00%	40%	0.12161	0.0325495	0.07995	1.1866%
BB-	1.50%	40%	0.10690	0.0379527	0.04935	1.3739%
B+	3.00%	40%	0.07549	0.0446520	0.10120	1.8258%
B	6.00%	40%	0.04592	0.0481643	0.03259	1.9828%
B-	10.00%	40%	0.03393	0.0537193	0.03128	2.1508%
CCC	18.29%	40%	0.03022	0.0686874	0.05355	**2.5186%**

In order to calculate capital requirements for SMEs as corporates, there is specific adjustments for size and maturity. The first part of the capital requirements formula is the same as the one for retail customers; however, there is adjustments for maturity in the second part. Moreover, the formula for correlation is different for SMEs as corporates, and an adjustment for size is appeared in the formula. *M* denotes the maturity for each sales class which I assumed three-year maturities for SMEs as small-sized corporates and five-year for SMEs as medium-sized corporates. Maturity adjustment is shown as (b) which is just a function of probability of default. For the size parameter (S) an average number of 10 million Euro (100 MMNOK) for small SMEs and 30 million Euro (300 MMNOK) for medium-sized SMEs are inserted in the formula.

$$Capital\ requirement\ (K) = \left\{LGD \cdot N\left[\frac{G(PD)}{\sqrt{(1-R)}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot LGD\right\} \cdot \frac{1+(M-2.5)\cdot b}{1-1.5\cdot b}$$

$$Correlation\ (R) = 0.12 \cdot \frac{1-e^{-50\cdot PD}}{1-e^{-50}} + 0.24 \cdot \left(1 - \frac{1-e^{-50\cdot PD}}{1-e^{-50}}\right) - \left(1 - \frac{S-5}{45}\right)$$

$$Maturity\ adjustment\ (b) = \{0.11852 - 0.05478 \cdot ln(PD)\}^2$$

Applying the one-year PD model on the same sample for one-year PD (as it has been done in previous researches by Altman and Sabato (2005, 2007)), and solving the capital requirements using above formula for SMEs as corporates, results in a capital requirement equal to 4.85% for small-sized SMEs and 6.77% for medium-sized SMEs. The result is represented in table 12; in the fifth column calculated maturity adjustment (*b*) for each rating class is represented; capital requirements (K_{sme}) relevant to each rating class is shown in the seventh column, and in the last column cumulative weighted capital requirements (Cum. Weighted K_{sme}) are shown. Relevant maturity (M_{eff}) is also shown in the sixth column of the table. As it mentioned earlier, according to the SMEs distribution in the population, only 10 percent of the SMEs in the real market are medium-sized SMEs; therefore, weighted capital requirements applying the one-year PD model for SMEs as corporate is 5.04%.

Table 12. Capital requirements for all SMEs as corporates

Rating	PD	LGD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	$K_{corp.}$	Weight	Cum. Weighted $K_{corp.}$
Sales 50 - 250 MMNok								
AAA	0.05%	40%	0.20148	0.28612	3	0.0128823	0.00260	0.0033%
AA	0.06%	40%	0.20090	0.27553	3	0.0143122	0.01080	0.0188%
A	0.11%	40%	0.19802	0.24177	3	0.0202048	0.15065	0.3232%
BBB+	0.15%	40%	0.19577	0.22535	3	0.0239768	0.13964	0.6580%
BBB	0.30%	40%	0.18773	0.19075	3	0.0343719	0.26109	1.5554%
BBB-	0.50%	40%	0.17790	0.16709	3	0.0435254	0.08730	1.9354%
BB	1.00%	40%	0.15723	0.13749	3	0.0566578	0.07995	2.3884%
BB-	1.50%	40%	0.14113	0.12151	3	0.0638678	0.04935	2.7036%
B+	3.00%	40%	0.11122	0.09648	3	0.0752869	0.10120	3.4655%
B	6.00%	40%	0.09042	0.07433	3	0.0914911	0.03259	3.7637%
B-	10.00%	40%	0.08525	0.05986	3	0.1112751	0.03128	4.1118%
CCC	18.29%	40%	0.08446	0.04477	3	0.1378624	0.05355	**4.8500%**
Sales 251 - 500 MMNok								
AAA	0.05%	40%	0.21926	0.28613	5	0.0216025	0.00260	0.0056%
AA	0.06%	40%	0.21868	0.27554	5	0.0237323	0.01080	0.0312%
A	0.11%	40%	0.21580	0.24178	5	0.0322700	0.15065	0.5174%
BBB+	0.15%	40%	0.21355	0.22536	5	0.0375667	0.13964	1.0420%
BBB	0.30%	40%	0.20551	0.19075	5	0.0516255	0.26109	2.3899%
BBB-	0.50%	40%	0.19568	0.16709	5	0.0634521	0.08730	2.9438%
BB	1.00%	40%	0.17501	0.13749	5	0.0796131	0.07995	3.5803%
BB-	1.50%	40%	0.15891	0.12151	5	0.0881052	0.04935	4.0151%
B+	3.00%	40%	0.12900	0.09648	5	0.1012378	0.10120	5.0396%
B	6.00%	40%	0.10820	0.07434	5	0.1197643	0.03259	5.4300%
B-	10.00%	40%	0.10303	0.05986	5	0.1413984	0.03128	5.8723%
CCC	18.29%	40%	0.10224	0.04477	5	0.1679729	0.05355	**6.7717%**

Having constructed specific models for different probability of default (three-year PD, and five-year PD models) together with data for almost the whole population (almost all registered firms in Norway), makes it possible to calculate capital requirements specifically for different groups and maturities. First, retail customers capital requirements are calculated using the one-year PD model, but in this case only SMEs as retail customers are included in the sample (i.e., all SMEs with sales less than 50 MMNOK). This sample includes 134,173 retail customers which 2,885 of them are defaulted SMEs. This time eleven rating classes based on probability of defaults which was explained earlier is created. Table 13 represents the results; a capital requirement of 2.54% is suggested by using only retail customers.

Table 13. Capital requirements specific for retail customers SMEs (over a sample only includes retail customers SMEs)

Rating	PD	LGD	R_{sme}	K_{sme}	Weight	Cum. Weighted K_{sme}
AAA	0.05%	40%	0.15774	0.0047140	0.00165	0.0008%
A	0.11%	40%	0.15509	0.0085094	0.16056	0.1374%
BBB+	0.15%	40%	0.15335	0.0106227	0.18864	0.3378%
BBB	0.30%	40%	0.14704	0.0169234	0.14680	0.5862%
BBB-	0.50%	40%	0.13913	0.0230124	0.15084	0.9334%
BB	1.00%	40%	0.12161	0.0325495	0.09608	1.2461%
BB-	1.60%	40%	0.10426	0.0387407	0.02560	1.3453%
B+	3.00%	40%	0.07549	0.0446520	0.12213	1.8906%
B	6.34%	40%	0.04413	0.0485110	0.01295	1.9534%
B-	9.99%	40%	0.03394	0.0537020	0.04864	2.2146%
CCC	19.93%	40%	0.03012	0.0712059	0.04610	**2.5429%**

For calculating capital requirements using specific default probability models, size constraints are applied on samples with cumulative default probabilities for three and five-year PDs. New samples include 20,397 (with 533 defaults) and 1,434 (with 37 defaults) for three-year and five-year capital requirements calculation respectively. The main reason to create rating classes exclusively for different size SMEs and different maturities is that an enterprise that has a loan with maturity in 5 years may default on other years than just first year; giving a high score to a firm that will default later than first year while decision needs to be made for a loan with maturity more than one year, does not make that much sense (even if we have correction for different maturities, as a firm that defaults right after first year in a 5-year loan may not receive a high score). For example, in the three-year cumulative probability sample only 27% of defaulted SMEs default on the first year. Using specific models for different maturities potentially can enable us to rank SMEs more accurately. For instance, all parameters hold the same, when the one-year PD model classifies a firm in the BB- class, the three-year PD model classifies the same firm in the B class; based on the three-year PD model (Table 12), linked capital requirement for the BB- class is around 6.4% while the same method results in 9.2% requirement for the class B. Obviously, using a one-year PD model for calculating longer maturities underestimates the outcome for possible capital requirements, which we will see later in this chapter.

Applying a three-year PD model with specific sample of small-sized SMEs, I could create 12 ranking classes (Table 14). Capital requirements for three-year maturity SMEs as and small-sized corporates turns out 6.06%. For five-year maturity and SMEs as medium-sized corporates, nine ranking classes is created, and capital requirements is 9.25%. Weighted capital requirements applying the specific PD models on exclusive samples for SMEs as small and medium-sized corporate is 6.38% (0.10 × 9.25% + 0.90 × 6.06).

Table 14. Capital requirements for all SMEs as corporates applying the new models specific for three-year and five-year PD for SMEs and specific samples for SMEs as small and medium-sized corporates.

Rating	PD	LGD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	$K_{corp.}$	Weight	Cum. Weighted $K_{corp.}$
Sales 50 - 250 MMNok								
AAA	0.05%	40%	0.20148	0.28612	3	0.0128823	0.00230	0.0030%
AA	0.06%	40%	0.20090	0.27553	3	0.0143122	0.00657	0.0124%
A-	0.15%	40%	0.19577	0.22535	3	0.0239768	0.03378	0.0934%
BBB+	0.20%	40%	0.19302	0.21064	3	0.0279657	0.09648	0.3632%
BBB	0.30%	40%	0.18773	0.19075	3	0.0343719	0.14904	0.8755%
BB+	0.75%	40%	0.16692	0.14942	3	0.0512377	0.18189	1.8074%
BB	1.00%	40%	0.15723	0.13749	3	0.0566578	0.07383	2.2258%
BB-	1.50%	40%	0.14113	0.12151	3	0.0638678	0.14012	3.1207%
B+	3.00%	40%	0.11122	0.09648	3	0.0752869	0.14693	4.2269%
B	6.00%	40%	0.09042	0.07433	3	0.0914911	0.07604	4.9226%
B-	10.00%	40%	0.08525	0.05986	3	0.1112751	0.04756	5.4518%
CCC	16.50%	40%	0.08448	0.04719	3	0.1336712	0.04545	**6.0593%**
Sales 251 - 500 MMNok								
AAA	0.05%	40%	0.21926	0.28613	5	0.0216025	0.00136	0.0029%
A	0.10%	40%	0.21637	0.24695	5	0.0307712	0.00272	0.0113%
BBB	0.30%	40%	0.20551	0.19075	5	0.0516255	0.02039	0.1166%
BB+	0.75%	40%	0.18470	0.14943	5	0.0730553	0.09109	0.7821%
BB	1.00%	40%	0.17501	0.13749	5	0.0796131	0.20326	2.4003%
BB-	1.50%	40%	0.15891	0.12151	5	0.0881052	0.23997	4.5146%
B+	3.00%	40%	0.12900	0.09648	5	0.1012378	0.37254	8.2861%
B-	10.00%	40%	0.10303	0.05986	5	0.1413984	0.04759	8.9589%
CCC	19.35%	40%	0.10223	0.04347	5	0.1365919	0.02107	**9.2468%**

Comparing the results from on-year PD model for retail customers capital requirements on the general sample (that includes different size SMEs) and specific sample for SMEs as retail customers, there is slightly more capital requirement using specific sample (0.02%). However, applying exclusive models and samples for different size of SMEs and maturities, this difference is noticeable with 1.34 percentage point higher requirements using specific samples and models (Table 15). Anyway, capital requirements using all models represent a lower percentage than the Basel III suggested ones (at least 0.42 percentage point).

Table 15. Capital requirements comparison based on different PD models and samples.

	Based on the general model	Based on specific models for different maturities
SME as retail	2.52%	2.54%
SME as corporate	5.04%	6.38%

According to the models developed in this research, all the capital requirements for SMEs whether as corporates or retail customers turned out to be lower than the one has been suggested by the Basel III. However, banks use a combination of SMEs in their portfolio. Table 16 shows capital requirements corresponds to different combinations of the SMEs as retail customers and corporates. Second column includes the capital requirements calculated based on the one-year probability of default model, and the third column represents capital requirements linked to specific models and samples. In the fourth column (Standardized) capital requirements are calculated using different risk weight suggested by the Basel III with various portfolio mixture. That is, 6% (0.75 × 8%) for retail customers and 6.8% (0.85 × 8%) for SMEs as corporates. Although applying exclusive PD models to estimate capital requirements results in higher percentages, it does not show higher requirements than using the latest Basel proposition.

Table 16. Breakeven analysis for capital requirements using models developed in this research compare to fixed capital requirements suggested by the latest Basel.

	Capital requirements			
Portfolio combination of SMEs as retail and as corporate	IRB based on the general model	IRB based on specific models	Standardized	Current
0% as retail 100% as corporate	**5.04%**	**6.38%**	6.80%	6.80%
10% as retail 90% as corporate	**4.79%**	**6.00%**	6.72%	6.80%
20% as retail 80% as corporate	**4.54%**	**5.61%**	6.64%	6.80%
30% as retail 70% as corporate	**4.28%**	**5.23%**	6.56%	6.80%
40% as retail 60% as corporate	**4.03%**	**4.84%**	6.48%	6.80%
50% as retail 50% as corporate	**3.78%**	**4.46%**	6.40%	6.80%
60% as retail 40% as corporate	**3.53%**	**4.08%**	6.32%	6.80%
70% as retail 30% as corporate	**3.28%**	**3.69%**	6.24%	6.80%
80% as retail 20% as corporate	**3.02%**	**3.31%**	6.16%	6.80%
90% as retail 10% as corporate	**2.77%**	**2.92%**	6.08%	6.80%
100% as retail 0% as corporate	**2.52%**	**2.54%**	6.00%	6.80%

Finally, for testing impacts of different LGD and maturities (as a fixed LGD of 40% for all SMEs whether classified as retail customers or corporates was assumed), I ran a sensitivity analysis for all constructed models. The different LGDs are 20%, 60%, and 80%; maturities of one-year and five-year for SMEs as small-sized corporates, and three-year and ten-year for SMEs as medium-sized corporates are used into the test. Table 17 shows a summary of the sensitivity test; detail results are available in appendix.

Table 17. Sensitivity analysis

		Based on the general model	Based on the specific models for different maturities
Results with LGD = 20%			
SME as retail		**1.26%**	**1.27%**
	Sales 50 -250	2.42%	2.51%
SME as corporate	Sales 251 -500	3.39%	3.96%
	Combination	**2.52%**	**2.65%**
Results with LGD = 60%			
SME as retail		**3.78%**	**3.81%**
	Sales 50 -250	7.27%	7.51%
SME as corporate	Sales 251 -500	10.16%	11.87%
	Combination	**7.56%**	**7.95%**
Results with LGD = 80%			
SME as retail		**5.04%**	**5.09%**
	Sales 50 -250	9.69%	10.01%
SME as corporate	Sales 251 -500	13.53%	15.82%
	Combination	**10.08%**	**10.59%**
Results with maturity of 1 and 3 years			
	Sales 50 -250	3.67%	4.75%
SME as corporate	Sales 251 -500	5.45%	7.69%
	Combination	**3.85%**	**5.04%**
Results with maturity of 5 and 10 years			
	Sales 50 -250	6.03%	7.37%
SME as corporate	Sales 251 -500	10.08%	13.14%
	Combination	**6.44%**	**7.95%**

5.4 PD Models Comparison

According to the Basel III, using IRB approach, banks need to calculate their capital requirement for SMEs based on one-year PD model regardless of SMEs classified as corporates or retail customers. Two highly important researches in this field have been done by Altman and Sabato (2005, 2007). In both, they calculated minimum capital requirements for all SMEs based on the same ranking classes resulted from a one-year PD model (for each country they only used one sample that included all SMEs). There is no sample limitation, for example, ranking only small-sized firms when the target capital requirement is only relevant to small-sized firms. In this study, first the same methodology as the one Altman and Sabato have used in their studies is applied. Then, contrary to the Basel III suggestion, models for three- and five-years default probabilities are estimated for ranking the SMEs as small and medium-sized corporates. The results showed (see Table 15) different minimum capital requirements based on the general model (one-year PD, and the same sample) and specific models (various PD models and specific size-filtered samples). Using specific sample of only retail customers shows a neglectable difference as most of the Norwegian SMEs can be assumed as retail customers (more than 90%). However, the capital requirements difference for SMEs as corporates is noticeable. The general model using the one-year PD model and sample (the conventional way of calculation) results in approximately 1.35 percentage point lower minimum capital requirements for SMEs.

For comparing the different approaches' outcomes, capital requirements based on a random sample which includes 5,000 SMEs (2,500 randomly selected retail customers, and 2500 randomly selected SMEs as corporates) is calculated using one-year PD ranking system and specific ranking system which have been resulted in different PD models. That is, I assumed an imaginary bank, that already has developed different models for their SMEs customers as corporates (those which are developed in this research), faces 5,000 new customers. Bank has no idea how many of them will default, but has access to different ranking systems (i.e., one-year, three-year, and five-year PD models). Assuming bank calculated the average LGD for them as 50 percent, capital requirements is calculated for each SMEs based on different ranking systems, once based on one-year PD rankings, and then based on size-filtered maturity specific models ranking system. Table 18 shows the average requirements obtained using different approaches (see appendix for more details).

Table 18. Capital requirements comparison

	# of firms	One-year PD	Specific PD Models	Difference
SMEs as retail customers	2,500	2.82%	2.82%	0%
SMEs as small-sized corporates	2,282	5.20%	7.00%	1.80%
SMEs as medium-sized corporates	218	6.86%	11.20%	4.34%
Portfolio of SMEs	5,000	4.08%	5.09%	**1.01%**

According to the results, there is no difference of using different ranking systems for retail customers. The main reason is that retail customers in Norwegian market are relatively more than SMEs that can be classified as corporates. For example, in the sample that has been used for setting up one-year ranking system more than 91% of the SMEs can be classified as retail customers. Thus, removing remaining SMEs as corporates would not have significant impact on future classification power of the one-year system for retail customers. However, calculating capital requirements for SMEs as corporate, there is significant difference between using the general one-year PD model and specific PD models for ranking. For small-sized SMEs as corporate, using the general ranking model gives us capital requirements of 1.8 percentage point lower than using three-year ranking system. This 1.8 percentage point lower is equal to 29.5% lower relative[56] capital requirements for small SMEs. Similarly, there is 4.34 percentage point (48% higher relative requirements) higher capital requirements for medium-sized SMEs using five-year PD model instead of one-year model. Finally, assuming bank intends to hold a balanced portfolio of 50 percent retail customers and 50 percent SMEs as corporates, using general model gives 1 percentage point lower capital requirements which can be translated into a difference of 22% relative difference between these two approaches. Thus, there is enough evidence that using a general one-year PD model significantly underestimates capital requirements for SMEs as corporates.

Additionally, SMEs as corporates have relatively greater exposure at default (EAD) compare to retail customers as they have access to loan with higher monetary values and longer maturities. This makes this difference even greater when it is translated to monetary value

56 $\frac{7\%-5.2\%}{(7\%+5.2\%)/2} = 29.51\%$

(RWA = K × 12.5 × EAD). One may argue that relative weight of SMEs as corporates in a bank portfolio can be smaller than relative weight of SMEs as retail customers. For testing that, we can assume that bank will keep its portfolio as close as possible to a real distribution of the SMEs market such that 90% retail customers, 9% small SMEs, and 1% medium-sized SMEs. Also, we assume that the bank just gives on average 1 million kroner to retail customers, 2 million to small-sized SMEs, and 4 million to medium-sized SMEs. Table 19 shows the result. The number of total SMEs in the portfolio set to 1,000 for calculation simplicity.

Table 19. Estimating risk weighted asset under two different systems.

	# of firms	K	EAD (Million)		RWA (Million)	
Under One-year PD ranking						
SMEs as retail customers	900	2.82%	NOK	1.00	NOK	0.35
SMEs as small-sized corporates	90	5.20%	NOK	2.00	NOK	1.30
SMEs as medium-sized corporates	10	6.86%	NOK	4.00	NOK	3.43
Portfolio of SMEs	1000	3.07%	NOK	1,120.00	NOK	**430.44**
Under specific PD ranking						
SMEs as retail customers	900	2.82%	NOK	1.00	NOK	0.35
SMEs as small-sized corporates	90	7.00%	NOK	2.00	NOK	1.75
SMEs as medium-sized corporates	10	11.20%	NOK	4.00	NOK	5.60
Portfolio of SMEs	1000	3.28%	NOK	1,120.00	NOK	**459.20**

Calculating risk weighted asset under two different ranking systems resulted in 6.5% higher RWA under the specific system even when portfolio was set up with 90% of SMEs as retail customers. This example shows a portfolio that contained high proportion of SMEs as retail customers. According to Altman and Sabato (2007), classifying a large proportion of SME's portfolio as retail customers may not be possible for some banks. Consequently, banks which have proportionally larger weight of SMEs as corporates in their portfolio would face relatively greater difference in their RWA based on different ranking methods.

6. Limitations

Although SNF has collected an outstanding high-quality data for Norwegian firms, there was a missing indicator for this study. There is no information for the credit line provided to a firm by a specific creditor. For estimating loss given default (LGD) and exposure at default (EAD) this indicator is essential. Thus, there was no way to calculate the minimum capital requirements rather than just using a fixed LGD that suggested by the Basel III. However, a sensitivity analysis has been done for estimating the minimum capital requirements under different LGDs. For EAD, following the Altman and Sabato (2005), the loan exposure amount assumed to be equal to EAD for on-balance sheet items.

Moreover, logistic regression is the main technique in this research as it turned out to be must popular method over more than 4 decades, and following Sabato (2010), no significant improvement has been obtained over the prediction accuracy of credit scoring models using other statistical techniques that have attempted to improve the logit prediction accuracy. However, during the previous 8 years, after 2010, some authors have attempted to apply new machine learning techniques on predicting failure. For example, Wang and Ma (2011) applied integrated ensemble machine learning (IEML) and concluded that IEML can be applied on corporate credit risk prediction problem as an alternative. Zhu et al. (2017) compared various machine learning approaches on SMEs credit risk prediction and concluded that RS-boosting performed better compare to other machine learning methods that they have tested. Thus, this research could also benefit from a more recent machine learning technique.

7. Conclusions

I have attempted to construct models for predicting defaults for Norwegian small and medium-sized enterprises (SME). Three different models have been developed throughout this research for different distances to default. Using logistic regression for analyses, I found that financial ratios together with company age, and industry indicator would predict probability of default (PD) for Norwegian SMEs with relatively high accuracy rates. Asset as a size indicator has shown to add statistically predictive power to models with longer than one-year default prediction. However, other qualitative variables such as employees count, geographical region, and the legal form of the business showed no significant increase in predictive power of models for SME default prediction.

After modelling default probabilities, I have calculated minimum capital requirements for Norwegian SMEs under the Basel III requirements with two different approaches. First, the approach suggested by the Basel that using one-year PD model for capital requirements calculations. Then, I have calculated minimum capital requirements based on specific PD models for longer maturities. That is, a three-year PD model has been developed for SMEs as small-sized corporates, and a five-year PD model for SMEs as medium-sized corporates. Using either approach categorizing SMEs as retail customers and corporates, minimum capital requirements for Norwegian SMEs turned out to be lower for banks under the IRB approach, of the Basel III than current required rate by the Basel III. This is always the case no matter what percentage of SME firms classified as corporates or retail customers. The main reasons for this lower capital requirements, confirming the Altman and Sabato (2007) finding, is that a specific SME credit risk model applied on a SME samples has higher discriminative power than a generic corporate model. Moreover, SMEs default rate is relatively lower in Norway (2.1%) comparing to other OECD countries.

For the third research question, I have investigated whether using a one-year PD model suggested by the Basel II and Basel III has the same outcome with using specific models for longer maturities. A random sample of Norwegian SMEs, consisting a specific combination of SMEs as retail customers and corporates, closest to the combination of the available SMEs in the market, has been drawn from the population, and the minimum capital requirements for an imaginary bank that has them in its portfolio is calculated. Confirming the result from the second research question, using specific models for different maturities resulted in higher capital requirements for the bank. The main reason is that on average a firm naturally has higher probability of default in longer periods than one year.

Considering that the minimum capital requirements calculated for Norwegian SMEs is lower than the current rate proposed by the Basel III, using specific models for longer maturities than one-year would decrease the potential credit risk for banks and would not negatively impact the SMEs. Moreover, banks should use more specific instruments, such as scoring and rating systems, exclusively designed for SMEs.

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Appendix

Table I. Data sets for the Norwegian SMEs in the three-year probability of default

Year		Balanced Sample		Non-Balanced Sample	
	Defaults	Non-defaults	Total sample	Non-defaults	Total sample
1995	249	249	498	5,680	5,929
1996	244	244	488	5,566	5,810
1997	347	347	694	7,915	8,262
1998	432	432	864	9,854	10,286
1999	678	678	1,356	15,465	16,143
2000	839	839	1,678	19,137	19,976
2001	975	975	1,950	22,239	23,214
2002	842	842	1,684	19,206	20,048
2003	592	592	1,184	13,503	14,095
2004	479	479	958	10,926	11,405
2005	462	462	924	10,538	11,000
2006	342	342	684	7,801	8,143
2007	642	642	1,284	14,644	15,286
2008	440	440	880	10,036	10,476
2009	754	754	1,508	17,198	17,952
2010	510	510	1,020	11,633	12,143
2011	541	541	1,082	12,340	12,881
2012	592	592	1,184	13,503	14,095
2013	469	469	938	10,698	11,167
2014	455	455	910	10,378	10,833
2015	325	325	650	7,413	7,738
Total	11,209	11,209	22,418	255,673	**266,882**

Table II. Data sets for the Norwegian SMEs in the five-year probability of default

Year	Defaults	Balanced Sample		Non-Balanced Sample	
		Non-defaults	Total sample	Non-defaults	Total sample
1995	252	252	504	2,748	3,000
1996	247	247	494	2,693	2,940
1997	358	358	716	3,904	4,262
1998	435	435	870	4,744	5,179
1999	679	679	1,358	7,404	8,083
2000	844	844	1,688	9,204	10,048
2001	984	984	1,968	10,730	11,714
2002	846	846	1,692	9,225	10,071
2003	604	604	1,208	6,586	7,190
2004	503	503	1,006	5,485	5,988
2005	548	548	1,096	5,976	6,524
2006	352	352	704	3,838	4,190
2007	656	656	1,312	7,154	7,810
2008	449	449	898	4,896	5,345
2009	780	780	1,560	8,506	9,286
2010	518	518	1,036	5,649	6,167
2011	545	545	1,090	5,943	6,488
2012	598	598	1,196	6,521	7,119
2013	471	471	942	5,136	5,607
2014	455	455	910	4,962	5,417
2015	325	325	650	3,544	3,869
Total	11,449	11,449	**22,898**	124,848	**136,297**

Figure I. Distribution of sales, number of employees, and total asset in the Norwegian SMEs' sample for three-year PD

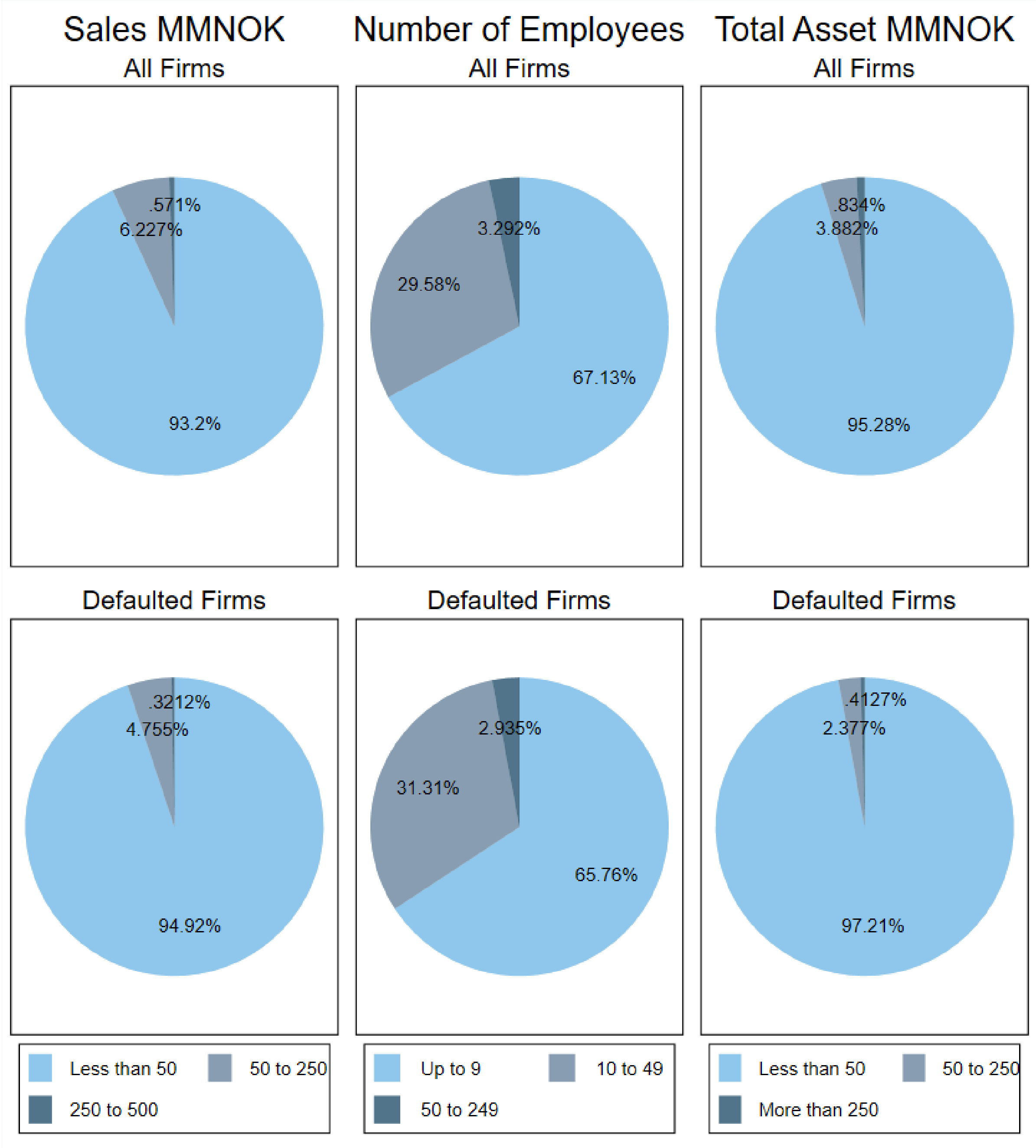

Figure II. Distribution of sales, number of employees, and total asset in the Norwegian SMEs' sample for five-year PD

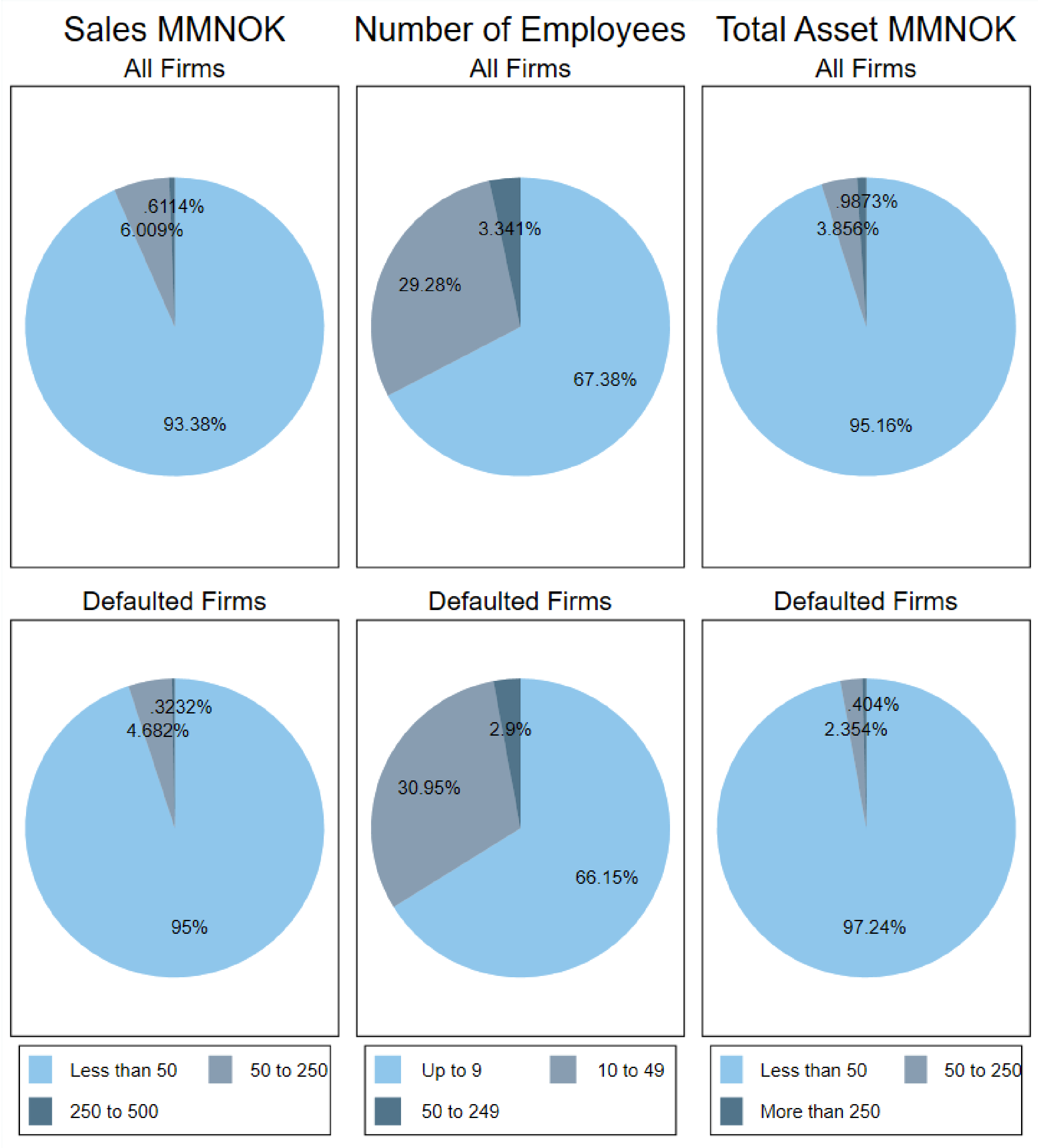

Figure III. Distribution of sales, number of employees, and total asset in the Norwegian SMEs' sample for one-year, three-year, and five years for cumulative probabilities samples (2.1% for one-year PD, 4.2% for three-year PD, and 8.4% for five-year PD)

Table III. Geographical Region

No.	Region	Counties
1	Østviken	Østfold, Oslo Akershus
2	Innlandet	Hedmark, Oppland
3	Vestviken	Buskerud,Vestfold Telemark
4	Sørlandet	Aust-Agder, Vest-Agder
5	Vestlande	Rogaland, Hordaland, Sogn og Fjordane, Møre og Romsdal
6	Trøndelag	Sør-Trøndelag, Nord-Trøndelag
7	Nord-Norge	Nordland, Troms, Finnmark

Table IV. Variables univariate analysis three-year PD

Variable			Population means		Univariate
No.	Category*	Name	Defaulted	Non-Defaulted	*F*-test
1	3	Net Pre-Tax Income/Total Assets	-0.1910	0.0902	2881.1
2	3	EBITDA/Total Assets	-0.0900	0.1392	2663.2
3	3	Net Income/Total Assets	-0.1913	0.0610	2294.7
4	2	Cash/Total Assets	0.0835	0.1947	2272.8
5	2	Working Capital/Total Assets	-0.1378	0.1405	1800.9
6	1	Liabilities/Total Assets	1.1509	0.7558	1354.6
7	8	Age of the Firm (Based of registration date)	9.4129	14.4442	1098.1
8	1	Public Charges/Total Assets	0.1171	0.0677	1092.2
9	7	Natural Logarithm of Total Assets in 2015	8.5249	8.9286	770.3
10	5	Payroll Expenses/Total Asset	0.6788	0.4828	609.4
11	5	Sales/Total Assets	2.5057	1.9536	357.2
12	5	Total Income/Total Assets	2.5615	2.0085	355.2
13	1	Total interest-bearing liabilities/Total Assets	0.4906	0.3201	269.0
14	4	EBITDA/Interest Expenses	-14.2312	116.2136	236.4
15	2	Intangible Assets/Total Assets	0.0407	0.0226	200.7
16	4	EBIT/Interest Expenses	-22.6316	94.3529	151.9
17	5	Depreciation/Total Asset	0.0487	0.0397	96.6
18	5	Trade Debtors/Liabilities	0.2262	0.2541	50.9
19	1	Public Charges/Total Income	0.0888	0.0459	11.8
20	1	Equity/Liabilities	0.0625	1.6889	8.2
21	5	Trade Creditors/Total Income	1.6999	0.1319	4.1
22	6	ROA(t)- ROA(t-1)	-0.0522	0.0122	3.7
23	5	Trade Creditors/Sales	2.0454	0.5401	3.5
24	2	Cash/EBIT	0.1230	1.6796	1.8
25	3	Net Income/Sales	-21.6789	2.3042	1.7
26	3	EBIT/Sales	-19.9100	1.3086	1.6
27	3	Net Income/Total Income	-20.7026	1.1698	1.5
28	3	EBIT/Total Income	-19.2429	-0.0786	1.3
29	1	Short Term Debt/Equity	2.3795	2.7284	0.1

*Categories:
1 = Leverage
2 = Liquidity
3 = Profitability
4 = Coverage
5 = Activity
6 = Growth
7 = Size
8 = Age

Table V. Variables univariate analysis five-year PD

Variable			Population means		Univariate
No.	Category*	Name	Defaulted	Non-Defaulted	*F*-test
1	3	Net Pre-Tax Income/Total Assets	-0.1883	0.0901	2839.5
2	3	EBITDA/Total Assets	-0.0875	0.1378	2561.8
3	3	Net Income/Total Assets	-0.1888	0.0608	2263.1
4	2	Cash/Total Assets	0.0836	0.1929	2251.8
5	2	Working Capital/Total Assets	-0.1360	0.1388	1771.2
6	1	Liabilities/Total Assets	1.1481	0.7566	1317.3
7	8	Age of the Firm (Based of registration date)	9.3799	14.3719	1144.6
8	1	Public Charges/Total Assets	0.1164	0.0672	1112.4
9	7	Natural Logarithm of Total Assets in 2015	8.5222	8.9466	845.5
10	5	Payroll Expenses/Total Asset	0.6725	0.4798	609.0
11	5	Sales/Total Assets	2.4874	1.9509	345.7
12	5	Total Income/Total Assets	2.5434	2.0056	344.1
13	1	Total interest-bearing liabilities/Total Assets	0.4899	0.3223	259.8
14	2	Intangible Assets/Total Assets	0.0406	0.0221	221.6
15	4	EBITDA/Interest Expenses	-13.7068	122.2304	128.3
16	5	Depreciation/Total Asset	0.0486	0.0392	106.0
17	4	EBIT/Interest Expenses	-21.9924	102.1916	95.9
18	1	Equity/Liabilities	0.0671	1.0622	55.4
19	5	Trade Debtors/Liabilities	0.2264	0.2548	51.6
20	1	Public Charges/Total Income	0.0919	0.0578	5.9
21	6	ROA(t)- ROA(t-1)	-0.0514	0.0143	4.2
22	5	Trade Creditors/Total Income	1.6727	0.1578	4.0
23	2	Cash/EBIT	0.1208	1.0096	3.8
24	5	Trade Creditors/Sales	2.0163	0.4877	3.8
25	3	Net Income/Sales	-21.2388	1.5196	1.6
26	3	EBIT/Sales	-19.5015	0.4564	1.5
27	3	Net Income/Total Income	-20.2825	0.5347	1.4
28	3	EBIT/Total Income	-18.8512	-0.3631	1.3
29	1	Short Term Debt/Equity	1.9520	2.9285	1.1

*Categories:

1 = Leverage

2 = Liquidity

3 = Profitability

4 = Coverage

5 = Activity

6 = Growth

7 = Size

8 = Age

Table VI. Summary statistics

	count	mean	sd	min	max
One-year balanced sample					
Liabilities/Total Assets	6144	.9936044	1.273422	.0130132	92.09808
Public Charges/Total Assets	6144	.0945064	.1230976	-.1585213	2.083148
Cash/Total Assets	6144	.1341958	.1811553	-.2732483	.9998707
Intangible Assets/Total Assets	6144	.0328799	.0974732	0	.9879017
Public Charges/Total Assets	6144	.0945064	.1230976	-.1585213	2.083148
Net Pre-Tax Income/Total Assets	6144	-.0840879	.4302367	-9.9063	1.607584
Payroll Expenses/Total Asset	6144	.5987132	.5982336	-.057648	6.991107
Depreciation/Total Asset	6144	.0465308	.0657699	-.0853425	1.733464
age	6144	12.14274	12.11368	2	147
One-year sample with cumulative default probability (2.1% defaulted firms)					
Liabilities/Total Assets	146285	.7719053	.4702377	-.9780084	92.09808
Public Charges/Total Assets	146285	.0681376	.0758841	-.3848797	2.083148
Cash/Total Assets	146285	.1879524	.2081065	-1.158826	1.046482
Intangible Assets/Total Assets	146285	.0223677	.0756816	-.2079431	1
Public Charges/Total Assets	146285	.0681376	.0758841	-.3848797	2.083148
Net Pre-Tax Income/Total Assets	146285	.0761034	.3465645	-46.34418	59.18118
Payroll Expenses/Total Asset	146285	.4850911	.5105022	-.3928345	14.08798
Depreciation/Total Asset	146285	.0403963	.0574539	-1.338519	8.548088
age	146285	14.1257	12.9678	2	200
Three-year balanced sample					
Public Charges/Total Assets	22418	.0923986	.114562	-.2857591	2.083148
Interest-bearing liabilities/Total Assets	22418	.4053661	.7827522	-.2840571	91.76883
Cash/Total Assets	22418	.139091	.1832934	-1.91021	1.334192
Net Pre-Tax Income/Total Assets	22418	-.0503899	.4166074	-16.77073	2.78886
Total Income/Total Assets	22418	2.284985	2.214115	.0000329	56.40635
age	22418	11.92854	11.64118	2	200
Log of Assets	22418	8.726721	1.10757	7.37149	16.59284
Three-year sample with cumulative default probability (4.2% defaulted firms)					
Public Charges/Total Assets	266882	.069587	.0781205	-.524879	2.083148
Interest-bearing liabilities/Total Assets	266882	.3327662	.4574654	-1.933238	91.76883
Cash/Total Assets	266882	.1869101	.2076917	-1.91021	1.334192
Net Pre-Tax Income/Total Assets	266882	.0711861	.3278579	-59.85431	7.970566
Total Income/Total Assets	266882	2.028349	1.990151	-.5472936	100.4482
age	266882	14.04531	12.77224	2	200
Log of Assets	266882	8.919843	1.193128	7.37149	17.21949
Five-year balanced sample					
Public Charges/Total Assets	22898	.0917871	.1141862	-.2857591	2.083148
Interest-bearing liabilities/Total Assets	22898	.4060752	.7910804	-.4058243	91.76883
Cash/Total Assets	22898	.1382571	.1827484	-1.91021	1.334192
Net Pre-Tax Income/Total Assets	22898	-.0491333	.4191154	-16.77073	3.985066
Sales/Total Assets	22898	2.219172	2.19943	.0000336	56.40529
age	22898	11.87593	11.43945	2	144
Log of Assets	22898	8.734418	1.124523	7.37149	17.38017
Five-year sample with cumulative default probability (8.4% defaulted firms)					
Public Charges/Total Assets	136297	.0714944	.0823198	-.2857591	2.634236
Interest-bearing liabilities/Total Assets	136297	.3393307	.497197	-1.428158	91.76883
Cash/Total Assets	136297	.1817036	.2051239	-1.91021	1.334192
Net Pre-Tax Income/Total Assets	136297	.0609601	.3178794	-40.81054	11.47103
Sales/Total Assets	136297	1.99748	2.036122	3.29e-06	100.3486
age	136297	13.90602	12.73846	2	189
Log of Assets	136297	8.901311	1.189642	7.37149	17.53878

Table VII. Logistic regression results (odds ratio) with and without control for industry type Using original form of accounting ratios.

	One-year PD		Three-year PD		Five-year PD	
	Without Industry	With Industry	Without Industry	With Industry	Without Industry	With Industry
Liabilities/Total Assets	13.36***	14.11***				
	(16.23)	(16.14)				
Public Charges/Total Assets	135.6***	156.5***	122.7***	172.0***	119.2***	144.4***
	(8.38)	(8.56)	(21.55)	(22.53)	(21.46)	(21.94)
Cash/Total Assets	0.0478***	0.0552***	0.0268***	0.0330***	0.0262***	0.0300***
	(-10.87)	(-10.29)	(-26.94)	(-25.10)	(-27.49)	(-26.19)
Intangible Assets/Total Assets	4.947***	5.138***				
	(4.01)	(3.95)				
Net Pre-Tax Income/Total Assets	0.0120***	0.0135***	0.0276***	0.0311***	0.0311***	0.0342***
	(-19.96)	(-19.35)	(-37.34)	(-35.91)	(-37.33)	(-36.14)
Payroll Expenses/Total Asset	1.392***	1.400***				
	(3.42)	(3.42)				
Depreciation/Total Asset	0.0552***	0.0278***			0.0360***	0.0289***
	(-4.09)	(-4.77)			(-10.14)	(-10.43)
age	0.978***	0.977***	0.966***	0.964***	0.964***	0.962***
	(-6.97)	(-7.35)	(-19.93)	(-20.62)	(-20.82)	(-21.37)
Total interest-bearing liabilities/Total Assets			1.205***	1.237***	1.211***	1.234***
			(3.46)	(3.83)	(3.42)	(3.73)
Working Capital/Total Assets			0.446***	0.393***	0.412***	0.381***
			(-13.55)	(-15.08)	(-14.74)	(-15.65)
Sales/Total Assets			1.098***	1.062***		
			(10.20)	(6.21)		
Log of Assets			0.841***	0.847***	0.813***	0.814***
			(-10.82)	(-10.09)	(-13.24)	(-12.75)
Total Income/Total Assets					1.097***	1.069***
					(10.17)	(6.90)
Agriculture		1		1		1
		(.)		(.)		(.)
Offshore/Shipping		0.816		1.186		1.156
		(-0.66)		(1.14)		(1.00)
Transport		1.544		0.942		1.157
		(1.69)		(-0.50)		(1.23)
Manufacturing		1.520*		1.221		1.245*
		(1.99)		(1.90)		(2.11)
Telecom/IT/Tech		1.103		0.898		0.874
		(0.33)		(-0.78)		(-1.01)
Electricity		0.249		0.414*		0.184***
		(-1.62)		(-2.32)		(-4.38)
Construction		0.682		0.852		0.789*
		(-1.92)		(-1.64)		(-2.45)
Wholesale/Retail		0.990		1.258*		1.099
		(-0.05)		(2.31)		(0.96)
Other services		0.647*		0.792*		0.830
		(-2.12)		(-2.31)		(-1.88)
Observations	6144	6144	22418	22418	22898	22898
AIC	5273.3	5212.4	22460.4	22347.8	23037.1	22928.4
BIC	5333.8	5326.7	22532.6	22484.1	23117.5	23073.1

Exponentiated coefficients; *z* statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table VIII. Logistic regression results (odds ratio) with quadratic and cubic terms.

	One-year PD	Three-year PD	Five-year PD
Liabilities/Total Assets	14.38^{***}		
	(16.21)		
Public Charges/Total Assets	106.6^{***}	102.6^{***}	65.08^{***}
	(7.89)	(19.40)	(17.70)
Cash/Total Assets	0.0535^{***}	0.0331^{***}	0.0311^{***}
	(-10.40)	(-24.97)	(-25.89)
Intangible Assets/Total Assets	5.178^{***}		
	(3.97)		
Net Pre-Tax Income/Total Assets	0.0136^{***}	0.0312^{***}	0.0336^{***}
	(-19.30)	(-35.80)	(-36.34)
Payroll Expenses/Total Asset	2.208^{***}		
	(4.83)		
(Payroll Expenses/Total Asset)^2	0.826^{***}		
	(-3.57)		
Depreciation/Total Asset	0.0214^{***}		0.0192^{***}
	(-5.12)		(-12.00)
age	0.976^{***}	0.964^{***}	0.962^{***}
	(-7.38)	(-20.52)	(-21.34)
Total interest-bearing liabilities/Total Assets		1.318^{***}	1.362^{***}
		(4.76)	(5.34)
Working Capital/Total Assets		0.381^{***}	0.356^{***}
		(-15.46)	(-16.53)
Sales/Total Assets		1.239^{***}	
		(8.22)	
(Sales/Total Assets)^2		0.981^{***}	
		(-5.62)	
(Sales/Total Assets)^3		1.000^{***}	
		(3.93)	
Log of Assets		0.852^{***}	0.820^{***}
		(-9.69)	(-12.17)
Total Income/Total Assets			1.413^{***}
			(11.54)
(Total Income/Total Assets)^2			0.958^{***}
			(-9.34)
(Total Income/Total Assets)^3			1.001^{***}
			(7.73)
Agriculture	1	1	1
	(.)	(.)	(.)
Offshore/Shipping	0.822	1.149	1.102
	(-0.63)	(0.92)	(0.67)
Transport	1.453	0.885	1.057
	(1.45)	(-1.02)	(0.46)
Manufacturing	1.433	1.151	1.120
	(1.70)	(1.33)	(1.08)
Telecom/IT/Tech	1.036	0.880	0.833
	(0.12)	(-0.93)	(-1.37)
Electricity	0.249	0.431^{*}	0.199^{***}
	(-1.60)	(-2.21)	(-4.19)
Construction	0.665^{*}	0.839	0.761^{**}
	(-2.04)	(-1.79)	(-2.81)
Wholesale/Retail	0.935	1.162	0.968
	(-0.34)	(1.50)	(-0.33)
Other services	0.629^{*}	0.777^{*}	0.793^{*}
	(-2.24)	(-2.51)	(-2.33)
Observations	6144	22418	22898
AIC	5203.3	22305.7	22830.0
BIC	5324.3	22458.0	22990.8

Exponentiated coefficients; z statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure IV. Margin plots for three-year PD

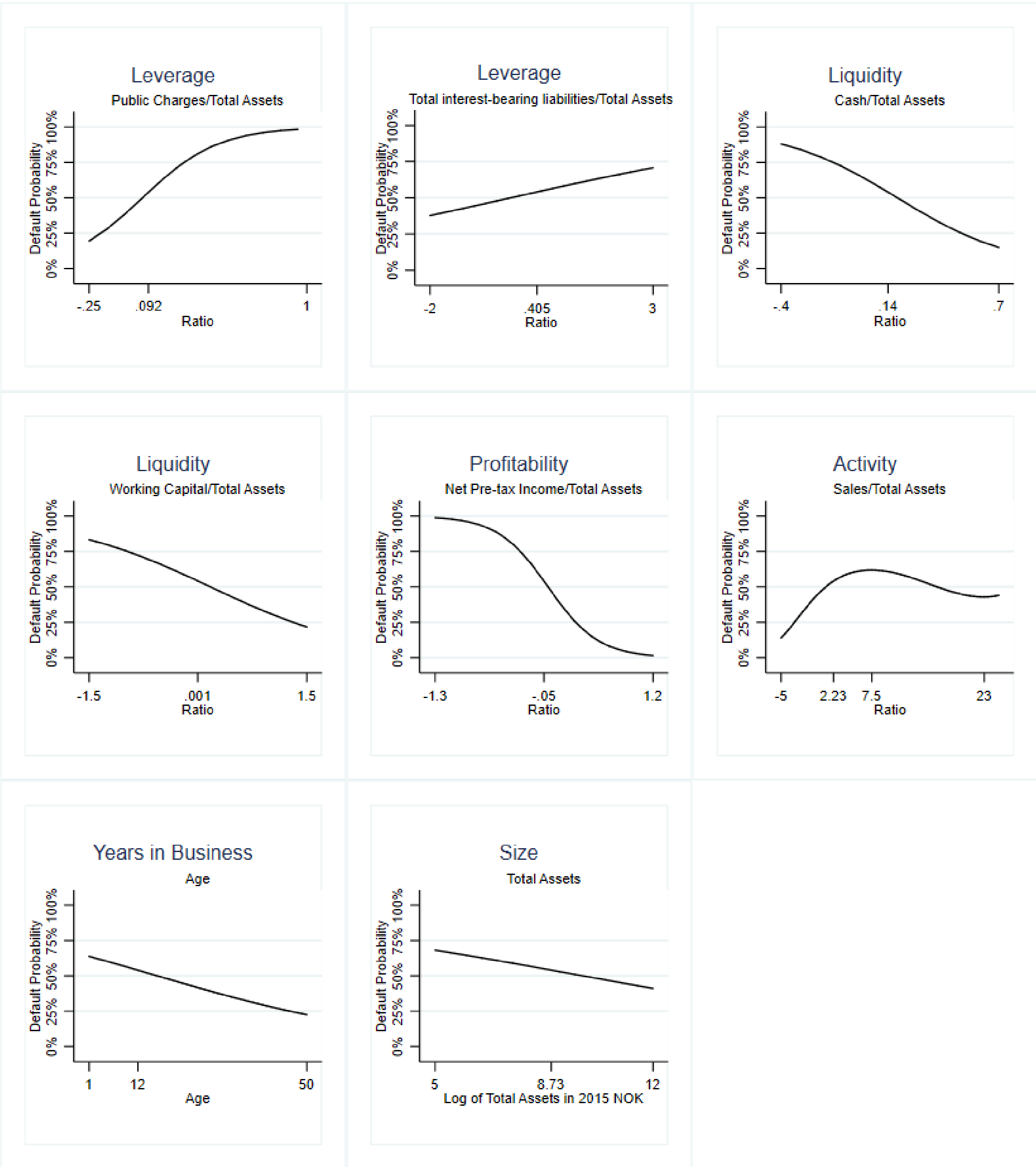

Figure V. Margin plots for five-year PD

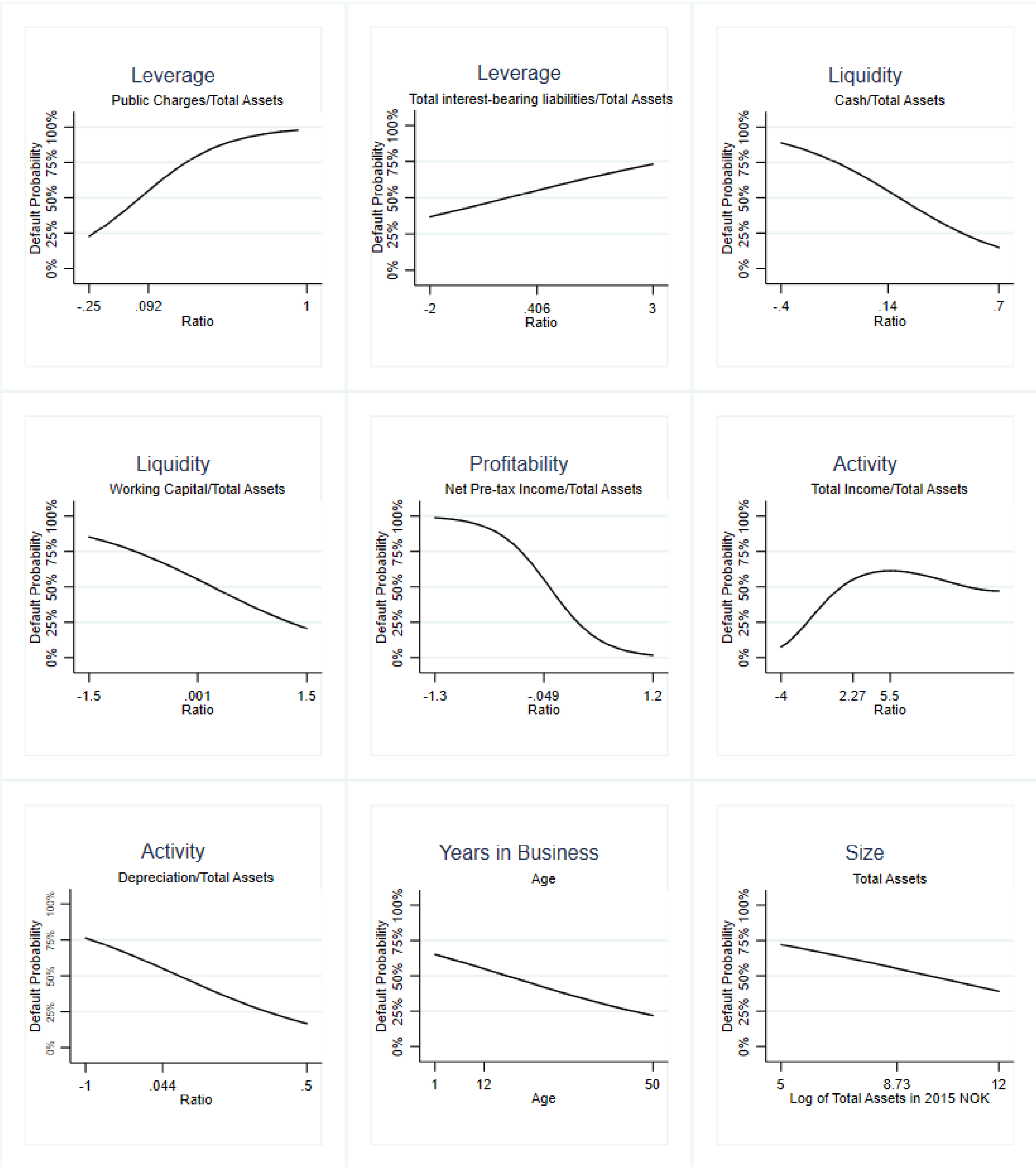

Figure VI. Receiver operating characteristic (ROC) curves comparison three-year PD

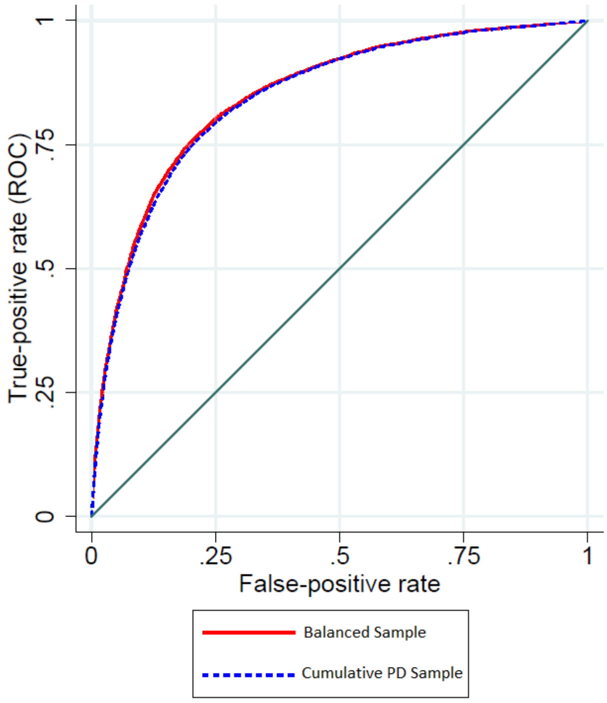

Figure VII. Receiver operating characteristic (ROC) curves comparison five-year PD

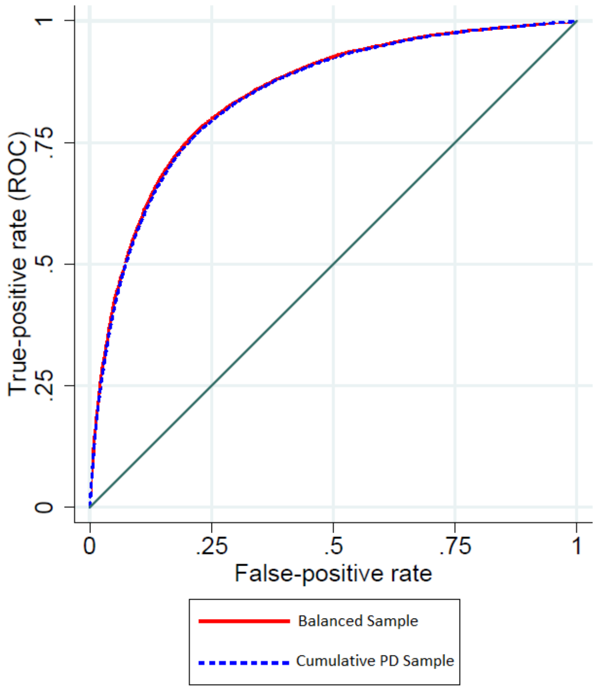

Table IX. Variance inflation factor (VIF) results

One-year PD		Three-year PD		Five-year PD	
Variable	VIF	Variable	VIF	Variable	VIF
Payroll Expenses/Asset	1.95	Working Capital/Total Assets	1.84	Working Capital/Total Assets	1.83
Public Charges/Total Assets	1.88	Interest-bearing liabilities/Total Assets	1.44	Interest-bearing liabilities/Total Assets	1.46
Net Pre-Tax Income/Total Assets	1.24	Net Pre-Tax Income/Total Assets	1.41	Net Pre-Tax Income/Total Assets	1.44
Liabilities/Total Assets	1.12	Public Charges/Total Assets	1.26	Public Charges/Total Assets	1.26
Cash/Total Assets	1.09	Log (Total Assets in 2015 NOK)	1.17	Log (Total Assets in 2015 NOK)	1.17
Depreciation/Asset	1.08	Sales/Total Assets	1.16	Total Income/Total Asset	1.17
Intangible Assets/Total Assets	1.05	Cash/Total Assets	1.10	Cash/Total Assets	1.11
Age	1.02	Age	1.06	Depreciation/Asset	1.07
				Age	1.06

Table X. Functional misspecification test

Variables	One-Year PD	Three-Year PD	Five-Year PD
Predicted	1.00090 (0.000)	0.99907 (0.000)	1.03764 (0.000)
Predictedsq	0.01270 (0.808)	0.00376 (0.955)	0.05908 (0.235)
Constant	-0.03134 (0.888)	-0.00530 (0.976)	-0.07934 (0.636)

P-values in parentheses

Table XI. Sensitivity analysis for retail customers using different LGDs (General Sample)

Rating	PD	LGD	R_{sme}	K_{sme}	Weight	Cum. Weighted K_{sme}
AAA	0.05%	20%	0.15774	0.0023570	0.00260	0.0006%
AA	0.06%	20%	0.15730	0.0027098	0.01080	0.0035%
A	0.11%	20%	0.15509	0.0042547	0.15065	0.0676%
BBB+	0.15%	20%	0.15335	0.0053113	0.13964	0.1418%
BBB	0.30%	20%	0.14704	0.0084617	0.26109	0.3627%
BBB-	0.50%	20%	0.13913	0.0115062	0.08730	0.4632%
BB	1.00%	20%	0.12161	0.0162747	0.07995	0.5933%
BB-	1.50%	20%	0.10690	0.0189763	0.04935	0.6869%
B+	3.00%	20%	0.07549	0.0223260	0.10120	0.9129%
B	6.00%	20%	0.04592	0.0240821	0.03259	0.9914%
B-	10.00%	20%	0.03393	0.0268597	0.03128	1.0754%
CCC	18.29%	20%	0.03022	0.0343437	0.05355	**1.2593%**
AAA	0.05%	60%	0.15774	0.0070711	0.00260	0.0018%
AA	0.06%	60%	0.15730	0.0081294	0.01080	0.0106%
A	0.11%	60%	0.15509	0.0127642	0.15065	0.2029%
BBB+	0.15%	60%	0.15335	0.0159340	0.13964	0.4254%
BBB	0.30%	60%	0.14704	0.0253851	0.26109	1.0882%
BBB-	0.50%	60%	0.13913	0.0345186	0.08730	1.3895%
BB	1.00%	60%	0.12161	0.0488242	0.07995	1.7799%
BB-	1.50%	60%	0.10690	0.0569290	0.04935	2.0608%
B+	3.00%	60%	0.07549	0.0669780	0.10120	2.7387%
B	6.00%	60%	0.04592	0.0722464	0.03259	2.9741%
B-	10.00%	60%	0.03393	0.0805790	0.03128	3.2262%
CCC	18.29%	60%	0.03022	0.1030312	0.05355	**3.7779%**
AAA	0.05%	80%	0.15774	0.0094281	0.00260	0.0024%
AA	0.06%	80%	0.15730	0.0108392	0.01080	0.0142%
A	0.11%	80%	0.15509	0.0170189	0.15065	0.2705%
BBB+	0.15%	80%	0.15335	0.0212453	0.13964	0.5672%
BBB	0.30%	80%	0.14704	0.0338469	0.26109	1.4509%
BBB-	0.50%	80%	0.13913	0.0460248	0.08730	1.8527%
BB	1.00%	80%	0.12161	0.0650990	0.07995	2.3732%
BB-	1.50%	80%	0.10690	0.0759054	0.04935	2.7478%
B+	3.00%	80%	0.07549	0.0893040	0.10120	3.6515%
B	6.00%	80%	0.04592	0.0963285	0.03259	3.9655%
B-	10.00%	80%	0.03393	0.1074387	0.03128	4.3016%
CCC	18.29%	80%	0.03022	0.1373749	0.05355	**5.0372%**

Table XII. Sensitivity analysis for retail customers using different LGDs (Specific Sample for Retail Customers)

Rating	PD	LGD	R_{sme}	K_{sme}	Weight	Cum. Weighted K_{sme}
AAA	0.05%	20%	0.15774	0.0023570	0.00165	0.0004%
A	0.11%	20%	0.15509	0.0042547	0.16056	0.0687%
BBB+	0.15%	20%	0.15335	0.0053113	0.18864	0.1689%
BBB	0.30%	20%	0.14704	0.0084617	0.14680	0.2931%
BBB-	0.50%	20%	0.13913	0.0115062	0.15084	0.4667%
BB	1.00%	20%	0.12161	0.0162747	0.09608	0.6231%
BB-	1.60%	20%	0.10426	0.0193703	0.02560	0.6726%
B+	3.00%	20%	0.07549	0.0223260	0.12213	0.9453%
B	6.34%	20%	0.04413	0.0242555	0.01295	0.9767%
B-	9.99%	20%	0.03394	0.0268510	0.04864	1.1073%
CCC	19.93%	20%	0.03012	0.0356030	0.04610	**1.2715%**
AAA	0.05%	60%	0.15774	0.0070711	0.00165	0.0012%
A	0.11%	60%	0.15509	0.0127642	0.16056	0.2061%
BBB+	0.15%	60%	0.15335	0.0159340	0.18864	0.5067%
BBB	0.30%	60%	0.14704	0.0253851	0.14680	0.8793%
BBB-	0.50%	60%	0.13913	0.0345186	0.15084	1.4000%
BB	1.00%	60%	0.12161	0.0488242	0.09608	1.8692%
BB-	1.60%	60%	0.10426	0.0581110	0.02560	2.0179%
B+	3.00%	60%	0.07549	0.0669780	0.12213	2.8360%
B	6.34%	60%	0.04413	0.0727666	0.01295	2.9302%
B-	9.99%	60%	0.03394	0.0805529	0.04864	3.3220%
CCC	19.93%	60%	0.03012	0.1068089	0.04610	**3.8144%**
AAA	0.05%	80%	0.15774	0.0094281	0.00165	0.0016%
A	0.11%	80%	0.15509	0.0170189	0.16056	0.2748%
BBB+	0.15%	80%	0.15335	0.0212453	0.18864	0.6756%
BBB	0.30%	80%	0.14704	0.0338469	0.14680	1.1725%
BBB-	0.50%	80%	0.13913	0.0460248	0.15084	1.8667%
BB	1.00%	80%	0.12161	0.0650990	0.09608	2.4922%
BB-	1.60%	80%	0.10426	0.0774814	0.02560	2.6906%
B+	3.00%	80%	0.07549	0.0893040	0.12213	3.7813%
B	6.34%	80%	0.04413	0.0970221	0.01295	3.9069%
B-	9.99%	80%	0.03394	0.1074039	0.04864	4.4293%
CCC	19.93%	80%	0.03012	0.1424118	0.04610	**5.0859%**

Table XIII. Sensitivity analysis for corporates using different LGDs (General Sample)

						LGD = 20%		**LGD = 60%**		**LGD = 80%**	
Rating	PD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	Weight	$K_{corp.}$	Cum. Weighted $K_{corp.}$	$K_{corp.}$	Cum. Weighted $K_{corp.}$	$K_{corp.}$	Cum. Weighted $K_{corp.}$
Sales 50 - 250 MMNok											
AAA	0.05%	0.20148	0.28612	3	0.002598	0.6441%	0.00%	1.9323%	0.01%	2.5765%	0.00%
AA	0.06%	0.20090	0.27553	3	0.010801	0.7156%	0.01%	2.1468%	0.03%	2.8624%	0.03%
A	0.11%	0.19802	0.24177	3	0.150651	1.0102%	0.16%	3.0307%	0.48%	4.0410%	0.64%
BBB+	0.15%	0.19577	0.22535	3	0.139638	1.1988%	0.33%	3.5965%	0.99%	4.7954%	1.31%
BBB	0.30%	0.18773	0.19075	3	0.261093	1.7186%	0.78%	5.1558%	2.33%	6.8744%	3.10%
BBB-	0.50%	0.17790	0.16709	3	0.087295	2.1763%	0.97%	6.5288%	2.90%	8.7051%	3.86%
BB	1.00%	0.15723	0.13749	3	0.079954	2.8329%	1.19%	8.4987%	3.58%	11.3316%	4.77%
BB-	1.50%	0.14113	0.12151	3	0.049349	3.1934%	1.35%	9.5802%	4.06%	12.7736%	5.40%
B+	3.00%	0.11122	0.09648	3	0.1012	3.7643%	1.73%	11.2930%	5.20%	15.0574%	6.92%
B	6.00%	0.09042	0.07433	3	0.032594	4.5746%	1.88%	13.7237%	5.65%	18.2982%	7.52%
B-	10.00%	0.08525	0.05986	3	0.031281	5.5638%	2.06%	16.6913%	6.17%	22.2550%	8.22%
CCC	18.29%	0.08446	0.04477	3	0.053546	6.8931%	**2.42%**	20.6794%	**7.27%**	27.5725%	**9.69%**
Sales 251 - 500 MMNok											
AAA	0.05%	0.21926	0.28613	5	0.002598	1.0801%	0.00%	3.2404%	0.01%	4.3205%	0.00%
AA	0.06%	0.21868	0.27554	5	0.010801	1.1866%	0.02%	3.5599%	0.05%	4.7465%	0.05%
A	0.11%	0.21580	0.24178	5	0.150651	1.6135%	0.26%	4.8405%	0.78%	6.4540%	1.02%
BBB+	0.15%	0.21355	0.22536	5	0.139638	1.8783%	0.52%	5.6350%	1.56%	7.5133%	2.07%
BBB	0.30%	0.20551	0.19075	5	0.261093	2.5813%	1.19%	7.7438%	3.58%	10.3251%	4.77%
BBB-	0.50%	0.19568	0.16709	5	0.087295	3.1726%	1.47%	9.5178%	4.42%	12.6904%	5.88%
BB	1.00%	0.17501	0.13749	5	0.079954	3.9807%	1.79%	11.9420%	5.37%	15.9226%	7.15%
BB-	1.50%	0.15891	0.12151	5	0.049349	4.4053%	2.01%	13.2158%	6.02%	17.6210%	8.02%
B+	3.00%	0.12900	0.09648	5	0.1012	5.0619%	2.52%	15.1857%	7.56%	20.2476%	10.07%
B	6.00%	0.10820	0.07434	5	0.032594	5.9882%	2.71%	17.9647%	8.14%	23.9529%	10.85%
B-	10.00%	0.10303	0.05986	5	0.031281	7.0699%	2.94%	21.2098%	8.81%	28.2797%	11.73%
CCC	18.29%	0.10224	0.04477	5	0.053546	8.3986%	**3.39%**	25.1959%	**10.16%**	33.5946%	**13.53%**

Table XIV. Sensitivity analysis for corporates using different LGDs (Specific PD models and Samples)

						LGD = 20%		LGD = 60%		LGD = 80%	
Rating	PD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	Weight	$K_{corp.}$	Cum. Weighted $K_{corp.}$	$K_{corp.}$	Cum. Weighted $K_{corp.}$	$K_{corp.}$	Cum. Weighted $K_{corp.}$
Sales 50 - 250 MMNok											
AAA	0.05%	0.20148	0.28612	3	0.002598	0.644%	0.002%	1.932%	0.005%	2.576%	0.000%
AA	0.06%	0.20090	0.27553	3	0.010801	0.716%	0.009%	2.147%	0.028%	2.862%	0.031%
A-	0.15%	0.19577	0.22535	3	0.150651	1.199%	0.190%	3.597%	0.570%	4.795%	0.753%
BBB+	0.20%	0.19302	0.21064	3	0.139638	1.398%	0.385%	4.195%	1.156%	5.593%	1.534%
BBB	0.30%	0.18773	0.19075	3	0.261093	1.719%	0.834%	5.156%	2.502%	6.874%	3.329%
BB+	0.75%	0.16692	0.14942	3	0.087295	2.562%	1.058%	7.686%	3.173%	10.248%	4.224%
BB	1.00%	0.15723	0.13749	3	0.079954	2.833%	1.284%	8.499%	3.852%	11.332%	5.130%
BB-	1.50%	0.14113	0.12151	3	0.049349	3.193%	1.442%	9.580%	4.325%	12.774%	5.760%
B+	3.00%	0.11122	0.09648	3	0.101200	3.764%	1.823%	11.293%	5.468%	15.057%	7.284%
B	6.00%	0.09042	0.07433	3	0.032594	4.575%	1.972%	13.724%	5.915%	18.298%	7.880%
B-	10.00%	0.08525	0.05986	3	0.031281	5.564%	2.146%	16.691%	6.437%	22.255%	8.577%
CCC	16.50%	0.08448	0.04719	3	0.053546	6.684%	**2.504%**	20.051%	**7.511%**	26.734%	**10.008%**
Sales 251 - 500 MMNok											
AAA	0.05%	0.21926	0.28613	5	0.002598	1.080%	0.003%	3.240%	0.008%	4.320%	0.011%
A	0.10%	0.21637	0.24695	5	0.010801	1.539%	0.019%	4.616%	0.058%	6.154%	0.078%
BBB	0.30%	0.20551	0.19075	5	0.150651	2.581%	0.408%	7.744%	1.225%	10.325%	1.633%
BB+	0.75%	0.18470	0.14943	5	0.139638	3.653%	0.918%	10.958%	2.755%	14.611%	3.673%
BB	1.00%	0.17501	0.13749	5	0.261093	3.981%	1.958%	11.942%	5.873%	15.923%	7.831%
BB-	1.50%	0.15891	0.12151	5	0.087295	4.405%	2.342%	13.216%	7.027%	17.621%	9.369%
B+	3.00%	0.12900	0.09648	5	0.079954	5.062%	2.747%	15.186%	8.241%	20.248%	10.988%
B-	10.00%	0.10303	0.05986	5	0.049349	7.070%	3.096%	21.210%	9.288%	28.280%	12.383%
CCC	19.35%	0.10223	0.0434743	5	0.101200	8.495%	**3.956%**	25.486%	**11.867%**	33.981%	**15.822%**

Table XV. Sensitivity analysis for corporates using different maturities (General Sample)

Rating	PD	LGD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	$K_{corp.}$	Weight	Cum. Weighted $K_{corp.}$
Sales 5 - 25 MMNok								
AAA	0.05%	40%	0.20148	0.28612	1	0.00643	0.00260	0.0017%
AA	0.06%	40%	0.20090	0.27553	1	0.00738	0.01080	0.0096%
A	0.11%	40%	0.19802	0.24177	1	0.01149	0.15065	0.1827%
BBB+	0.15%	40%	0.19577	0.22535	1	0.01426	0.13964	0.3819%
BBB	0.30%	40%	0.18773	0.19075	1	0.02240	0.26109	0.9668%
BBB-	0.50%	40%	0.17790	0.16709	1	0.03010	0.08730	1.2296%
BB	1.00%	40%	0.15723	0.13749	1	0.04208	0.07995	1.5660%
BB-	1.50%	40%	0.14113	0.12151	1	0.04924	0.04935	1.8090%
B+	3.00%	40%	0.11122	0.09648	1	0.06143	0.10120	2.4306%
B	6.00%	40%	0.09042	0.07433	1	0.07838	0.03259	2.6861%
B-	10.00%	40%	0.08525	0.05986	1	0.09834	0.03128	2.9937%
CCC	18.29%	40%	0.08446	0.04477	1	0.12579	0.05355	**3.6673%**
Sales 26 - 50 MMNok								
AAA	0.05%	40%	0.21926	0.28613	3	0.01440	0.00260	0.0037%
AA	0.06%	40%	0.21868	0.27554	3	0.01599	0.01080	0.0210%
A	0.11%	40%	0.21580	0.24178	3	0.02254	0.15065	0.3606%
BBB+	0.15%	40%	0.21355	0.22536	3	0.02674	0.13964	0.7340%
BBB	0.30%	40%	0.20551	0.19075	3	0.03829	0.26109	1.7337%
BBB-	0.50%	40%	0.19568	0.16709	3	0.04850	0.08730	2.1570%
BB	1.00%	40%	0.17501	0.13749	3	0.06332	0.07995	2.6633%
BB-	1.50%	40%	0.15891	0.12151	3	0.07168	0.04935	3.0170%
B+	3.00%	40%	0.12900	0.09648	3	0.08550	0.10120	3.8823%
B	6.00%	40%	0.10820	0.07434	3	0.10475	0.03259	4.2237%
B-	10.00%	40%	0.10303	0.05986	3	0.12667	0.03128	4.6200%
CCC	18.29%	40%	0.10224	0.04477	3	0.15445	0.05355	**5.4470%**
Sales 5 - 25 MMNok								
AAA	0.05%	40%	0.20148	0.28612	5	0.01933	0.00260	0.0050%
AA	0.06%	40%	0.20090	0.27553	5	0.02124	0.01080	0.0280%
A	0.11%	40%	0.19802	0.24177	5	0.02892	0.15065	0.4637%
BBB+	0.15%	40%	0.19577	0.22535	5	0.03369	0.13964	0.9341%
BBB	0.30%	40%	0.18773	0.19075	5	0.04634	0.26109	2.1441%
BBB-	0.50%	40%	0.17790	0.16709	5	0.05695	0.08730	2.6412%
BB	1.00%	40%	0.15723	0.13749	5	0.07124	0.07995	3.2108%
BB-	1.50%	40%	0.14113	0.12151	5	0.07850	0.04935	3.5981%
B+	3.00%	40%	0.11122	0.09648	5	0.08915	0.10120	4.5003%
B	6.00%	40%	0.09042	0.07433	5	0.10461	0.03259	4.8412%
B-	10.00%	40%	0.08525	0.05986	5	0.12421	0.03128	5.2298%
CCC	18.29%	40%	0.08446	0.04477	5	0.14994	0.05355	**6.0326%**
Sales 26 - 50 MMNok								
AAA	0.05%	40%	0.21926	0.28613	10	0.03962	0.00260	0.0103%
AA	0.06%	40%	0.21868	0.27554	10	0.04309	0.01080	0.0568%
A	0.11%	40%	0.21580	0.24178	10	0.05658	0.15065	0.9093%
BBB+	0.15%	40%	0.21355	0.22536	10	0.06464	0.13964	1.8119%
BBB	0.30%	40%	0.20551	0.19075	10	0.08497	0.26109	4.0303%
BBB-	0.50%	40%	0.19568	0.16709	10	0.10084	0.08730	4.9107%
BB	1.00%	40%	0.17501	0.13749	10	0.12034	0.07995	5.8729%
BB-	1.50%	40%	0.15891	0.12151	10	0.12916	0.04935	6.5103%
B+	3.00%	40%	0.12900	0.09648	10	0.14059	0.10120	7.9330%
B	6.00%	40%	0.10820	0.07434	10	0.15730	0.03259	8.4457%
B-	10.00%	40%	0.10303	0.05986	10	0.17821	0.03128	9.0032%
CCC	18.29%	40%	0.10224	0.04477	10	0.20179	0.05355	**10.0837%**

Table XVI. Sensitivity analysis for corporates using different maturities (Specific PD models and Samples)

Rating	PD	LGD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	$K_{corp.}$	Weight	Cum. Weighted $K_{corp.}$
Sales 5 - 25 MMNok								
AAA	0.05%	40%	0.20148	0.28612	1	0.00643	0.00230	0.0015%
AA	0.06%	40%	0.20090	0.27553	1	0.00738	0.00657	0.0063%
A-	0.15%	40%	0.19577	0.22535	1	0.01426	0.03378	0.0545%
BBB+	0.20%	40%	0.19302	0.21064	1	0.01731	0.09648	0.2215%
BBB	0.30%	40%	0.18773	0.19075	1	0.02240	0.14904	0.5554%
BB+	0.75%	40%	0.16692	0.14942	1	0.03699	0.18189	1.2282%
BB	1.00%	40%	0.15723	0.13749	1	0.04208	0.07383	1.5389%
BB-	1.50%	40%	0.14113	0.12151	1	0.04924	0.14012	2.2288%
B+	3.00%	40%	0.11122	0.09648	1	0.06143	0.14693	3.1314%
B	6.00%	40%	0.09042	0.07433	1	0.07838	0.07604	3.7273%
B-	10.00%	40%	0.08525	0.05986	1	0.09834	0.04756	4.1950%
CCC	16.50%	40%	0.08448	0.04719	1	0.12135	0.04545	**4.7465%**
Sales 26 - 50 MMNok								
AAA	0.05%	40%	0.21926	0.28613	3	0.01440	0.00136	0.0020%
A	0.10%	40%	0.21637	0.24695	3	0.02137	0.00272	0.0078%
BBB	0.30%	40%	0.20551	0.19075	3	0.03829	0.02039	0.0859%
BB+	0.75%	40%	0.18470	0.14943	3	0.05716	0.09109	0.6066%
BB	1.00%	40%	0.17501	0.13749	3	0.06332	0.20326	1.8936%
BB-	1.50%	40%	0.15891	0.12151	3	0.07168	0.23997	3.6138%
B+	3.00%	40%	0.12900	0.09648	3	0.08550	0.37254	6.7989%
B-	10.00%	40%	0.10303	0.05986	3	0.12667	0.04759	7.4017%
CCC	19.35%	40%	0.10223	0.04347	3	0.13659	0.02107	**7.6896%**
Sales 5 - 25 MMNok								
AAA	0.05%	40%	0.20148	0.28612	5	0.01933	0.00230	0.0045%
AA	0.06%	40%	0.20090	0.27553	5	0.02124	0.00657	0.0184%
A-	0.15%	40%	0.19577	0.22535	5	0.03369	0.03378	0.1322%
BBB+	0.20%	40%	0.19302	0.21064	5	0.03862	0.09648	0.5049%
BBB	0.30%	40%	0.18773	0.19075	5	0.04634	0.14904	1.1956%
BB+	0.75%	40%	0.16692	0.14942	5	0.06549	0.18189	2.3867%
BB	1.00%	40%	0.15723	0.13749	5	0.07124	0.07383	2.9126%
BB-	1.50%	40%	0.14113	0.12151	5	0.07850	0.14012	4.0126%
B+	3.00%	40%	0.11122	0.09648	5	0.08915	0.14693	5.3224%
B	6.00%	40%	0.09042	0.07433	5	0.10461	0.07604	6.1178%
B-	10.00%	40%	0.08525	0.05986	5	0.12421	0.04756	6.7085%
CCC	16.50%	40%	0.08448	0.04719	5	0.14600	0.04545	**7.3720%**
Sales 26 - 50 MMNok								
AAA	0.05%	40%	0.21926	0.28613	10	0.03962	0.00136	0.0054%
A	0.10%	40%	0.21637	0.24695	10	0.05426	0.00272	0.0201%
BBB	0.30%	40%	0.20551	0.19075	10	0.08497	0.02039	0.1934%
BB+	0.75%	40%	0.18470	0.14943	10	0.11279	0.09109	1.2209%
BB	1.00%	40%	0.17501	0.13749	10	0.12034	0.20326	3.6671%
BB-	1.50%	40%	0.15891	0.12151	10	0.12916	0.23997	6.7666%
B+	3.00%	40%	0.12900	0.09648	10	0.14059	0.37254	12.0039%
B-	10.00%	40%	0.10303	0.05986	10	0.17821	0.04759	12.8520%
CCC	19.35%	40%	0.10222	0.04347	10	0.13659	0.02107	**13.1398%**

Table XVII. Retail customers capital requirement for simulation purpose (conventional method)

Rating	PD	LGD	R_{sme}	K_{sme}	Weight	Cum. Weighted K_{sme}
AAA	0.05%	50%	0.157744791	0.0058926	0.0052	0.0031%
AA	0.06%	50%	0.157298465	0.0067745	0.0128	0.0117%
A	0.11%	50%	0.155090122	0.0106368	0.2056	0.2304%
BBB+	0.15%	50%	0.153351062	0.0132783	0.15	0.4296%
BBB	0.30%	50%	0.147042188	0.0211543	0.2704	1.0016%
BBB-	0.50%	50%	0.139129413	0.0287655	0.0816	1.2363%
BB	1.00%	50%	0.121609452	0.0406869	0.0728	1.5325%
BB-	1.50%	50%	0.106902197	0.0474409	0.0332	1.6900%
B+	3.00%	50%	0.075491907	0.0558150	0.0704	2.0830%
B	6.00%	50%	0.045919336	0.0602053	0.022	2.2154%
B-	10.00%	50%	0.03392566	0.0671492	0.0256	2.3873%
CCC	18.29%	50%	0.030215679	0.0858593	0.0504	**2.8201%**

Table XVIII. Corporates capital requirement for simulation purpose (conventional method)

Rating	PD	LGD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	$K_{corp.}$	Weight	Cum. Weighted $K_{corp.}$
Sales 50 - 250 MMNok								
AAA	0.05%	50%	0.20148	0.28612	3	0.01610	0.00263	0.0042%
AA	0.06%	50%	0.20090	0.27553	3	0.01789	0.00263	0.0089%
A	0.11%	50%	0.19802	0.24177	3	0.02526	0.16477	0.4251%
BBB+	0.15%	50%	0.19577	0.22535	3	0.02997	0.18273	0.9727%
BBB	0.30%	50%	0.18773	0.19075	3	0.04296	0.31420	2.3227%
BBB-	0.50%	50%	0.17790	0.16709	3	0.05441	0.08151	2.7662%
BB	1.00%	50%	0.15723	0.13749	3	0.07082	0.07362	3.2875%
BB-	1.50%	50%	0.14113	0.12151	3	0.07983	0.04032	3.6094%
B+	3.00%	50%	0.11122	0.09648	3	0.09411	0.07669	4.3311%
B	6.00%	50%	0.09042	0.07433	3	0.11436	0.02323	4.5967%
B-	10.00%	50%	0.08525	0.05986	3	0.13909	0.01578	4.8161%
CCC	18.29%	50%	0.08446	0.04477	3	0.17233	0.02191	**5.1937%**
Sales 251 - 500 MMNok								
AAA	0.05%	50%	0.21926	0.28613	5	0.02700	0.00000	0.0000%
AA	0.06%	50%	0.21868	0.27554	5	0.02967	0.00000	0.0000%
A	0.11%	50%	0.21580	0.24178	5	0.04034	0.17431	0.7031%
BBB+	0.15%	50%	0.21355	0.22536	5	0.04696	0.18807	1.5863%
BBB	0.30%	50%	0.20551	0.19075	5	0.06453	0.36697	3.9544%
BBB-	0.50%	50%	0.19568	0.16709	5	0.07932	0.08716	4.6457%
BB	1.00%	50%	0.17501	0.13749	5	0.09952	0.08257	5.4674%
BB-	1.50%	50%	0.15891	0.12151	5	0.11013	0.00917	5.5684%
B+	3.00%	50%	0.12900	0.09648	5	0.12655	0.06422	6.3811%
B	6.00%	50%	0.10820	0.07434	5	0.14971	0.00917	6.5185%
B-	10.00%	50%	0.10303	0.05986	5	0.17675	0.01376	6.7617%
CCC	18.29%	50%	0.10224	0.04477	5	0.20997	0.00459	**6.8580%**

Table XIX. Retail customers capital requirement for simulation purpose (unconventional method)

Rating	PD	LGD	R_{sme}	K_{sme}	Weight	Cum. Weighted K_{sme}
AAA	0.05%	50%	0.157744791	0.0058926	0.0032	0.0019%
A	0.11%	50%	0.155090122	0.0106368	0.2204	0.2363%
BBB+	0.15%	50%	0.153351062	0.0132783	0.2076	0.5120%
BBB	0.30%	50%	0.147042188	0.0211543	0.1576	0.8454%
BBB-	0.50%	50%	0.139129413	0.0287655	0.1400	1.2481%
BB	1.00%	50%	0.121609452	0.0406869	0.0812	1.5785%
BB-	1.60%	50%	0.104257178	0.0484259	0.0180	1.6656%
B+	3.00%	50%	0.075491907	0.0558150	0.0836	2.1322%
B	6.34%	50%	0.04413331	0.0606388	0.0088	2.1856%
B-	9.99%	50%	0.033939424	0.0671274	0.0360	2.4273%
CCC	19.93%	50%	0.030121485	0.0890074	0.0436	**2.8153%**

Table XX. Corporates capital requirement for simulation purpose (unconventional method)

Rating	PD	LGD	$R_{corp.}$	$(b)_{corp.}$	$M_{eff.}$	$K_{corp.}$	Weight	Cum. Weighted $K_{corp.}$
Sales 50 - 250 MMNok								
AAA	0.05%	50%	0.20148	0.28612	3	0.01610	0.002629273	0.0042%
AA	0.06%	50%	0.20090	0.27553	3	0.01789	0.00832603	0.0191%
A-	0.15%	50%	0.19577	0.22535	3	0.02997	0.041630149	0.1439%
BBB+	0.20%	50%	0.19302	0.21064	3	0.03496	0.127081507	0.5881%
BBB	0.30%	50%	0.18773	0.19075	3	0.04296	0.176599474	1.3469%
BB+	0.75%	50%	0.16692	0.14942	3	0.06405	0.192813322	2.5818%
BB	1.00%	50%	0.15723	0.13749	3	0.07082	0.069237511	3.0722%
BB-	1.50%	50%	0.14113	0.12151	3	0.07983	0.137598598	4.1707%
B+	3.00%	50%	0.11122	0.09648	3	0.09411	0.113935145	5.2429%
B	6.00%	50%	0.09042	0.07433	3	0.11436	0.063540754	5.9696%
B-	10.00%	50%	0.08525	0.05986	3	0.13909	0.031551271	6.4084%
CCC	16.50%	50%	0.08448	0.04719	3	0.16709	0.035056968	**6.9942%**
Sales 251 - 500 MMNok								
AAA	0.05%	50%	0.219259412	0.28613	5	0.02700	0	0.0000%
A	0.10%	50%	0.216369753	0.24695	5	0.03846	0	0.0000%
BBB	0.30%	50%	0.205507179	0.19075	5	0.06453	0.013761468	0.0888%
BB+	0.75%	50%	0.184696936	0.14943	5	0.09132	0.128440367	1.2617%
BB	1.00%	50%	0.175005901	0.13749	5	0.09952	0.275229358	4.0007%
BB-	1.50%	50%	0.158906209	0.12151	5	0.11013	0.256880734	6.8298%
B+	3.00%	50%	0.128997841	0.09648	5	0.12655	0.275229358	10.3127%
B-	10.00%	50%	0.103030776	0.05986	5	0.17675	0.041284404	11.0424%
CCC	19.35%	50%	0.102229762	0.04347	5	0.17074	0.009174312	**11.1990%**