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TCPOP: Valuation and Optimal Strategy for Option Contracts in the Shipping Industry

by

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Master thesis

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Abstract

This thesis is an analysis of a special type of option contract often found in the shipping industry, a time charter with purchase option (TCPOP). The TCPOP is a time charter (TC) agreement with an embedded purchase option.

The freight rate is modeled using the two mean reverting stochastic processes: Ornstein Uhlenbeck (OU) and Geometric Mean Reversion (GMR). Both models are estimated from historical spot freight rate data on Suezmax tankers, using OLS. Based on the stochastic processes I specify a one factor model for vessel values. The model is calibrated to historical prices for 5 year old vessels, by approximating the risk premium, using a numerical least squares method. GMR seems to perform better than OU in predicting the distribution of future freight rates and vessel values.

Using Monte Carlo simulation and applying the least square Monte Carlo approach (LSM) proposed by Longstaff & Schwartz (2001), I specify procedures for approximating values of option contracts with different complexity, where a Suezmax vessel is the underlying asset. GMR consistently predicts higher vessel values and option values than OU.

Acknowledgments

I would like to thank my supervisor, Professor Steinar Ekern, for providing invaluable advice during the process. Ekern, together with Professor Petter Bjerksund, should also be acknowledged for their early work on contingent claims in shipping, which has been an important building block in this thesis. I would also like to thank Tom Johan Austrheim, CFO at Kristian Gerhard Jebsen Skipsrederi, for introducing me to the Bermudan TCPOP. In addition, I would like to thank Roar Adland for providing me with data.

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1 Introduction

When discussing shipping markets it is important to clarify what the ambiguous term freight rate means. The freight rate is often quoted in USD per day and means the market price level for buying a freight service for one day. Similarly, a TC rate is the daily cost for buying a freight service over a longer time interval. Unless otherwise stated, the term spot freight rate refers to the time charter equivalent (TCE) spot freight rate[1].

1.1 TCPOP

Risk management in the shipping industry is an evolving activity. The modern developments stem from the 1985 development of the Baltic Freight Index (BFI) in connection with the opening of the Baltic International Freight Futures Exchange (BIFFEX). BIFFEX was the first exchange that enabled agents in the shipping industry to participate in hedging and speculation activities through trading of financial derivatives. In addition to the contracts offered at official exchanges, derivative contracts are being traded over the counter (OTC) between parties in many types of transactions in the industry. One such contract is the time charter with purchase option (TCPOP), which is a time charter agreement with an embedded purchase option.

A time charter (TC) is similar to a lease. The shipowner gives the charterer the right to operate a vessel for a predetermined time period and price. During the TC period, the shipowner still takes care of management of the vessel, such as crew and operation, while the charterer orders how the vessel should be employed. The shipowner still pays the operation and capital costs, such as crew-, maintenance-, insurance-, management- and interests-costs, while the charterer pays, in addition to the TC-rate, the voyage and cargo handling costs, which normally include bunkers, canal tolls and port dues. The price of a TC is often quoted in USD per day. TC's can have durations from weeks to several years.

[1] TCE freight rate is voyage income less voyage costs. See section 1.2 for an extended discussion.

A purchase option, where the underlying is a vessel, is defined as the right, but not the obligation to buy the vessel at a predetermined price and time. A European option can only be exercised at one specific predetermined point in time, while an American option can be exercised at any time during a predetermined time interval. In between is the Bermudan option, which can be exercised at several predetermined points in time.

A Bermudan TCPOP is thus a TC of a vessel, where the vessel can be purchased at several predetermined points in time, and thus end the TC.

As the literature on TCPOP is rather scarce, there is not much information on how these contracts are normally priced. The general impression, as can be seen from the quotes below, is that the general knowledge about the valuation and treatment of such options is limited.

Siri Strandenes claims that

> *These (TCPOPs) can have substantial value, but are often given away*[2].

while Bjerksund & Ekern says:

> *Det kan virke som verdsettingen av slike realopsjoner innen shipping kan være temmelig tilfeldig. Et skipsverft kan gi fra seg en nybyggingsopsjon nærmest gratis, som et ekstra smøringsmiddel for å få dagens kontraheringskontrakt i havn. På sin side kan en reder være i stor tvil om hvilket tillegg til gjeldende TC-rate en forlengelsesopsjon il befrakter burde medføre. Likevel synes slike opsjonstrekk rutinemessig å inngå som tillegg til andre kontrakter i shippingmarkedene*[3].

Based on personal experience it seems practitioners often lack the knowledge on how to treat such options, both in valuation and in determining optimal

[2] Notes from lecture on risk in the course "Shipping economics" at NHH, spring 2008.
[3] Bjerksund & Ekern (1992)

exercise strategies. Seemingly, the option contracts are "given" away in an attempt to sweeten deals, but lack of knowledge might give unintentional consequences for both parties. This is unfortunate as investment decisions in shipping seems to be extremely important, and should thus be dealt with in a consistent and rational manner. Kavussanos & Visvikis (2006, p. 63) for instance, claims that asset play has a much stronger impact on balance sheets in general than the operational profits.

1.2 Objectives and outline

This thesis will deal with three main topics:

1. Modeling and calibration of the freight rate dynamics using OU and GMR.

2. Valuation and choice of optimal exercise strategy for a Bermudan TCPOP

3. A comparison of how the GMR performs compared to OU.

In section 2 I deal with the specification of a freight rate model. I will briefly present some of the available research that relates to the topic, then discuss the most important features that characterizes shipping markets, and point out what a freight rate model should take into account. From there I will present two stochastic models, OU and GMR, to represent the freight rate process.

In section 3 I use historical data from the Suezmax market to estimate the parameters in OU and GMR.

In Section 4 I present a one factor freight rate model for vessel values. Because of bad model performance, I attempt a calibration procedure by estimating a variable risk premium. The variable risk premium should according to theory be dependent on market conditions, so I use the freight rate as a proxy variable and apply a numerical least square method to estimate a linear model.

In section 5 I propose how one can use the vessel value models, combined with Monte Carlo simulation, to price a set of options contracts. The option contracts vary in complexity, with the Bermudan TCPOP as the most complex one. For the path dependent[4] option contracts I apply the LSM method from Longstaff & Schwartz (2001).

A note on the TCE freight rate

The TCE freight rate is defined as voyage income less voyage cost. Voyage cost incorporate bunker cost, canal fees, port dues, etc. Especially bunker costs are volatile. This might represent a source of bias in the models, as the TCE spot freight rate in fact is influenced by uncertainties from both cost and revenue. For further discussions on the TCE, see Aadland (2003, pp 26-27).

1.3 Current literature

The literature on TCPOP contracts is not very extensive. Jørgensen & De Giovanni (2009) is to my knowledge the only publication that deals with the pricing of TCPOP contracts in specific. However, since a TCPOP is similar to a leasing contract with an embedded option, we can utilize the already available literature on this topic. Jørgensen & De Giovanni provides the following overview:

> *To the authors' knowledge this paper is the first which considers the valuation and optimal management of the common T/CPOP contract in shipping markets. However, since T/CPOPs is a very specific form of lease contract with an option to purchase the underlying (real) asset we can indeed point the interested reader to previous literature which treats the subject of lease contracts with options from a more general perspective. For example, McConnell & Schallheim (1983) set up a discrete time*

[4] Path dependence implies that decisions made at a certain point in time, is affected by decisions made at earlier points in time (e.g an American option).

> *model in which they consider the general valuation of a variety of different types of asset leasing contracts. Examples of such contracts are 1) leases that grant the lessee an option to purchase the leased asset at a fixed price at the maturity date of the lease, 2) leases that grant an option to the lessee to purchase the leased asset at a prespecified price anytime during the life of the lease, and, 3) leases which grant the lessee an option to extend the life of the lease. Another key reference in this respect is Trigeorgis (1996) which deals with the numerical valuation of leasing contracts with a variety of complex embedded operating options including purchase options, exit options, and options to extend.*[5]

Jørgensen & De Giovanni use an OU process to value different types of TCPOP contracts, for instance a Bermudan TCPOP similar to the one under consideration in this thesis. However, the use of OU as a freight rate model have some known fallacies. In their summary they point them out, and suggest alternative solutions for future research.

> *Our analysis has some limitations and there are therefore some obvious directions for future research. First, although we argued that mean reversion is supported in freight rate data, the Ornstein-Uhlenbeck process admittedly also has some less desirable properties. It implies future freight rates that are Gaussian although actual freight rates often appear to be skewed, it has a constant rate of volatility although some empirical research (e.g. Aadland (2000)) has found evidence of a volatility rate that increases with the level of freight rates, and it implies a positive probability of negative freight rates at any future point in time (although this probability is usually negligible). To alleviate the above-mentioned deficiencies, Tvedt (1997) has proposed the Geometric Mean Reversion process as an alternative to the Ornstein-Uhlenbeck process and it would indeed be an interesting*

[5] Jørgensen & De Giovanni (2009)

> *subject for future research to investigate the use of this and other alternative processes in pricing TC-POPs and other freight rate derivatives.*[6]

Jørgensen & De Giovanni does not attempt to estimate their model parameters, and thus cannot judge how their model performs in predicting future vessel value distributions. To determine option values they use a finite difference (FD) algorithm. While FD might be a numerical more convenient way to solve the problem, it has a disadvantage in being somewhat hard to understand and implement. LSM on the other hand, allows one to bypass many of the mathematical and theoretical obstacles encountered using FD.

The literature on freight rate dynamics is more extensive. In broad terms there are two types of models that are dominant. Tvedt (2003), and Aadland (2003) gives a thorough overview of the two. First, we have the equilibrium models, that specify freight rates as a function of traditional demand and supply. Second, we have the stochastic process models, that specify the freight rates as a stochastic process. Later developments in the area deals with how to combine the two types of models. See for instance Tvedt (2003) and Aadland & Strandenes (2004). Aadland (2003) provides another approach where he specifies a non-parametric model for the spot freight rate.

Bjerksund & Ekern (1995) created an important building block for contingent claims analysis in shipping when they specified a freight rate model using the OU process. Tvedt (1997) did a comparison on VLCC values using the OU process, but also introduced the GMR model from Schwartz (1997). As mentioned, the GMR model has some desirable features that OU lacks. Schwartz used GMR to model the behavior of commodity prices. The GMR model is sometimes refered to as the "Schwartz model 1".

The literature on valuation under uncertainty is extensive. This research area has received much attention in financial academia. Hull (2006) is highly

[6] Jørgensen & De Giovanni (2009) pp.34-35

regarded and covers a wide range of techniques and theory for option pricing. Longstaff & Schwartz (2001) suggests a simple and intuitive numerical method based on simulation and ordinary least squares; the "least square Monte Carlo approach"(LSM), to value path dependent options.

2 Freight rate modeling

As mentioned in the introduction, the modeling of freight rates has traditionally been dominated by two different approaches. The earlier models have relied on traditional equilibrium theory, where the specification of supply and demand functions plays the most important role. In more recent year, stochastic market models have prevailed. These models have to some extent dominated the field in freight rate modeling. The current innovations in freight rate modeling have evolved around either combining the equilibrium models with the stochastic ones, or developing new stochastic ones. Much effort have been put into examination of the risk premium in shipping markets, see for instance Aadland (2003), Adland & Strandenes (2006) and Kavussanos & Alizadeh-M (2002). However, any model chosen should be well founded in economic theory, so a brief discussion of the characteristics of the shipping freight market is neccesary.

2.1 Market equilibrium

The bulk markets are normally considered to be a good example of a perfect competitive market. The demand side is primarily driven by the global demand for goods in combination with geographical patterns. In the short run the demand curve is relatively inelastic, due to the lack of alternative freight methods[7]. In the long run however, for high freight rates, people tend to adjust by importing or buying goods that are geographically closer.

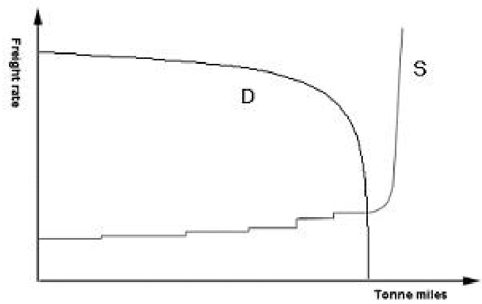
The supply side of the equation is mainly influenced by the available vessels on the market.[8] In the short run, shipowners have three choices that may influence the supply. If freight rates are low, they may stop operating the vessel to avoid operation and voyage costs, or even scrap vessels to re-

[7] Airplanes and railroad are the only real alternatives, but can obviously not match the scale of a bulk vessel. Pipelines however, can be a economically feasible substitute for transport of oil and gas.

[8] Usually measured in tonnemiles, i.e the aggregated freight capacity in the market, measured in tonnes, times the aggregated vessel travel capacity, measured in miles.

ceive a residual value[9]. If freight rates are high they can increase the vessel speed. An increase in vessel speed will be costly, due to increased tear, and as the speed reaches a physical maximum, the supply curve becomes almost perfectly inelastic, as we can see from figure 2.1. In the long run, the supply is influenced by the amount of newbuildings entering the market. For a bulk carrier the delivery time is normally minimum one year. Because of the relative long building times, the supply side is rather slow in adapting to booming conditions. Because of the slow adaption shipping markets have a distinctive business cycle.[10]

Figure 2.1: Typical supply and demand in bulk shipping markets



2.2 Stochastic modeling

In this section I will examine two candidate models for the freight rate process. First I will present the OU model from Bjerksund & Ekern (1995), and then an alternative model, the GMR model from Tvedt (1997) and Schwartz (1997). As mentioned earlier, the main problem with OU is that it may predict negative freight rates, and that the volatility is constant for all freight rate levels. However, even though the GMR process can account for these characteristics, the crucial benefit from working with the OU process is its available analytical solutions. Where the OU process offers easy calculations, the GMR relies on using time and computer-consuming procedures, which we will encounter later.

[9] Vessels are sold to demolition yards.
[10] Figure 2.1 is from Aadland & Strandenes (2004)

the stochastic models for the freight rate are similar to those often used for commodities. However, freight is not a commodity, but a service. As a service cannot be stored, an important distinction is that we cannot use standard cost-of-carry arguments[11].

A stochastic model of the freight rate should account for the features that stems from the discussion on market equilibrium above. However, it is not straight forward to identify what these features are. Aadland (2003) gives an extensive discussion and stylizes four main characteristics.

Mean reversion

Mean reversion is maybe the most distinctive feature for the freight rate. It implies that if freight rates are high, they tend to go down, and if freight rates are low, they tend to go up. The feature is governed by the fact that when freight rates are high, shipowners will postpone scrapping of old vessels and increase vessel speed, while newbuildings will keep entering the market. This will shift the supply curve to the left, and over time push freight rates downward. For low freight rates, shipowners will lay up and scrap vessels. There is a lower freight rate boundary for profitable operation. Over time, vessel scrapping will shift the supply to the left increse the freight rate.

Short run momentum

In the short run, freight rate trends often show persistence. Aadland claims that since the freight rate itself cannot be traded or stored, there is no possibility to exploit trends in the market in the short run. This will naturally lead to short run market imbalances. Neither OU or GMR accounts for this, except through the mean reversion effect.

[11] A future price is a function of todays price and the cost of holding the underlying over the time interval

Level effect in variance

As we can see from figure 2.1, the short run run supply is close to perfectly elastic as demand for transport approaches zero. At low freight rates, vessels lay up. One can argue that the freight rate will stay at a minimum until the last unemployed vessel get chartered. In a competitive market price fluctuations should be very low until freight rates reach a level where the entire fleet is employed. At higher freight rates, when the fleet is fully employed, we know that supply turns close to perfectly inelastic, and the freight rate is subject to large fluctuations, for small shifts in the demand. OU fails to account for this, while for GMR, the variance increase as the freight rate increase.

Lag effect in variance

Kavussanos (1996) finds empirical lag effects in the freight rate volatility. Aadland however, claims that ARCH models doesnt account for a relationship between freight rates and volatility, and thus that the lagged volatility is merely picking up the level effect. Aadland also argue that a lagged volatility effect probably is present, but that a potential model must control for the level and the lag effect simultaneously. Neither OU nor GMR controls for lagged effects in the volatility, but a consolidation can be found in Aadlands chapter 3, where he concludes that the lag effects seems to be of less importance for larger vessel types. The Suezmax vessel considered in this thesis can be characterized as a larger vessel type.

2.3 Ornstein Uhlenbeck

From Bjerksund & Ekern (1995), I start by stating the instant cash flow generated by an operating ship as:

$$D(t)dt = a(S(t) - b)dt, \tag{2.1}$$

where $S(t)$ is the freight rate per unit of cargo at time t, a is the amount of cargo carried by the vessel, and b is the costs involved with operating the vessel. The data used in this thesis represents $S(t)$ as a daily spot rate, rather than a per unit of cargo rate, I set $a = 1$, and return to b later, in section 4. Bjerksund & Ekern further assume that the spot freight rate $S(t)$ follows the mean-reverting Ornstein-Uhlenbeck-process,

$$dS(t) = \kappa(\phi - S(t)) + \sigma dW(t) \tag{2.2}$$

where κ represents the strength of the mean reversion, ϕ is a long term average freight rate level, σ is its standard deviation and $dW(t)$ is the standard Wiener process with expectation 0 and variance dt. The solution to the stochastic differential equation [2.2] can be shown to be

$$S(T) = S(t)e^{-\kappa(T-t)} + \phi(1 - e^{-\kappa(T-t)}) + \sigma \int_t^T e^{-\kappa(u-t)} dW(u) \tag{2.3}$$

The stochastic integral term can be rewritten as

$$\sigma \int_t^T e^{-\kappa(T-u)} dW(u) = \sigma \sqrt{\frac{1 - e^{-2k(T-t)}}{2k}} dW(t) \tag{2.4}$$

$S(T)$ will now be normally distributed with the time t conditional expectation and variance:

$$E_t\{S(T)\} = S(t)e^{-\kappa(T-t)} + \phi(1 - e^{-\kappa(T-t)}) \tag{2.5}$$

$$Var_t\{S(T)\} = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(T-t)}) \tag{2.6}$$

2.4 Geometric mean reversion

As with the OU process we start out with the stochastic differential equation describing the increment for the freight rate ($S(t)$ is still the spot freight rate):

$$dS(t) = \kappa(\phi - lnS(t))S(t)dt + \sigma S(t)dW(t) \tag{2.7}$$

Following the procedure from Schwartz (1997), we denote $X = lnS(t)$ and apply Itos Lemma (see appendix A for details), which enables us to rewrite the equation to

$$dX = \kappa(\alpha - X)dt + \sigma dW(t) \qquad [2.8]$$

where

$$\alpha = \phi - \frac{\sigma^2}{2\kappa}. \qquad [2.9]$$

dX can be interpreted as a percentage change in $S(t)$ in an infinite small time step.

We see that X follows a OU process. The solution to equation [2.8] can be written as

$$X(T) = e^{-\kappa(T-t)}X(t) + (\phi - \frac{\sigma^2}{2\kappa})(1 - e^{\kappa(T-t)}) + \sigma \int_t^T e^{-\kappa(T-u)}dW(u), \qquad [2.10]$$

where the stochastic integral term can be rewritten for Monte Carlo simulation as in equation [2.4]. $X(T)$ will be normally distributed with the time t conditional expectation and variance,

$$E_t\{X(T)\} = e^{-\kappa(T-t)}X(t) + (\phi - \frac{\sigma^2}{2\kappa})(1 - e^{-\kappa(T-t)}) \qquad [2.11]$$

$$Var_t\{X(T)\} = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(T-t)}). \qquad [2.12]$$

We can now formulate an expression for the freight rate, $S(T)$, at time T conditional on the information available at time t

$$S(T) = e^{X(T)} \qquad [2.13]$$

$$= exp\Big(e^{-\kappa(T-t)}X(t) + (\phi - \frac{\sigma^2}{2\kappa})(1 - e^{-\kappa(T-t)}) + \sigma \int_t^T e^{-\kappa(T-u)}dW(u)\Big) \qquad [2.14]$$

Since $X(T)$ is normally distributed, $e^{X(T)} = S(T)$ will be lognormal distributed. From the properties of the lognormal distribution we know the time t conditional expectation and variance.

$$E_t[S(T)] = exp\Big(E_tX(T) + \frac{1}{2}Var_tX(T)\Big) \quad [2.15]$$

$$= exp\Big(e^{-\kappa(T-t)}X(t) + (\phi - \frac{\sigma^2}{2\kappa})(1 - e^{-\kappa(T-t)}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})\Big) \quad [2.16]$$

$$Var_t[S(T)] = exp\Big(2E_t[X(T)] + Var_t[X(T)]\Big)\Big(exp\big(Var_t[X(T)]\big) - 1\Big) \quad [2.17]$$

$$= exp\Big(2e^{-\kappa(T-t)}ln(S(t)) + 2(\phi - \frac{\sigma^2}{2\kappa})(1 - e^{\kappa(T-t)})(e^{\zeta} - e^{\frac{\zeta}{2}})\Big) \quad [2.18]$$

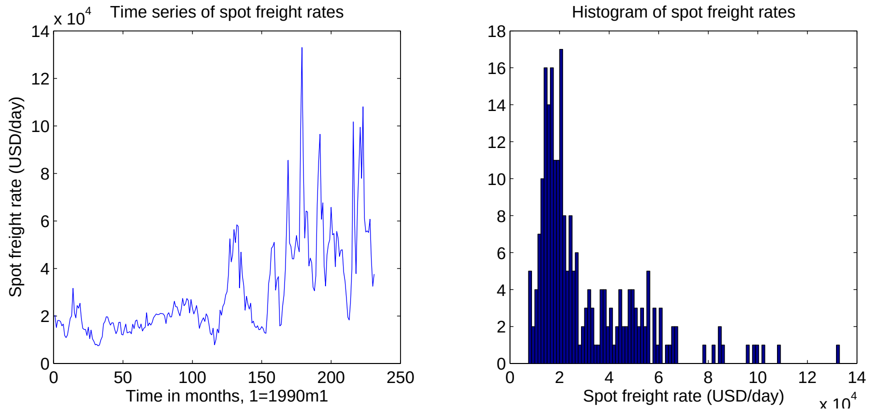
where $\zeta = \frac{\sigma^2(1-e^{-2\kappa(T-t)})}{\kappa}$.[12]

[12] Derivation in Tvedt (1997, p.14)

3 Model calibration

To estimate the parameters in the freight rate models I have gathered data on the monthly (TCE) spot freight rate for Suezmax tankers, from January, 1990 to March, 2009. Figure 3.1 shows the TCE spot freight rate in this period. The freight rate is quoted in USD per day.

Figure 3.1: Data presentation



As we can see, the freight rate seems to have a low volatility in the period from 1990 to 2000 compared to the period from 2000 until 2009. In the latter period we can observe freight rates near 150000 USD, which in a historical perspective is fairly high. It is also worth noting the distribution of the freight rate. It is clearly skewed to the right, and does not seem to be normally distributed as implied in OU. This suggests that the lognormal GMR might be a better fit.

From Enders (2003, p.171) we also know that applying OLS on non-stationary variables often lead to spurious regressions[13]. To determine whether the observed freight rates are in fact stationary, we can apply the Augmented Dickey Fuller Unit Root Test (ADF). ADF test the null hypothesis that the process contains a unit root, which is equivalent testing for stationary. For details

[13] Spurious regression occurs when OLS discovers a relationship which is really not there, and can often lead to a very high R^2 and strong autocorrelation in the residuals.

on ADF, see Wooldridge (2006, pp 639-645). We apply ADF on both $S(t)$ and $lnS(t)$, as both will be used in later regressions.

Table 3.1: Augmented Dickey Fuller

	Lags	Test statistic	P-value	H_0
S	12	-4.8960	0.0001	Non-stationary
log(S)	12	-4.2741	0.0041	

From table 3.1, ADF rejects the null of non-stationarity in both cases. This means that we can ease our worry about applying OLS. The results are also in line with our a priori assumptions. In fact, we have already assumed stationarity through OU and GMR which are both stationary. As we will see in the next section, both models can be rewritten as a classical AR(1) model.

If we had no a priori intuition about how the freight rate process should be modeled, we could utilize the Box-Jenkins (BJ) methodology to select a specification. An important input in BJ is the use of autocorrelation- and partial autocorrelation-functions (ACF and PACF). Enders (2003, pp. 60-72) gives an explanation on how to identify a model based on the shape of its ACF and PACF. For an AR(1) process, such as OU and GMR, the ACF should decline slowly towards zero, while the PACF will be significant for lag 1, and insignificant for all other lags. The ACF and the PACF for the sample of Suezmax freight rates can be seen in figure B.1. Both for the price and the log of the price, the ACF and PACF acts according to a AR(1) process, which is in line with the OU and GMR assumptions.

3.1 Ornstein Uhlenbeck

To calibrate the freight process, we transform equation [2.3] to a discrete form. Since our data is monthly we impose a new parameter $\delta = \frac{1}{12}$, which is the time step length measured in years. We can now rewrite equation [2.3] to

$$S(t+\delta) = S(t)e^{-\kappa\delta} + \phi(1 - e^{-\kappa\delta}) + \epsilon. \tag{3.1}$$

where ϵ is normal distributed with expectation 0 and standard deviation $\sigma_\epsilon = \sigma\sqrt{\frac{1-e^{-2k\delta}}{2k}}$.

Setting $\alpha = \phi(1-e^{-\kappa\delta})$, $\beta = e^{-\kappa\delta}$ we can form the equation

$$S_{t+\delta} = \alpha + \beta S_t + \epsilon, \tag{3.2}$$

which can be estimated by the use of OLS. It is straightforward to show that[14]

$$\begin{aligned} \kappa &= -\frac{ln\beta}{\delta} \\ \phi &= \frac{\alpha}{1-\beta} \\ \sigma &= \sigma_\epsilon\sqrt{\frac{-2ln\beta}{\delta(1-\beta^2)}} \end{aligned} \tag{3.3}$$

3.2 Geometric mean reversion

We hold on to $\delta = \frac{1}{12}$, and rewrite equation [2.10] to

$$lnS(t+\delta) = e^{-\kappa\delta}lnS(t) + (\phi - \frac{\sigma^2}{2\kappa})(1-e^{\kappa\delta}) + \epsilon, \tag{3.4}$$

where ϵ is normal distributed with expectation 0 and standard deviation $\sigma_\epsilon = \sigma\sqrt{\frac{1-e^{-2\kappa\delta}}{2\kappa}}$.

Setting $\alpha = (\phi - \frac{\sigma^2}{2\kappa})(1-e^{\kappa\delta})$ and $\beta = e^{-\kappa\delta}$, we can form the equation

$$lnS_{t+1} = \alpha + \beta lnS_t + \epsilon \tag{3.5}$$

[14] See C

which can also be estimated by OLS. It is straightforward to show that[15]

$$
\begin{aligned}
\kappa &= -\frac{ln\beta}{\delta} \quad \text{(same as OU)} \\
\phi &= \frac{\alpha}{1-e^{-\kappa\delta}} + \frac{\sigma^2}{2\kappa} \\
\sigma &= \sigma_\epsilon \sqrt{\frac{-2ln\beta}{\delta(1-\beta^2)}} \quad \text{(same as OU)}
\end{aligned}
\tag{3.6}
$$

3.3 Estimation results

By using OLS I obtain the estimates for both models, with the results in table 3.2.

Table 3.2: Estimation results

	OU	**GMR**
Parameter	**Estimate** (Std. Err.)	
α	4744.484**	0.838**
	(1307.989)	(0.269)
β	0.848**	0.918**
	(0.035)	(0.026)
$std.\hat{dev}(\epsilon)$	11271.179	0.240
N	230	230
R^2	0.719	0.841
$F_{(1,228)}$	583.704	1204.221

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

Using the estimated parameters we can calculate the structural parameters from equations [3.3] and [3.6]. The results are shown in table 3.3,

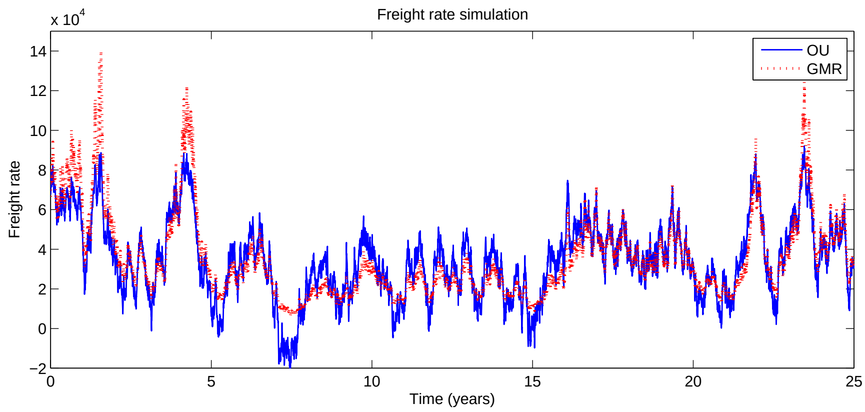
[15] See C

Table 3.3: Structural parameters

	OU	GMR
κ	1.98	1.03
ϕ	31213.71	10.56
σ	42303.78	0.87

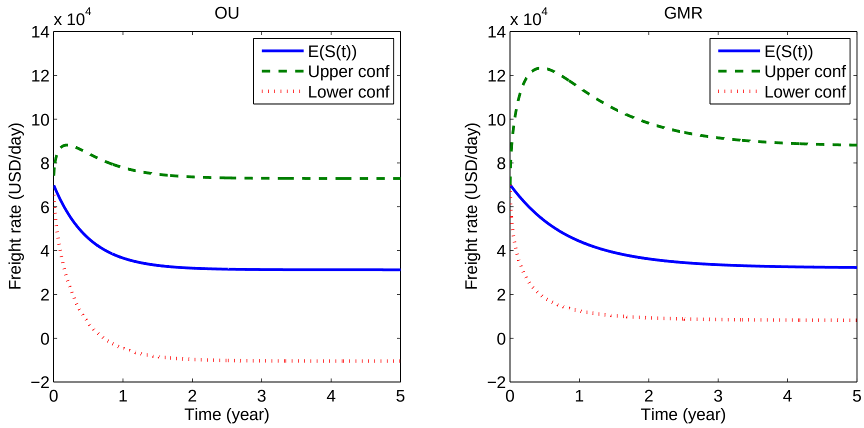
A visual inspection of the price process behavior using the estimated parameters is appropriate. Figure 3.2 shows a freight rate simulation. The simulation is based on the same random numbers for both processes. The figure illustrates some of the features pointed out earlier. First, we see that both processes revert to a mean. Second, we see that the GMR process, because of the lognormal distribution, can obtain much higher values than the normal distributed OU process. Third, we see that the OU process obtains negative values, while the GMR does not. Fourth, we see that the GMR process exhibits volatility relative to its value (i.e low volatility when rate is low and high when rate is high), while OU has a constant volatility. These properties are in line with expectations.

Figure 3.2: Freight rate simulation illustrated



Since there is limited information in watching single simulation paths, figure 3.3 shows the conditional expectations and it's 95% confidence intervals. We see that in the long run, the contitional variance converges. It is also worth noting that the lower confidence interval for the OU process becomes negative.

Figure 3.3: Conditional expectation



3.4 Estimation diagnostics

According to Wooldridge (2006), OLS is an unbiased estimator, $E(\beta) = \hat{\beta}$, for time series data if the following assumptions are fulfilled:

TS.1 (Linearity)	The relationship between the independent variable and the independent variable is linear.
TS.2 (No perfect collinearity)	No independent variable is constant nor a perfect linear combination of the others
TS.3 (Zero conditional mean)	$E(u_t\|X) = 0$. The expected value of the error term u_t, given the vector of variables $X = (x_{1t}, ..., x_{nt})$ is zero for all t.

When TS.1-TS.3 are fulfilled, OLS is an unbiased estimator. However, to be able to make inference there are additional requirements.

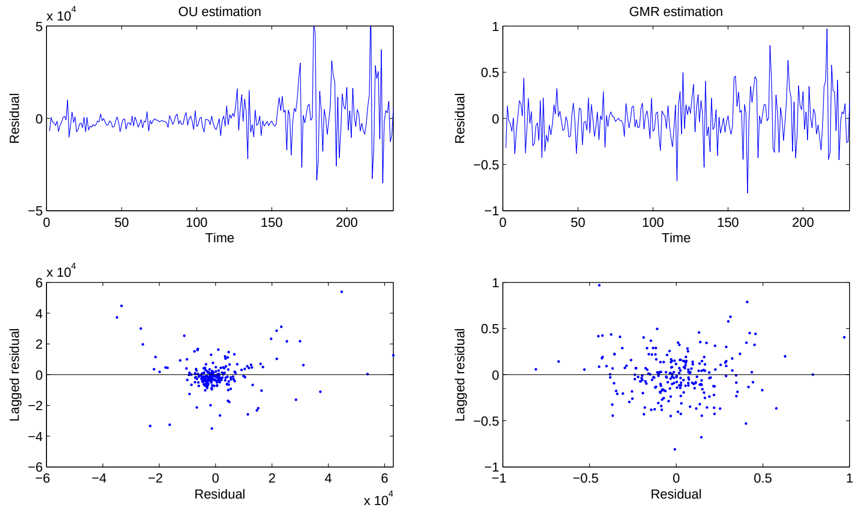
TS.4 (Homoskedasticity)	$Var(u_t\|X) = Var(u_t) = \sigma^2$ for all t. The variance for the error term must be the same for all t.
TS.5 (Autocorrelation)	$\rho(u_t, u_{t+i}) = 0$ for all i, (where $\rho(y_1, y_2)$ is the correlation coefficient for y_1 and y_2). The error term in one period must be uncorrelated with the error term in any other period.

When TS.1 through TS.5 holds, OLS is the best linear unbiased estimator, where $E(\sigma^2) = \hat{\sigma}^2$.

TS.1 is true by assumption. We have specified a linear model and thus TS.1 holds. TS.2 must hold since we only use a single explanatory variable. We can also see from figure 3.1 that the explanatory variable is not constant. TS.3 holds for both models. The mean of the residuals are both zero. Since TS.1 through TS.3 holds, our estimates are unbiased.

To check whether TS.4 and TS.5 holds we need to investigate the residuals. Figure 3.4 shows the residuals from both estimations.

Figure 3.4: Diagnostics: residual plots



From figure 3.4 we see that the residuals from the OU estimations clearly exhibits heteroscedasticity when plotted against time, while for GMR it is not straight forward to tell. From the plot of residuals versus lagged residuals, we see that none of the models exhibit any clear form of autocorrelation. The graphical examination is however not sufficient and we need to formally test TS.4 and TS.5. To test for homoscedasticity I use the Breusch-Pagan test[16]. To test for autocorrelation I use the Ljung-Box test[17].

Table 3.4: Diagnostics: Heteroscedasticity and autocorrelation

Model	Test	Test statistic	P-value	H_0
OU	Breusch-Pagan	248.23	0.0000	Constant variance
	Ljung Box Q (l=50)	75.029	0.0125	No autocorrelation
GMR	Breusch-Pagan	19.42	0.000	Constant variance
	Ljung Box Q (l=50)	50.1673	0.4668	No autocorrelation

As we can see, the tests rejects the null of homoscedasticity for both models, but fails to reject the null of no autocorrelation. The easiest, but possibly not

[16] See Wooldridge (2006, pp 280-283)
[17] See Enders (2003, p. 68)

the most sophisticated way to correct for heteroscedastisity, is robust OLS estimation. I will not present results from a robust estimation, but mention that all coefficients remain significant at a 1% level.

4 Vessel value

Having figured out a model for the freight rate dynamics, we can go on with our analysis and start by specifying a model for vessel value. As in Jørgensen & De Giovanni (2009), I specify a one factor model, where the only uncertain input is the cash flow income from receiving the freight rate. In practice, it is obviously unrealistic that the freight rate is the only uncertain cash flow from operating a vessel. While other factors such as interest rates, exchange rates, residual vessel value, operation costs etc., could be relevant[18], it is convenient not to include these. However, the freight rate income is obviously a significant, and probably the most important determinant for vessel values, and thus the simplification is reasonable.

4.1 Risk neutral valuation

A useful application in finance is the change of probability measure for the stochastic process. The processes we specified in equations [2.3] and [2.10] are representations of what we can refer to as the real world. To valuate such cash flows we need to account for subjective risk preferences in order to determine a specific discount rate, which is cumbersome. However, assuming standard no-arbitrage arguments, allows us to transform the stochastic process into a risk neutral one, and use the risk free interest rate as a discount rate for valuation.

Following the procedure from Jørgensen & De Giovanni (2009, p. 7), we assume that the standard no arbitrage arguments holds (i.e. the existence of a traded twin asset, risk free interest rates, no taxes and no transaction costs). Using Girsanovs theorem[19], we can transform the process to a risk neutral one.

$$dW(t) = dW^*(t) - \lambda dt,$$

[18] Aadland (2003)
[19] See Hull (2006) for details.

where λ is the market price of risk.

$$\begin{aligned} dS &= \kappa(\phi - S)dt + \sigma dW(t) \\ &= \kappa(\phi - S)dt + \sigma(dW^*(t) - \lambda dt) \\ &= \kappa(\phi^* - S)dt + \sigma dW^*(t) \end{aligned} \quad [4.1]$$

Where $\phi^* = \phi - \frac{\sigma\lambda}{\kappa}$ can be interpreted as the long term mean under Q. $dW^*(t)$ is normal distributed with expectation 0, and variance dt.

4.1.1 Ornstein Uhlenbeck

The solution to the risk neutral differential equation is equivalent to the solution for the real process. By substituting ϕ with ϕ^*, and $dW(t)$ with $dW^*(t)$ our risk neutral OU process will be

$$S(T) = S(t)e^{-\kappa(T-t)} + \phi^*(1 - e^{-\kappa(T-t)}) + \sigma \int_t^T e^{-\kappa(u-t)} dW^*(t) \quad [4.2]$$

with a time t conditional expectation under the Q measure,

$$E_t^Q\{S(T)\} = S(t)e^{-\kappa(T-t)} + \phi^*(1 - e^{-\kappa(T-t)}). \quad [4.3]$$

4.1.2 Geometric mean reversion

We do the same for GMR which gives the freight rate under the Q measure

$$S(T) = exp\left(e^{-\kappa(T-t)} lnS(t) + (\phi^* - \frac{\sigma^2}{2\kappa})(1 - e^{\kappa(T-t)}) + \sigma \int_t^T e^{-\kappa(t-u)} dW^*(u)\right), \quad [4.4]$$

with a time t conditional expectation under Q,

$$E_t^Q\{S(T)\} = exp\left(e^{-\kappa(T-t)} lnS(t) + (\phi^* - \frac{\sigma^2}{2\kappa})(1 - e^{-\kappa(T-t)}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})\right), \quad [4.5]$$

4.1.3 Vessel valuation

Under the risk neutral measure we can now specify an expression for the value of receiving a stream of freight rate cash flows over time. The value at time t of receiving the spot rate $S(t)$ in the interval t to T can be stated as

$$\int_t^T E_t^Q S(u) e^{-r(u-t)} du, \tag{4.6}$$

where r is the risk free interest rate.

For OU, we denote this value as $V_{OU}^{flow}(S(t), t : T)$. From Bjerksund & Ekern (1995, 12.2112.23) we have the luxury of an available closed form solution.

$$V_{OU}^{flow}(S(t), t; T) = E_t^Q \int_t^T S(u) e^{-r(u-t)} du \tag{4.7}$$

$$= (S(t) - \phi^*) A(T - t, r + k) + \phi^* A(T - t, r) \tag{4.8}$$

$A(n, m) = \frac{1-e^{-mn}}{m}$[20] is an annuity factor, which multiplied with an annuity gives the present value of the annuity for n periods, with a discount rate of m per period.

For the GMR process we denote the value as $V_{GMR}^{flow}(S(t), t : T)$. Unfortunately no closed form solutions are available[21]. Because of this we will from now on rely on numerical methods[22] to solve the following integral.

$$\begin{aligned} V_{GMR}^{flow}(S(t), t; T) &= E_t^Q \int_t^T S(u) e^{-r(u-t)} du \\ &= \int_t^T \Big[exp\big(e^{-\kappa(u-t)} lnS(t) + (\phi - \frac{\sigma^2}{2\kappa})(1 - e^{\kappa(u-t)}) \\ &\quad + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(u-t)})\big) e^{-ru} du \Big] \end{aligned} \tag{4.9}$$

The above expressions describes the value of receiving the spot freight rate

[20] Note that the names of the variables differ from Bjerksund & Ekern (1995)
[21] See Tvedt (1997, p 168)
[22] Matlab offers, among others, the "trapz" command for numerical solving integrals

over time, However, to express vessel values there are some other factors we need to consider. To be able to compare results with Jørgensen & De Giovanni (2009), I use the similar assumptions.

Age

A vessel has a limited lifetime. For Suezmaxe vessels, it is reasonable to assume 25-30 years of operation. For the remaining of this analysis I assume an operating age of 25 years. We denote this $\hat{T} = 25$

Scrap value

When the vessel reaches it's maximum operation age it is normally sold of to a scrap yard that utilizes the hull for steel production. The demand and supply determining the price of the hull is normally closely related to the freight rate, as shipowners do continuous consideration of what is most profitable, scrapping or operating. When freight rates are high the shipowner will postpone the scrapping to utilize the vessel to generate income, while when the freight rates are low, shipowners might gain from scrapping. If markets are efficient the value from scrapping should equal the value from continuing to operate the vessel. One could model this relationship as done by Tvedt (1997), but for convenience I assume the residual value to be constant. Kavussanos & Visvikis (2006, p.67) shows that the mean scrap rate for Suezmax vessels in the period 1990 to 2005 was 4.53 million USD. For the remainder of this thesis I assume a constant residual value as $\hat{V} = 5.000.000$.

Operation cost

From equation [2.1], we know that we also have to consider relevant costs. Operation costs, often referred to as OPEX, consists of all the costs related to operating the vessel. Such costs include crew, maintenance, mangement, insurance etc. OPEX normally shows some degree of variability and goes up as the age of the vessel increases. If considered reasonable, and with available data, one could expand the analysis to also model OPEX. For the remaining of my analysis however, i will assume OPEX to be constant at a rate of 15000 USD per day[23]. Let $b = 15000$ denote OPEX.

Under the assumption that we know with certainty that the vessel will be scrapped at time $\hat{T}$ for $\hat{V}$, with an OPEX, b, the vessel value can be stated as, expanding [4.6],

$$\int_t^{\hat{T}} E_t^Q S(u) e^{-r(u-t)} du - bA(\hat{T} - t, r) + \hat{V} e^{-r(\hat{T}-t)} \qquad [4.10]$$

The value of a vessel can no be stated as in equations and , but with substituting T with $\hat{T}$, adding $\hat{V} e^{-r(\hat{T}-t)}$, and subtracting $bA(\hat{T} - t, r)$.

4.2 Model performance

The term market price of risk can be somewhat vague. A general definition is that it is the market's view on the reward that should be attached to the risk inherent in an uncertain cashflow. In short, the excess return over the risk free rate for taking on extra risk. In a shipping context the market price of risk is the markets view on the premium that should be attached to the freight rate to compensate for the risky cash flows the freight rate represents.

Many recent publications have investigated the application of the pure expectation hypothesis in shipping markets. The studies have not been able to prove that the PEH aplies, and have thus led to a general consensus that there is probably a time varying market price of risk in shipping markets (see for instance Aadland (2003). However, since it is not straight forward to observe and quantify it, a common approach has been to assume a zero market price of risk, $\lambda = 0$. Both Tvedt (1997) and Jørgensen & De Giovanni (2009) does this. If the market price of risk is zero, the freight rate process under the real measure P is identical to the one under the risk neutral measure Q. This implies that market participants are risk neutral and that we can use

[23] See Stopford (1997) for details on costs in shipping.

the risk free interest rate as a discount rate for the real process. To see if this assumption is reasonable, a visual inspection of the performance of the proposed models is appropriate.

Figure 4.1 shows the time t conditional expected vessel value, together with a 95% confidence interval, under the assumption that $Q = P$. The risk free rate is set to $r = 0.05$. Anyone familiar with Suezmax vessel prices, which I will present later, will see that the models clearly overestimates real world market prices. We can also see that the confidence intervals are rather narrow, compared to what one would expect. These observations indicates that the assumption might be inappropriate.

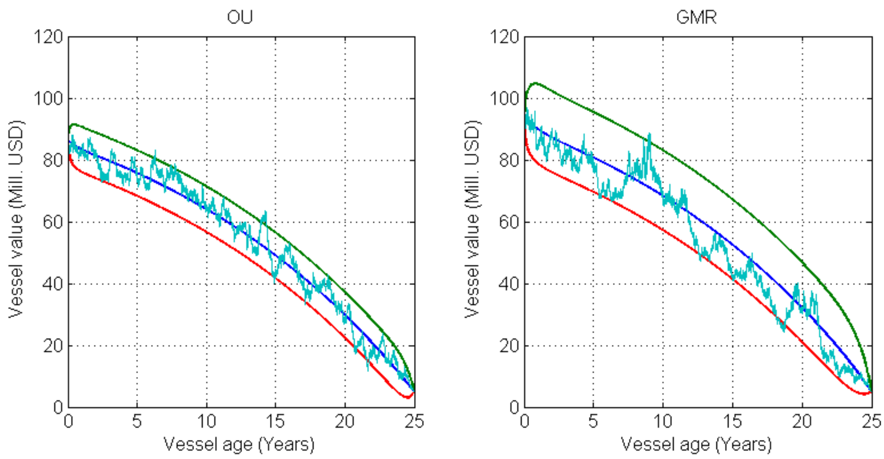


Figure 4.1: Vessel values under Q=P

To further investigate the assumption, I have gathered data on monthly vessel prices for the period from January 1990 to October 2008. Figure 4.2 shows prices for 5, 10 and 15 year old double hull Suezmax tankers. The sample shows for instance that five year old vessels had a market price of around 40 to 50 USD for a time interval of several years. Comparing with the models from figure 4.1, we see that this would be an extremely unlikely outcome. As the reader can see, the samples for 10 and 15 year old vessels are smaller than for 5 year old vessels. Therefore, only data for 5 year old vessels will be used for further analysis.

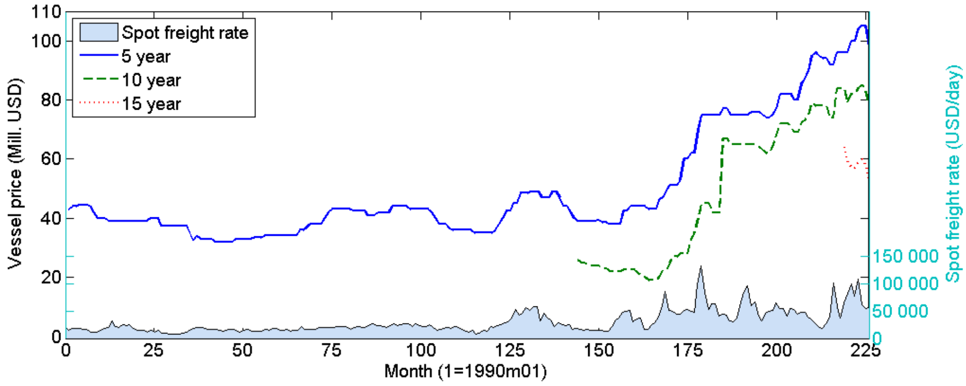


Figure 4.2: Historical Suezmax prices

Even more interesting is how the proposed models for vessel value perform, compared to the historical vessel prices, if we use the historical freight rate as input. As we see in figure 4.3, the models, not surprisingly, fails to fit the market vessel price. However, we should note that if we examine the first two thirds of the time interval, we can see that the movements in the price to some extent is captured, but the levels are totally off. In the end of the period, both models seems to fail completely in both movement and level.

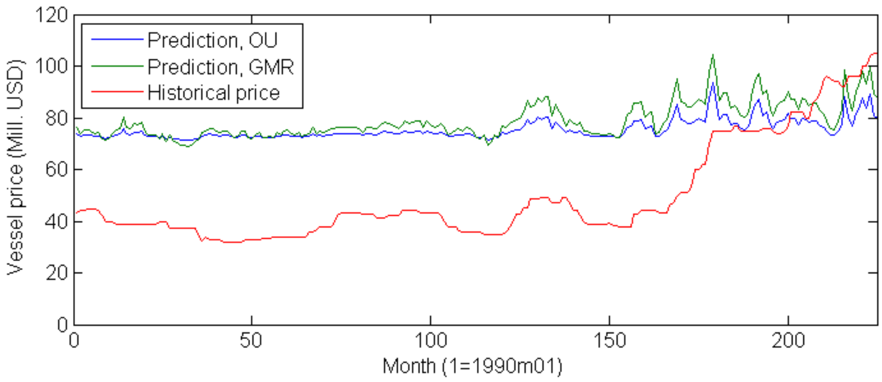


Figure 4.3: Model performance, predicting 5 year old Suezmax price

So, what conclusions can we draw from this? There are mainly two sources for the observed errors. First, the models, the estimated parameters and the assumed parameters may be wrong. If this was the case, we would probably not be able to obtain the good fit in the regressions from table 3.2. The estimated parameters are also significant. The fact that the predictions from figure 4.3 seems to capture some of the movements in the actual price, may be an indication that the model specification is correct. For the assumed parameters, which rely on (qualified) guesses, we could obviously put more effort into a more rigorous specification (i.e. a stochastic interest rate and OPEX, dynamic decision for scrapping, vessel utilization, etc). There is also a possibility that there are factors outside of my knowledge that should be included.

Second, the assumption of a zero market price of risk may be wrong. On the basis of the conclusions drawn from Aadland (2003), this is the most likely source of error. As mentioned in the beginning of this section, there is consensus on the existence of a non-zero market price of risk in shipping markets. If such a risk premium exists, it is not unlikely to cause errors in the scale that we have observed.

4.3 Estimating market price of risk

Given that the error in fact lies in the zero market price of risk, a correction is needed. The correction lies in obtaining an estimate for the risk premium. This is not straight forward, as the risk premium is not observed in the freight rate. However, given that the models and the parameters are correct, the observed difference in the prediction versus the real price of the vessel should reveal information about some implied market price of risk. Aadland states

> The implied risk premium is defined as the difference between the published period charter rate and the model-implied period charter rate when the market price of freight risk is equal to zero.

Equivalently, the difference between the published vessel price, and the model implied vessel price should be the implied risk premium.

Aadland (2003) argues that the market price of risk is non-zero and depend on the market conditions. To obtain an approximate estimate, I propose a simple linear model, where the market price of risk is a function of the freight rate level. The freight rate serves as a proxy variable for market conditions[24].

$$\lambda_t = \beta_0 + \beta_1 S_t \tag{4.11}$$

To estimate equation [4.11], I apply a numerical least squares routine[25], using the data for 5 year old vessels values and the vessel value models. I minimize the sum of squared residuals from the model predictions with respect to β_0 and β_1.

$$Min \sum (V_t - \hat{V}_t)^2, \text{ wrt } \beta_0 \text{ and } \beta_1 \tag{4.12}$$

V_t is the historical vessel price for five year old vessels, while $\hat{V}$ is the predicted vessel value from the model. $\hat{V}$ is calculated using equation [4.10],

$$\phi^* = \phi - \frac{\sigma\lambda}{\kappa} \tag{4.13}$$

where we substitute equation 4.11 so that

$$\phi^* = \phi - \frac{\sigma(\beta_0 + \beta_1 S_t)}{\kappa} \tag{4.14}$$

The numerical least square procedure finds the solution to [4.12] by trying different combinations of β_0 and β_1, based on an initial guess. In the estimation, the freight rate is given in 1000 USD, for shorter coefficients. The reason for using such a simple model is based on the fact that I am reliant on numerical solutions for estimation.

From the estimation we obtain the following parameters[26]

[24] An observed variable that is related but not identical to an unobserved variable, Wooldridge (2006)

[25] The Matlab code is provided in appendix D

[26] See Wooldridge (2006) for details on R^2

Table 4.1: Market price of risk parameters

	OU	GMR
β_0	0.428	0.335
β_1	-0.0051	-0.0031
R^2	0.511	0.514
SST	84889	84889
SSE	41516	41259

The estimated market price of risk is presented visually in figure 4.4. We see the market price of risk is decreasing to the freight rate, which is in line with the expectations based on Aadland (2003, Chapter 4). We see that the market price of risk lies approximately between 20-40% during "normal" market condition (i.e. freight rates between 10000 and 50000). This seems like a reasonable estimate. We see that for very high freight rates, the risk premium turns negative, indicating that market participants become risk seekers in good market conditions. From Aadlands discussion on the risk premium, this might also be a reasonable estimate. OU requires a higher premium at lower values than GMR, which probably is due to the negative value problem.

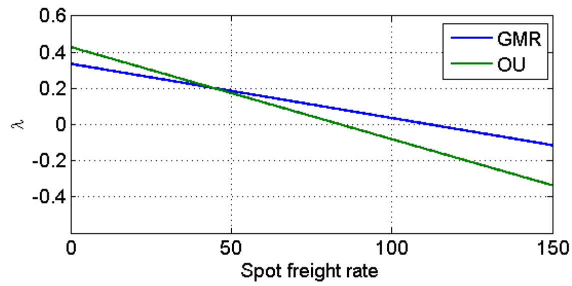


Figure 4.4: Estimated market price of risk

To see how the model performs under the variable risk premium we plot the predictions against the historical vessel prices in figure 4.5. Under the variable risk premium, OU and GMR performs almost identical for the given sample. The predictions perform fairly well in the first part of the sample, while it seems to perform worse in the latter. Graphical representations of the residuals can be studied in appendix D.2. As just mentioned, the residuals show bad performance in the end of the sample period. However, the plot against the explanatory variable, the freight rate, the residuals seems to fluctuate around zero. This indicates that the estimates are unbiased. From the ACF and PACF, we see that the residuals exhibits strong autocorrelation (seemingly AR(1). This suggests that the model is underspecified.

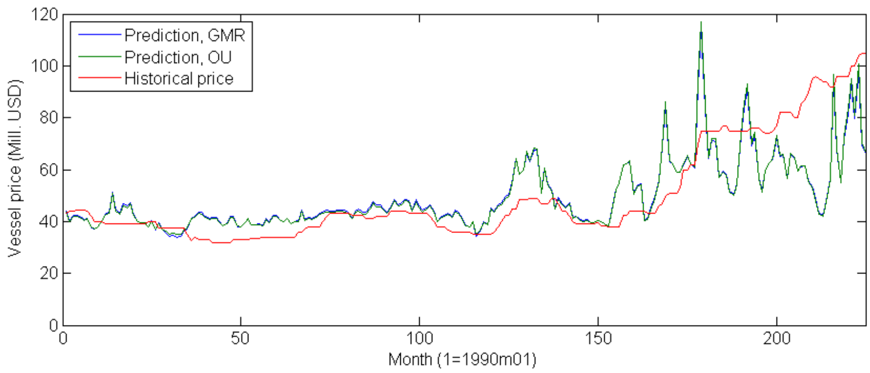


Figure 4.5: Model performance, after adjusting for variable risk premium

For further comparison, I have shown the conditional expected vessel value in figure 4.6, and some distributions in figure 4.7. We see that the expected vessel values are significantly lower now, than under the assumption of a zero risk premium. The new expectations are much more in line with actual market conditions. We also see that the confidence intervals have widened, which also seems to be in line with real conditions. If we compare OU and GMR, we see that they differ slightly in expectation, but that the confidence intervals differ significantly, a natural consequence of the model properties.

See also figure E.1 for illustration of the relationship between vessel value and spot freight rate. We can see that OU is linear to the freight rate, while GMR shows non-linearity.

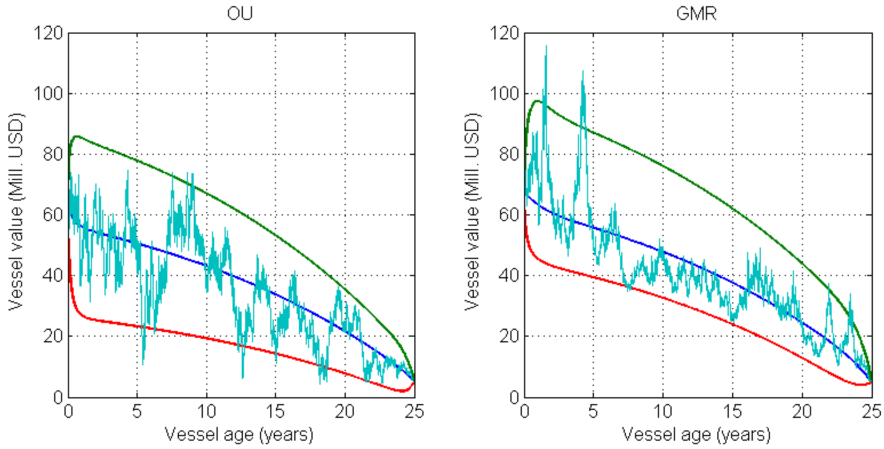


Figure 4.6: Expected vessel values, controlled for risk premium

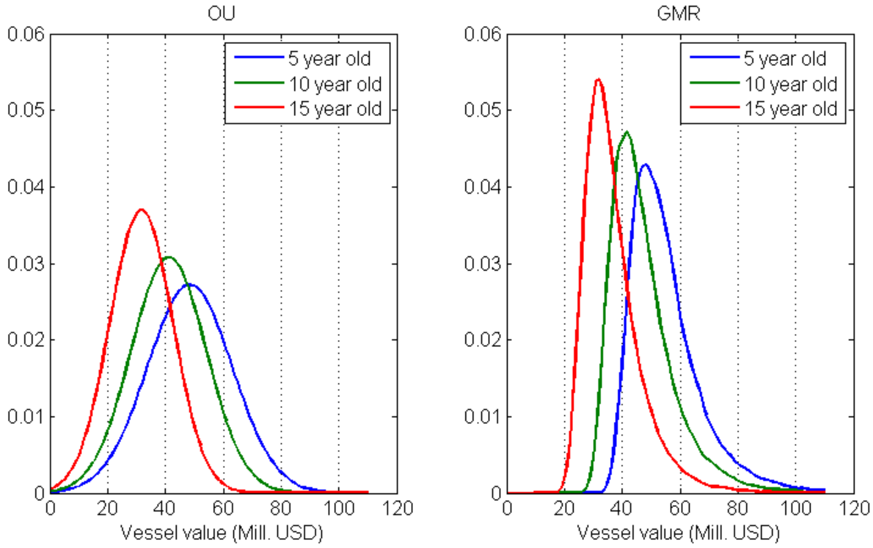


Figure 4.7: Distribution of vessel values for age 5, 10 and 15 (years)

Note on the estimation

The estimation of a potential risk premium is a topic that deserves much more attention than feasible in the scope of this thesis, and the estimates should not be interpreted without a high degree of caution. The results should not be considered to be anything else than an approximation to account for the seemingly unrealistic assumption of a zero market price of risk.

First, the estimation relies on the assumed parameters in the vessel value function. If for instance OPEX turns out to be higher, the estimated constant term for the risk premium will be lower. This will again lead the expected future vessel values to change. Second, one would expect the risk premium to also depend on the age of the vessel. If the estimation were done on historical values of ten or five year old vessels instead, the estimated risk premium might be different. Third, the simple linear model shows clear signs of being underspecified. Especially in the last part of the sample, the model fails. In addition, we find strong autocorrelation in the residuals, which the model should account for. The attempt to estimate the risk premium from Aadland (2003) finds a significant AR(1) relationship for the risk premium.

However, based on the models for freight rates, I have now specified a model for the value of a vessel during its lifetime. Under the equivalent martingale measure Q, a time varying risk premium has also been specified. We can now move on to the pricing of option contracts in the shipping industry.

5 Option pricing

In this section I will present methods, using numerical examples, and results for the pricing of four different types of option contracts on Suezmax vessels. The first three contracts works as natural building steps toward the fourth and last, which is the contract of original interest, the Bermudan TCPOP.

1. European purchase option on a vessel

2. TC contract with embedded European purchase option on a vessel

3. Bermudan purchase option on a vessel

4. TC contract with embedded Bermudan purchase option on a vessel

5.1 European purchase option

5.1.1 Numerical example

Consider a European call option on a Suezmax tanker with expiration in year T and a strike, K. At expiration, the vessel will be T years old. The value of the option at expiration is

$$Max\{V_T - K, 0\}, \tag{5.1}$$

so the value of the option c_t at time t is

$$c_t = E_t^Q\left[Max(V_T - K, 0)e^{-r(T-t)}\right] \tag{5.2}$$

To approximate the option value I simulate n vessel values, V_T at time T. Suppose that $T = 15$ and $K = 30$. Table shows an example of 10 simulations.

Table 5.1: Numerical method example, European call

i	V_T	$Max(V_T - K, 0)$
1	27	0
2	60	30
3	32	2
4	30	0
5	32	2
6	26	0
7	31	1
8	25	0
9	44	14
10	42	12

To find the approximate option value, we discount the realized cash flows (i.e the option value at time T) back to time t, and find the mean.

$$\hat{c}_t = \frac{\sum_{i=1}^{n}(Max(V_{T,i} - K, 0))e^{-r(T-t)}}{n}, \qquad [5.3]$$

where $i \in [1, 2, ..., n]$ is the number of the simulation path. Table 5.2 shows the discounted cash flows. The estimated option value will in this case be

$$\hat{c}_t = \frac{14 + 1 + 1 + 7 + 6}{10} = 2.9 \qquad [5.4]$$

Table 5.2: Numerical method example, European call, discounted cash flows

i	$Max(V_T - K, 0)e^{-r(T-t)}$	
1	0×0.47=	0
2	30×0.47=	14
3	2×0.47=	1
4	0×0.47=	0
5	2×0.47=	1
6	0×0.47=	0
7	1×0.47=	0
8	0×0.47=	0
9	14×0.47=	7
10	12×0.47=	6

5.1.2 Analysis

Figure 5.1 shows the option price estimate, $\hat{c}_t$, as a function of the strike price K, for 5, 10 and 15 year expirations. The graph is in line with what we would expect. The option price based on the GMR is consistently higher than the one based on OU. We see that the highest difference in the option price estimate is approximately 5 mill. USD for a 5 year old vessel where K is close to 0.

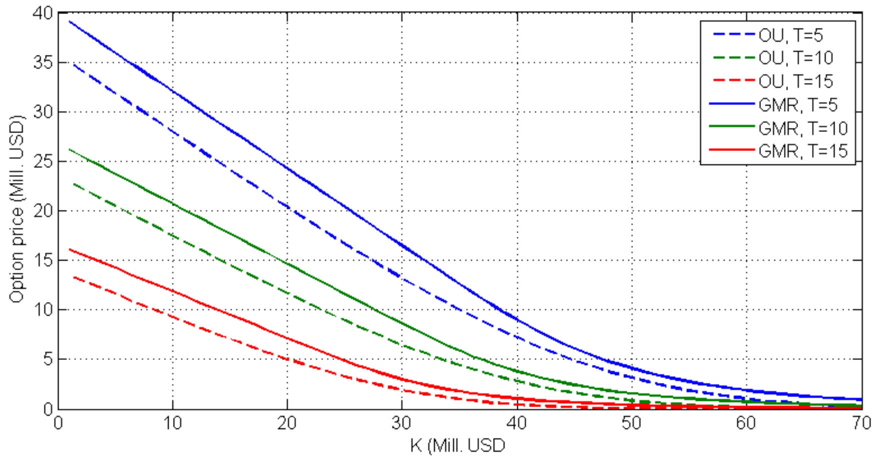


Figure 5.1: Price of a European call determined by K

5.2 TC with European purchase option

This contract is the European equivalent to the Bermudan TCPOP, and can thus be called a European TCPOP. The value of such an agreement, which consists of a TC contract from time t to T, with an embedded European call option the vessel at time T ca be written

$$V^{flow}(S(t), t; T) - TC_{t,T}A(T-t, r) + E_t^Q\left[Max(V_T - K, 0)e^{-rT}\right] \quad [5.5]$$

$V^{flow}(S(t), t; T)$ is the expected present value, under Q, of the revenue cash flow from operating in the spot market from time t to T. $TC_{t,T}$ is the daily TC rate from time t to T. $E_t^Q\left[Max(V_T - K, 0)e^{-rT}\right]$ is the expected value of the embedded European call option, identical to the one in equation [5.2] from the previous section.

As we can see, the valuation of this agreement can be decomposed into two components, where each component can be priced independently from the other. The first component is the TC contract and the second component is the option contract. Since the option contract has been covered in the previous section, the rest of this section will focus on the value of the TC contract. The analytical solutions presented below is based on Bjerksund & Ekern (1995) and Jørgensen & De Giovanni (2009).

From the shipowners point of view a TC contract is used to remove the risk of low freight rates and thus low revenues, while for the charterer it is used to remove the risk of high freight rates and thus high costs.

The TC component of the agreement however has not been treated yet, and deserves some attention.

In a complete market we can assume that the TC rate for a contract from time t to T should equal the expected value under Q, of receiving the spot rate for the same period.

$$\begin{aligned} TC_{t,T}A(T-t,r) &= V^{flow}(S(t),t;T) \\ &\Rightarrow \\ TC_{t,T} &= \frac{V^{flow}(S(t),t;T)}{A(T-t,r)} \end{aligned} \qquad [5.6]$$

Let $\hat{TC}_{t,T}$ denote the fair TC rate which satisfies equation [5.6]. This is the certainty equivalent rate of the stochastic spot rate.

Figure 5.2 shows how the fair TC rate relates to the current spot rate and the duration of the contract. Notice, when the length of the contract (T-t) increases the fair TC rate becomes less sensitive to spot freight rate. Though not observable in this figure, there is a spot freight rate level where the fair

TC rate is equal for all contract durations. This occurs when the spot freight rate equals the risk adjusted unconditional expected freight rate level, $E^Q\phi^*$. Notice also that the OU model is linear in S(t), while GMR is not. See table F.1 for further comparisons.

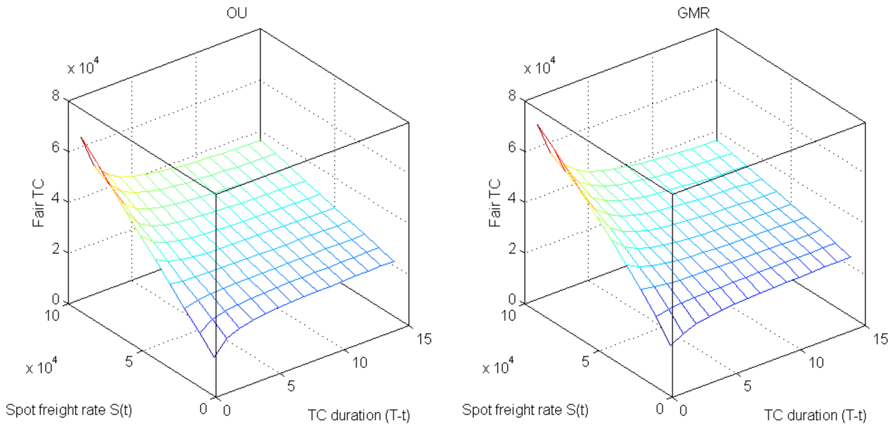


Figure 5.2: Fair TC rate

If the TC rate is set at the fair rate, the TC agreement will have an expected value of 0. However, sometimes TC agreements are made prior to the start date, and the TC rate is set before we know what the fair rate will be at that time. If the TC rate differs from the fair rate, the agreement will have an expected non-zero value. The expected value of a TC contract with rate TC from time t to T can be denoted

$$V_{TC} = V^{flow}(S(t), t; T) - TC_{t,T}A(T-t, r) \qquad [5.7]$$

Substituting from equation [5.6], we get

$$V_{TC} = \left[\hat{TC}_{t,T} - TC_{t,T}\right]A(T-t, r) \qquad [5.8]$$

Note that these values are for the charterer. The shipowner, as the counterpart, will naturally expect the opposite value.

The expected value of a 5 year TC, for different freight rate levels at the contract starting date, can be seen in figure 5.3. We see that such contracts can be of significant value if the right conditions are met. For the OU process the TC value is linear in the freight rate , while GMR it is non-linear. We can also see that the GMR consistently values the contract higher than OU.

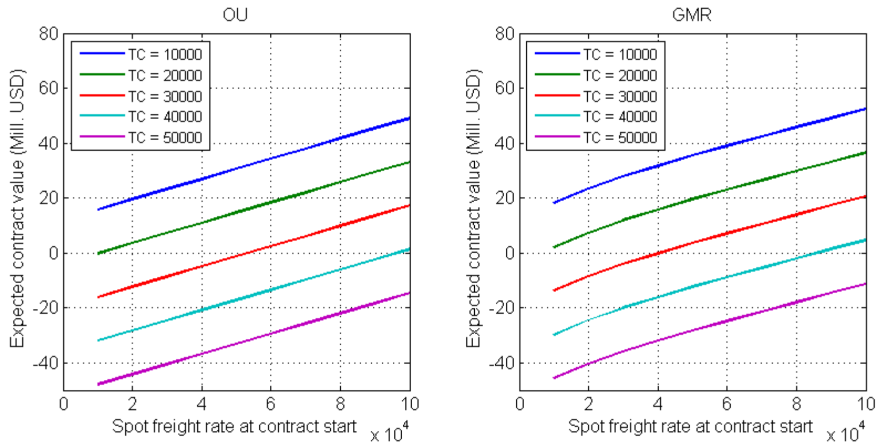


Figure 5.3: Expected value from TC

5.3 Bermudan purchase option

The Bermudan call option is characterized by multiple exercise dates in the future. The challenge when pricing such options is the creation of a stopping rule that determines if the option should be exercised, or if we should wait to the next exercise date. To do this, I apply the LSM approach. The contract we consider is a Bermudan purchase option on a new vessel built at $t = t_0$, with expiration in year t_3. The option can be exercised at $t = t_1, t_2$ and t_3. At each exercise date the strike price is K_t.

5.3.1 Implementation of LSM

1. Simulate n price paths.

2. At each exercise date, for each simulated path, store the vessel value.

3. Starting in the last period, create a cash flow matrix (($n \times$ number of exercise dates)), where for each simulation path we store the intrinsic value of the option at time t_3, in the last column.

4. Go back to the previous exercise date, t_2. Find the continuation value. Exercise if the intrinsic value is higher than the continuation value. If the option is exercised store the intrinsic value in the column for t_2, and update column t_3.

5. Go back to the previous exercise date, t_1 and repeat the procedure.

6. Discount all cash flows from the cash flow matrix back to t_0 and use the average as an approximation of the TCPOP-value.

5.3.2 Numerical example

In the following example I have simulated 10 sample vessel price paths, shown in table 3(a). Suppose that the strike prices are $K_{t_1} = 50$ $K_{t_2} = 40$ $K_{t_3} = 30$, the exercise dates are $t_1 = 5$, $t_2 = 10$, $t_3 = 15$, and that the risk free rate is $r = 0.05$. Based on the realized vessel values, the options intrinsic value,

$$Max(V_t - K_t, 0), \tag{5.9}$$

at each exercise date, will be as shown in table 3(b)

Table 5.3: Bermudian purchase option, numerical example

(a) Simulated vessel values

i \ t	t_1	t_2	t_3
1	46	45	28
2	55	42	36
3	44	33	39
4	49	45	50
5	65	42	29
6	37	59	36
7	70	58	35
8	66	46	24
9	46	42	23
10	65	30	35

(b) Intrinsic values

i \ t	t_1	t_2	t_3
1	0	5	0
2	5	2	6
3	0	0	9
4	0	5	20
5	15	2	0
6	0	19	6
7	20	18	5
8	16	6	0
9	0	2	0
10	15	0	5

At the last exercise date, t_3, the optionholder chooses to exercise if the option is in the money, and not exercise if it is out of the money. We start the algorithm at time t_3, and create a payoff matrix with the intrinsic option values as shown in table 5.4.

Table 5.4: Payoff matrix

i \ t	t_1	t_2	t_3
1	$\cdots$	$\cdots$	0
2	$\cdots$	$\cdots$	6
3	$\cdots$	$\cdots$	9
4	$\cdots$	$\cdots$	20
5	$\cdots$	$\cdots$	0
6	$\cdots$	$\cdots$	6
7	$\cdots$	$\cdots$	5
8	$\cdots$	$\cdots$	0
9	$\cdots$	$\cdots$	0
10	$\cdots$	$\cdots$	5

We now move one exercise date back in time, to t_2. At t_2, the optionholder has to decide whether the option should be exercised now or not. If exercised, the optionholder receives the intrinsic value, $V_{t_2} - K_{t_2}$, if not, he receives nothing, but is eligible to receive a cash flow in the future. The optionholder chooses Using LSM to estimate the continuation value, *using only in the money paths at* t_2, we regress the intrinsic value at t_2 on the discounted intrinsic value at t_3 for the corresponding simulation path. We specify a linear regression equation of the form

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 \tag{5.10}$$

where X is the t_2 intrinsic value, and Y is the discounted t_3 intrinsic value.

$$Y = (V_{t_3} - K_{t_3})e^{-r(t_3-t_2)} \tag{5.11}$$

$$X = (V_{t_2} - K_{t_2}) \tag{5.12}$$

Table 5.5 shows the variables used in the regression. For path 1, $X = 5$ is found in table 3(b) under t_2. $Y = 0 \times 0.78$ is the intrinsic value at t_3, 0, from the initial payoff matrix in table 5.4, multiplied with a discount rate $0.78 = e^{-0.05(15-10)}$.

Table 5.5: LSM regression variables, t_2

i	Y	X
1	0×0.78	5
2	6×0.78	2
3	...	...
4	20×0.78	5
5	0×0.78	2
6	6×0.78	19
7	5×0.78	18
8	0×0.78	6
9	0×0.78	2
10	...	...

We estimate equation [5.10] by OLS, and get

$$\hat{Y} = 4.9607 - 0.0044X + 0.0001X^2. \tag{5.13}$$

$\hat{Y}$ now serve as an estimate for the continuation value. To decide whether the option will be exercised or not we estimate the continuation value for all simulation paths and compare it to the intrinsic value. The estimated continuation values are shown in table 6(a). We can now update the cash flow payoff matrix. Since the option can only be exercised once, if the option is exercised at t_2, we must set the cash flow at t_3 to 0.

Table 5.6: Payoff at t_2

(a) Continuation choice att_2

i	$V_{t_2} - K_{t_2}$	$\hat{Y}$	Decision
1	5	5	Exercise
2	2	5	Continue
3	0	5	Continue
4	5	5	Exercise
5	2	5	Continue
6	19	5	Exercise
7	18	5	Exercise
8	6	5	Exercise
9	2	5	Continue
10	0	5	Continue

(b) Updated payoff matrix

i \ **t**	t_1	t_2	t_3
1	⋯	5	0
2	⋯	0	6
3	⋯	0	9
4	⋯	5	0
5	⋯	0	5
6	⋯	19	0
7	⋯	18	0
8	⋯	6	0
9	⋯	0	0
10	⋯	0	5

We now move a step further back in time, to t_1, and repeat the procedure. We estimate an equation identical to the one in equation [5.10], but this time, X is the intrinsic value at t_1, *only for paths that are in the money*, and Y is the discounted value from the cash flow matrix.

$$Y = Max\left[(V_{t_3} - K_{t_3})e^{-r(t_3-t_1)}, (V_{t_2} - K_{t_2})e^{-r(t_2-t_1)}\right] \quad [5.14]$$

$$X = (V_{t_1} - K_{t_1}) \quad [5.15]$$

For path 2, $Y = 6 \times 0.61$. We find 6 from the updated payoff matrix in table 6(b), under t_3. 0.61 is the discount factor $e^{-0,05(15-5)}$. $X = 5$ is the intrinsic value at t_1, found in table 3(b).

Table 5.7: LSM regression variables, t_1

i	**Y**	**X**
1	⋯	⋯
2	6×0.61	5
3	⋯	⋯
4	⋯	⋯
5	5×0.61	15
6	⋯	⋯
7	18×0.78	20
8	6×0.78	16
9	⋯	⋯
10	5×0.61	15

Estimating the regression equation gives us[27]

$$\hat{Y} = 4.94 - 0.0035X + 0.0001X^2. \qquad [5.16]$$

Table 5.8: Payoff at t_1

(a) Continuation choice at t_1

i	$V_{t_2} - K_{t_2}$	$\hat{Y}$	Decision
1	6	5	Exercise
2	15	5	Exercise
3	4	5	Continue
4	9	5	Exercise
5	25	5	Exercise
6	0	5	Continue
7	30	5	Exercise
8	26	5	Exercise
9	6	5	Exercise
10	25	5	Exercise

(b) Updated payoff matrix

i \ t	t_1	t_2	t_3
1	6	0	0
2	15	0	0
3	0	0	9
4	9	0	0
5	25	0	0
6	0	19	0
7	30	0	0
8	26	0	0
9	6	0	0
10	25	0	0

We now have a complete payoff matrix for all exercise dates. We can approximate the option value, by discounting all cash flows back to t_0 and take the average. Discounting the cash flows from the payoff matrix would give present values shown in table 5.9. Simulation path 3 has a t_0 value equal to 4. This is calculated by finding the cash flow in the updated payoff matrix in table 8(b), and multiply with the discount rate, $9e^{-0.05(15-0)} = 4$

i	**PV**
1	5
2	12
3	4
4	7
5	20
6	12
7	23
8	20
9	5
10	20

Table 5.9: Discounted cash flows

[27] Based on 1000 simulations

The approximated option value will be

$$\hat{b} = \frac{5+12+4+7+20+12+23+20+5+20}{10} = 12.8 \qquad [5.17]$$

5.3.3 Analysis

Sticking with the option contract in the numerical example, we can do a more thorough analysis. I simulate 50.000 vessel values to estimate the option price. Figure 4.7 shows the distribution of the simulated vessel values at the different exercise dates. Figure 5.4 shows the estimated continuation value compared to the intrinsic value at each possible exercise date. The first graph, for time t_3, shows only the intrinsic value of the options, because the continuation value, as discussed earlier, is 0. The other graphs show the intrinsic value, as well as the estimated continuation value, using both OU and GMR, at time t_2 and t_1. We see that the GMR continuation value is higher than the OU process for all relevant vessel values, except for very high values at t_2.

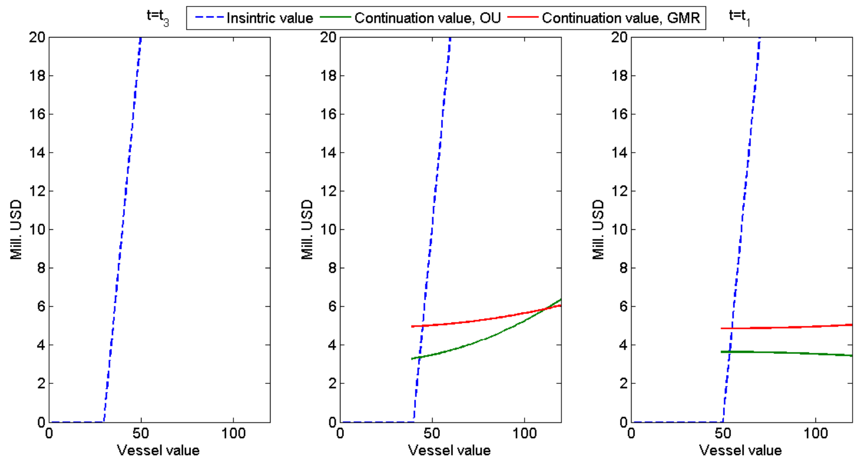


Figure 5.4: Continuation value illustrated

The option price and the corresponding standard deviation[28] for OU and GMR is given in table 5.10. We see that the GMR predicts a significantly

[28] $\frac{\sigma_{sample}}{\sqrt{n}}$

higher option price than OU.

Table 5.10: Bermudan purchase option value (Mill. USD)

	OU	GMR
Estimate	6.81	8.91
Std.dev	0.0261	0.037

5.4 TCPOP - TC with Bermudan purchase option

Finally, we've reached the Bermudan TCPOP. The Bermudan TCPOP can be seen as a combination of all the previous priced options. To find the value at the last period we use the methodology for option 1, the European purchase option. To find the value of the TC contract we can[29] use the results from option 2, the European TCPOP, and to account for the Bermudan component we apply LSM, as for option 3.

There are two things that makes this option more complex than the standard Bermudan purchase option (as considered in the previous section). First, at any possible exercise date, except for the last, the option may be out of the money, but still eligible for exercise. If the contract has a high enough TC rate, the charterer may be better of by buying the vessel with a loss, than by sticking to a bad TC rate where one expects an even greater loss. This means that there is a potential downside to the agreement, unlike a pure option, and the charterer may be forced into taking on a loss. Second, we have to consider two factors for the continuation, the expected option value at next exercise date, and the expected value of the TC agreement through the next period.

Taking the TC into account, the optimal strategy at each point is still to choose the alternative which yields highest expected profit.

$$Max(V_t - K_t, E_t^Q[V^{Continuation}]), \qquad [5.18]$$

[29] My approach is based on simulating this value.

but the expected continuation value can now be split into two components: first, the expected value from keep paying the TC and receiving the spot rate, and second, the expected option value at the next exercise date. The challenge lies in estimating the combined continuation value. To do this, the LSM methodology is applied.

5.4.1 Implementation of LSM

The following example will serve as an explanation of the implementation of LSM:
The contract we consider is similar to the one from (Jørgensen & De Giovanni, 2009, Section 5.2), a TCPOP on a new vessel for t_3 years, split into three periods, starting at $t = t_0$ with exercise dates at $t = t_1, t_2$ and t_3. At each exercise date the strike price is K_t. Between all the exercise dates the charterer pays a TC rate TC_p, where $p = 1, 2, 3$ is the period under consideration (e.g $p = 1$ is the first period between t_0 and t_1). Let L_p be the accumulated profit/loss from receiving the spot rate S_t, simulated under the Q measure, and paying TC_p, assuming daily settlement. Also assume that any daily profit/loss will be invested/borrowed at the risk free rate. L_p is the accumulated profit/loss evaluated at the end of the period (e.g L_1 is the value evaluated at time t_1, and L_2 is the value evaluated at time t_2).

1. Simulate i price paths.
2. At each exercise date, for each simulated path, store the spot rate, the vessel value, and the present value of the accumulated cash flow from the TC agreement in the previous period.
3. Starting in the last period, create a matrix (($i \times$ number of exercise dates)) where for each simulation path we store the intrinsic value of the option at the last column, representing time t_3.
4. Go back to the previous exercise date, t_2. Find the continuation value. Exercise if the intrinsic value is higher than the continuation value. If

the option is exercised store the intrinsic value in the coloumn for t_2 and update column t_3.

5. Go back to the previous exercise date, t_1 and repeat the procedure.

6. Generate a cash flow matrix (still ($i \times$ number of exercise dates)) and sum the cash flow from exercising, and the cash flows from the TC component, L_p. That is, if the option is exercised at t_3, include L_1,L_2 and L_3. If exercised at t_2, only include L_1 and L_2. If exercised at t_1, include only L_1. If not exercised, same as for exercise at t_3.

7. Discount all cash flows back to t_0 and use the average as an approximation of the TCPOP-value.

5.4.2 Numerical example

Table 11(a) shows outcomes for vessel values, while table 11(b) shows the net value of the TC payments, L_p, during period p. Table 11(c) shows the corresponding spot freight rate, S_t. Suppose that the strike prices are $K_{t_1} = 45$, $K_{t_2} = 35$, $K_{t_3} = 25$, the exercise dates are $t_1 = 5$, $t_2 = 10$, $t_3 = 15$, and that the risk free rate is $r = 0.05$.

Table 5.11: Simulation example (Mill. USD)

(a) Vessel value, V_t

i \ t	t_1	t_2	t_3
1	41	63	21
2	44	48	15
3	37	76	22
4	60	32	42
5	40	34	21
6	54	43	15
7	59	29	36
8	30	36	34
9	55	23	30
10	20	31	35

(b) TC values, L_p

i \ p	1	2	3
1	-11	-19	23
2	22	-17	-25
3	-8	-2	-3
4	14	-2	20
5	14	1	-21
6	-0	30	-12
7	7	-4	-5
8	32	-28	2
9	-10	-0	-51
10	-10	-25	31

(c) Spot freight rate, S_t (1000 USD)

i \ t	t_1	t_2	t_3
1	24	72	14
2	29	46	3
3	18	92	15
4	51	20	56
5	23	23	15
6	43	39	2
7	50	15	45
8	8	27	40
9	44	6	32
10	-6	18	42

We start by creating an option cash flow matrix and initiate the calculations at the last period, t_3. At t_3, we know that the value of the option is $Max(V_{t_3} - K_{t_3}, 0)$. We use this to create an initial payoff matrix, represented in table 5.12.

i \ t	t_1	t_2	t_3
1	$\cdots$	$\cdots$	0
2	$\cdots$	$\cdots$	0
3	$\cdots$	$\cdots$	0
4	$\cdots$	$\cdots$	17
5	$\cdots$	$\cdots$	0
6	$\cdots$	$\cdots$	0
7	$\cdots$	$\cdots$	11
8	$\cdots$	$\cdots$	9
9	$\cdots$	$\cdots$	5
10	$\cdots$	$\cdots$	10

Table 5.12: Payoff matrix

Moving a step backwards in time, to t_2, the expected value of continuing will involve not only the value of exercising the option in the next period, but also the value of the TC component, L_3. The regression must therefore include both. To find the continuation value I propose using the spot rate as an explanatory variable. This is reasonable since both the expected vessel value and the expected TC contract value depends on the current spot freight rate.

Our regression specification will be of the form

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2, \tag{5.19}$$

where Y will be the discounted simulated value of continuing, and X, the spot freight rate.

$$Y = (Max[V_{t_3} - K_{t_3}, 0] + L_3)e^{-r(t_3 - t_2)} \tag{5.20}$$

$$X = S_{t_2} \tag{5.21}$$

Table 5.13 shows the variables used in the regression. For path 1, $X = 72$ is the spotrate at t_2, found in table 11(c). For Y, $(0 + 23)$ is the insintric value at t_3 in table 5.12, plus the value of the TC at time t_3, found in table 11(b), while $0.78 = e^{-0.05(15-10)}$ is the discount rate.

i \ Var	Y	X
1	(0+23)×0.78	72
2	(0-25)×0.78	46
3	(0-3)×0.78	92
4	(17+20)×0.78	20
5	(0-21)×0.78	23
6	(0-12)×0.78	39
7	(11-5)×0.78	15
8	(9+2)×0.78	27
9	(5-51)×0.78	6
10	(10+31)×0.78	18

Table 5.13: Regression variables

Applying OLS, our estimated equation will be approximately[30]:

$$\hat{Y} = -6,67 + 0,00012X + 6,02 * 10^{-10}X^2. \qquad [5.22]$$

We can now estimate the continuation value, $\hat{Y}$, at time t_2. In table 14(a) we compare the estimated continuation value against the intrinsic value, $V_{t_2} - K_{t_2}$, of the option. If $(V_{t_2} - K_{t_2}) > \hat{Y}$, the option is exercised, if not, the TC is continued. Based on this decision algorithm, we can update the payoff matrix from 5.12. Since the option can only be exercised once, for any path where the option is exercised at t_2, we must set the payoff at t_3 to 0. Table 14(b) shows us the updated payoff matrix. Notice that for path 4, 8 and 10, the exercise decision has changed compared to the initial decision. Notice also that for path, 4, 5 and 10, the option has been exercised even though the payment is higher than the ships value. This is a consequence of, as discussed earlier, the possibility to get out from a bad TC contract.

[30] Based on a simulation of 1000 paths

Table 5.14: Payoff at t_2

(a) Continuation choice at t_2

i	$V_{t_2} - K_{t_2}$	$\hat{Y}$	Decision
1	28	5	Exercise
2	13	0	Exercise
3	41	10	Exercise
4	-3	-4	Exercise
5	-1	-3	Exercise
6	8	-1	Exercise
7	-6	-5	Continue
8	1	-3	Exercise
9	-12	-6	Continue
10	-4	-4	Exercise

(b) Updated payoff matrix

i \ **t**	t_1	t_2	t_3
1	$\cdots$	28	0
2	$\cdots$	13	0
3	$\cdots$	41	0
4	$\cdots$	-3	0
5	$\cdots$	-1	0
6	$\cdots$	8	0
7	$\cdots$	0	11
8	$\cdots$	1	0
9	$\cdots$	0	5
10	$\cdots$	-4	0

We now move further one step back in time, to t_1, and repeat our procedure to estimate the continuation value. In our regression equation, the explanatory variable, X, will still be the spot freight rate, but the dependent variable Y will change slightly. Y will now be the discounted values from the updated payoff matrix added to the discounted value of the TC agreement, L_2.

$$Y = Max[\text{Payoff}_{t_3} e^{-r(t_3-t_1)}, \text{Payoff}_{t_2} e^{-r(t_2-t_1)}] + L_2 e^{-r(t_2-t_1)} \quad [5.23]$$

$$X = S_{t_1} \quad [5.24]$$

Using path 7 in table 5.15 as an example, we see that $X = 24$ can be found in table 11(c). For Y, 11×0.61 is the value from the updated payoff matrix in table 14(b), times the discount factor $0.61 = e^{-0.05(15-5)}$. -4×0.78 is the value from the TC value in table 11(b), times the discount factor $0.78 = e^{-0.05(10-5)}$.

i \ Var	Y	X
1	(28-19)×0.78	24
2	(13-17)×0.78	29
3	(41-2)×0.78	18
4	(-3-2)×0.78	51
5	(-1+1)×0.78	23
6	(8+30)×0.78	43
7	11×0.61 -4×0.78	50
8	(1-28)×0.78	8
9	5×0.61 -0×0.78	44
10	(-4-25)×0.78	-6

Table 5.15: Regression variables

The estimated continuation value coefficients at t_1 is approximately[31]:

$$\hat{Y} = -5,29 + 0,00019X - 1,19 * 10^{-10}X^2. \quad [5.25]$$

In table 16(a) we have estimated the continuation value for all simulation paths, and to determine whether to exercise or not, the continuation value is compared to the intrinsic value. Based on the comparison, we decide whether to exercise or not. As before, the option can only be exercised once, so we exercise at time t_1, the values in the following periods must be set to zero in the updated payoff matrix, shown in table 16(b) .

Table 5.16: Payoff at t_1

(a) Continuation decision at t_1

i	$V_{t_1} - K_{t_1}$	$\hat{Y}$	Decision
1	-4	-1	Continue
2	-1	0	Continue
3	-8	-2	Continue
4	15	4	Exercise
5	-5	-1	Continue
6	9	3	Exercise
7	14	4	Exercise
8	-15	-4	Continue
9	10	3	Exercise
10	-25	-6	Continue

(b) Updated payoff matrix

i \ t	t_1	t_2	t_3
1	0	28	0
2	0	13	0
3	0	41	0
4	15	0	0
5	0	-1	0
6	9	0	0
7	14	0	0
8	0	1	0
9	10	0	0
10	0	-4	0

[31] Still based on only 1000 simulations

We have now finished determining when the option will be exercised. However, to obtain an approximation for the value of the contract, we also have to consider the cashflows from the TC until the option is exercised. Table 5.17 shows the accrued cash flows from the TC until the option is exercised or expires. For path 1, we see from the payoff matrix in table 16(b) that it is exercised at time t_2. The accrued cash flows from the TC in period 1 and 2, can be found in table 11(b).

i \ t	t_1	t_2	t_3
1	-11	-19	0
2	22	-17	0
3	-8	-2	0
4	14	0	0
5	14	1	0
6	-0	0	0
7	7	0	0
8	32	-28	0
9	-10	0	0
10	-10	-25	0

Table 5.17: Accrued cash flows from the TC components

To approximate the TCPOP value we discount the cash flows from the TC (table 5.17) and the cash flows from the updated payout matrix (table 16(b)) back to t_0, and sum for each path. For path 1 this we see that this is -3. The discounted cash flows from the TC is $-11e^{-0.05(5-0)} - 19e^{-0.05(10-0)} \approx -20$, where -11 and -19 are taken from table 5.17. The discounted cash flow from exercising the option is $28e^{-0.05(10-0)} \approx 17$, where 28 is taken from table 16(b). Summing, we see that $-20 + 17 = 3$.

i	**NPV**
1	-3
2	15
3	18
4	22
5	11
6	7
7	16
8	9
9	-0
10	-25

Table 5.18: t_0 value of total cash flow

An approximate value for the TCPOP, $\hat{c_{t_0}}$, will now be the average over all paths. In our case, the t_0 value for each path is shown in table 5.18. The average, and our approximate option value, is

$$\hat{c_{t_0}} = \frac{-3+15+18+22+11+7+16+9+0-25}{10} = 7 \qquad [5.26]$$

5.4.3 Analysis

To validate the method, I compare my results with the results found in Jørgensen & De Giovanni (2009, table 7). I find that my LSM method, given the same inputs, provides identical results to the finite difference approach.

For an application, i consider the same contract specified in the numerical example, and further specify the predetermined TC rate, $TC_1 = TC_2 = TC_3 = 22500$. I use 50000 simulations. The price of the contract, and the standard deviation[32] of the estimate is given in table 5.19. We see that, as expected, the GMR contract price is significantly higher than the one for OU.

Table 5.19: TCPOP contract value (Mill. USD)

	OU	GMR
Estimate	23.37	27.03
Std.dev	0.1046	0.1209

32 $\frac{\sigma_{sample}}{\sqrt{n}}$

A visual presentation of the simulated contract values is shown in figure 5.5. We see that the GMR predicts a skewed distribution for the TCPOP value.

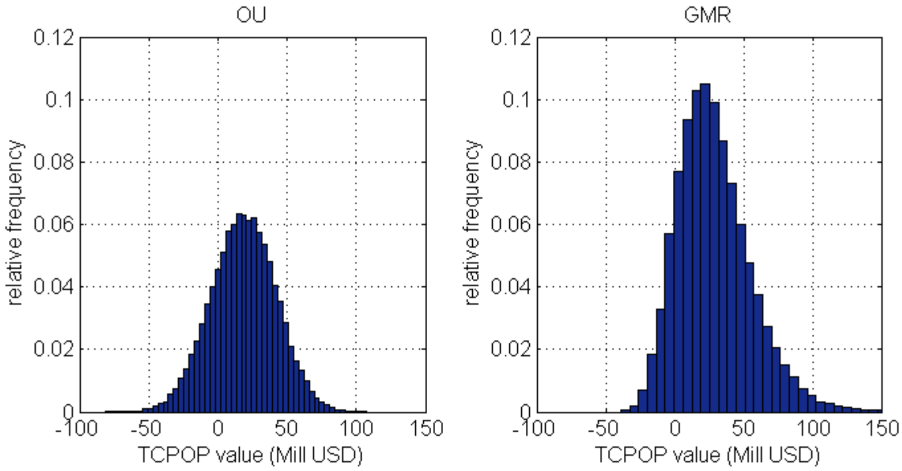


Figure 5.5: TCPOP value, relative frequency histogram

6 Concluding remarks

This thesis presents a simple, and easy to implement, method for the determination of price, and optimal continuation strategy for a set of option contracts in the shipping industry. I use Ornstein Uhlenbeck and Geometric Mean Reversion to model the freight rate. The freight rate processes are again used to create models for a vessels value through its lifetime. The vessel value models are calibrated under the assumption that there exists a variable risk premium. Based on the vessel value models, I use Monte Carlo methods to price the option contracts.

When modeling the freight rate process there are certain characteristics a model should control for, whereas mean reversion is probably the most important one. The OU process offers analytical, easy-to-work-with solutions, but has some undesirable characteristics, where the most prominent one is the possibility of negative predicted freight rates. While the GMR process has some desirable properties that corrects for what OU lacks, it can be tedious to work with as analytical solutions are not available. Model selection is therefore a tradeoff between easy computations and theoretical correctness. If results from the two processes are similar, the choice of model should be based on a parsimonious criteria, and OU should be chosen. Otherwise, the more theoretical correct GMR should be chosen. In this thesis I have found that the difference in the results from the two models can be of economic significance. GMR consistently estimate vessel values, and prices of options on vessels, to be higher than OU. On the basis of this, GMR should be the preferred choice of model. However, the tradeoff between tractability and theoretical correctness still applies, and for problems that are more complex than those considered here, one should of course reconsider.

An obvious weakness in my analysis is the determination of the market price of risk. First, the estimation is done under assumptions about the costs that are not very robust. Second, the proposed linear model shows clear signs of being underspecified. Third, it is not very realistic to assume that the risk premium is the same for different aged vessels. However, the model seems to capture some of the variation, and can probably be a better approximation than assuming a zero market price of risk. The identification and determination of a risk premium would be an interesting topic for future research, as it is crucial factor in the valuation of contingent claims.

Appendices

A Itos lemma

If x follows the process

$$dS = a(S,t)dt + b(S,t)dz \qquad [A.1]$$

Where dz is the wiener process. Itos lemma show that a function G of x and t follows the process

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}bdz \qquad [A.2]$$

If we define $G = lnS$

$$\begin{aligned} \frac{\partial G}{\partial x} &= \frac{1}{S} \\ \frac{\partial G}{\partial t} &= = 0 && [A.3] \\ \frac{\partial^2 G}{\partial x^2} &= -\frac{1}{S^2} && [A.4] \end{aligned}$$

From equation [2.7] we have that

$$\begin{aligned} a &= \kappa(\omega - lnS)S \\ b &= \sigma S \end{aligned}$$

Substituting into equation [A.2] we get

$$dG = (\frac{1}{S}\kappa(\phi - lnS)S + \frac{1}{2}(-\frac{1}{S^2})\sigma^2 S^2)dt + \sigma\frac{1}{S}Sdz$$

Simplifying gives us

$$dG = \kappa(\phi - lnS - \frac{\sigma^2}{2\kappa})dt + \sigma dz$$

Setting $\alpha = \phi - \frac{\sigma^2}{2\kappa}$ we get the following

$$dG = \kappa(\alpha - G)dt + \sigma dz$$

which is equivalent to equation [2.8]

B Estimation diagnostics

B.1 Price processes

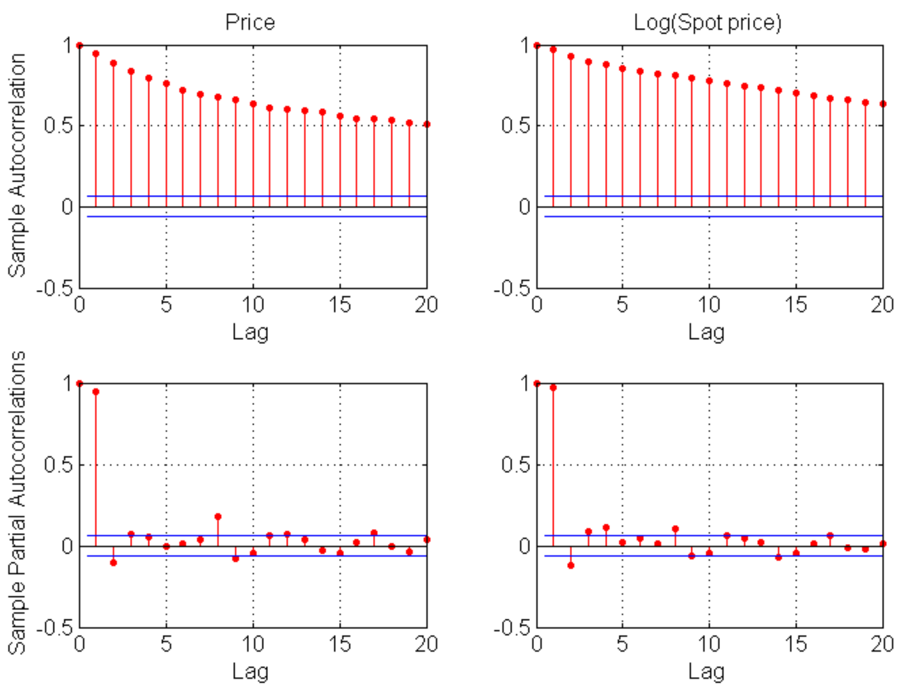


Figure B.1: ACF and PACF for the price and the log of the price

C Structural parameters

C.1 Ornstein Uhlenbeck

We have that:

$$\alpha = \phi(1 - e^{-\kappa\delta}) \quad [C.1]$$

$$\beta = e^{-\kappa\delta} \quad [C.2]$$

$$\sigma_\epsilon = \sigma\sqrt{\frac{1 - e^{-2k\delta}}{2k}} \quad [C.3]$$

Derivation of κ:

$$\beta = e^{-\kappa\delta} \quad [C.4]$$

Taking the logarithm on both sides gives

$$ln\beta = lne^{-\kappa\delta} = -\kappa\delta \quad [C.5]$$

$$\Rightarrow$$

$$\kappa = -\frac{ln\beta}{\delta} \quad [C.6]$$

Derivation of ϕ:

$$\alpha = \phi(1 - e^{-\kappa\delta}) \quad [C.7]$$

$$\phi = \frac{\alpha}{1 - e^{-\kappa\delta}} \quad [C.8]$$

where we can substitute from equation [C.2]

$$\phi = \frac{\alpha}{1 - \beta} \quad [C.9]$$

Derivation of σ

$$\sigma_\epsilon = \sigma\sqrt{\frac{1-e^{-2\kappa\delta}}{2k}} \quad [C.10]$$
$$\Rightarrow$$
$$\sigma = \sigma_\epsilon\sqrt{\frac{2\kappa}{1-e^{-2\delta}}} \quad [C.11]$$

Substituting from equation [C.2] and [C.6] gives

$$\sigma = \sigma_\epsilon\sqrt{\frac{2\frac{-ln\beta}{\delta}}{1-\beta^2}} \quad [C.12]$$

Multiplying with δ gives

$$\sigma = \sigma_\epsilon\sqrt{\frac{2\delta\frac{-ln\beta}{\delta}}{\delta(1-\beta^2)}} = \sigma_\epsilon\sqrt{\frac{-2ln\beta}{\delta(1-\beta^2)}} \quad [C.13]$$

C.2 Geometric mean reversion

We have that:

$$\alpha = (\phi - \frac{\sigma^2}{2\kappa})(1-e^{\kappa\delta}) \quad [C.14]$$
$$\beta = e^{-\kappa\delta} \quad [C.15]$$
$$\sigma_\epsilon = \sigma\sqrt{\frac{1-e^{-2\kappa\delta}}{2\kappa}} \quad [C.16]$$

Derivation of ϕ:

$$\alpha = (\phi - \frac{\sigma^2}{2\kappa})(1-e^{\kappa\delta}) \quad [C.17]$$
$$= \phi(1-e^{\kappa\delta}) - \frac{\sigma^2}{2\kappa}(1-e^{\kappa\delta}) \quad [C.18]$$
$$\Rightarrow$$
$$\phi = \frac{\alpha}{(1-e^{\kappa\delta})} + \frac{\sigma^2}{2\kappa} \quad [C.19]$$

Substituting from [C.15]

$$\phi = \frac{\alpha}{(1-\beta)} + \frac{\sigma^2}{2\kappa} \tag{C.20}$$

κ and σ: Same as for OU.

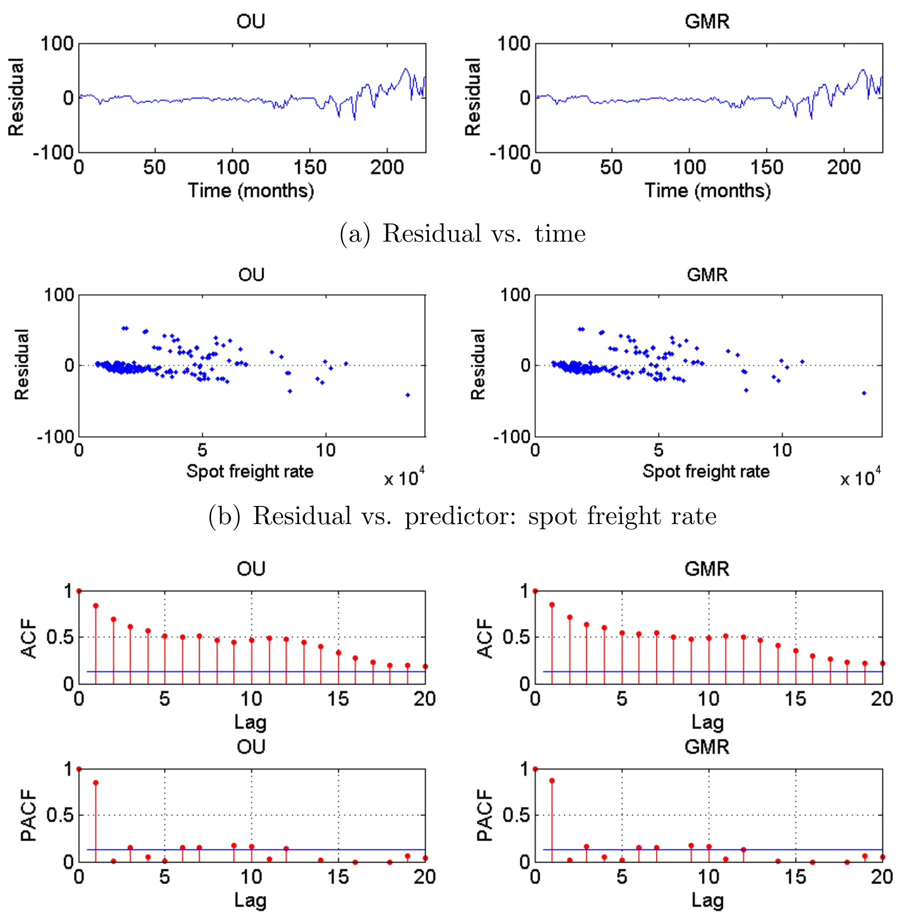
D Risk premium estimation

D.1 MATLAB code

```
%aGMR=beta_0
%bGMR=beta_1
%rate=historical freight rate
%omegaGMR=phi
%kGMR=kappa
%sigGMR=sigma

t=[0:50:That-T5];
aGMR=[0.32:0.005:0.36];
bGMR=[-0.004:0.0001:-0.001];

for n=1:length(aGMR)
for m=1:length(bGMR)

for j=1:length(rate)
v5GMR(j,1)=trapz(t,exp(t.*-r*dt).*(exp(log(rate(j,1))*exp(-kGMR*
    t.*dt)+(omegaGMR-(aGMR(n)+bGMR(m)*(rate(j,1)/1000))-((sigGMR
    .^2)/(2*kGMR)))*(1-exp(-kGMR*t.*dt))+(((sigGMR.^2)/(4*kGMR))
    *(1-exp(-2*kGMR*t.*dt))))-opex))+(Vhat*exp(-r*(That-T5)*dt));
end

resGMR(n,m)=sum((v5GMR/1000000-rv5).^2);
end
end


[j,i]=find(resGMR==(min(min(resGMR))))
aGMR=aGMR(j)
bGMR=bGMR(i)
```

D.2 Residuals

Figure D.1: Residual plots



(a) Residual vs. time

(b) Residual vs. predictor: spot freight rate

(c) ACF and PACF for residuals

E Vessel values illustrated

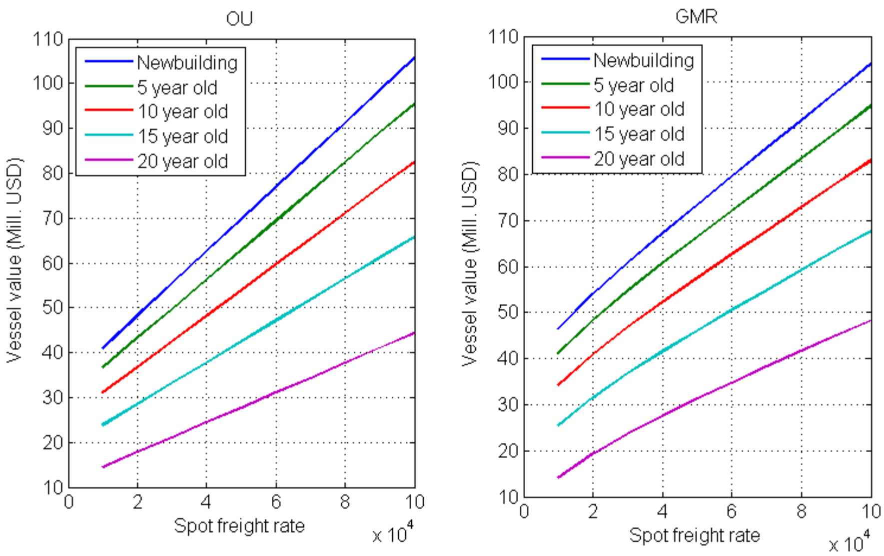


Figure E.1: Expected vessel values as in relation to spot freight rate

F TC values

Table F.1: Fair TC rates, in 1000 USD

(a) Fair TC, OU

S(t) \ T-t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10000	10	16	18	19	20	20	21	21	21	21	21	22	22	22	22
20000	16	20	21	22	22	23	23	23	23	23	23	23	23	23	23
30000	22	24	24	24	25	25	25	25	25	25	25	25	25	25	25
40000	28	27	27	27	27	27	27	27	27	27	27	27	27	27	27
50000	34	31	30	30	29	29	29	29	29	28	28	28	28	28	28
60000	40	35	33	32	32	31	31	31	30	30	30	30	30	30	30
70000	46	39	36	35	34	33	33	32	32	32	32	32	32	32	31
80000	52	43	39	37	36	35	35	34	34	34	34	33	33	33	33
90000	58	47	42	40	38	38	37	36	36	36	35	35	35	35	35
100000	64	51	45	43	41	40	39	38	38	37	37	37	37	36	36

(b) Fair TC, GMR

S(t) \ T-t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10000	15	18	20	21	21	22	22	22	23	23	23	23	23	23	23
20000	23	24	24	25	25	25	25	25	25	25	25	25	25	25	25
30000	29	29	28	28	27	27	27	27	27	27	27	27	27	27	27
40000	36	33	32	31	30	29	29	29	29	29	28	28	28	28	28
50000	42	37	35	33	32	32	31	31	30	30	30	30	30	30	30
60000	47	41	38	36	34	34	33	32	32	32	32	31	31	31	31
70000	53	45	41	38	37	36	35	34	34	33	33	33	33	32	32
80000	58	49	44	41	39	37	37	36	35	35	34	34	34	34	34
90000	63	52	46	43	41	39	38	37	37	36	36	36	35	35	35
100000	69	56	49	45	43	41	40	39	38	38	37	37	37	36	36

References

Aadland, Roar. 2000. *A non-parametric model of the timecharter-equivalent spot freight rate in the very large crude carrier market,*. Working paper.

Aadland, Roar, & Strandenes, Siri P. 2004. A discrete-time stochastic partial equilibrium model of the spot freight market. *Snf report*, **04**(April).

Aadland, Roar Os. 2003. *The stochastic behavior of spot freight rates and the risk premium in bulk shipping.* Ph.D. thesis, Massachusetts Institute of Technology. Dept. of Ocean Engineering.

Adland, Roar, & Strandenes, Siri. 2006. Market efficiency in the bulk freight market revisited. *Maritime policy and management*, **33**(2), 107–117.

Bjerksund, Petter, & Ekern, Steinar. 1992. Prinsipper for opsjonsbaserte analyser innen shipping. *Snf-rapport 13/92.*

Bjerksund, Petter, & Ekern, Steinar. 1995. Contingent claims evaluation for mean-reverting cash flows in shipping. *Pages 207–219 of:* Trigeorgis, Lenos (ed), *Real options in capital investment: Models, strategies, and applications.* Praeger Publishers.

Enders, Walter. 2003. *Applied econometric time series, 2nd edition.* Wiley.

Hull, John C. 2006. *Options, futures and other derivatives (6th edition).* Prentice Hall.

Jørgensen, Peter L., & De Giovanni, Domenico. 2009. Time charters with purchase options in shipping: Valuation and risk management. *Ssrn elibrary.*

Kavussanos, Manolis G. 1996. Comparisons of volatility in the dry cargo freight market ship sector. *Journal of transport economics and policy*, **30**(1), 67.

Kavussanos, Manolis G., & Alizadeh-M, Amir H. 2002. The expectations hypothesis of the term structure and risk premiums in dry bulk shipping freight markets. *Journal of transport economics and policy*, **36**(2), 267–304.

KAVUSSANOS, MANOLIS G., & VISVIKIS, ILIAS D. 2006. *Derivatives and risk management in shipping.* 1st edn. Witherbys publishing.

LONGSTAFF, FRANCIS A, & SCHWARTZ, EDUARDO S. 2001. Valuing american options by simulation: A simple least-squares approach. *Review of financial studies*, **14**(1), 113–47.

MCCONNELL, J. J., & SCHALLHEIM, J. S. 1983. Valuation of asset leasing contracts. *Journal of financial economics*, **12**(2), 237–261.

SCHWARTZ, EDUARDO S. 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of finance*, **52**(3), 923–73.

STOPFORD, MARTIN. 1997. *Maritime economics (2nd edition)*. Routledge.

TRIGEORGIS, LENOS. 1996. Evaluating leases with complex operating options. *European journal of operational research*, **91**(2), 315–329.

TVEDT, JOSTEIN. 1997. Valuation of vlccs under income uncertainty. *Maritime policy and management*, **24**(2), 159–174.

TVEDT, JOSTEIN. 2003. Shipping market models and the specification of freight rate processes. *Maritime economics and logistics*, **5**(4), 327–346.

WOOLDRIDGE, J.M. 2006. *Introductory econometrics: A modern approach.* 3nd edn. South-Western, Thomson Learning.