## NHH

## Two-Sided Social Networks

The Impact of Network Effects on Strategic Differentiation

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Master thesis in economic analysis

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#### Abstract

In this thesis, we examine the presence of direct network effects in two-sided markets. Online social networks like Facebook are examples of firms which exhibit both direct and indirect network effects. These effects have important implications for firms' incentives to strategically differentiate. While the literature on each type of network effect is extensive, studies of firms who exhibit both these characteristics are few. We survey the literature on direct network effects and two-sided markets separately. We then add to the literature by presenting a Hotelling model with endogenous location where both types of network effects are present. We present two versions of the model, a one-sided duopoly model with direct network effects, and a two-sided model with direct network effects and duopoly competition on one side. We find that both direct and indirect network effects incentivize the firms to differentiate less, implying fiercer competition.


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## 1 Introduction

Few inventions have transformed society as profoundly as the introduction of the Internet to the general public. From changing the way people navigate their daily lives, to disrupting markets for goods and services, the Internet has undoubtedly brought the world closer together. As part of this, online social networks have become a large part of people's lives, as well as serving as in increasingly important marketplace for goods and services. 3.2 billion people use social media daily, spending an average of 2 hours and 22 minutes per day on social networks and messaging (Bayindir \& Kavanagh, 2018). Consumers' attention is in scarce supply, and advertisers are more than keen to tap into this use.

The largest online social network is Facebook, with an impressive 2.3 billion monthly active users (Statista, 2019c). Despite its dominant position, the platform does not have monopoly; among its top competitors, five other social media platforms have more than a billion active users. ${ }^{1}$ One thing these platforms have in common, is that their services for the most part are free to their users. Yet they run large profits. Facebook's revenue in 2018 was 55.8 billion US dollars (Statista, 2019b). The main source of this revenue is the selling of advertisement space on the platform. Facebook's value to advertisers is directly connected to its vast user base; advertisers pay Facebook for the chance to reach these 2.3 billion potential consumers. Facebook profit precisely from connecting advertisers to users. A burgeoning literature on two-sided markets examines the economics of firms that makes their profits this way. In this literature, Facebook is dubbed a platform, serving two distinct market sides: the user market and the advertiser market. The fact that advertisers care about the number of users Facebook has, is called the cross-group network effect, and this effect has been shown to have great consequences for the strategic behavior of platforms who deal with them. At the same time, firms like Facebook attract users by having users; the value of the network to a single user increases with the number

[^0]of other users that are connected to it. This is referred to as the within-group network effect.

With the foothold firms like Facebook have on our time, attention and money, we believe that understanding their strategic behavior is more important than ever. Examining the two distinct types of network effects that characterize many of the Internet giants of our day, we aim to offer insight into the way such firms behave as they compete for users, advertisers, and ultimately, profits. To achieve this, we survey the literature on both within-group and cross-group network effects. While each of these properties are examined extensively by now, few scholars have studied the interaction of both effects. ${ }^{2}$ The within-group network effect is present in a range of industries. The same is true for the cross-group network effect. Platforms where both are present at the same time, seem to us to have become more important in the age of the Internet. We add to the literature by examining the combination of these effects.

Spatial differentiation models are a popular way to study strategic differentiation. We look at the linear model as it was presented by Harold Hotelling in 1929, and later modified by several scholars. We offer a discussion on its strengths and shortcomings, before we utilize the framework in our own version. We present a linear differentiation model with convex transportation costs and direct network effects. In the first iteration of the model, two firms compete for consumers in a sequential game, before reaching a symmetric equilibrium outcome. In the second iteration of the model, we include a second market side, and allow the firms to collect revenue from selling ads. With this, we try to answer the following research question:

How does the presence of within-group and cross-group network effects affect firms' incentives to strategically differentiate?

The rest of the text is structured as follows. In chapter 2, we offer an overview of the existing economic literature on product differentiation, network effects, and two-sided

[^1]markets. In chapter 3, we present a spatial differentiation model with direct network effects, that extends to a two-sided model. In chapter 4, we discuss the results we get. Chapter 5 concludes.

## 2 Literature review

In this literature review we begin with a brief exploration of oligopoly price competition, with an emphasis on horizontal differentiation as a way to escape the Bertrand paradox. In chapter 2.1, we present a simple Hotelling model for spatial differentiation.

Network effects has been a popular topic of research since Jeffrey Rohlf's pioneering work in the 1970s. With the rise of the Internet, the subject took on a new relevance, as many used network effects to explain the rapid growth of some successful Internet companies. In chapter 2.2, we concern ourselves with relevant literature on direct network effects. Since Tirole and Rochet introduced the concept of two-sided markets in 2002, there has been published a great deal of work on the subject. Chapter 2.3 reviews the relevant theory on two-sided markets.

### 2.1 Product differentiation

The theory of how industries are organized often start out with models of either price or quantum competition, commonly based on the seminal works of economists Joseph Bertrand and Antoine Augustine Cournot. Where basic microeconomic theory predominately examines cases of monopoly and perfect competition to describe economic fundamentals like price setting, most real life markets lie somewhere in between these two extremes. Real-life firms typically face some competition, yet the competition is not perfect; they are able to charge prices above marginal costs, thus maintaining a positive profit margin. Oligopoly models like the Cournot-Nash model and the Bertrand model describe how firms with some degree of market power determine the combination of price and output that maximizes its profits.

Under price competition, assuming homogeneous products, the Bertrand paradox describes a situation where prices will quickly converge to marginal costs, and no firm will make a profit. In reality, of course, many firms that compete in prices make positive
profits. In other words, they find ways to escape the Bertrand paradox. One such way is through differentiation. Chamberlin introduced the idea of product differentiation in 1933, stating that for the available products within an industry, consumers may have different preferences (Beath \& Katsoulacos, 1991, p. 2). Utilizing these preferences for heterogeneity, businesses may achieve market power, and earn profits. Generally, goods can be differentiated in two ways, vertically and horizontally. Vertical differentiation refers to a strategy where goods vary in some aspect, e.g. quality, in a way that can be objectively graded from best to worst. Horizontal differentiation implies heterogeneous preferences among buyers, hence the goods are differentiated along a dimension of more subjective grading.

### 2.1.1 Hotelling's linear city

A customary way to illustrate horizontal differentiation is by use of spatial models. Hotelling's linear model of spatial differentiation, as well as Salop's circular model, are most frequently used in the literature. In Hotelling's model, often times referred to as Hotelling's linear city, two firms (firm 1 and firm 2) compete for consumers uniformly distributed along a line $l$. This line is commonly normalized to have length $1 .{ }^{3}$

In the linear city, heterogeneity in preferences is simply modeled as the distribution of consumers on a line. Usually, the consumers are modeled to be uniformly distributed. The firms can choose their ideal level of differentiation by choosing the optimal location on the line. The further away from each other they locate, the more differentiated they are. If both firms locate in point $1 / 2$, at the very middle of the line, the Bertrand paradox is fulfilled.

In the simplest version of the model, consumer preference is a function of only prices and transportation costs, a cost dependent on the length they have to travel along the line in order to reach the firm from which they buy the good. Each consumer along

[^2]the line purchases exactly one good. Whether or not a consumer buys from firm 1 or firm 2, thus depends not only on the prices the two firms charge for their goods, but also on the firms' and the consumer's location, along with the consumer's aversion to travel. If the goods are otherwise homogeneous, the consumer behaves according to her subjective prices $s_{i}=p_{i}+t\left(d_{i}\right)$, where $p_{i}$ is the price firm $i$ charges, and the travel cost $t\left(d_{i}\right)$ is a function of the distance $d_{i}$ from the consumer to firm $i$. Assuming that firm 1 is located (weakly) to the left of firm 2, their locations, $x_{1}$ and $x_{2}$, can be expressed as their respective distance, $a$ and $b$, to point 0 and 1 on the line.

Minimizing aggregate travel cost for all consumers on the line yields the socially optimal location of firm 1 and 2 in the points $1 / 4$ and $3 / 4$, respectively. Figure 1 is a graphical depiction of the linear city with with this outcome. $D_{1}$ and $D_{2}$ denotes the demand of the two firms.


Figure 1: Hotelling's linear city

Assuming that each consumer has an intrinsic value $v$ of participating in the market, the consumer's utility function from consuming firm $i$ 's good is $u_{i}=v-s_{i}$. The demand for each good is found by identifying the consumer who is indifferent between the two firms. Mathematically, this is done by letting $u_{1}=u_{2}$, and solving for $x$. Denoting this particular $x$ value $\bar{x}$, we establish the location of the indifferent consumer. The area spanning from $\bar{x}$ to each side of the line, represents each firm's demand, which is given by $D_{i}=\frac{1}{2}-\frac{p_{i}-p_{j}}{2 t}$.

The game is solved in two stages. In the first stage, the firms decide their optimal location on the line, and in stage two, the firms compete by setting their optimal price. Figure 2 illustrates the two competing forces in this game, which will be at the center of our discussion in this text. These are the direct effect and the strategic effect. ${ }^{4}$ The direct

[^3]effect describes a firm's incentive to lower its price in order to steal consumers from its rival. In the linear city, this involves moving closer to its rival, "pulling" the firms to locate more towards the center of the line (as illustrated by the solid arrows in the figure). The opposing effect is the strategic effect, which is the incentive firms have to differentiate from its rival, in order to maintain a positive profit margin. This effect "pushes" the firms apart on the line (as illustrated by the dashed arrows in the figure). The relative strength of these effects determine the firms' equilibrium location.


Figure 2: The direct versus the strategic effect

When Harold Hotelling proposed his model in 1929, he assumed linear transportation costs, $t\left(d_{i}\right)=t(1-a-b)$. Solving his model, the equilibrium outcome states that it is rational for the firms to locate themselves at the middle of the line, suggesting minimal differentiation ( $a=b=1 / 2$ ). In this outcome, the Bertrand paradox arises: prices are equal to marginal costs, and the firms make no profit. This was pointed out by d'Aspremont, Gabszewicz and Thisse (1979), who disputed Hotelling's result, claiming it has stability issues. By applying quadratic transportation costs, $t\left(d_{i}\right)=t(1-a-b)^{2}$, they found that in equilibrium, the firms go for a maximal differentiation strategy, that is locating at the extremes of the line - even outside of the line if possible (i.e. allowing $a, b<0$ ). Using the quadratic transportation costs, it can be showed that the firms will indeed locate at $a=b=-1 / 4$. The result of d'Aspremont et al. was later confimred by Economides, who also pointed out that it holds only for specifically quadratic forms of the transportation cost term (1986). Allowing the transportation cost function to be less convex than d'Aspremont et al., but not linear (i.e. $t\left(d_{i}\right)=t(1-a-b)^{\alpha}$, with $1<\alpha<2$ ), he showed that for some range of $\alpha$, the equilibrium outcome involves interior locations on the line.
effect. We use these terms interchangeably.

Generally, the larger parameter $t$ is, the stronger preferences the consumer has, granting market power to the firms. It is this market power over the consumers that enables them to take higher prices by strategic differentiation. In the extreme case where $t=0$, any functional form of the travel cost will yield the Bertrand paradox as an outcome. Applying a convex travel cost function implies that the cost of travel increases in distance, at an increasing rate. For any location other than $a=b=1 / 2$, this puts a larger emphasis on the travel cost term than in the linear version, emphasizing the firm's incentives to strategically differentiate.

Figure 3 illustrates how convex and linear transportation costs affect the consumer's subjective prices $s_{i}=p_{i}+t(1-a-b)^{\alpha}$. For the consumer located in point $a$ (alt. $b$ ), the subjective price faced by consuming good $1(2)$ is simply $p_{1}\left(p_{2}\right)$. As the consumer is located further from the firm's location, the subjective price increases. This increase is greater when the transportation costs are convex than when they are linear, as illustrated in the figure by the two subjective price graphs $s_{i}^{c}$ and $s_{i}^{l} \cdot \bar{x}$ is the location of the consumer where $s_{1}=s_{2}$, that is the consumer who is indifferent between the two firms.


Figure 3: Quadratic versus linear transportation cost

When using space as a metaphor for preference, it makes sense to talk about $t\left(d_{i}\right)$ as a transportation cost. Of course, depending on the context, the preference could be for
actual proximity, but it is also worth noting that it may just as well represent distances in terms of taste, like the distance from the far left to the far right on a political spectrum. It could also be the preference for a professionally oriented social network like LinkedIn, over a more general social network like Facebook. The transportation cost represents the disutility the consumer experiences from deviating from her preferences. The larger parameter $t$ is, the greater the disutility is. This implies strong preferences among the consumers. Another way to put it, is that switching costs are high. It is not unreasonable to assume that $t$ should vary across types of industries.

### 2.2 The network effect

Since the 1970s, the economic literature became more focused on network effects, as new technology made it easier for people and firms to connect to each other and gain value from the other users of the product or service. A classic and often used example is the telecommunication market. Online network communities such as Facebook, LinkedIn, and MySpace are more recent phenomena where network effects are present. Network effects, also referred to as network externalities, will often have an impact on the strategic behavior of firms because of the altered dynamics on the demand side. The literature in this field mainly concerns three different research topics: 1. The technology adaption, 2. The compatibility decision and 3. Decisions among incompatible technology and products (Majumdar \& Venkataraman, 1998). This literature review will focus on the latter, which includes different factors that are related to the consumer's choice among firms that offer incompatible technologies. The organization of this chapter is as follows. In chapter 2.2.1, we provide a definition of the network effect. In chapter 2.2.2, we look at some of the literature regarding the demand side of the market and the implications for the different market equilibria. The rest of the chapters are focused on the supply side of the market in a competitive setting. Chapter 2.2 .3 sums up the implications of an incentive-based contract between leader and manager with and without network effects, and chapter 2.2.4 concerns the startup problems firms tend to have in such markets, as
well as the coexistence issue often discussed. Lastly, in chapter 2.2.5, we look at some empirical findings in industries which exhibits network effects.

### 2.2.1 Defining the network effect

Katz and Shapiro (1985) describe network effects like this: "There are many products for which the utility that a user derives from consumption of the good increases with the number of other agents consuming the good." To exemplify this concept, they use a classic network industry: the telephone market. Say that only one person in the world owns a phone, he could not benefit from the phones first intended function: calling other people. As more people enter the phone market, the more people he can call, which makes the phone more valuable to him, as well as to the other owners and future buyers. Thus, the increase in consumer mass will benefit other current owners of the product, at the same time as it increases the incentive to buy the product for consumers currently outside the market. Farrell and Klemperer (2006) denote the first effect as the total effect, while the latter they call the marginal effect of an increase in consumers of the network good.

The telephone market is an example of a market where the network effect is positive, as an additional user will increase the perceived value of the product. Conversely, a consumer leaving the market will decrease the recognized value for consumers in the network and for potential buyers. A market that exhibits a positive network effect will consequently experience a self-fulfilling upward spiral in the perceived value when new consumers enter the market. If, however, the consumers do not believe in the product, the value will quickly decrease as the consumers switch product or leave the market altogether. Other markets can exhibit negative network effects. If, for example, many people own and drive a car, the roads can be overcrowded at some point, which can negatively affect the next consumer buying a car. Rohlfs (1974) argues that the telephone market can at some point exhibit negative network effect if too many annoying sellers are in possession of a phone. We will in the rest of this literature review refer to network effects as positive since it is typically modeled this way.

Although network externalities and network effects are used interchangeably in the literature, Liebowitz and Margolis (1994) argue for a distinction between these two terms. The network effect, they suggest, is a description of the change in the net value of a product if more agents buy or uses the product, while they reserve network externalities for a specific type of network effect, where in equilibrium there are unexploited gains. Therefore, the term network externality can be used to describe network effects that cause market failure. However, since most of the literature does not care for this distinction, we use both of the terms Liebowitz and Margolis describe as the network effect.

Another, more important, distinction is between the direct and the indirect network effect (Katz \& Shapiro, 1985; Liebowitz \& Margolis, 1994). The direct network effect is the direct change in the value or quality of a product or service of an additional consumer in that network. Such as an increase in the value of social networks, like Facebook and LinkedIn, as more people join the network. A different, perhaps more descriptive, term for the direct network effect is the within-group effect (Sun \& Tse, 2007). When the change in value or quality is not directly associated with more consumers buying and using the product, but this growing user mass has an impact on the value indirectly, it is called an indirect network effect. This is seen in the market for video games, where the consumers do not directly benefit from more consumers of video games, but as the consumer group increases, game developers have incentives to make more and better games. This will ultimately be beneficial for the consumers using the video game platform. This is also an example of a two-sided market, where the indirect network effect plays a crucial role. Further definition and discussion of economic theory regarding two-sided markets is provided in chapter 2.3.

### 2.2.2 Demand-side dynamics and market equilibrium

Another term used for network effects is economics of scale on the consumer side (Shapiro \& Varian, 1999, p. 14). As mentioned in subsection 2.2.1, the demand side exhibits direct network effects when an inflow (outflow) of consumers increase (decrease) the perceived
value of the product or service for agents in the same network. The main difference in a market with network effects, compared to a non-network market, lies on the consumers' perceived value which alters the characteristics of the demand curve. This change in the demand curve will consequently have some interesting implications of the firms' behavior and market equilibrium in various market structures.

In microeconomic theory, the demand curve is usually a downward sloping function of the price. Simply put, if the price decreases, more people will have a willingness to pay (WTP) that is above the market price, and they will purchase the product. ${ }^{5}$ In a market which exhibits economics of scale on the consumer side, the demand curve can look quite different, depending on the assumptions that are made. Rohlfs (1974) created a model of uniform calling pattern where the number of subscribers affects the value of the service, but the subscribers do not care about who the other subscribers are. He argues that this might be a good assumption as the subscriber will not necessarily know all the people he will interact with through a phone in the future. From this, the number of subscribers can serve as an indication for the incremental value he or she derives from the service. In the world of uniform calling pattern he constructs a demand curve which has a somewhat peculiar shape and interpretation. Initially, the demand is upward sloping in price, until the market has reached a certain number of subscribers, where it turns into the normal declining demand curve, in an inverted U-shape. To illustrate the idea of the inverted U-shaped demand curve, Shapiro and Varian (1998) made a very simple model which is presented here to supplement and make the verbal discussion of the characteristics in the network market more precise.

Suppose there are $\phi$ people in this market. ${ }^{6}$ Let $v$ be the reservation price or the intrinsic value of the network good which is distributed: $v=[1,2 \ldots, \phi]$. For each person this means that they will buy the good if their intrinsic value is higher than the price: $\hat{v}-p \geq 0$. Here, $\hat{v}$ is the marginal individual's intrinsic value of the good. The first person has an

[^4]intrinsic value of 1 and is willing to pay a maximum of $\$ 1$ for the good, the next person's maximum price is $\$ 2$ and so on. From this structure it is implied that if the price is $\$ 100$ the first 100 people would not buy the good and we get the normal downward sloping demand curve: $n=\phi-p$ where n is the number of people purchasing the good and $\phi$ is the last person in the market with the highest WTP. However, in the market of a network good, the total value of the good also depends on the number of people purchasing the good. This suggests that the WTP for such a good is a product of both the intrinsic value and the number of people in the market.
$p=\hat{v} n$

The term above denotes the indifferent person where the price he must pay for the good is equal to the total value of the good. This means that all the people with a higher intrinsic value than this indifferent person $(v>\hat{v})$ would want to buy the product.
$n=\phi-\hat{v}$

Rewrite this with respect to the intrinsic value and we get a function of price by the number of consumers in the market:
$p=n(\phi-n)$

Figure 4 below is an illustration of the demand expression.

To see how the price is changing with the number of consumers, we can look at the derivative of the price function:
$\frac{\partial p}{\partial n}=\phi-2 n$

From this we can observe an increasing demand for small n i.e. if $\phi>2 n$, and if n is bigger than $\frac{\phi}{2}$, the demand curve turns negative, as in the normal case without network effects. A simple explanation behind this inverted U-shape is that the consumers are uniformly distributed from the highest to the lowest intrinsic value in the diagram. From the


Figure 4: Demand with network effects
demand function we know that $p(0)=0$, however, if only one consumer joins the market, the value of the good increases exponentially and the price follows. As people with a lowering intrinsic value join the market the demand curve will eventually turn downward as the positive network effect will be dominated by the consumers' low intrinsic value. This kind of market with positive network effects can exhibit multiple equilibria, or none, depending on the size of the marginal cost. In figure 5 we can see an example of a market with multiple equilibrium when the marginal cost of producing is constant at $c_{1}$.


Figure 5: Multiple equilibria with network effects

When the marginal cost is $c_{1}$, it is evident from figure 5 that there exists three possible equilibria in $n=\left[0, n_{1}^{\text {low }}, n_{2}^{\text {high }}\right]$, where $n_{1}^{\text {low }}$ is the only unstable equilibrium. If the firm reaches a consumer mass just above $n_{1}^{\text {low }}$ more people will join because the next consumer's
willingness to pay is higher than the price, and the consumer mass will jump to the stable equilibrium in $n_{1}^{\text {high }}$. On the other hand, if the consumer mass does not reach $n_{1}^{\text {low }}$, it will quickly spiral down to $n=0$. This is why the point $n=n_{1}^{\text {low }}$ is typically referred to as the critical mass (Shapiro \& Varian, 1998; Economides \& Himmelberg, 1995). This might explain why small firms in networks markets are rarely observed. How to get a consumer mass beyond the critical mass is, therefore, one of the principal problems in a market with network effects. In figure 5 we can see that a lowering of the marginal cost will lower the critical consumer mass, which can make it easier to get beyond the critical mass and become self-sufficient. Another approach could be to offer the product for free a certain period of time to quickly reach a sufficiently high consumer mass (Rohlfs, 1974).

An essential part when discussing the equilibrium outcome (possible multiple equilibria) in these markets, is the role of consumer expectations about the future development in the market. A common notion to use is rational, or fulfilled expectations among the consumers (see e.g. Katz \& Shapiro, 1985; Rohfs, 1974). When the consumers exhibit fulfilled expectations, they can perfectly predict how many participants, on the demand side, there will be in forthcomming periods, except for in equilibrium (Katz \& Shapiro, 1992; Economides \& Himmelberg, 1995). The role of consumer expectations is not explicitly mentioned in the simple model above. Nevertheless, it is the consumer expectations that depict which of the three equilibria is reached.

In the model above, and many other models on network effects, it is assumed that the consumers derive the same additional utility regardless of who is entering the network market. Rohlfs (1974), in addition to the model of uniform calling pattern, constructed a model where the consumers get increased utility only if someone in their community enters the market, and derives no extra utility if people outside their community enters the market. It can be reasonable to assume consumers' additional utility is dependent on whether the next consumers is someone they know or not, at least in some network markets. Take social networks as an example: a user of Facebook (or another social network) will benefit more from people whom she has some kind of relationship with. An
implication of this assumption is that the critical mass is reached with fewer consumers as long as they are the right consumers. Rohlfs argues that allowing for different preferences makes the modelling work more complicated. However, we will see later in this chapter and in our own model that it is possible in a Hotelling framework.

Figure 5 illustrates a perfectly competitive market which presents three different equilibria where the price is equal to marginal cost. ${ }^{7}$ From the previous discussion, it is known that a small network size $n=n_{1}^{\text {low }}$ will not be sustainable because of its unstable nature. A small network will, therefore, not be observed in this market as the network either increases to $n=n_{1}^{\text {high }}$ or spirals down to zero. This finding is supported by Economides and Himmelberg (1995). They also find that a perfectly competitive market will provide a network size smaller than the optimal size seen from a social planner's point of view. This happens as the marginal social benefit of having one more consumer in the market is higher than the firms' marginal benefits. Economides and Himmelberg (1995) discover an interesting result regarding the equilibrium network size in a monopoly market. Even if the monopolist can influence the consumers' beliefs about future network size, it will choose a smaller network size compared to the case of perfect competition. ${ }^{8}$ That is if price-discrimination is not an option. An oligopoly market, they find, supports a network size smaller than in a perfectly competitive market, and higher than in a monopoly market.

Perhaps the most interesting, and probably the most investigated type of market, is an oligopolistic market. Katz and Shapiro (1985) were early to analyze such a market. They developed a model in an attempt to identify the implications of the network effect in an oligopolistic setting, where firms sells homogeneous products and compete in quantity. It is evident that in this model, where the consumers have fulfilled expectations, the firms' reputation plays an important role as the consumers' expectations determine which of

[^5]the multiple equilibria is reached.

Another approach to describe the network effects' impact is done by Navon, Sky and Thisse (1995). They use a Hotelling model to allow for heterogeneity among the consumers, with an endogenous location. They add a network effect parameter $\alpha$ to the number of consumers buying from store $i$ : $n_{i} .{ }^{9}$ The utility function for the consumer buying from store $i$ is as follows: $U_{i}=\alpha n_{i}-p_{i}-\tau\left|x-x_{i}\right| . \tau$ represent the transportation cost, $p_{i}$ is the price charged by firm $i . x_{i}$ is the location of firm $i$. Navon et.al. find that with negative or moderately positive network effects, i.e. when the network effect does not overpower the transportation cost $\tau$, there exists an equilibrium where both firms survive. They also find that positive network effects induce a tougher price competition where the firms have incentives to underbid each other to capture more consumers. If the network effect is strong enough to overpower the transportation cost, it is likely for the firm that can charge the lower price to corner the market. A relatively small change in the network effect can therefore turn a duopolistic competition to a monopoly market. A further discussion on whether it is possible for coexistence in a market with network effects is provided in chapter 2.2.4. In the next chapters we will continue to focus on oligopolistic markets and the implications the network effect have on the competition.

### 2.2.3 Strategic delegation

In the literature about strategic delegation, it is argued that a separation between ownership and management can have implications on the strategic behavior of a firm (Sklivas, 1987; Vickers, 1985). The owner of the firm can offer an incentive-based contract that has the intention to induce a behavior from the manager which might differ from pure profit maximization. ${ }^{10}$ They argue that in order to gain a strategic advantage it is not necessarily enough to establish whether the strategic variable in a duopoly market is price, quantity or some other instrument. It is also important to understand the firm's internal

[^6]organization, i.e. the manager's incentives. Furthermore, significant network effects also have an impact on the strategic positioning in the market. We will mainly look at the literature regarding the price adaption in an oligopoly market, with and without network effects.

Some of the leading authors in this field, Sklivas (1987), Vickers (1985), and Freshtman and Judd (1987), examines a two-stage delegation game where the owner offers the manager a publicly announced incentive-based, before competition in prices or quantity in the second stage. It is important that the contract is credible for it to have any behavioral implications on the other firms in the market (Freshtman \& Judd, 1987). They use a contract which is a linear combination of profit and cost to investigate a possible equilibrium that is not pure profit maximizing. ${ }^{11}$ If the firms compete in quantity, the manager will be offered a contract to augment sales, while in price competition the owner will offer a contract which gives more compensation when reducing cost. The result in a quantity competition game is stronger competition between the firms. The manager's incentives to increase the production rises, price will lower, as will the profits of the firms, and the consumer surplus will increase. The opposite is true in a price competition game. As the incentive-contracts soften the competition between the firms, the equilibrium price will increase as well as each of the firms' profit. A higher price will consequently lower the consumer surplus.

Other authors have tested different contracts in a similar delegation game, where the objective is to investigate the consequence on the competing firms' behavior of using different types of incentive-contracts. Jansen, van Lier and van Witteloostuijn (2007) look at the market share case, where the manager is offered a linear combination contract between profits and market share. ${ }^{12}$ As in the profit/cost contract, the profit/market share contract induces a more aggressive manager in the quantity competition. This is because the manager will receive a compensation for increasing the production as they

[^7]gain market share when the competitive firm answers by lowering their quantity, all else equal. A similar result is obtained when the firms are competing in prices where the manager is incentivized to increase the price and soften the competition.

Another type of contract used in a strategic delegation game is the relative performancebased incentive scheme. Miller and Pazgal (2001) investigates an incentive scheme where the managers are compensated based on a linear contract between the firm's own profit and the rival firm's profit. They obtain a similar qualitative result as the previously mentioned incentive schemes. They also obtain an equivalence result between price and quantity competition. That is, for a single value of the weight the owner puts on the profit of the other firm, the equilibrium price outcome will be the same whether the firms compete in price or quantity. The intuition behind this result is that a lowering in prices from the manager's perspective, is partially offset by the lowering in the rivaling firm's profit as the rivaling firm also will respond by lowering prices. If the firms instead compete in quantity, the manager knows that an increase in the production (which leads to a lower price) will induce the other firm to decrease their production to increase the profit of the rivaling firm. Thus, if the owner has adequate influence toward the manager, the profit gained whether the two firms compete in prices or quantity will be the same.

While the results from the models mentioned above are interesting, none of the authors above have discussed the impact of significant network effects on these incentive schemes and the equilibrium in the market. Hoernig introduces the direct network effect component in his paper from 2012. His model is based on the price competition model from Sklivas' (1987) paper, where the owner offers the manager an incentive-based contract, a linear combination of profits and revenues. Under the assumption that the consumers have fulfilled expectations, he finds that strong network effects can strengthen the price competition in two firms where an incentive-based contract is offered to the manager. This implies that in the presence of network effects, the managers have incentives to underbid each other. The same result is obtained by Pal (2015). He investigates whether the equivalence result from Miller and Pazgal (2001) also holds if there are significant
network effects on the consumer side. With a linear combination between one firm's profit and the rival firm's profit, he find that the equivalence result does not hold when the network effect is strong enough, regardless of whether the network effect is positive or negative.

### 2.2.4 The start-up problem and coexistence

It is commonly believed that it is difficult for new companies to enter a market in the presence of strong network effects. An essential part for a startup in any kind of market is to attract enough consumers to make the firm viable. In a network market, however, a startup may encounter what is referred to as the "chicken and egg" dilemma: to attract new consumers the the firm must have some initial users, but consumers will not join unless there aleready are other users in the network (Economides \& Himmelberg, 1995; Caillaud \& Jullien, 2003). ${ }^{13}$ Whether a firm manages to enter the network market or not relies on several different factors. By using qualitative models, both dynamic and static, researchers have identified some of these factors.

Farrell and Saloner (1986) developed a dynamic model to look at the consumers' incentives to adapt new incompatible technologies. They suggest that in a market that exhibits significant network effects, the timing of the announcement of new technology can determine whether or not this technology will be adopted by the consumers and exceed the old technology. The main question is if the consumers tend to be biased towards old technology, denoted excess inertia, or if they tend to rush into new, incompatible technology, leaving the installed base of the old technology stranded, denoted excess momentum. Both cases can lead to a loss in efficiency. This can be because neither buyers nor sellers properly consider the loss in value of the old technology if new consumers adopt to the new technology rather than the old. Another explanation is that early adopters of the new technology do not take into account that they increase the incentives for later adopters to buy the new technology. To show this they assume the consumers to be divided into

[^8]two groups: the old users who make up the installed base of the old technology, and the new users who after the arrival of the new technology must decide between the two. The equilibrium outcome depends on the size of the installed base in the old technology at the time the new technology is introduced, how quickly the network benefits from the new technology, and relative payoffs the consumers receive from adopting either of the technologies. The findings in this model suggest that the pioneering firm can have an advantage in entering first when allowing for an installed base. If, however, the network effect is not sufficiently strong, there might exist a first-mover advantage in introducing new technology. ${ }^{14}$ It is a conventional belief that in markets with network effects firstmover advantages can be long-lasting if the firm can establish an installed base before competitors enters the market (Shapiro \& Varian, 1999).

Building on the previous model, Katz and Shapiro (1992) have a somewhat different approach. While Farrell and Saloner take the date of the introduction of new technology to be exogenous, Katz and Shapiro develop a model where the introduction of new technology is decided endogenously. They also introduce the pricing strategy aspect by considering two technologies which are not perfect competitors, causing the price to be decided endogenously in the model. They assume that the technological progress is the continuous reduction of the marginal cost or, equivalently, the increase in the quality of the product. This implies that a new company which enter at a later date will have a lower marginal cost or higher quality caused by technological progress. At the same time, the established firm will have a higher user base the later the entry of the new firm. Another important assumption in this model is that the consumers exhibit fulfilled expectations. They provide several conditions in which it is feasible to enter a market with network externalities. First, if the new firm's underlying advantage, measured as the cost advantage and initial network relative to the established firm, is higher than the consumers expectation, measured as the benefits for the consumers buying now if all future consumers buy from the new company, then there exists a unique equilibrium

[^9]where the new firm sells to all current and future buyers. Say that the new firm has such an underlying advantage, and the consumers expect that the new firm will win in the future, then it will be profitable for the new firm to enter, even for a range of fixed cost of entry. The entry might lower social welfare. However, this is ignored by the new firm.

In the two papers cited above, it is assumed that the consumers have fulfilled expectations which lead to cornering of the market over time. That is, where either of the companies will overtake the whole market in a winner-takes-it-all scenario. With the assumptions used in these models, it is implied that coexistence of competing, incompatible technologies in the same market is not plausible. This might be a misleading result as we have seen a coexistence of competing technologies such as the telephone operating systems iOS and Android. Lee, Lee, and Lee (2006) revise some of the typical assumptions made in a market with network effects which leads to cornering of the market. They suggest that the former literature has, to an excessive degree, focused on the installed base and the consumers' valuation of connecting to a large network. While this is an important characteristic in a network market, they suggest that the consumers may also be influenced by the choice of their acquaintances. That is, the consumers are, to a greater degree, affected by the choice of friends and colleagues. As people tend to have a smaller number of acquaintances, it is possible that some of these will choose the lagging technology as opposed to the leading one. This is a phenomenon Lee et. al. (2006) addresses as local biases. This would leave room for smaller networks and act as a break on the winner-takes-it-all process. If local biases persist, that is, if the consumers only interact with people they know and if this is a main source of the consumer's benefit, it could be a source of coexistence of technologies.

Shapiro and Varian (1999, pp. 187-189) identify two other forces which can determine whether it is more or less likely for the market to tip to only one technology. The first is economics of scale. It is noted here that the term economics of scale contains both the supply side as well as the demand side economics of scale. While economics of scale will, in any market, make it harder for different competing technology to coexist, it will
also amplify the economics of scale on the demand side in a market with network effects, making it even more difficult. ${ }^{15}$ The other force is the demand for variety. This could be if the consumers have heterogeneous preferences, or if the consumers prefer to buy more than one product. Table 1 below illustrates the criteria of when the market is more likely to tip.

|  | Low Economics of Scale | High Economics of Scale |
| :---: | :---: | :---: |
| Low Demand for Variety | Unlikely | High |
| High Demand for Variety | Low | Depends |

Table 1: Likelihood of market tipping (Shapiro \& Varian, 1999, p. 188).

If there exists an advantage of producing higher quanta at the same time as the networks effects are strong, the market is more likely to tip. If the consumers have a high demand for variety, the likelihood of standardization of technologies are less likely because with heterogeneous preferences there exists a market for more than one competing technology. It is, in other words, the balance between these two forces that determines whether it is likely or not for technologies to coexist in the market.

Another factor that may contribute to market tipping and make it harder for new entrances, is the collective switching cost the consumers experience from the network effect (Farrell \& Klemperer, 2006; Shapiro \& Varian, 1999). Take social networking sites as an example. There is a social cost associated with switching to another site if the consumer has built a large community of friends and acquaintances. A general assumption in literature is that the consumer's decision to participate is considered to be irreversible, which can be a way of incorporating large switching costs. However, this can cause the leading firm to get an unstoppable competitive advantage which is suspected to overstate the effect of switching costs (Evans \& Schmalensee, 2010).

[^10]
### 2.2.5 Empirical findings

Until now, most of the literature reviewed in this chapter has been grounded in qualitative, theoretical models. The models that are used to explain the network effect implications in the market are highly simplified to easily capture the network effect implications. However, it rarely gives us the whole, real world picture. While the theoretical research in this field is plentiful, the empirical studies are rather scarce.

Many of the historical examples of market failure due to network effects are concerned with indirect network effects, e.g. the format war between Blu-ray and HD-DVD. In that case, Blu-ray cornered the market, and HD-DVD went extinct despite the fact that HD-DVD was offered cheaper and entered the market before the Blu-ray format (den Uijl \& de Vries, 2013). Some of the factors that are believed to explain why Blu-ray was able to overtake the market was the exclusive support Blu-ray received from Warner Bros, which triggered exclusive support from other companies, as well as the short time window between the entries of the two formats (den Uijl \& de Vries, 2013; Daidj et al., 2010). This format war showed the importance of having the right alliances. Another example often used when looking at the (indirect) effects a network has, is the QWERTYkeyboard design. David (1985) argue that the collective switching cost of changing the design is too high. This, he suggests, have caused better keyboard designs to fail to enter the market at a later point in time. There are, however, some disagreement in whether some of the opponents to the QWERTY-keyboard actually were superior (Liebowitz \& Margolis, 1994). This complicates the question of whether the network effect is guilty of the cornering of the market or if it is also due to other external factors.

A more recent example of two competing companies where direct network effects are present, is the competition between online social networking sites such as Facebook and MySpace. MySpace was launched in 2003, three years before Facebook opened for public use (Boyd \& Ellison, 2008). At its peak, it attracted close to 76 million users, making the world's most visited social networking site. Already in 2008, Facebook overtook MySpace's place as the most popular social network site, and the users of MySpace kept
declining; in 2011 they lost 10 million unique users in the matter of one month (Barnet, 2011, March 21). In an attempt to survive, MySpace relaunched in 2013 with a new design and an emphasis on music streaming, as a way to differentiate from competitors like Facebook (Knopper, 2013, June 12). Today MySpace is still active but the big rush of new consumers it still yet to come. With only 8 million monthly users in the last quarter of 2018, compared to Facebook's 2.3 billions, it does not seem like MySpace's relaunch has successfully turned the tables (Armstrong, 2019, March 18; Statista, 2019c).

A different approach in empirical studies is to construct an econometric model in an attempt to isolate the network effect. A challenge linked to this kind of empirical research is to be sure to have captured the network effect as opposed to other unrelated changes (Manski, 1993). Nevertheless, Asvanund, Clay, Krishnan and Smith (2004) attempted to construct an econometric model which goal is to investigate the direct network effect in a peer-to-peer ( P 2 P ) music sharing network. A general assumption in a typical P2P network is that the performance improves as the network grows. That is, when more people join, more people upload different content, making the service more attractive. Asvanund et. al. argue that the effect can also be negative if the consumption is too high in scarce network resources or if people tend to free ride in larger networks. They find that the consumers contribute to the network at a decreasing rate, and inflict a cost at an increasing rate. Optimal network size is therefore bounded. This finding is in contrast to many of the reviewed theoretical models where the network strength is included as a linear parameter, i.e. all consumers who joins the network contribute equally to the value.

Another empirical study conducted by Srinivasan, Lilien, and Rangaswamy (2004) examine how network externalities affect the lifetime of different firms. They disprove the conventional belief that the pioneering firm often has a first-mover advantage in network markets. In markets with strong network effects they find that network externalities have a negative effect on time of survival for the pioneering firm. The exception is for more radical and technologically intense products, where a strengthened network effect
is associated with increased survival duration.

### 2.3 Two-sided markets

In a two-sided market, value is added by an intermediary agent - a platform - who facilitates the interaction of two (or more) user groups that exhibit cross-group network effects; one user group's expected gain from participating on the platform is dependent on participation from the other user group. ${ }^{16}$ A key assumption for a market to be deemed as two-sided, is that the two sides can not successfully coordinate this exchange without the intermediary (Evans \& Schmalensee, 2005). A familiar example of such a platform is a publication with both editorial content and advertisement printed on its pages. The intermediary, in this case the publication, caters to two customer groups: readers and advertisers. The more readers the publication has, the more valuable a slot of advertising space is for an advertiser. This is the indirect network effect. ${ }^{17}$ This effect is internalized through the intermediary and expressed through the distribution of prices faced by the different user groups. A common feature of two-sided markets is the subsidization of one of these groups at the expense of the other, often to the point where one side pays below-marginal cost prices, even zero or negative prices. For example, a publication may have a shelf price that is below the marginal cost of printing (it may even give away its content for free). Given the presence of an ad market, such a subsidy may still be the profit maximizing outcome. In a nutshell, this is how two-sided markets work.

### 2.3.1 The birth of a theory

Print publications are, of course, not a new invention, nor is the strategic behavior suggested in the example above. However, the rise of the Internet seems to have spurred a growth of these types of intermediary companies. Many of the online-based companies

[^11]we interact with on a daily basis have one user group looking to consume some type of content or service, while at the same time catering to an ad market. In many cases, the content or service that is provided, is free to the user, completely supported by ad revenue. Internet users have gotten used to this trade-off: we get to read news, search for products, or use social networking sites to share thoughts, pictures or videos, all for free - granted we accept the presence of ads.

Although two-sided markets are now recognized by economists to have existed for thousands of years, most economic theory on the subject has been developed after the turn of the millennium. ${ }^{18}$ In a paper that began circulating among Industrial Organization scholars in 2002, Jean-Charles Rochet and Jean Tirole for the first time generalized the idea of two-sided markets, exemplified by the economics of payment cards. ${ }^{19}$

As noted by Roson (2005), certain two-sided markets had in fact been studied for years by the time Rochet and Tirole started writing about it. Some of the characteristics of two-sidedness had even been noted, such as in Baxter's (1983) pioneering work on credit cards. As a series of antitrust cases concerning the international credit card industry triggered a debate on the economic theories of credit card networks, economists such as Tirole and Rochet (2002), Katz (2001), Gans and King (2003), Schmalensee (2002) and Wright (2003a; 2003b; 2004) contributed. Sparked by this debate, similarities with other industries were recognized, and the general theory of two-sided markets was formed. Following this seminal work, there have been important contributions that focuses on topics such as media markets (Ferrando et al., 2004; Reisinger, 2004; Kaiser \& Wright, 2006), as well as on electronic intermediaries (Caillaud \& Jullien, 2003; Jullien, 2005).

Evans and Schmalensee (2005) divide two-sided platforms into four main categories: exchanges (e.g. auction houses, dating sites, book publishers, employment agencies or other intermediaries), advertiser-supported media, transaction devices (e.g. credit cards), and

[^12]software platforms (e.g. video game consoles). This categorization seem to fit the current body of literature on two-sided markets well.

In much of the early writing on the subject, there is a striking lack of a clear definition of two-sided markets. Although the "getting both sides on board" characteristic can be useful (as discussed in chapter 2.3.6), this really applies for any market, as buyers and sellers needs to be brought together for gains from trade to be realized (Rochet \& Tirole, 2006).

Several attempts at a general definition of two-sided markets has been made. Roson (2005) presents a simple definition in his survey: "A market is two-sided if platforms serve two groups of agents, such that the participation of at least one group raises the value of participating for the other group." This definition is less restrictive than the one Tirole and Rochets (2004) pose, requiring also that "prices of each side (which can be zero or negative) have direct influence on market participation on the other side". In the current literature on two-sided markets, there seem to be consensus on the fact that at least three criteria must hold for a market to be two-sided. Inspired by Evans and Schmalensee (2008), we sum them up as follows: i) there has to be (at least) two distinct user groups, ii) there exists some positive indirect network effects between these groups, going at least one way, and iii) there exists a platform that internalizes these network effects when setting its prices.

It has been pointed out that the general theory, as introduced by Rochet and Tirole, does not limit itself to only two sides. Following this insight, Evans and Schmalensee have begun to consistently refer to businesses such as the one described above as multisided platforms, rather than two-sided ones. We are not rejecting this terminology. However, for simplicity and consistency, and without too great a loss of generality, we will in this text stick to the most common terminology of two-sided markets.

### 2.3.2 Price structures in two-sided markets

One of the early attempts to create a general model for a two-sided market is Armstrong's (2006). While his paper is closely related to earlier contributions by Chaillaud and Jullien (2003) and Rochet and Tirole (2003), it extends to discuss platforms facing both monopoly and competition, as well as both single-homing and multi-homing consumers. In its simplicity, the basic model he presents captures the essence of a two-sided market well, and it serves as a good starting point for further discussion of pricing schemes. Following the categorization by Evans and Schmalensee (2005), it is also well suited for our discussion of platforms with advertisers, as it is developed to fit newspapers, whereas Chaillaud and Jullien's article is better suited to fit exhanges, and Rochet and Tirole's apply to transaction devices. ${ }^{20}$ We reproduce part of it in this chapter. Note that we make some changes in the notation. This is to ensure consistency with the rest of our text, and to avoid confusion from the models we use. ${ }^{21}$

The most defining feature of a two-sided market is the added benefit an agent from one side of the market derives from the participation of an agent on the other side - the crossgroup network effect. This benefit is modeled as $\beta^{i}$ where $i \in A, B$ represents two distinct groups of agents, such as readers and advertisers. Assuming first a platform monopoly with homogeneity among agents within each group, the utility of an agent in any of the the two groups is simply $u^{A}=\beta^{A} D^{B}-p^{A}$ and $u^{B}=\beta^{B} D^{A}-p^{B}$ respectively, where $p^{i}$ is the fee associated with participating on the platform for each agent of group $i$, and $D^{i}$ is the number of members from each groups connected to the platform. In other words, each user's utility is a function (only) of the price she faces, and - in some way - the other group's demand for the shared platform. We define a market as two-sided as long as $\beta^{i}$

[^13]

Figure 6: A two-sided market
is positive for users of at least one of the groups. ${ }^{22}$ Figure 6 depicts the market.

In the monopoly setting, Armstrong assumes that demand is simply increasing in utility by some function $D^{i}=\phi^{i}\left(u^{i}\right)$, where $D^{i}$ is the number of users from side $i$ that joins the platform. Assuming some marginal cost $c^{i}$ of serving users of each group, the platform's profit function is $\pi=\left(p^{A}-c^{A}\right) D^{B}+\left(p^{B}-c^{B}\right) D^{B}$. ${ }^{23}$ By inverting the utility function to represent prices, $p^{i}=\beta^{i} D^{j}-u^{i}$, profit can be expressed in terms of utility:
$\pi=\left(\beta^{A} D^{B}-u^{A}-c^{A}\right) D^{A}+\left(\beta^{B} D^{A}-u^{B}-c^{B}\right) D^{B}$

Further substituting $D^{i}$ for $\phi^{i}\left(u^{i}\right)$ yields:
$\pi\left(u^{A}, u^{B}\right)=\phi^{A}\left(u^{A}\right)\left[\beta^{A} \phi^{B}\left(u^{B}\right)-u^{A}-c^{A}\right]+\phi^{B}\left(u^{B}\right)\left[\beta^{B} \phi^{A}\left(u^{A}\right)-u^{B}-c^{B}\right]$

By maximizing the profit function and solving for $p^{i}$, the monopoly model yields the following optimal prices:

[^14]$p^{A}=c^{A}-\beta^{A} D^{B}+\frac{\phi^{A}\left(u^{A}\right)}{\phi^{A \prime}\left(u^{A}\right)}$ and $p^{B}=c^{B}-\beta^{B} D^{A}+\frac{\phi^{B}\left(u^{B}\right)}{\phi^{B \prime}\left(u^{B}\right)}$
The monopoly outcome prices can be explained as follows: $c^{i}$ is simply the marginal cost of providing service to group $i$, passed on to the customer by the monopolist. The prices are pushed downward by the positive externality the group's participation exerts on the other group, $\beta^{i} D^{j}$. The last part of the expression, $\frac{\phi^{i}\left(u^{i}\right)}{\phi^{i}\left(u^{i}\right)}$, expresses the elasticity of participation. Technically, the expression tells us that as long as demand is increasing in utility, and utility can not be negative, for any (marginal) increase in utility that agents of one group receives from participating on the platform, prices for this group will be pushed down. In other words, if price elasticity is high, prices tend to be low.

It can be useful to compare the monopoly outcome to one by some benevolent social planner. This is achieved by establishing an aggregate consumer surplus $v^{i}\left(u^{i}\right)$ for each group $i$ that satisfies the envelope condition $v^{i \prime}\left(u^{i}\right) \equiv \phi^{i}\left(u^{i}\right) .{ }^{24}$ Total welfare (profit plus consumer welfare) can thus be expressed as a single function of utilities $w=\pi\left(u^{A}, u^{B}\right)+$ $v^{A}\left(u^{A}\right)+v^{B}\left(u^{B}\right)$. From its first-order conditions, this can easily be solved for a socially optimal set of utilities $u^{A}=\left(\beta^{A}+\beta^{B}\right) D^{B}$ and $u^{B}=\left(\beta^{A}+\beta^{B}\right) D^{B}$. When inserting for the utility functions, it gives us the set of socially optimal prices $p^{A}=c^{A}-\beta^{B} D^{B}$ and $p^{B}=c^{B}-\beta^{A} D^{A}$.

Thus, in a social optimum, prices are defined only by cost and cross-group externalities. In other words, as long as cross-group externalities are present, below-cost prices will occur.

The monopoly outcome can also involve subsidizing of one group - say, group 1-if at least one of two things occur: either that group 1's price elasticity is high, or that group 1's participation involves high external benefit to group $2\left(\beta^{B} D^{B}\right.$ is high). However, monopoly prices will always be higher than the socially optimal ones.

The monopoly model extends easily to a duopoly model. While other papers assume duopoly competition on only on side of the market (see for example Kind et al., 2013),

[^15]Armstrong (2006) assumes a two-sided duopoly, i.e. that two platforms competes for both sides of the marked, and he employs a Hotelling style framework for duopoly competition. In his analyzis, he assumes single-homing consumers on both sides, with two platforms, 1 and 2 , competing for uniform demand from the two sides A and B. Now, utility for the two sides is denoted as $u_{i}^{A}$ and $u_{i}^{B}$, with $i \in 1,2$. These take a similar form as in the monopoly model, with $u_{i}^{A}=\beta^{A} D_{i}^{A}-p_{i}^{A}$ and $u_{i}^{B}=\beta^{B} D_{i}^{A}-p_{i}^{B}$ with $\left\{p_{i}^{A}, p_{i}^{B}\right\}$ being the prices charged to each group by each platform $i$.

In line with standard Hotelling competition models, the demand for platform $i$ by the two groups is given by functions $D_{i}^{A}=\frac{1}{2}+\frac{u_{i}^{A}-u_{j}^{A}}{2 t^{A}}$ and $D_{i}^{B}=\frac{1}{2}+\frac{u_{i}^{B}-u_{j}^{B}}{2 t^{B}}$, where $t^{A}$ and $t^{B}$ are transport costs for group A and B , respectively. Assuming $D_{j}^{A}=1-D_{i}^{A}$ and inserting the utility functions, gives demand expressions $D_{i}^{A}=\frac{1}{2}+\frac{\beta^{A}\left(2 D_{i}^{B}-1\right)-\left(p_{i}^{A}-p_{j}^{A}\right)}{2 t^{A}}$ and $D_{i}^{B}=\frac{1}{2}+\frac{\beta^{B}\left(2 D_{i}^{A}-1\right)-\left(p_{i}^{B}-p_{j}^{B}\right)}{2 t^{A}}$.

The expressions show that, keeping prices fixed, one extra agent from a given group on a platform, attracts $\beta^{i} / t^{i}$ agents of the other group to the same platform.

For there to exist a market-sharing equilibrium, the following condition must hold: $4 t^{A} t^{B}>\left(\beta^{A}+\beta^{B}\right)^{2}$. In other words, cross-group externalities must be weak relative to brand preference for the market to be shared. Intuitively, this makes sense; if brand preferences were weak, and externalities strong, we would expect one platform to corner both sides of the market.

Solving the set of demand equations yield following results:
$D_{i}^{A}=\frac{1}{2}+\frac{1}{2} \frac{\beta^{A}\left(p_{j}^{B}-p_{i}^{B}\right)+t^{B}\left(p_{i}^{A}-p_{i}^{A}\right)}{t^{A} t^{B}-\beta^{A} \beta^{B}}$ and $D_{i}^{B}=\frac{1}{2}+\frac{1}{2} \frac{\beta^{B}\left(p_{j}^{A}-p_{i}^{A}\right)+t^{A}\left(p_{j}^{B}-p_{i}^{B}\right)}{t^{A} t^{B}-\beta^{A} \beta^{B}}$
The model outcome implies that a platform will target one group more aggressively than the other if that group is (i) on the more competitive side of the market and/or (ii) causes larger benefits to the other group than vice versa.

Comparing the duopoly outcome with the monopoly outcome, Armstrong's model shows that the duopolist puts twice the emphasis on the external benefit as the monopolist. This
makes sense, as the platform competes for consumers at both sides at once. Increasing group 1's price to the point where one group 1-agent leaves, has different consequences in the two scenarios. For the monopolist, this means that the agent leaves the market. For the duopolist, it means that the agent moves to the rival, making it harder for the platform to attract and keep group 2-agents than it is for the monopolist.

Although elegant in its simplicity, it is worth noting that Armstrong's model is ill-suited to discuss welfare implications. The fixed group sizes make imply that prices are simply transfers between agents, not affecting total surplus. Most of all, the paper illustrates what Armstrong argues are the three main factors affecting the pricing structure in a two-sided market.

One is the indirect network externalities and their relative sizes, i.e. the respective sizes of $\beta_{1}$ and $\beta_{2}$. The example he uses is the much cited one of nightclubs as an arena for heterosexual matchmaking: if men gain more from interacting with women than vice versa, one would expect a pricing scheme skewed towards subsidizing women.

In the model framework presented above, this is illustrated by setting a uniform price p. ${ }^{25}$ By solving platform $i$ 's profit $\left(p^{i}-f\right)\left(n_{1}^{i}+n_{2}^{i}\right)$, the equilibrium price is $p=f+2 \frac{t_{1} t_{2}-\beta_{1} \beta_{2}}{t_{1}+t_{2}+\beta_{1}+\beta_{2}}$, which lies between the two non-uniform equilibrium prices $p_{1}=f_{1}+f_{2}-\beta_{1}$ and $p_{2}=$ $f_{2}+t_{2}-\beta_{2}$ if $\beta_{1} \neq \beta_{2}$.

Another factor is the pricing scheme itself. Rochet and Tirole (2006) discuss different price structures of two-sided platforms. They distinguish between subscription fees and transaction fees; does an agent on either side have to pay a fee to the platform simply to have access to it, does a fee incur only in cases of a successful match of agents from opposing sides, or both? The type of fee levied on agents seems to vary among the different types of platforms, sometimes also among the different groups connected to a platform. For some payment cards, for instance, the cardholder may pay a yearly fee, while the merchant pays a fee (typically a percentage amount) for each transaction that

[^16]is made using the card. Armstrong (2006) shows how platform profits will be higher when it is charging per-transaction, due to weaker inter-group externalities. The reason is that part of the marginal benefit of an additional user is then eroded by the increased payment incurred. Lastly, another important factor affecting platform pricing is whether one or both sides multi-home, which we discuss in more depth in chapter 2.3.4.

### 2.3.3 Policy implications

The peculiarities of two-sided markets have some important implications for public policy. In a Canadian study of mergers in the newspaper industry, Chandra and Collard-Wexler (2009) present a model showing how mergers in two-sided markets may not lead to higher prices for either side of the market, contrary to what one usually expects of mergers in one-sided markets. Another paper examines the effects of value added taxes (VAT) on advertisement-supported newspapers, and finds the paradoxical result that the consequences of a low-tax regime might be the opposite of the intention (Kind et al., 2013). While a policy may intend to reduce prices on newspapers by lowering the tax rate, it could in fact lead to higher prices. The authors present a three stage game, where in stage 1 the firms decide both their locations on the Hotelling line, and their level of investments into journalistic quality. In stage 2, they set the optimal level of advertisement, and in stage 3, they compete in prices. The outcome of the game indicates two opposing effects regarding the impact of VAT on newspaper prices. When the marginal cost of newspapers is positive, the price of newspapers will increase when the VAT rate increases. ${ }^{26}$ The added impact the presence of an ad market has on the direct effect, however, draws the firms to compete more intensely, driving the price down. The net effect depends on the relative size of these two effects.

The relative sizes of the two market sides also have important implications for the degree of differentiation. When the strategic effect dominates over the demand effect, the result is

[^17]the principle of maximum differentiation (Tirole, 1988). A larger ad market will increase the demand effect relative to the strategic effect, as the increased revenue from the ad market will make it relatively more profitable for the newspaper to locate closer to its rival. A sufficiently large advertiser side may give rise to the principle of minimum differentiation (Gabszewicz, Laussel, \& Sonnac, 2001, 2002). ${ }^{27}$

The results of Kind et al. (2013) show that lowering the VAT rate on newspapers tends to increase newspaper differentiation. As a result, if the advertising market is relatively small, the newspapers might invest too little in journalism and be too differentiated from a social point of view, making taxes a welfare-enhancing instrument.

### 2.3.4 Multi-homing

In the standard Hotelling model, it is assumed that each buyer consumes one and only one good. This is arguably a major weakness of the model. In reality, horizontal differentiation may just as easily incline consumers to buy both goods, rather than choosing between one or the other. Said differently, in reality, some consumers value variety. In the spatial differentiation literature, this property has been termed multi-homing. Conversely, when consumers are assumed only to buy one good, they are said to single-home. Multi-homing is relevant also for studies of two-sided markets; when studying the consumption of newspapers in the Washington DC area, Gentzkow finds that a third of his sample reads more than one paper (2007). A Finnish study on payment systems, find that informed consumers use more payment systems than uninformed ones (Hyytinen \& Takalo, 2004). This suggests that consumers find some incremental utility from consuming a good, even if they are already consuming a rivaling good. As a consequence of this, the demand of a firm facing multi-homing consumers is composed of those who choose to buy only from said firm, and of those who choose to multi-home. In a Hotelling framework, this can be illustrated as follows:

Consumers on the Hotelling line will buy as long as they get a positive utility. Depending

[^18]on prices, travel costs, and their location on the line, they will have a preference for one of the two firms, except for the consumer who is indifferent, located in point $\bar{x}$. Assuming consumers experience some positive incremental value, say $\theta v$ (assuming $0<\theta<1$ ), from consuming a second good, they will have an incremental utility function for buying the second good $j$, having already bought the first good $i$ : $u_{i j}=\theta v_{j}-p_{j}-t\left(d_{j}\right)^{2}$.

Figure 7 illustrates the make-up of the demand of two firms located in point 0 and 1 . There are three indifferent consumers in the figure: the consumer in point $\bar{x}$ is the indifferent consumer in the single-homing case, the consumer in $x_{12}$ is indifferent between buying only from firm 1, and buying from both firms, while the consumer in $x_{21}$ is indifferent between buying only from firm 2, and buying from both firms.


Figure 7: Hotelling's linear city with multi-homers

The weakness of the Hotelling model with single-homing is addressed by Kim and Serfes (2006), who find that by allowing multi-homing in the model, firms may differentiate less than when only single-homing is allowed. They show that when the incremental utility of consuming a second good exceeds a certain threshold, the equilibrium shifts from a singlehoming outcome to an outcome where a fraction of the consumers buys both products. Under certain conditions, they find that with multi-homing, spatial differentiation will be minimal, i.e. Hotelling's pricinple of minimal differentiation is restored. ${ }^{28}$

The intuition behind their result can be described as follows: in a Hotelling model with multi-homing consumers, the competition over consumers is not a zero-sum game. Each firm has a demand that is made up by a base of exclusive consumers, and a base of multi-homers. In this framework, each firm's total demand is not a function of the rival's price, as it is in the single-homing Hotelling model. However, each firm's price affects

[^19]the composition of the rivaling firms demand; by lowering its price, a firm can increase the base of multi-homers, and as a result reduce the rivaling firm's base of exclusive consumers. As a consequence of this property, the strategic effect is weaker relative to the business stealing effect in the case of multi-homing, compared to the single-homing case. To use Kim and Serfes' (2006) words, the multi-homing consumers "act as a buffer, altering the nature of price competition significantly".

Armstrong (2006) introduces the concept of "competitive bottlenecks" in two-sided markets where one side multi-homes and the other single-homes. He shows how if, for example, media consumers single-home, while advertisers multi-home, advertisers suffer from the platform's exclusive market power when it comes to delivering consumers. He proposes that for any equilibrium outcome, the number of advertisers is chosen by maximizing the joint surplus of consumers and platform, ignoring the interest of advertisers. ${ }^{29}$ The result is too few advertisers from a social point of view. With Wright, Armstrong develops a framework to show that when platforms are viewed as homogenous by one group (e.g. advertisers) but heterogeneous by another (e.g. consumers), these competitive bottlenecks arise endogenously. In other words, advertisers will choose to multi-home, even though they do not receive any of the gains from trade (Armstrong \& White, 2007).

Peitz and Reisinger (2015) offer a relevant application of the competitive bottleneck theory: in the market for television, consumers engage in single-homing, as they cannot view more than one channel at the same time, but advertisers multi-home (they can place ads on multiple channels). They argue that under this assumption, an advertiser will choose to broadcast his ads at the exact same time on multiple channels, thus avoiding paying for the same viewers several times. The basis of this argument is one often used in two-sided models of media markets: revenue from the advertiser side is given by the number of exclusive consumers, implying that potential multi-homing consumers are worthless to advertisers. Some papers challenge this. Anderson, Foros and Kind (2017) develop and present a model of incremental pricing, where each platform is able

[^20]to price to advertisers only the value of its exclusive consumers plus the incremental value associated with multi-homers. The implication of this model is that advertisers experience some value from second impressions, but not as great as that of first impressions. They further show how an increase in the incremental value of multi-homers incentivizes the platforms to locate closer to each other, as it becomes relatively less profitable to chase exclusive consumers. A paper analysing the effect of tying in two-sided markets where consumers multi-home, finds that tying induces more consumers to multi-home (Choi, 2010). By making platform-specific exclusive content available to more consumers, which is beneficial to content providers, tying can thus be welfare-enhancing when multi-homing is possible.

### 2.3.5 Negative network effects

So far, we have discussed the presence of positive cross-group network effects. There is however, no reason to believe that agents in one group can not experience a disadvantage from connecting with agents from another group. The most cited example we find, is consumers' distaste for advertising in media markets. Advertisement in the economic literature if often divided into two types, informative and persuasive. This categorization is relevant for whether advertisement is considered a good or a bad in terms of consumer utility. Kaldor (1950) argued that advertisement is a social bad, if it offers no extra benefit when information is perfect. Others argues that advertising provides information to consumers indirectly by signaling the quality of the goods advertised (see e.g. Nelson, 1974; Kihlstrom \& Riordan, 1984; Horstman \& MacDonald 1994). Empirical studies also provide some conflicting answers. Studying the market for magazines in Germany between 1992 and 2004, Kaiser and Song (2009) find little evidence of readers disliking advertising; indeed, in some segments readers seem to appreciate ads. Their study suggests that the more informative ads are, the more they are appreciated by readers. ${ }^{30} \mathrm{~A}$ similar study of US magazines provide ambiguous results to the research question "is advertising a good

[^21]or a bad?" (Depken II \& Wilson, 2004). Studies focusing on television however, suggests that viewers perceive ads as a nuisance (Moriarty \& Everett, 1994; Wilbur, 2008).

While some two-sided media models have simply assumed that consumers are indifferent to the presence of ads (Gabszewicz et al., 2001, 2002; Kind et al., 2013), some have included a nuisance for ads. Gabszewicz (2004), for instance, show that in a duopoly competition between two two-sided platforms, if viewers dislike ads, minimal differentiation will not occur, but that the strategic effect is less effective (making the platforms locate closer) as advertising aversion becomes stronger.

Peitz and Valetti (2008) also present a stylized model for platform competition between two television networks when consumers have a distaste for ads. Investigating two payment schemes - pay-tv, where the platform gets revenue from both viewers and advertisers, and free-to-air, where the platform gets all its revenue from advertisers - they find that if consumers strongly dislike advertising, the free-to-air payment scheme will have a higher advertisement intensity than the pay-tv scheme. They also find that the platforms differentiate maximally in the pay-tv scheme, while under free-to-air, the firms only differentiate maximally if viewers have a particularly strong distaste for ads (modelled as a high $\delta$ ). Discussing the welfare implications of this, they find that for a lower level of $\delta$, free-to-air can provide the socially optimal differentiation level (i.e. $a=b=1 / 4$ using the notation of our paper). Further, they find that for a range of $\delta$, provision of ads is socially better under free-to-air, while above a certain level of $\delta$, pay-tv is the socially optimal payment scheme.

A related paper studies asymmetric competition between two media platforms where one platform charges consumers subscription fees, while the other is free for consumers but charges advertisers to have access to its platform (Dietl et al., 2013). Here, disutility from ads is expressed by the negative term $\gamma a_{i}$ in the consumer's utility function, where $a_{i}$ is platform $i$ 's ad volume and $\gamma$ is the consumers sensitivity to ads (if $\gamma=1$, the consumer is as sensitive to an increase in ad volume as she is an increase in price). The paper explicitly compares advertisement pricing under two different pricing schemes -
lump-sum and per-consumer - and find that disutility of ads needs to be sufficiently low for aggregate platform profits to be higher under the lump-sum scheme than under the per-user scheme.

Another implication of having $\gamma>0$ (disutility of ads), is that in the ad market, prices are strategic substitutes - contrary to in a one-sided model. The intuition is that if one platform increases its ad price, the demand for ads will drop, making the platform relatively more attractive to readers. It follows that the rival platform is relatively less attractive to readers, making it relatively less attractive to advertisers. The rival platform's best response is thus to reduce its ad price - as the ad market has become less important relative to the reader market. Further, the paper show how, due to $\gamma>0$, ad volume increases with $t$, the travel cost parameter, which signals increased market power. These results are robust for both pricing schemes.

Armstrong (2006) also shows how readers preferences for ads affect the platform's preference for pricing schemes. In his competitive bottleneck framework, when readers like (dislike) ads, the equilibrium price and profit for the platform is higher (lower) if advertisers are charged on a lump-sum basis, rather than a per-user basis. Contrary to Dietl et al. (2013) however, he studies a symmetrical duopoly.

### 2.3.6 Dynamics

As the two-sided platform is dependent on both sides in order to create value, the problem of getting "both sides on board" is an important one. Chaillaurd and Jullien (2003) were, to our knowledge, the first to dub the dynamic problem of two-sided markets as a "chicken and egg" problem. Especially for platforms where the positive indirect network effect runs both ways, such as the case is for a dating club for heterosexuals, the problem is that in order to attract female users, you need to offer a selection of male users, and vice versa. Most models simplify from this dynamics problem, and assume simultaneous arrival from both sides, based on rational expectations about the attendance of the other side.

There are examples of industries where one side has to be on board in order to attract the other. The most cited one, we find, is the case of video game consoles. For the console, the two market sides are the end-users (gamers) and the game developers. In order to appear on the market, a console must come completely equipped with a wide range of games, in other words, the developers need to come on board first. This is usually overcome by offering a sweet deal to the side that has to move first, in this case the developers. Evans (2003) mentions another way to overcome the dynamics problem, which is for the platform company itself to directly supply the one side (the example he uses is the software side of the hand held computer Palm), thus offering a sufficiently rich supply of applications for the platform to attract users. Then, after market penetration is ensured, the platform company (Palm) withdraws from said market side, leaving room for developers to supply new software for an already existing consumer base.

Static equilibrium models with network effects often make assumptions about fulfilled expectations or perfect foresights by the agents involved. As Sun and Tse (2007) argue, although convenient, this is often a weak representation of reality. They model a network dynamic based on agents deciding their participation after observing the actual size of the network at the time they decide. This approach is arguably more realistic in modelling consumer behavior, but it also complicates the modelling work greatly, without necessarily offering any extended insight into the mechanisms we are interested in studying.

### 2.3.7 Empirical studies of two-sided markets

The literature on two-sided markets has grown substantially since the turn of the millennium, evolving from anecdotal studies to general theoretical models describing the special characteristics of these markets. The body of literature is predominantly heavy on the theory side, leaving empirical research for future studies. There are however some notable exceptions. Chandra and Chllard-Wexler's (2009) present first a theoretical model studying mergers in a two-sided market, before they test their theory on a sample of Canadian newspapers in the 1990s. Both theoretically and empirically, they find that increased market concentration did not lead to higher prices for either subscribers or advertisers.

Wilbur (2008), uses a two-sided empirical model of the American television market to highlight some interesting results. His study suggests that not only are viewers indeed averse to advertisement, but that the price elasticity of advertisement demand has increased over the last 30 years; a $10 \%$ decrease in ad time for a highly rated network, suggests a median increase of $25 \%$, all else equal. Further, he finds that networks cater more to advertisers' preferences than viewers', and that giving viewers access to technology that enables them to avoid ads, tends to increase the equilibrium volume of ads, and decrease platform revenue.

Kumar, Lifshits and Thompson (2010) perform an empirical analysis of the value accruing to members of each side of a two-sided market, based on the presence of the other side. They use the online question-answer sites Stack Overflow and Yahoo! Answers to study the rate of arrival of question-askers, as a function of the number of answer-providers to identify the cross-group network effect between the two groups.

Evans (2003) conducts an empirical study of the two-sided platform companies Microsoft and Palm OS, and find similarities in their business models that he uses to generalize for multi-sided platforms in general. These include the use of differential pricing to get both sides on board, and maintaining both sides, that multihoming occurs, and that a successsful solution to the pricing complexity is to start with a small but scalable platform.

### 2.4 Remarks on the literature review

The literature on spatial differentiation examines how firms strategically differentiate in order to maintain higher profits. This is a way of softening competition. We have studied two strains of literature on network effects that challenge this strategy by different mechanisms.

The theoretical literature regarding direct network effects often suggests that positive network effects strengthen the competition between firms. Because of the special dynamics where the product's value increase with the number of new consumers, the firms are
more willing to lower the prices in order to attract enough consumers, as the objective is to become self-sufficient. It is also commonly believed that the market dynamics will make it harder for new firms to enter the market, and if a firm successfully does so there is not always room for them both to exist, especially in markets where the network effect is strong. This is also something we have seen historically: new firms have entered the market successfully and after a while have formed monopolies.

The growing theory on two-sided markets investigates firms that cater to two different customer groups at the same time. The literature emphasizes the need to get both sides on board, often by use of complex pricing schemes. Indirect network effects between the groups affect consumer welfare in a way that necessitates the use of two-sided models when examining policy issues such as taxing and antitrust acts. The literature on two-sided markets suggests that the indirect network effect contribute to increased competition in duopoly models, as the profitability of reaching the next consumer increases. Whether or not consumers multi-home, is crucial for the outcomes of such models.

The literature on both direct and indirect network effects is dominated by theoretical studies. This is also the part of the literature to which we contribute with this text. We do however recognize the need for further empirical work.

## 3 A spatial model with network effects

In this chapter we present a Hotelling model with endogenous location and direct network effects. The model is inspired by d'Aspremont et al. (1979) and Navon et al. (1995). We first conduct a two-stage game between two firms that exhibit positive network effects. In chapter 3.2 we expand the model to a three-stage game between two firms where both direct and cross-group network effect are positive. The direct network effect is implemented additively in the utility function, as in the model of Katz and Shapiro (1985). The consumer's expected utility of purchasing the good from firm $i$ (with $i \in 1,2$ ) is:

$$
U_{i}=v-p_{i}-t\left(d_{i}\right)^{2}+\alpha D_{i}^{e}
$$

Here, $v$ is an intrinsic value of consuming any of the two goods, $p_{i}$ is the price of good $i$, and $\alpha$ is a parameter measuring the strength of the direct network effect. We assume the direct network effect to be positive: $\alpha>0 . D_{i}^{e}$ is the consumer's expectations of the demand of firm $i$. $t\left(d_{i}\right)^{2}$ is the transportation cost, where $d_{i}=\left|x-x_{i}\right|$ measures the distance a consumer has to travel to get to firm $i$ when $x_{i}$ is the location of firm $i$ and the consumer is located in point $x$ (see figure 8).

The choice of quadratic transportation costs is a practical one; lending from the model of d'Aspremont et al. (1979), allows us to compare our results to theirs in a straightforward fashion. Based on what we know from the literature on network effects, we expect their presence in the model to amplify the business stealing effect, which ceteris paribus leads to less differentiation. Absent the term $\alpha D_{i}^{e}$ in the utility function, our model is identical to that of d'Aspremont et.al., where the firms' optimal choice is to differentiate at $a=$ $b=-\frac{1}{4}$. This serves as a useful benchmark for our discussion.

From Shapiro and Varian (1998), as discussed in chapter 2.2.2, we assume that the network effect has a concave from. This is supported by the empirical study conducted by Asvanund et. al. (2004) mentioned in section 2.2.5. They find that the strength of
the network effect is decreasing and the cost inflicted on the network is increasing in the number of consumers. In our model however, the effect is included linearly. While a concave form might be more realistic in a network market, we consider it to make the model overly complicated for our purpose.

The location of the two firms is expressed by their respective distances to the end of the line in the direction of the center. As illustrated by figure 8 , the distance from firm 1 to point 0 is called $a$, while the distance from point 1 to firm 2 is called $b$. Firm 1 is thus located at point $x_{1}=a$ on the Hotelling line, while firm 2 is located in point $x_{2}=1-b$. Allowing for $a, b<0$ implies that also locations outside the line is possible.


Figure 8: Location of the network firms

### 3.1 One-sided model

In standard Hotelling fashion, we start out with establishing demand functions by identifying the consumer who is indifferent between the two firms. Letting $U_{1}=U_{2}$ and solving for $\bar{x}$ gives us the location of the indifferent consumer, which can be interpreted as the demand for the left-most firm on the line. ${ }^{31}$

$$
\begin{equation*}
\bar{x}=D_{1}=a+\frac{1-a-b}{2}-\frac{p_{1}-p_{2}}{2 t(1-a-b)}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{2 t(1-a-b)} \tag{1}
\end{equation*}
$$

Further, demand for firm 2 is

$$
\begin{equation*}
1-\bar{x}=D_{2}=b+\frac{1-a-b}{2}-\frac{p_{2}-p_{1}}{2 t(1-a-b)}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{2 t(1-a-b)} \tag{2}
\end{equation*}
$$

[^22]The last term in these expressions has an interesting interpretation. Note that if the consumers expect one firm to have a higher demand than the other, this creates a selffulfilling increase in that firm's demand. The strength of this effect is measured by $\alpha$.

The firm's objective is to maximize profits. For now, we consider a one-sided market, where each firm has a profit function $\pi_{i}=\left(p_{i}-c\right) D_{i}$. Assume, for simplicity, that the two firms face identical marginal costs: $c_{1}=c_{2}=c$. This indicates that the firms are symmetric, which simplifies the calculation. As we are not interested in the implications of different marginal cost, we do not lose significant insight by assuming this.

### 3.1.1 Results from the one-sided model

This is a two-stage game where the optimal location of the firms is decided in stage 1 before the price competition occurs in stage 2 . The game is solved by backward induction, where we first solve for the optimal price in stage 2 , taking $a$ and $b$ as given.

Stage 2
The firms maximize profits by setting $\frac{\partial \pi_{i}}{\partial p_{i}}=0$. From the first-order condition, we attain the firm's reaction functions $p_{1}\left(p_{2}\right)$ and $p_{2}\left(p_{1}\right)$, respectively:

$$
\begin{align*}
& p_{1}=\frac{t(1-a-b)(3-b+a)}{2}+\frac{p_{2}+c}{2}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{2}  \tag{3}\\
& p_{2}=\frac{t(1-a-b)(3-a+b)}{2}+\frac{p_{1}+c}{2}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{2} \tag{4}
\end{align*}
$$

From these reaction functions we establish that the prices are strategic complements $\left(\frac{\partial p_{i}}{\partial p_{j}}>0\right)$. If one of the firms decreases its price, the other firm's best response is to also decrease its price, in order to not lose profits.

From the two reaction functions, we solve for the firms' optimal prices:

$$
\begin{equation*}
p_{1}^{*}=c+\frac{t(1-a-b)(3-b+a)}{3}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{3} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
p_{2}^{*}=c+\frac{t(1-a-b)(3-a+b)}{3}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{3} \tag{6}
\end{equation*}
$$

When inserting the equilibrium prices, we attain the demand functions:

$$
\begin{align*}
& D_{1}=\frac{3-b+a}{6}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{6 t(1-a-b)}  \tag{7}\\
& D_{2}=\frac{3-a+b}{6}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{6 t(1-a-b)} \tag{8}
\end{align*}
$$

Following Katz and Shapiro (1985), we assume that the consumers have fulfilled expectations. An essential feature of our model as opposed to other models we have seen, is the timing of when the consumers' expectations are formed. The consumers form their expectations after observing the firms' location. Then the firms set their price. Because it is more time-consuming for the firms to change their location (e.g. by changing their design or features) than the price, we find the timing for the consumers' expectations reasonable. Based on this assumption, as well as assuming full market coverage, we have $D_{1}^{e}=D_{1}$ and $D_{2}^{e}=D_{2}=1-D_{1}$, allowing us to solve the set of equations for the following demand functions:

$$
\begin{align*}
& D_{1}=\frac{t(1-a-b)(3-b+a)-\alpha}{2[3 t(1-a-b)-\alpha]}  \tag{9}\\
& D_{2}=\frac{t(1-a-b)(3-a+b)-\alpha}{2[3 t(1-a-b)-\alpha]} \tag{10}
\end{align*}
$$

Looking at the demand functions (9) and (10), $\alpha$ has a seemingly ambiguous effect on the demand of any given firm. However, by taking the difference in the demand between the two firms, we can identify the net effect of an increase in $\alpha$.

$$
\begin{equation*}
D_{1}-D_{2}=t(a-b) \frac{1-a-b}{3 t(1-a-b)-\alpha} \tag{11}
\end{equation*}
$$

We require $a+b<1$, therefore, if $a>b$, an increase in $\alpha$ will increase the demand of
firm 1. A higher $a$, we can see from (11), is equivalent with an increase in the demand for firm 1, all else equal. The firm with the higher demand will benefit from a stronger network effect. This is also easily seen from equation (7) and (8), as a consequence of the self-fulfilling increase in demand.

The optimal prices for the firms when we include fulfilled expectations are:

$$
\begin{align*}
& p_{1}^{*}=c+\frac{t(1-a-b)(3-b+a)}{3}+\frac{t(a-b)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha  \tag{12}\\
& p_{2}^{*}=c+\frac{t(1-a-b)(3-a+b)}{3}+\frac{t(b-a)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha \tag{13}
\end{align*}
$$

The last term in the price function reveals the network effect. Considering equation (12), as $\alpha$ increases, $p_{1}$ increases if $a>b-$ as long as $3 t(1-a-b)>\alpha$. This means that if firm 1 has the higher demand of the two, it can take a higher price compared to in a duopoly market without network effects. ${ }^{32}$ Another interesting result is that the firm with the higher demand can continue to increase the price as it captures more consumers from the other firm. This result can be traced back to the utility function, where the utility of the consumers increases as the expected demand increases. The firm can therefore take a higher price because of the increased market power. This only holds for values of $\alpha<3 t(1-a-b)$; when the network effect is too powerful, the result is reversed. This is a common result of models including network effects. A strong network effect can consequently create a winner-takes-it-all situation (see e.g. Navon et. al. 1995).

In a symmetric outcome however, the last term in the price expression is equal to zero, implying that $\alpha$ has no direct impact on the optimal prices. This result is obtained as a consequence of the timing of the consumers' expectations. When the consumers already know that the firms are of equal size, none of the firms can create an advantage by decreasing the price, thus losing their incentives to compete more intensely.

## Stage 1

[^23]In stage 1, the firms choose their locations. Both firms have a general profit function:

$$
\pi_{i}=\left[p_{i}(a, b, \alpha, t)-c\right] D_{i}\left[a, b, p_{i}(a, b, t, \alpha), p_{j}(a, b, t, \alpha), \alpha, t\right]
$$

They maximize this by choosing the optimal location. For firm 1 (2), this involves setting the optimal level of $a(b)$. Symmetry in the model specification makes it superfluous to analyze both firms individually. Moving forth, we use firm 1 as the focus of discussion. Qualitatively, however, the analysis holds just as well for firm 2.

The first-order condition can be expressed as:

$$
\begin{equation*}
\frac{\partial \pi_{1}}{\partial a}=\frac{\partial p_{1}}{\partial a} D_{i}+\left(p_{1}-c\right)\left[\frac{\partial D_{1}}{\partial a}+\frac{\partial D_{1}}{\partial p_{1}} \frac{\partial p_{1}}{\partial a}+\frac{\partial D_{1}}{\partial p_{2}} \frac{\partial p_{2}}{\partial a}\right]=0 \tag{14}
\end{equation*}
$$

By rewriting expression (14), we identify the envelope theorem. The first term in (15) can be recognised as the first-order condition from the maximisation problem in stage 2 , where we let $\frac{\partial \pi_{i}}{\partial p_{i}}=0$. We can, therefore, set this term equal to 0 .

$$
\begin{equation*}
\frac{\partial \pi_{1}}{\partial a}=\underbrace{\left[D_{1}+\left(p_{1}-c\right) \frac{\partial D_{1}}{\partial p_{1}}\right]}_{=0} \frac{\partial p_{1}}{\partial a}+\left(p_{1}-c\right)[\underbrace{\frac{\partial D_{1}}{\partial a}}_{\text {Direct effect }}+\underbrace{\frac{\partial D_{1}}{\partial p_{2}} \frac{\partial p_{2}}{\partial a}}_{\text {Strategic effect }}]=0 \tag{15}
\end{equation*}
$$

We can separate the first-order condition into two different effects: the direct effect and the strategic effect. The strategic effect describes how an increase in $a$ will affect firm 2's price, and how this price change will in turn affect the demand for firm 1. $\frac{\partial p_{2}}{\partial a}$ will always be negative, as less differentiation implies fiercer price competition among the firms, driving prices down. $\frac{\partial D_{1}}{\partial p_{2}}$ will conversely always be positive. The reason is that when firm 2 increases its price, all else equal, firm 1 becomes relatively more attractive to consumers. As a consequence, the strategic effect is negative; moving closer to each other will decrease profits for the firms. The direct effect describes how firm 1's demand is directly affected by the change in $a$, holding all else equal. In the standard Hotelling model, this term is positive, as the firm, by moving closer to the center of the line, can capture more consumers who are averse to travel. In our model, this depends on the
strength of the network effect.

The model is highly responsive to changes in the values of $t$ and $\alpha$. This makes it challenging to obtain the optimal location for each of the firms. The second-best approach is to conduct a sensitivity analysis. By evaluating the profit function's derivatives around the outcome of d'Aspremont et al., (1979), we can compare our model to the similar version without network effects, and see how the two outcomes differ. We evaluate our results for values of $t$ close to $\alpha$, as well as the knife-edge case where $t=\alpha$, to investigate how these opposing effects influence the firms' equilibrium location. We discuss three cases below.

Case 1: Transportation cost equal to the network effect, $t=\alpha$.

$$
\left.\frac{\partial \pi_{1}}{\partial a}\right|_{a=b=-\frac{1}{4}}>0,\left.\quad \frac{\partial \pi_{1}}{\partial a}\right|_{a=b=0}=0
$$

For the knife-edge case where $t=\alpha$, the optimal location for the firms is $a=b=0$. In other words, they will locate at point 0 and 1 on the Hotelling line. This is a less differentiated outcome than the model of d'Aspremont et al. (1979) yields. Adding the network effect incentivizes the firms to locate closer, as the value of capturing the next consumer increases. This strengthens the demand effect relative to the strategic effect. The impact the quadratic travel cost function has on the firms' incentives to differentiate, is still substantial however, as the firms will locate at the end points of the line. The second-order condition holds for this result.

Case 2: Transportation cost higher than the network effect, $t>\alpha$

$$
\left.\frac{\partial \pi_{1}}{\partial a}\right|_{a=b=-\frac{1}{4}}>0,\left.\quad \frac{\partial \pi_{1}}{\partial a}\right|_{a=b=0}<0
$$

If the transportation cost parameter is larger than the network effect parameter, the firms will differentiate more than in the case of $t=\alpha$, but less compared to the results of d'Aspremont et al. (1979). The reason is that the transportation cost, which grants the
firms the power to strategically differentiate, pushes the firms appart on the Hotelling line. $t$ thus represents the opposite mechanism of $\alpha$. Nevertheless, the presence of $\alpha>0$, leads to less differentiation than the benchmark case without $\alpha$. Evaluating the first-order condition around $a=b=0$, reveals that the outcome still entails locations outside the Hotelling line. The second-order condition holds for this result.

Case 3: Transportation cost lower than the network effect, $t<\alpha$

$$
\left.\frac{\partial \pi_{1}}{\partial a}\right|_{a=b=-\frac{1}{4}}>0,\left.\quad \frac{\partial \pi_{1}}{\partial a}\right|_{a=b=0}>0
$$

When the strength of the network effect is greater than the transportation cost we see that the firms will differentiate less than for the two cases above. A higher $\alpha$ increases the relative value of business stealing. This will, as we have discussed earlier in this chapter, benefit the larger network as the network effect contributes to a self-fulfilling increase in the demand. Further increasing $\alpha$ will amplify this effect. When the consumers have more homogeneous preferences in the form of a lower $t$ the firms will consequently seek to be more similar to each other. When testing the model for different values of $t$ and $\alpha$, we find that for a too high value of $\alpha$ relative to $t$, the second-order condition does not hold. ${ }^{33}$

In all three cases we find that the firms' optimal location is closer than than the equilibrium outcome in the model of d'Aspremont et. al. (1979) when we include network effects in the model. Where they locate in equilibrium depends on the relative strength of the transportation cost and the network effect. For the knife-edge case $t=\alpha$, we find a unique equilibrium outcome at the end points of the Hotelling line. Increasing (decreasing) the strength of $\alpha$ relative to $t$ leads to less (more) differentiation.

[^24]
### 3.2 Two-sided model

We now extend our model to account for duopoly competitions between two platforms selling network goods in a two-sided market. We follow Armstrong's (2006) notation, as presented in chapter 2.3.2, but simplify the model to address duopoly competition à la Hotelling on only one side, in the style of Kind et al. (2013), Peitz and Valletti (2004), and Anderson and Coate (2005). We model the direct network effect by use of parameter $\alpha^{i}$, as before, and the cross-group network effect by use of $\beta^{i}$, with $i \in A, B$. Superscript A and B denotes the two sides of the market. The market analyzed in chapter 3.1, now accounts for side A in the extended model.

For illustrative purposes, imagine the platform has customer group A similar to the user base of a social network service. Customer group B consists of advertisers trying to reach the members of group A. Suppose group A agents are indifferent to the presence of group B members, i.e. $\beta^{A}=0$. Group $B$ agents, on the other hand, care about the participation of group A members, i.e. $\beta^{B}>0$. In other words, cross-group network effects run only one way in our model. ${ }^{34}$ The parameter $\beta^{B}$ measures the amount of advertisement per side A agent on the platform, thus serving as a measure for the cross-group network effect. Further, we assume that the within-group network effect is prevalent only in group A, i.e. $\alpha^{A}>0, \alpha^{B}=0 . \alpha^{A}$, as before, measures the strength of the network effect in marked side A. The two-sided market is illustrated in figure 9.

In the two-sided model, the game is in three stages. In stage 1, as before, the firm decides its location. In stage 2, it sets the optimal volume of advertisement on its platform. In stage three, the two firms compete for users in prices.

With $\beta^{A}=0$, the utility a group A member attains from purchasing each good $(i \in 1,2)$ is:
$U_{i}^{A}=v^{A}-p_{i}^{A}-t\left(d_{i}^{A}\right)^{2}+\alpha^{A} D_{i}^{A e}$

[^25]

Figure 9: Illustration of the two-sided model
Note that this is identical to the original utility function presented at the beginning of chapter 3 . We have simply added the superscript denoting group affiliation.

Each platform $i$ has a profit $\Pi_{i}=\pi_{i}^{A}+\pi_{i}^{B}$, where $\pi_{i}^{A}=\left(p_{i}^{A}-c^{A}\right) D_{i}^{A}$ is the profit derived from side A of the market, while $\pi_{i}^{B}=\left(p_{i}^{B}-c^{B}\right) D_{i}^{B}$ is the profit derived from side B. $c^{A}$ and $c^{B}$ denotes the marginal cost associated with serving side A and B respectively, which remains identical for the two firms. Group A' demand for platform $i, D_{i}^{A}$, is given by the Hotelling specification derived in the chapter 3.1. For group B, the demand function is simply:
$D_{i}^{B}=\beta^{B} D_{i}^{A}$
In other words, the ad market's demand for platform access is linearly dependent on the amount of ad exposure it gets from the users of the platform.

On the ad side of the market, side B, we follow Peitz and Valletti (2008), Anderson and Coate (2005), and Kind et al. (2013) in assuming a simple downward-sloping demand for ads, given by:
$p^{B}=\rho-\sigma \beta^{B}$

Here, $\rho, \sigma>0$ represent the size of the ad market ( $\rho$ being the reservation price for ads and $\sigma$ representing the slope of the demand for ads). $\beta^{B}$ can be interpreted as the amount of influence the ad market has on the side A users, measured by the number of ads each side A user is exposed to. $D_{i}^{B}$ is thus the total demand for ads, composed by the amount of ads each side A agent is exposed to, multiplied by the number of side A agents the platform is able to attract.

Substituting into the profit function, this yields a total profit for platform $i$ :
$\Pi_{i}=\left(p_{i}^{A}-c^{A}\right) D_{i}^{A}+\left(\rho-\sigma \beta^{B}-c^{B}\right) \beta^{B} D_{i}^{A}$
Or, slightly re-written:
$\Pi_{i}=\left[\left(p_{i}^{A}-c^{A}+\beta^{B}\left(\rho-\sigma \beta^{B}-c^{B}\right)\right] D_{i}^{A}\right.$
It is easy to see that the platform's total profit depends on the amount of side A users it is able to attract. This is ultimately decided in stage 3 of the game (duopoly price competition), but the outcomes of stage 1 (setting the optimal differentiation level), and stage 2 (setting the optimal ad level) will matter for the outcome in this final stage.

### 3.2.1 Results from the two-sided model

The three-stage game is solved by backward induction.

## Stage 3

In stage 3, the platforms compete for group A consumers. This is equivalent to the second stage in the model in chapter 3.1. Hence, platform 1's optimal price for group A is:

$$
\begin{equation*}
p_{1}^{A *}=c+\frac{t(1-a-b)(3-b+a)}{3}+\frac{t(a-b)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha \tag{16}
\end{equation*}
$$

For platform 2, the optimal group A price is:

$$
\begin{equation*}
p_{2}^{A *}=c+\frac{t(1-a-b)(3-a+b)}{3}+\frac{t(b-a)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha \tag{17}
\end{equation*}
$$

## Stage 2

Inserting the optimal prices into $\Pi_{i}$ and maximizing the profit function with regards to $\beta^{B}$, allows us find the platform's optimal level of advertising:

$$
\begin{equation*}
\frac{\partial \Pi_{i}}{\partial \beta^{B}}=0=>\beta^{B *}=\frac{\rho-c^{B}}{2 \sigma} \tag{18}
\end{equation*}
$$

From equation (18), we see that the optimal level of advertising is increasing in the size of the ad market (increasing in $\rho$, decreasing in $\sigma$ ), and decreasing in the marginal costs, as should be expected.

## Stage 1

In stage 1, the platforms decide their optimal location. For platform 1, after inserting the results from stage 2 and 3 into the profit function $\Pi_{1}$, this entails solving the first-order condition $\frac{\partial \Pi_{1}}{\partial a}=0$ for $a$, while for platform 2, we solve the first-order condition $\frac{\partial \Pi_{2}}{\partial b}=0$ for $b$.

For platform 1, inserting $\beta^{B *}$ into the B-side profit function, yields:

$$
\begin{equation*}
\pi_{1}^{B}=\frac{\left(\rho-c^{B}\right)^{2}}{4 \sigma} D_{1}^{A}=\left[\frac{\left(\rho-c^{B}\right)^{2}}{4 \sigma}\right]\left[\frac{t(1-a-b)(3-b+a)-\alpha}{2[3 t(1-a-b)-\alpha]}\right] \tag{19}
\end{equation*}
$$

The platform's profit from side A is:

$$
\pi_{1}^{A}=\left[\frac{t(1-a-b)(3-b+a)}{3}+\frac{\alpha t(a-b)(1-a-b)}{3[3 t(1-a-b)-\alpha]}\right]\left[\frac{t(1-a-b)(3-b+a)-\alpha}{2[3 t(1-a-b)-\alpha]}\right]
$$

In stage 1, the platform decides its optimal location by the first-order condition:

$$
\frac{\partial \Pi_{1}}{\partial a}=\frac{\partial \pi_{1}^{A}}{\partial a}+\frac{\partial \pi_{1}^{B}}{\partial a}=0
$$

For the result of this derivation, see the appendix. What we are interested in at this point, is to compare the result with the one from chapter 3.1. By evaluating $\frac{\partial \Pi_{1}}{\partial a}$ around
the same values as $\frac{\partial \pi_{i}^{A}}{\partial a}$, we can obtain some insight into how introducing an ad market affects the firms' location. In all the 3 cases studied in the one-sided model, the outcome was less differentiation than the one of d'Aspremont et al. (1979) ( $\left.\frac{\partial \pi_{1}}{\partial a}\right|_{a=b=-\frac{1}{4}}>0$ ). In line with our expectations of an amplified business stealing effect, we find that the same holds for the two-sided model. We move on to check if the results differ when we evaluate the derived function around $a=b=0$. When dealing with this many variables, we have simulated the model at $\rho=2, c^{B}=0$ and $\sigma=1$ yielding a cross-group network effect $\beta^{B}=\frac{\rho-c^{B}}{2 \sigma}=1$.

Case 1: Transportation cost equal to the network effect, $t=\alpha$

$$
\left.\frac{\partial \Pi_{1}}{\partial a}\right|_{a=b=0}>0
$$

In the first case, the result deviates from that in chapter 3.1: facing two market sides, the firms are incentivized to differentiate less than they would in the one-sided model. In other words, the business stealing effect is indeed amplified in the presence of cross-group network effects. For this case, the second-order condition holds for all values of $t=\alpha>1$.

Case 2: Transportation cost higher than the network effect, $t>\alpha$

$$
\left.\frac{\partial \Pi_{1}}{\partial a}\right|_{a=b=0} \lessgtr 0
$$

When evaluating the first-order condition over different values of $\alpha$ and $t$, we find that for values of $t$ close to $\alpha$, but nevertheless higher, the firms will differentiate less (locating inside the Hotelling line) This is in contrast to the case 2 result we found in chapter 3.1.1. A higher degree of consumer heterogeneity will normally lead to a higher degree of differentiation between the firms. In this case, however, when we have both direct and cross-group network effects, the transportation cost must be sufficiently high for the firms' strategic decision to be more differentiated. When testing the model given the parameters, we find that the firms will differentiate more than $a=b=0$ if $\alpha \in\langle 0, t-0.5\rangle$ and less if $\alpha \in\langle t-0.5, t\rangle$.

Case 3: Transportation cost lower than the network effect, $t<\alpha$

$$
\left.\frac{\partial \Pi_{1}}{\partial a}\right|_{a=b=0}>0
$$

In this case we obtain the same results as in case 3 in chapter 3.1. It is difficult to establish in our model whether they will locate closer or not compared to the same case in the one-sided model. Nevertheless, because of the results obtained in case 1 and 2 above, it is probable that the firms indeed would differentiate less in this case compared to the same case in the one-sided model for the same values of $t$ and $\alpha$. Also in this case the second-order condition holds for values of $t$ close to $\alpha$ when $t>1.5 .^{35}$

[^26]
## 4 Discussion

We constructed a model to offer theoretical insight on the effect of network effects on strategic differentiation. Although direct network effects have been studied extensively by economists for decades, adding more recent contributions on two-sided markets to the mix allows us to investigate the strategic behavior of firms that share key characteristics with some of the Internet giants of our days: two-sided social media platforms. We restrict our discussion to one about differentiation and its implications for prices in a symmetric equilibrium outcome. As most economic models do, ours greatly simplifies the reality we set out to investigate. We discuss the limitations of our approach in this chapter. Nevertheless, we find that by holding our results up against the existing literature, we are able to draw some conclusions about how network effects influence firms' incentives to strategically differentiate.

In our one-sided model, we find that adding the direct network effect term $\alpha D_{i}^{e}$ to the consumer's utility function leads to less differentiation. The intuition behind this result is that the network effect increases the relative importance of capturing the market for the firm. This comes from the positive feedback loop the expected demand causes on the utility function. The timing of the game is essential. The consumers form their expectations based on the firms' location, which is decided in stage 1. The firms know this, therefore they will place relatively more emphasis on increasing its demand in stage 1 than they would absent the $\alpha D_{i}^{e}$ term. The outcome we find, which is less differentiation, implies a more intense price competition among the two firms, and we expect prices and profits to decrease as a result.

We study a symmetric equilibrium outcome $(a=b)$. When the firms equally share the market, we see from our price expressions, equations (12) and (13), that the last term cancels out. Without this term, our price expressions would be identical to those of of d'Aspremont et al. (1979). Therefore, at the face of it, it appears that $\alpha$ does not affect prices in our model. It does however have an indirect effect, as it affects the location of
the firms in stage 1. This is evident from the second term of the price expressions, and it confirms our expectation that a direct network effect leads to a reduction in prices. ${ }^{36}$ As the two firms share the market equally in equilibrium, reduced prices also implies reduced profits for both firms.

If we were to consider an asymmetric outcome (e.g. $a>b$ ), we see from equation (12) that an increased network effect $(\alpha \uparrow)$ will increase the price for firm 1 . This is a due to the timing of the consumers' expectations. When the consumers observe that one firm has a larger demand, a stronger network effect will amplify the market power of the larger firm as it creates a self-fulfilling increase in the demand. Because the consumers' expectations are formed before the firms set their prices, the firms lose their incentives to decrease their prices in order to capture more consumers.

We perform our analysis on three different cases to highlight the opposing effects $t$ and $\alpha$ has in the model. Generally, the larger the transportation cost, the more important is the strategic effect. After all, $t>0$ is what enables the firms to escape the Bertrand paradox in the Hotelling model; if $t=0$, the two firms are perfect substitutes, and will always locate at point $1 / 2$ on the line. Remember that $t$ represents the strength in consumers' preferences, or the switching cost. A dominant $t$ thus leads to a more differentiated equilibrium outcome than the cases where $t=\alpha$, or $\alpha>t$. The presence of a network effect term, however, has the opposite effect of $t$. As the value of the next consumer increases, so does the business-stealing effect, incentivizing the firms to locate closer to the center. A dominant $\alpha$ thus leads to less differentiation than cases where $t=\alpha$ or $t>\alpha$, but the model only holds for positive values of $\alpha-t$ up to a certain point. ${ }^{37}$

Extending the model from one-sided to two-sided, we expected the firms to locate even closer to the center of the Hotelling line. The intuition is that the value of the next consumer increases when the platform can profit from her on two market sides at once, further increasing the business-stealing effect relative to the strategic effect. The results

[^27]from our two-sided model confirms our expectation: adding a positive cross-group network effect to the model increases the relative importance of the business-stealing effect. ${ }^{38}$ From the platform's profit function, we see that the total profit margin for the platform is larger in the two-sided market whenever $\beta^{B}>0$ and $\rho-\sigma \beta^{B}>c^{B}$, compared to in the one-sided model. We have established that $\beta^{B}>0$ and we assume that $\rho-\sigma \beta^{B}>c^{B}$ must hold as well. ${ }^{39}$ The profitability from reaching the next A -side user increases relative to in the one-sided model, incetivizing the firms to locate closer to each other on the Hotelling line. This finding confirms those of similar studies of differentiation in two-sided markets (see e.g. Kind et al., 2013).

We have assumed identical marginal costs for our two platforms. Allowing for different marginal costs would complicate our model without adding significant insight to the questions we are interested in answering. In our numerical simulations, we assume marginal costs equal to zero, as this simplification does not take away from our analysis. Our choice in this is supported by previous literature on this field (Dietl et al., 2013; Kind et al., 2013). Furthermore, keeping online social networks in mind, we consider this simplification to be sufficiently close to reality.

We added the network effect as a linear term in the consumer's utility function. Theoretical and empirical studies of network effects suggest that a more realistic approach would be for the term to have a concave form (Shapiro \& Varian, 1998; Asvanund et al., 2004). We do not set out to establish the magnitude of the network effect in our model. Including the effect as a concave function (e.g. $\alpha \sqrt{D_{i}^{e}}$ ) would indeed yield qualitatively the same results, that is a reduction in the firms' differentiation compared to the benchmark. Therefore, we accept this shortcoming in terms of realism, as it allows us to present a less complicated model.

In chapter 3, we discuss why we use of quadratic travel costs. Convex transportation costs imply that the negative impact increased travel has on a consumer's utility, is increasing

[^28]in distance. We are not convinced that this is an accurate assumption. One empirical study suggests that the consumer's travel cost may in fact have a concave shape, but more extensive research on this area would be of interest (Davis, 2006). A theoretical analyzis of the impact of a concave travel cost function, we leave for future research. A similar study to ours, using linear transportation $\operatorname{costs}\left(\tau d_{i}\right.$, with $\left.\tau \geq 0\right)$, also finds that positive network effects makes competition more fierce, driving prices down for the two firms (Navon et al., 1995).

### 4.1 Model limitations

It is a weakness of our model that we are unable to solve stage 1 analytically. The expression we obtain for the first-order condition, is so messy it does not solve for one optimal value of $a$ or $b .^{40}$ This is true for both the one-sided version and the twosided version of the model. A numerical approach allows us nonetheless to evaluate the first-order condition around certain values of $a, b$, and to check when the second-order condition holds (when $\frac{\partial^{2} \pi_{i}}{\partial a^{2}}<0$ ). We employ this approach for the three different cases we examine: $t=\alpha, t>\alpha$ and $t<\alpha$. This enables us to obtain results for analysis, even in the absence of one optimal solution. Through numerical analysis, we do find an optimal solution for case 1 in the one-sided model, where we find that the optimal location for the two firms is $a=b=0$.

An implication of using a model with quadratic transportation costs, is that it fails to show an outcome where one firm corners the market. If $\alpha$ is sufficiently strong to reverse the result in our model (when $\alpha>3 t(1-a-b)$ ) we find that the second-order condition does not hold. We can therfore not draw any conclusions for when the network effect is overpowering the transportation cost. It is however reasonable to believe that strong network effects can lead to a monopolization of the market as Navon et. al. (1995) prove in their paper.

[^29]In our model, the firms move simultaneously, and only once. A dynamic framework might have opened up for different, perhaps more realistic, strategies, such as a firm allowing negative profits for one period in order to capture market share, enabling the firm to profit from a dominant position in later periods. Allowing for potential asymmetric outcomes by e.g. keeping one of the firms' location fixes, could yield interesting interpretations. First-mover advantages are assumed to exist when network effects are sufficiently strong, as discussed in chapter 2.2.4. From the historical examples in chapter 2.2 .5 we have seen that it is not always crucial to be the first to enter a market with network effects, both in one-sided and two-sided markets. Further developing our model to account for asynchronous market entry could be useful in investigating these mechanisms.

### 4.2 Possible extensions of the model

We assume in our model that each consumer on the Hotelling line buys exactly one product. In the context of social networks, this assumption is likely to be flawed. In fact, in 2017, the average user had 7.6 different social media accounts (Statista, 2019a). An interesting extension of our model, would be to allow for multi-homing, as we discuss in chapter 2.3.4. Based on other Hotelling models that allow for multi-homing, we expect that the level of differentiation would be even less compared to our results, as the relative value of exclusive consumers decreases (Anderson et al., 2017; Kim \& Serfes, 2006).

Another interesting extension would be to extend our model to account also for a vertical dimension. Allowing the firms to invest in some quality parameter, similar to what Kind et al. (2013) does in the first stage of their model, could allow us to investigate how network effects affect not only the level of horizontal differentiation, but also the firms' incentive to vertically diffentiate by investing in quality.

Finally, we have not included negative cross-group network effects like ad aversion in our model. Empirical studies indicate that this assumption rings true, at least for some industries. In the case of social media, there may be reason to do so. According to a
survey, $74 \%$ of respondents dislike the presence of ads on social media platforms (Gitlin, n.d.). Facebook's CEO alledgedly told his investors that the company is "nearing the limit of the number of ads it can show in people's News Feeds before they abandon the app or website in search of something with fewer commercial interruptions" (Levy, n.d.). It is worth noting that most social networks that are two-sided today, started out without advertisement. Facebook, for instance, waited until it had 50 million users - three and a half years - before ads where introduced on the platform (Facebook, n.d.). In line with the theory on critical mass in networks, it is not unreasonable to assume that Facebook waited until the mass was sufficiently large for the network to sustain, before introducing ads to the platform.

There are two possible explanations for such a strategy. First of all, Facebook overcame the "getting both sides on board" problem two-sided platforms face by concentrating solely on users the first few years. Second, if users are averse to ads, it may be in Facebook's interest to ensure a large enough demand that the net effect on utility is positive after ads are introduced to the platform. To elaborate on the last point, we can imagine a two-period game where the platform is one-sided in the first period, and two-sided in the second period. While we assumed perfect foresight in the static model, it would be natural to assume the demand expectation for period 2 to be based the observed demand in period 1 (in the style of Sun \& Tse, 2007). Recalling the user's utility function from chapter 3.2 , including a term denoting aversion for ads, $\beta^{A}$, we get utility function: $U_{i}^{A}=v^{A}-p_{i}^{A}-t\left(d_{i}^{A}\right)^{2}+\alpha^{A} D_{i}^{A e}-\beta^{A} \beta^{B}$. Now, $\beta^{B}$ is still the amount of ads faced by each side-A user, and $\beta^{A}$ is the disutility a side-A user experiences from exposure to ads. In such a two-period framework, the platform could make sure that $\beta^{A} \beta^{B}<\alpha^{A} D_{i}^{A e}$ in the second period, by affecting $D_{i}^{A e}$ in the first. In other words, if the potential user number 50 million-and-one observed Facebook's demand, and expected it to continue to have a large demand in the future, the positive within-group network effect may outweigh the negative cross-group network effect from the introduction of ads. The start-up dynamics of two-sided platforms with both positive and negative cross-group externalities is indeed a topic that would be interesting to investigate further.

## 5 Conclusion

Our aim with this thesis is to add to the literature on two-sided markets by investigating platforms where direct network effects are present. We believe that the dominant position online social networks has come to have, both in people's lives, and in the market for advertisement, necessitates investigation of their strategic behavior. We survey the general theory on network effects, and we present a theoretical model that is not industry-specific. Still, it is interesting to discuss the relevance our model and results have for social media platforms. Studying the worlds' most used networks, these are differentiated along several dimensions. China's censoring of global giants like Facebook, allows for some degree of geographical variation in use. But even among the platforms with global reach, we see tendencies of horizontal differentiation. Specializing a platform to particular uses, like YouTube's focus on videos, or WhatsApp, Messenger and WeChat focusing primarly on chatting and online calling, is one example. Another is catering to certain user groups, like the way LinkedIn separates from e.g. Facebook by focusing on professional networks. In other words, minimal differentiation does not seem to be the case. In fact, the presence of the direct network effect may indeed necessitate such differentiation in order to overcome the winner-takes-it all outcome, as discussed in the case of Facebook and MySpace in chapter 2.2.5.

Our results suggest that including advertisement on the platform further incentivizes the firms to differentiate less. This implies that social networks, evolving from one-sided to two-sided platforms, are becoming more like each other. A possible tendency for social networks to offer increasingly similar features, may support our findings. Take the popular feature stories as an example. ${ }^{41}$ Snapchat was the first platform to launch stories in 2013, but following its polularity, Instagram followed in 2016, and Facebook in 2017 (Newton, 2017, March 28). Other examples of the dispersion of popular features, include how previously chat-oriented networks like WhatsApp and Facebook Messenger

[^30]have introduced the opportunity to make online calls and videochat, a position formerly dominated by Skype. ${ }^{42}$ Conversely, picture-and-video-sharing platforms Instagram and Snapchat have come to allow users to chat verbally on their platforms.

When studying social media platforms, we find that including particularly multi-homing and negative cross-group network effects to the model could be benefitial for the analysis. Other theoretical models which include these features suggest that this may lead competing firms to further decrease their level of differentiation. We leave these model extensions for future studies.

[^31]
## Appendix

## One-sided model: calculations

The consumers' utility function buying from firm 1:

$$
U_{1}=v-p_{1}-t(x-a)^{2}+\alpha D_{1}^{e}
$$

The consumers' utility function buying from firm 2:

$$
U_{2}=v-p_{2}-t(1-b-x)^{2}+\alpha D_{2}^{e}
$$

Calculating demand functions by finding the indifferent consumer in point $\bar{x}$ :

$$
U_{1}=U_{2}=>v-p_{1}-t(\bar{x}-a)^{2}+\alpha D_{1}^{e}=v-p_{2}-t(1-b-\bar{x})^{2}+\alpha D_{2}^{e}
$$

Solving this equation for $\bar{x}$, yields the demand for firm 1:

$$
\bar{x}=a+\frac{1-a-b}{2}-\frac{p_{1}-p_{2}}{2 t(1-a-b)}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{2 t(1-a-b)} \equiv D_{1}
$$

Demand for firm 2:

$$
1-\bar{x}=b+\frac{1-a-b}{2}-\frac{p_{2}-p_{1}}{2 t(1-a-b)}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{2 t(1-a-b)} \equiv D_{2}
$$

Inserting the demand function, yields firm 1's profit function:

$$
\pi_{1}=\left(p_{1}-c\right)\left[a+\frac{1-a-b}{2}-\frac{p_{1}-p_{2}}{2 t(1-a-b)}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{2 t(1-a-b)}\right]
$$

Firm 2' profit function:

$$
\pi_{2}=\left(p_{2}-c\right)\left[b+\frac{1-a-b}{2}-\frac{p_{2}-p_{1}}{2 t(1-a-b)}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{2 t(1-a-b)}\right]
$$

Stage 2

First-order condition:

$$
\frac{\partial \pi_{1}}{\partial p_{1}}=a+\frac{1-a-b}{2}-\frac{p_{1}-p_{2}}{2 t(1-a-b)}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{2 t(1-a-b)}-\frac{p_{1}-c}{2 t(1-a-b)}=0
$$

Solving the f.o.c. for $p_{1}$ :

$$
p_{1}\left(p_{2}\right)=\frac{t(1-a-b)(3-b+a)}{2}+\frac{p_{2}+c}{2}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{2}
$$

Similarly, for firm 2, the f.o.c. is:

$$
\frac{\partial \pi_{2}}{\partial p_{2}}=b+\frac{1-a-b}{2}-\frac{p_{2}-p_{1}}{2 t(1-a-b)}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{2 t(1-a-b)}-\frac{p_{2}-c}{2 t(1-a-b)}=0
$$

Solving for $p_{2}$ :

$$
p_{2}\left(p_{1}\right)=\frac{t(1-a-b)(3-a+b)}{2}+\frac{p_{1}+c}{2}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{2}
$$

Establishing that prices are strategic complements:

$$
\frac{\partial p_{1}\left(p_{2}\right)}{\partial p_{2}}=\frac{\partial p_{2}\left(p_{1}\right)}{\partial p_{1}}=\frac{c}{2}>0
$$

Solving the equation set $p_{1}\left(p_{2}\right)$ and $p_{2}\left(p_{1}\right)$ for $p_{1}$ and $p_{2}$ yields:

$$
p_{1}^{*}=c+\frac{t(1-a-b)(3-b+a)}{3}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{3}
$$

and

$$
p_{2}^{*}=c+\frac{t(1-a-b)(3-a+b)}{3}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{3}
$$

Inserting $p_{1}^{*}$ and $p_{2}^{*}$ into the demand functions, yield:

$$
D_{1}=\frac{3-b+a}{6}+\frac{\alpha\left(D_{1}^{e}-D_{2}^{e}\right)}{6 t(1-a-b)}
$$

and

$$
D_{2}=\frac{3-a+b}{6}+\frac{\alpha\left(D_{2}^{e}-D_{1}^{e}\right)}{6 t(1-a-b)}
$$

Allowing $D_{1}^{e}=D_{1}$, gives:

$$
D_{1}=\frac{t(1-a-b)(3-b+a)-\alpha}{2[3 t(1-a-b)-\alpha]}
$$

Allowing $D_{2}^{e}=D_{2}=1-D_{1}$, gives:

$$
D_{2}=\frac{t(1-a-b)(3-a+b)-\alpha}{2[3 t(1-a-b)-\alpha]}
$$

Letting $D_{1}^{e}=D_{1}$ and $D_{2}^{e}=D_{2}$ in the price functions, yield the optimal prices:

$$
p_{1}^{*}=c+\frac{t(1-a-b)(3-b+a)}{3}+\frac{t(a-b)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha
$$

and

$$
p_{2}^{*}=c+\frac{t(1-a-b)(3-a+b)}{3}+\frac{t(b-a)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha
$$

Stage 1

Inserting the price function from stage 2 into firm 1's profit functions:

$$
\pi_{1}=\left[\frac{t(1-a-b)(3-b+a)}{3}+\frac{t(a-b)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha\right] \frac{t(1-a-b)(3-b+a)-\alpha}{2[3 t(1-a-b)-\alpha]}
$$

Firm 1's first-order condition for stage 1:

$$
\begin{gathered}
\frac{\partial \pi_{1}}{\partial a}=\frac{t a^{2}\left(t a^{2}+2 t a+\alpha+(1-b)(b-3) t\right)\left(9 t^{2} a^{3}+5 t \alpha+(21 b-15) t^{2}\right)}{2[3 t(1-a-b)-\alpha]^{3}} \\
\frac{\left.+\left[(4 b-1) t \alpha+3 t^{2}(b-1)(5 b-1)\right] a+\alpha^{2}+(1-b)(b-4) t \alpha+3(b-1)^{2}(b+1) t^{2}\right)}{2[3 t(1-a-b)-\alpha]^{3}}=0
\end{gathered}
$$

## Two-sided model: calculations

The calculations in stage 3 in the two-sided model is identical to stage stage 2 in the one-sided model. See stage 2 in one-sided calculation above. ${ }^{43}$

Stage 2:

Platform 1's profit function in stage 2:

$$
\Pi_{1}=\left[p_{1}^{A *}-c^{A}+\beta^{B}\left(\rho-\beta^{B} \sigma-c^{B}\right)\right] D_{1}^{A}\left(p_{1}^{A *}, p_{2}^{A *}\right)
$$

Platform 1 find the optimal ad-level:

$$
\begin{gathered}
\frac{\partial \Pi_{1}}{\partial \beta^{B}}=\left(\rho-\beta^{B} \sigma-c^{B}-\beta^{B} \sigma\right) D_{1}=0 \\
\beta^{B *}=\frac{\rho-c^{B}}{2 \sigma}
\end{gathered}
$$

Stage 1

Platform 1's profit function in stage 1:

$$
\Pi_{1}=\left[\frac{t(1-a-b)(3+a-b)}{3}+\frac{t(a-b)(1-a-b)}{3[3 t(1-a-b)-\alpha]} \alpha+\frac{(\rho-c)^{2}}{4 \sigma}\right] \frac{t(1-a-b)(3+a-b)-\alpha}{2[3 t(1-a-b)-\alpha]}
$$

[^32]Platform 1's first-order condition:

$$
\begin{gathered}
\frac{\partial \Pi_{1}}{\partial a}=\frac{3 t^{2}(1-a-b)(3+a-b-\alpha)\left(\frac{t(1-a-b)(3+a-b)}{3}+\frac{\alpha t(1-a-b)(a-b)}{3(3 t(1-a-b)-\alpha)}+\frac{(\rho-c)^{2}}{4 \sigma}\right)}{2(3 t(1-a-b)-\alpha)^{2}} \\
-\frac{t(3+a-b-\alpha)\left(\frac{t(1-a-b)(3+a-b)}{3}+\alpha t(1-a-b)(a-b) 3(3 t(1-a-b)-\alpha)+(\rho-c)^{2} 4 \sigma\right)}{2(3 t(1-a-b)-\alpha)} \\
+\frac{t(1-a-b)\left(\frac{t(1-a-b)(3+a-b)}{3}+\frac{\alpha t(1-a-b)(a-b)}{3(3 t(1-a-b)-\alpha)}+\frac{(\rho-c)^{2}}{4 \sigma}\right)}{2(3 t(1-a-b)-\alpha)} \\
+\frac{t(1-a-b)(3+a-b-\alpha)\left(-\frac{t(3+a-b)}{3}+\frac{\alpha t^{2}(1-a-b)(a-b)}{3 t(1-a-b)-\alpha)^{2}}-\frac{\alpha t(a-b)}{3(3 t(1-a-b)-\alpha)}\right)}{2(3 t(1-a-b)-\alpha)} \\
\quad+\frac{t(1-a-b)(3+a-b-\alpha)\left(\frac{\alpha t(1-a-b)}{3(3 t(1-a-b)-\alpha)}+\frac{t(1-a-b)}{3}\right)}{2(3 t(1-a-b)-\alpha)}=0
\end{gathered}
$$

## Simulation of case 3

Table 2 provide the biggest $\alpha$ for a given value of $t$ where the second-order condition hold in case $3(t<\alpha)$ in the one-sided model. Table 3 is the equivalent for the two-sided model for the parameter values $\rho=2, c^{B}=0$ and $\sigma=1$.

| $a=b=-0.25$ |  | $a=b=0$ |  |
| :---: | :---: | :---: | :---: |
| $t$ | $\alpha$ | $t$ | $\alpha$ |
| 0.1 | 0.16 | 0.1 | 0.12 |
| 0.5 | 0.8 | 0.5 | 0.6 |
| 1.0 | 1.6 | 1.0 | 1.2 |
| 1.5 | 2.5 | 1.5 | 1.8 |
| 2.0 | 3.3 | 2.0 | 2.4 |
| 2.5 | 4.2 | 2.5 | 3.1 |
| 3.0 | 5.0 | 3.0 | 3.7 |
| 3.5 | 5.9 | 3.5 | 4.3 |
| 4.0 | 6.7 | 4.0 | 4.9 |
| 4.5 | 7.5 | 4.5 | 5.6 |
| 5.0 | 8.4 | 5.0 | 6.2 |

Table 2: Case 3 one-sided model

| $a=b=0$ |  |
| :---: | :---: |
| $t$ | $\alpha$ |
| 0.1 | 0.16 |
| 0.5 | 0.8 |
| 1.0 | 1.6 |
| 1.5 | 2.5 |
| 2.0 | 3.3 |
| 2.5 | 4.2 |
| 3.0 | 5.0 |
| 3.5 | 5.9 |
| 4.0 | 6.7 |
| 4.5 | 7.5 |
| 5.0 | 8.4 |

Table 3: Case 3 two-sided model

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[^0]:    ${ }^{1}$ YouTube currently has 1.9 billion, WhatsApp has 1.6, Facebook Messenger has 1.3, WeChat has 1.1, and Instagram has 1 billion users (Statista, 2019c).

[^1]:    ${ }^{2}$ A notable exception is an essay by Evans and Schmalensee (2010).

[^2]:    ${ }^{3}$ We adhere to this practice in the following text, although keeping Hotelling's original notation would have worked just as well.

[^3]:    ${ }^{4}$ The direct effect is also called the demand effect, the market size effect, or the business stealing

[^4]:    ${ }^{5}$ Given that the price is the only criteria for buying or not
    ${ }^{6}$ Shapiro and Varian use 1000 instead of $\phi$, however, we choose to use $\phi$ to make the model applicable for a change in the market size without any loss in the interpretation of the results.

[^5]:    ${ }^{7}$ Note that in the zero size network equilibrium there is no production and therefore no marginal costs.
    ${ }^{8}$ A monopolist that cannot influence the consumers' expectation will naturally provide a smaller network size compared with a monopolist that have the power to influence its consumers (Economides, 1996).

[^6]:    ${ }^{9} \alpha<0$ allows for negative network effects in the model.
    ${ }^{10}$ The owner is the decision maker whose objective is to maximize expected profit of his firm, also known as the principal.

[^7]:    ${ }^{11}$ Sklivas (1987) and Freshtman and Judd (1987) use a contract between profit and revenue, but with linear cost these two contracts are equivalent.
    ${ }^{12}$ The market share is measured as the companies production divided by the total production in the market.

[^8]:    ${ }^{13}$ This is a dilemma also typically seen in a two-sided market, as discussed in chapter 2.3 .

[^9]:    ${ }^{14}$ There are more ways to achieve a first-mover advantage. In this case it can be obtained by either entering first, or be the first one to introduce new technology.

[^10]:    ${ }^{15}$ That is, if the economics of scale are specific to each technology.

[^11]:    ${ }^{16}$ At least one group must experience cross-group network effects for the market to be two-sided.
    ${ }^{17}$ In the literature, indirect network effects are referred to also as cross-group, inter-group, or bilateral network effects.

[^12]:    ${ }^{18}$ Evans (2005) points to physical currency as well as village markets as two-sided platforms facilitating interactions between buyers and sellers in early civilizations.
    ${ }^{19} \mathrm{~A}$ credit card provider is the intermediary between cardholders and the merchants who accept the card as payment.

[^13]:    ${ }^{20}$ Chaillaud and Jullien's model apply to competing matchmakers such as dating agencies, real estate agents and internet "business-to-business" websites. Although general in their analyzis, Rochet and Tirole's paper concerns itself with credit card markets.
    ${ }^{21}$ Specifically, we swap Armstrong's parameter $\alpha$ for $\beta$, as the former is used to express within-group network effects in our text. We also swap subscripts for superscripts, and refer to the two market sides as A and B , rather than 1 and 2 (and competing firms for 1 and 2, rather than A and B ). This is to ensure consistency with the notation used in one-sided duopoly models we use. For the same reason, we let $D$ denote demand, rather than $n$, and $c$ denote marginal cost, rather than $f$.

[^14]:    ${ }^{22}$ See chapter 2.3.5 for a discussion of cases where $\beta^{i}$ is zero or negative for one of the groups.
    ${ }^{23}$ In terms of cost structure, Armstrong's model differs from Rochet and Tirole's (2003), where costs are incurred on a per-transaction basis $\left(c D^{A} D^{B}\right)$, rather than per participant on the platform, $\left(c^{A} D^{A}+\right.$ $\left.c^{B} D^{B}\right)$.

[^15]:    ${ }^{24}$ The envelope theorem describe sufficient conditions for the value of a parameterized optimization problem to be differentiable in the parameter (Milgrom \& Segal, 2002).

[^16]:    ${ }^{25}$ A necessary assumption here is $f_{1}=f_{2}=f$; as Armstrong points out, it makes little sense to discuss price discrimination in the presence of asymmetric cost functions.

[^17]:    ${ }^{26}$ Note that this exempts online newspapers, where the marginal cost of reaching readers should be assumed to be zero.

[^18]:    ${ }^{27}$ Also known as Hotelling's law.

[^19]:    ${ }^{28}$ This result holds for both linear and quadratic transportation costs.

[^20]:    ${ }^{29}$ Note however that the model is too general for one to easily predict how this joint surplus is shared.

[^21]:    ${ }^{30}$ Ads in car magazines, as well as business and politics magazines, are deemed more informative that ads in adult magazine.

[^22]:    ${ }^{31}$ Note that for the model to hold, we require $a+b<1$.

[^23]:    ${ }^{32}$ Conversely, the same result holds for firm 2, if $b>a$.

[^24]:    ${ }^{33}$ See table 2 (p. 73) in the appendix for a range of values $t$ and the corresponding maximal $\alpha$ where the second-order condition holds.

[^25]:    ${ }^{34}$ Following Gabszewicz (2001; 2002) as well as Peitz and Valletti (2008).

[^26]:    ${ }^{35}$ See table 3 (p.73) in the appendix.

[^27]:    ${ }^{36}$ In a symmetric outcome, this term is $t(1-a-b)$ for both firms. Larger values of $a$ and $b$, thus lead to lower prices.
    ${ }^{37}$ See table 2 (p. 73) in the appendix for these values.

[^28]:    ${ }^{38}$ As we cannot obtain the equilibrium location we are unable to prove this for case 3. However, we do not see why this should not hold for $t<\alpha$ as well
    ${ }^{39}$ If $\rho-\sigma \beta^{B} \leq c^{B}$ it would be of no interest to the firms to include ads on their platform.

[^29]:    ${ }^{40}$ See appendix for first-order condition of stage 1 in both versions of the model.

[^30]:    ${ }^{41}$ Stories make it possible for you to share a video or picture available to all your friends for 24 hours before it disappear.

[^31]:    ${ }^{42}$ Snapchat also introduced group video calling in 2018.

[^32]:    ${ }^{43}$ The notation in the two-sided model is slightly different to distinguish between the two consumer groups. In the consumer price firm 1 set is denoted by $p_{1}^{A}$ and the profit $\Pi_{1}$.

