# Optimization of age－structured bioeconomic model：recruitment， weight gain and environmental effects 

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## DISCUSSION PAPER

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# Optimization of age-structured bioeconomic model: recruitment, weight gain and environmental effects 

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#### Abstract

More and more fishery researchers begin to acknowledge that one-dimensional biomass models may omit key information when generating management guidelines. For the more complicated age-structured models, numerous parameters require a proper estimation or a reasonable assumption.

In this paper, the effects of recruitment patterns and environmental impacts on the optimal exploitation of a fish population are investigated. Based on a discrete-time age-structured bioeconomic model of Northeast Atlantic mackerel, we introduce the mechanisms that generate 6 scenarios of the problem. Using the simplest scenario, optimizations are conducted under 8 different parameter combinations. Then, the problem is solved for each scenario and simulations are conducted with constant fishing mortalities.

It is found that a higher environmental volatility leads to more net profits but with a lower probability of achieving the mean values. Any parameter combination that favours the older fish tends to lend itself to pulse fishing pattern. The simulations indicate that a constant fishing mortality around 0.06 performs the best. A comparison between the optimal and the historical harvest shows that for more than $70 \%$ of the time, the optimal exploitation precedes the historical one, leading to $43 \%$ higher net profit and $34 \%$ lower fishing cost.


## Keywords

Age-structured, bioeconomic, recruitment, optimization

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### 1.1 Introduction

Many economic fishery studies have described the state of a population using biomass as the only variable (Schaefer, 1954). Such surplus production models use lumped parameters to describe the stock dynamics. In recent years, fishery biologists and economists widely begin to recognize that such one-dimensional models are too simple for developing realistic management guidelines (Tahvonen, 2008). One concern is about the dangerous tendency to catch small and immature fish (Pauly, Christensen, Dalsgaard, Froese, \& Torres, 1998). Another is that ever increasing fishing pressure may cause various systematic changes in the internal structure and evolution of fish populations (Anderson et al., 2008), which may have crucial economic consequences that may not be captured by the biomass approach.

The age-structured framework is pioneered by Baranov (1945), Beverton \& Holt (1957), and Leslie (1945) among others. Many extensions have been explored since and have dominated fishery management (Hilborn \& Walters, 1991). However, analysing the problem technically is still challenging. Clark (2010) has pointed out that an analytical solution for the general agestructured problem is unattainable. Many models lend themselves to Mathematical Programming (Operational Research) and simulations, but not much to analytical studies. Still, it is possible to formulate a proper model and simulate how different factors influence the optimal fishing mortalities numerically. Our model is inspired by, among others, Walters (1969), Hannesson (1975), Getz \& Haight (1989), Horwood (1987) and Tahvonen (2009).

NEA (Northeast Atlantic) Mackerel (Scomber scombrus) is a fish species with high commercial values. In the 1960s and 1970s, the annual catches of mackerel in the Northeast Atlantic, mainly North Sea, rose steeply, resulting in an extreme drop in the 1980s. Consequently, the mackerel stock has been at low level for many years with poor recruitment. ICES (International Council for the Exploration of the Sea) advices have been proposing limits on the fishing mortality or the size of the catch to improve the situation of the unsustainable harvest. But still in 2014, as in all years since 2008, unilateral quotas have been set higher than the TAC indicated by the ICES Management Plan (ICES, 2014). It is thus of high interest to investigate the management of NEA mackerel.

Among many environmental factors, food availability can be crucial and representative for all species. As typical plankton feeders, mackerel is affected by the abundance, distribution and composition of zooplankton to a large extent (Reid, Walsh, \& Turrell, 2001). Most zooplankton
species have a life span shorter than one year, therefore no age structure is applied for the zooplankton population. The characteristics such as density, average size and distribution of zooplankton that interact with the mackerel stock are many. To simplify the problem reasonably, a single zooplankton index is used to describe prey density or the food availability for mackerel. Note that due to the position of zooplankton as a primary producer in the low trophic level, several assumptions are implied, which may or may not fall into the category of standard predator-prey models (Yodzis, 1994). First, the consumption of zooplankton by mackerel this year will not influence the prey density next year. Due to the many predators that zooplankton has simultaneously, the sole impact from mackerel is difficult to quantify. Second, the food supply of mackerel depends entirely on zooplankton abundance. Other prey species are not included.

Random variations in the environment affect the dynamics of populations through changes in individual life histories (Benton, Lapsley, \& Beckerman, 2002). In this paper, two interactions between the environmental factor (zooplankton index) and the fish population (mackerel) are considered. The first interaction is the influence by the zooplankton abundance on the mackerel recruitment. Mackerel spawns between May and July, which coincides with the zooplankton boom (Lockwood, 1988). It is known that higher food availability can bring down the natural mortality especially of the small fish larva by allowing them to spend more time in the deeper, darker and safer area of the sea. Since this interaction mainly applies for the first-year juveniles, it can be translated to a constant natural mortality plus a varying recruitment influenced by the zooplankton abundance. The argument for the second interaction is the strong and positive connection between weight and price. On the final product market, a mackerel can be called 'large size' if the average weight is 400 to 600 grams (4-6 category); 'small size' if the average weight is 200 to 400 grams (2-4 category). The price differences between categories can be significant. Assuming one price for all age classes is not realistic.

There have been different approaches addressing the two interactions: recruitment and weight. To deal with recruitment, some chose an exogenous and constant recruitment such as (Beverton \& Holt, 1957) and (Clark, 2010). While some assumed that recruitment can be endogenous and stock-dependent, such as (Walters, 1969) and (Getz \& Haight, 1989). In this paper, both approaches will be applied. To deal with weight, some studies such as (Walters, 1969) referred to the von Bertalanffy function as the rule that determines the individual fish weight. This study links mackerel's annual weight gain to the exogenous zooplankton index, which can be either deterministic or stochastic under different scenarios.

Due to its complexity, the age-structured model can be sensitive to many parameters (Horwood, 1987). It is thus necessary to be cautious when it comes to combinations of parameters. In the literature, one issue has been connected to the parameter sensitivity: the choice between smooth, stable harvesting over time and periodic (oscillatory or pulse) fishing (Tahvonen, 2010). Clark (2010) argued that when there exists perfect selectivity, it is optimal to harvest each cohort at its maximum biomass, creating a stable harvest strategy. Hannesson (1975) pointed out that non-selective gear leads to pulse fishing. He also showed that discounting shortens the intervals between fishing periods. Tahvonen (2009) proves that when there are 2 age classes with endogenous recruitment, optimal harvest is pulse fishing under specific conditions such as nonselective gear. Steinshamn (2011) showed that pulse fishing becomes less attractive as the distribution of the species moves from uniform to schooling. Rocha, Gutiérrez, \& Antelo (2012) concluded that imperfect selectivity increases the optimal lifespan and the optimal pulse length.

The approaches employed in this paper are innovative in several aspects. First, while many predator-prey models have been focusing on higher trophic levels (Major, 1978; Von Westernhagen, Westernhagen, \& Rosenthal, 1976), this paper investigates the lower trophic level including the primary production. Second, a series of 8 parameter combinations has been examined before introducing the random environmental interactions, which offers some thought-provoking results. Third, based on a common framework of the model, 6 different scenarios have been applied in order to fully investigate the problem. Last but not least, based on the same population dynamics, both simulation and optimization are conducted, offering deeper understanding of the model.

### 1.2 Model

### 1.2.1 Model formulation

The population dynamics is:

$$
\begin{equation*}
x_{i+1, t+1}=x_{i, t} e^{-\left(m_{i}+s_{i} f_{t}\right)}(i=0,1, \ldots, n-1 ; t=1,2, \ldots, T-1) \tag{1.1}
\end{equation*}
$$

where $x_{i, t}$ is the number of fish individuals of age $i$ at time $t$ measured in millions; $x_{i, 1}$ is given by historical data as the initial status of the stock; $m_{i}$ is the natural mortality of age $i ; s_{i}$ is the selectivity parameter of age $i$ and $f_{t}$ indicates the fishing mortality at time $t$ and is also the decision variable of the model. The dynamics indicates that every year part of the cohort dies out of natural causes and another part is being harvested. Both events are assumed to happen instantaneously. The rest of the cohort survives the year and continue to grow and reproduce.

The maximum age of the fish in the model is denoted by $n$ and $T$ indicates the end period. It is assumed that all fish with age above $n$ will naturally die.

In order to describe the recruitment, the Spawning Stock Biomass (SSB) is calculated as following:

$$
\begin{equation*}
S_{t}=\sum_{i} u_{i} x_{i, t} w_{i, t} \tag{1.2}
\end{equation*}
$$

where $S_{t}$ is the SSB at time $t$ measured in million tonnes; $u_{i}$ is the maturity ogive (proportion of sexually matured individuals in that age class) and $w_{i, t}$ is the individual weight of the average fish of age $i$ at time $t$.

The endogenous recruitment can be generalized as a recruitment function: $x_{0, t+1}=\varphi\left(S_{t}\right)$.

Harvest can be obtained from the well-known Beverton-Holt model:

$$
\begin{equation*}
H_{t}=\sum_{i} h_{i, t}=\sum_{i} \frac{s_{i} f_{t}}{s_{i} f_{t}+m_{i}}\left[1-e^{-\left(m_{i}+s_{i} f_{t}\right)}\right] w_{i, t} x_{i, t} \tag{1.3}
\end{equation*}
$$

where $H_{t}$ is the harvest biomass measured in million tonnes at time $t$ and $h_{i, t}$ is the harvest biomass measured in million tonnes of age $i$ at time $t$.

The problem's objective is to maximize the following:

$$
\begin{equation*}
Z=\max _{f_{t}} \sum_{t=1}^{T} \sum_{i=0}^{n}(1+r)^{-t}\left(p_{i, t} h_{i, t}-c f_{t}\right) \tag{1.4}
\end{equation*}
$$

where $Z$ is the present value of the total net profit, $r$ is the discrete discount rate, $p_{i, t}$ indicates the unit weight price for age $i$ at time $t$ and $c$ is the calibrated cost parameter.

The objective function is subject to the following constraints:

1. Population dynamics: Eq. (1.1)
2. Sustainability constraint: SSB does not fall below a proposed biomass reference point in the end: $S_{T} \geq B_{\text {lim }}$
3. Non-negativity for variables $x_{i, t}, w_{i, t}$ and $f_{t}$
4. Effort restriction (admissible controls defined by harvest capacity): $f_{t} \in F$

The problem is solved in GAMS as a nonlinear programming problem using solver NLP.

According to Steinshamn (2011), the stock elasticity parameters for different fish species vary, resulting in various population dynamics and production functions. The model is concise and easy to analyse in the extreme cases where stock elasticity equals either zero or one. Zero stock elasticity lend itself to pure schooling fishery where production function is independent of the stock. Mackerel, which has a certain schooling behaviour, has a stock elasticity between zero and one. A larger stock level, even for schooling species, naturally leads to higher probability of finding the fish schools given the same level of searching effort. Thus, we believe the production function is not completely stock independent. In addition, another study of a schooling species Norwegian Spring-Spawning Herring uses a similar model (Bjørndal, Gordon, Lindroos, \& Kaitala, 2000). Therefore, a stock elasticity of one is applied in the model.

Optimizing with respect to fishing mortality is equivalent to finding the optimal effort if we follow $f_{t}=q E_{t}$, where $q$ is the catchability parameter. The cost parameter $c$ in the model can be reckoned as the unit cost of effort multiplied by the catchability parameter. Both paradigms are interchangeable common practices. This paper focuses on optimizing the fishing mortality.

### 1.2.2 Two interactions

We introduce the zooplankton index $\rho_{t}$ which is assumed to take the form of a Bounded Random Walk (BRW) (Nicolau, 2002; Vandromme et al., 2011). This specified process ${ }^{1}$ has a mean reverting property around 1 but behaves like a random walk in the range of [0.6,1.4]. Another constraint of $\rho_{t} \in[0.5,1.5]$ is forced in order to avoid extreme outliers.

The first interaction of the zooplankton influencing mackerel's average weight gain goes as follows:

$$
\begin{equation*}
w_{i+1, t+1}=w_{i, t}+\left[0.036\left(\rho_{t}-1\right)+0.055\right] \quad w_{0, t} \approx 0 \tag{1.5}
\end{equation*}
$$

Usually growth rates are difficult to determine from catches because schools are sorted by size and their mobility prevents representative sampling (Skagen, 1989). A small difference between weight of catch and weight of stock exists in the data. It is ignored in the model for simplicity. It is assumed that the weight for the first age class is virtually zero. As the same cohort

[^1]accumulates its weight over time, the zooplankton index $\rho_{t}$ decides how much weight is gained each year for all cohorts.

Fig. 1.1 displays the historical average individual weight of the fish. When it is assumed that $\rho_{t}=1$ for all $t$, the weight gain is constant every year, creating a linear weight development pattern that can be written as:

$$
\begin{equation*}
w_{i+1, t+1}=w_{i, t}+0.055 \tag{1.6}
\end{equation*}
$$

(a) Stock
(b) Catch

Fig. 1.1 Minimum and maximum weight of mackerel by age class from year 1980 to 2014

The possible maximum and minimum individual weights respectively are 0.83 kg and 0.49 kg at the age of 12 in the model. There exist cases where maximum weight is reported to be 3.5 $\mathrm{kg}^{2}$. Such extreme values will not be considered in the model.

The second interaction is about zooplankton affecting mackerel's recruitment. Under different scenarios, recruitment can be: first, exogenously given and fixed as 4500 million, which is the historical mean recruitment from 1980 to 2014; second, governed by a recruitment function; third, exogenous and random from a normal distribution $N(4500,2000)$ based on historical data (ICES, 2014). For the first and third case, the behaviour of zooplankton index does not alter the recruitment. For the second case, we choose the Ricker formula (Ricker, 1954) as an estimation of the recruitment function and include the zooplankton index in a multiplicative form:

$$
\begin{equation*}
\varphi\left(S_{t}, \rho_{t}\right)=\rho_{t} \alpha S_{t} e^{-\beta S_{t}} \tag{1.7}
\end{equation*}
$$

where $\alpha=6.37$ and $\beta=0.52$ for NEA mackerel (Simmonds, Campbell, Skagen, Roel, \& Kelly, 2011).

[^2]

Fig. 1.2 Ricker recruitment function and the historical recruitment data
We see from Fig. 1.2 that the historical recruitment data (square dots) exhibits rather high volatility and spreads widely around the estimated curve. The recruitment curve seems to have limited explanatory power about the relationship between the SSB and the recruitment next year.

### 1.2.3 Scenarios illustration

The zooplankton index can either be deterministic and fixed as 1 (denoted as D) or stochastic as a BRW process (denoted as S). Recruitment can be fixed as 4500 million (denoted as F ) or governed by recruitment curve as in Eq. (1.7) (denoted as C) or random from a normal distribution $N(4500,2000)$ (denoted as R). All scenario combinations are listed in Table 1.1. In scenarios DF , DC and DR , weight gain is governed by Eq. (1.6) while in scenarios SF , SC and SR by Eq. (1.5).

Table 1.1. Scenario illustrations

| Zooplankton index |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  | Deterministic | Stochastic |
| Fixed | DF | SF |  |
| Curve | DC | SC |  |
| Random | DR | SR |  |

### 1.3 Parameters

This section applies the simplest scenario DF where the zooplankton index is deterministic as 1 throughout the entire period. Recruitment is fixed as the historical mean. Individual weight is assumed to have a constant yearly gain as in Eq. (1.6).

### 1.3.1 Parameter specifications

It is a common practice to set the age classes of mackerel from 0 to 12 , where the 0 age class is the recruitment of that year. So $i \in[0,1,2, \ldots 12]$ and $n=12$. The whole modelling period is set to be 100 years in order to gain long term insights of the problem. Thus $t \in[1,2, \ldots T]$ and $T=100$. The original status of the stock for each age class in the model comes from the historical data in year 1980. Discount rate $r$ is set to $5 \%$. The effort constraint can be chosen as $f_{t} \in[0,10]$, which has a high enough upper bound for fishing mortality (Horwood, 1987).

Fishing costs $c$ is calibrated to 23000 in order to obtain a cost-revenue ratio around $70 \%$ : a number that has been observed for pelagic fisheries such as mackerel, herring, blue whiting and capelin (Fiskeridirektoratet). Minimum SSB $B_{\text {lim }}$ is 1.84 million tonnes (ICES, 2014). Maturity ogives of mackerel $u_{i}$ are presented in Table. 1.2.

The three varying parameters are price, natural mortality and selectivity. For such parameters, we either apply a constant number, which is what usually has been done, or utilize the age structure of the model by assigning age-specific or weight-dependent parameter values.

Price of mackerel in Norwegian kroner per kilogram ( $\mathrm{NOK} / \mathrm{kg}$ ) is either constant as the mean price $p=8.46 \mathrm{NOK} / \mathrm{kg}$ or linear as a function of weight: $p_{i, t}=19.87 w_{i, t}$ (Zimmermann \& Heino, 2013). Natural mortality of mackerel is assumed to be a constant $m=0.15$ for all age classes (ICES, 2014) or age-specific: $m_{i}=0.32-0.02 i$.

Selectivity $s_{i}$ is assumed to be either knife-edge selective (denoted $s_{i}^{\prime}$ ) where only classes above a certain age are harvest or non-selective (denoted $s_{i}^{*}$ ) where every age class lend itself to some fishing mortality. Both cases are shown in Table. 1.2. Note that age-class zero is of no interest for harvesting. The non-selective case can be calculated as the age-specific fishing mortality divided by maximum fishing mortality in the same year. According to ICES (2014), there has been a slow shift from selecting older classes to younger classes over time. To embody this shift, we apply the average historical values on the first 35 years and use the values in year 35 for the remaining periods.

Table 1.2 Parameter values for maturity ogive and selectivity

| $i$ | $u_{i}$ | $s_{i}^{\prime}$ | $s_{i}^{*}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $t \in[1, T]$ | $t \in[1,35]$ | $t \in[36, T]$ |
| 0 | 0 | 0 | 0.03 | 0.01 |
| 1 | 0.106 | 1 | 0.1 | 0.04 |
| 2 | 0.539 | 1 | 0.18 | 0.18 |
| 3 | 0.913 | 1 | 0.37 | 0.43 |
| 4 | 0.998 | 1 | 0.64 | 0.72 |
| 5 | 0.999 | 1 | 0.73 | 0.82 |
| 6 | 0.999 | 1 | 0.9 | 0.83 |
| 7 | 1 | 1 | 1 | 1 |
| 8 | 1 | 1 | 1 | 1 |
| 9 | 1 | 1 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 1 | 1 |

### 1.3.2 Parameter combinations

The three varying parameters are combined and explored: constant vs. weight-dependent price; knife-edge selective vs. non-selective; constant vs. age-specific natural mortality. This gives a total of 8 different combinations. It is found that the optimization results are highly sensitive to parameter assumptions. In other words, a small change in parameter combination may lead to rather distinct results.

It seems that weight-dependent price $p_{i, t}$, knife-edge selectivity $s_{i}^{\prime}$ and constant natural mortality $m$ lend themselves to pulse fishing. A possible explanation is that the weight-specific price structure puts higher value on older classes, justifying the waiting period before harvesting. With selectivity $s_{i}^{\prime}$, younger age classes are more vulnerable towards harvesting and this creates a relatively lower fishing pressure for the older age classes. Selectivity parameters applied have in fact very limited difference: both $s_{i}^{\prime}$ and $s_{i}^{*}$ are the same above age 7. However, it induces obvious changes of the results. This paper numerically illustrates the scale of the issue, which should never be underestimated. In many studies, natural mortality is assumed to be constant for all. When the bigger fish has a higher probability to survive, this also gives incentive to wait for the stock to accumulate.


Fig. 1.3 Optimal fishing mortalities for 8 parameter combinations
To sum up, any parameter choice that favours the older age classes, for example by assigning higher value or decreasing the chance of death of older classes, tends to lend itself to pulse fishing pattern.

### 1.4 Optimization

In this section, we choose the parameter combination: price $p_{i, t}$ is weight-dependent, selectivity is $s_{i}^{*}$ and natural mortality is a constant $m$. This combination has a modest tendency towards favouring the pulse fishing pattern. All other parameter values are specified as in Section 1.3.1. All scenarios in Table. 1.1 will be explored and summarized in this section.

### 1.4.1 Mean results

For each scenario of SF, SC and SR, 1000 realizations of the zooplankton index are drawn randomly. Each represents a possible outcome of the environmental development path during the 100 years. The model is treated as a deterministic nonlinear programming problem under each path. Optimization is conducted for each realization and mean results are obtained by taking the average.

Several indicators of the results are used for interpretation. Net profits $Z$ and fishing costs $C$ are measured in million dollars and calculated as the mean from the 1000 scenarios. The fishing $\operatorname{cost} C$ for the whole period is calculated as: $C=\sum_{t}(1+r)^{-t} c f_{t}$.

The average time series of harvest $\bar{H}_{t}$ and stock biomass $\bar{B}_{t}$ are measured in million tonnes and are obtained as the average from the 1000 scenarios. $\bar{H}, \bar{B}$ and $\bar{x}_{0, t}$ are the average harvest, stock biomass and recruitment respectively. Only periods from $t 18$ to $t 90$ are used to calculate the mean results in order to avoid the adjusting phases in the beginning and at the end of the model, which have extremely high volatility. $\sigma_{Z}^{*}$ denotes the standard deviation of the sample for net profits and $\sigma_{C}^{*}$ denotes the standard deviation of the sample for fishing costs.

As illustrated in Table. 1.3 the only modelling difference between scenario DF and DR is the randomness of recruitment. Scenario DR has a higher profit margin on average but with a lower probability of actually reaching the mean. Note that random recruitment in scenario DR is from a symmetric probability distribution around the same mean as DF. The model seems to be able to efficiently capture and utilize the extremely high recruitment to reach higher average profits. It is also noticeable that $\sigma_{C}^{*}$ is generally larger than $\sigma_{Z}^{*}$. This may be explained by the way fishing costs and sales revenues are calculated. On one hand, many elements are involved in determining the sales revenue such as individual weight and stock size, thus smoothing out potential variances. On the other hand, the calculation of total fishing cost is purely linked to fishing mortality, which may have high volatility.

Scenario DC results in the lowest net profit, lowest cost and poorest harvest and stock biomass. This is mainly due to the poor recruitments. If the recruitments generated by the model strictly follow the estimated Ricker curve, it ends up with only $74 \%$ of what has been the historical average. With the fact that recruitment data usually is rather volatile, letting a single recruitment function to take over seems an unreliable approach leading to a pessimistic scenario.

Table 1.3 Mean results
( $Z$ and $C$ are net profit and fishing cost for the whole period. $\sigma^{*}$ represents the standard deviation of the sample. $\bar{H}$ is the average harvest in million tonnes. $\bar{B}$ is the average stock biomass in million tonnes. $\bar{x}_{0, t}$ is the average recruitment in millions. Mean values are obtained from $t 18$ to $t 90$.)

| Indicators | Scenario | DF | DC | DR | SF | SC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Z$ | 12504 | 8761 | 14222 | 12955 | 9491 | 14596 |
| $C$ | 29817 | 23733 | 30843 | 29954 | 24442 | 30821 |
| $\sigma_{Z}^{*}$ | $/$ | $/$ | 1497 | 2228 | 1851 | 2811 |
| $\sigma_{C}^{*}$ | $/$ | $/$ | 1884 | 2860 | 4199 | 3164 |
| $\bar{H}$ | 0.2 | 0.1 | 0.22 | 0.19 | 0.13 | 0.23 |
| $\bar{B}$ | 5.17 | 4.15 | 5.16 | 5.19 | 4.14 | 5.16 |
| $\bar{x}_{0, t}$ | 4500 | 3308 | 4509 | 4500 | 3290 | 4498 |

Compared to DF, scenario SF lends itself to slightly higher values of average net profit and fishing cost. The improvements are rather trivial. It seems that a nonlinear individual weight development path may not create huge differences in the results. However, note that the increases in $\sigma_{Z}^{*}$ and $\sigma_{C}^{*}$ are much higher between scenario SF and DF than between DR and DF , which is mainly caused by varying annual weight gains. This implies that when recruitment is fixed, the randomness of weight gain, which may be presumably small, is transferred to the volatility of the value as well as cost of harvesting.

Similar to DC, scenario SC has a poor performance: in more than $70 \%$ of the time, scenario SC leads to lower profits than SR. However, scenario SC has a higher net profit than DC on average due to the introduction of the random environmental factor. Scenario SR, similar to DR, has the highest profit and cost on average all the scenarios. The varying annual weight gain almost doubles $\sigma_{Z}^{*}$ and $\sigma_{C}^{*}$ by switching from DR to SR , which is not a surprise given two sources of randomness in the model.

To sum up, higher volatility of the zooplankton index, implying either varying recruitment or volatile weight gains, leads to higher net profits on average but together with a lower probability of actually hitting the mean values. It can be interpreted as the risk upon the fishing industry brought by nature. When recruitment is fixed, volatile weight gains cause considerable increase of $\sigma_{Z}^{*}$ and $\sigma_{C}^{*}$. When weight gain is constant, random recruitment also lends itself to larger volatility of profits and costs. Strong and extremely good recruitment can be utilized by the model to reach better profits. Net profits usually have smaller variances than fishing costs mainly due to the structure of the model. Recruitment governed by a recruitment function tends to lead to the weakest zero age-class, thus the poorest overall performance.

### 1.4.2 Time series results

Fig. 1.4 displays the mean estimated optimal fishing mortality time series of 1000 realizations under each scenario. Scenarios DF, DC, SC and SF exhibit various scales of pulse fishing pattern while DR and SR have a more stable harvest. For DR and SR, the normally distributed recruitments are generated with a rather large standard deviation, leading to widely distinctive optimization results. With all the realizations, they simply cancel out and smoothed out when the mean time series are presented.

The volatility of simulated recruitments in scenario SC is roughly 600 while this number is about 2000 in DR and SR. When the volatility of recruitment is zero (scenario SF) or relatively small (scenario SC), the pulse fishing patter clearly remains even after taking the average. With a fixed recruitment as in SF , the suggested fishing activity is periodic and reaches a peak about every 9 years. Yet the fishing mortality peaks are of moderate levels. For scenario SC, the recruitment is comparatively weaker, so the waiting time or moratorium period is longer. About every 14 years, the time series of scenario SC suggest quite a strong harvesting action.

A similar conclusion could be drawn when comparing scenario DF and DC. A guaranteed recruitment offers the possibility for more frequent fishing activities and lower fishing mortality in general.

While the time series results for scenario DC and SC are extremely alike, more distinctions can be found between DF and SF. When the weight gain is stochastic, the 'waiting period' is not strictly moratorium and the strong harvest years becomes more conservative.


Fig. 1.4 Mean estimated optimal fishing mortalities

### 1.5 Simulation

A straightforward policy that is easy to implement in reality is a constant fishing mortality. In this section, fishing mortalities from 0.02 to 0.1 are assessed through simulations of the stock in a period of 100 years under the 6 scenarios. All parameters are the same as in Section 1.4.

Simulating with a constant fishing mortality has been a tool for analysing such bioeconomic problems. Bjørndal, Ussif, \& Sumaila (2003) investigated the Norwegian spring spawning herring (NSSH) and found that with a time horizon of 20 years, a constant fishing mortality of 0.15 is economically optimal. They assumed that the price of the fish is a constant and applied 20 years as the simulation period, which is much shorter compared with this study.

As shown in Fig. 1.5, regardless of the choice of scenarios, net profit is maximized when fishing mortality is around 0.06 . In the management plan simulations of ICES advice 2015, the NEA
mackerel stock is simulated with different target fishing mortalities from 0.2 to 0.35 . No economic elements are accounted for in these simulations. This paper promotes a much lower fishing mortality than what is being evaluated in the ICES advice. One aspect that potentially contributes to this difference is the weight-specific price structure. This assumption includes crucial economic aspects that could be neglected in pure biological studies. When the fishing mortality is high and constant, the composition of the entire population may shift towards smaller individuals, leaving the cohorts limited time to accumulate body weight in time and therefore lose its value.
(a) Net profits
(b) Harvest biomass

Fig. 1.5 Simulated net profits and harvest biomass under different fishing mortalities for each scenario

The level of net profits is largely influenced by the overall recruitment and the random environmental factor. In scenario DC and SC where recruitment is determined by the Ricker function, the recruitments are systematically lower, leading to smaller profits and smaller optimal fishing mortality. In the other scenarios, DF has the lowest profits, which goes in line with the findings from Section 1.4.1. Compared to Table. 1.3, simulations with a fixed fishing mortality lead to average net profits that are at least $1 \sigma_{Z}^{*}$ lower than the optimization results.

The curve of total harvest biomass against fishing mortality in Fig. 1.5(b) is slightly concave. Since the fishing mortality is kept constant for years in the simulation, a heavier harvesting gives rise to a smaller stock biomass and smaller individuals. When the stock is harvested to a poor level, even large fishing effort will still have little return. This may explain what can be observed in Fig. 1.5(a) and (b): the more we manage to harvest, the worse it becomes regarding net profits once the fishing mortality goes beyond the optimal level.

### 1.6 Historical vs. optimal harvest

It is of interest to apply some real data to the model and make comparison between historical harvest and optimal harvest offered by the optimization model. Similar to scenario SC, this section assumes that the zooplankton index is stochastic and recruitment is governed by a recruitment curve as in Eq. (1.7), in order to fully capture the two interactions of recruitment and weight gain. Parameter $T$ is changed to 40 years in order to cover the available data from 1980 to 2014. Fishing cost $c$ is adjusted to 11000 .


Fig. 1.6 Results for scenarios $H T$ and $O P$
We use ' $H T^{\prime}$ ' to indicate the results of historically applied harvest and ' $O P^{\prime}$ ' for the optimal harvest. Both have the same random number generator seed. Since the stock has never been managed under a pulse fishing regime with consecutive years of strict moratorium, the optimal exploitation requires certain constraint in order to remain practical and comparable. To offer relevant proposals, an extra constraint of $k_{1} \leq \frac{f_{t+1}}{f_{t}} \leq k_{2}$ is added in the optimization, where
$k_{1}=0.75$ and $k_{2}=1.25$ are the minimum and maximum annual change rate respectively from historical data.

As shown in Table. 1.4, HT leads to $43 \%$ lower profit, $34 \%$ higher cost and a larger cost-revenue ratio on average. In addition, with more than twice the fishing mortality and 1.6 times the harvest biomass of OP, HT maintains $32 \%$ lower stock biomass level. The historical exploitation is economically inefficient and biologically unsustainable.

As presented in Fig. 1.6, it is no surprise that the results of HT show that NEA mackerel stock had been harvested unsustainably. The stock biomass kept decreasing to a minimum level around 2.4 million tonnes in year 2005. In year 1994 and 2003, fishing mortality peaked to 0.37 and 0.46 respectively. After the second peak, fishing mortality came down to around 0.22 , leading to a slight recover in stock biomass.

The fishing mortalities in Fig. 1.6(b) can be the proposal for management plan of the NEA mackerel from our model. It not only leads to a higher net profit but also a higher and more stable stock biomass, which is crucial for a healthy stock structure and sustainable fishery resource management. The proposed harvest policy secures both biological and economical potential of the stock, diminishing the possibility of potential population collapse.

Table 1.4 Results for scenarios $H T$ and $O P$
( $Z$ and $C$ are net profit and fishing cost for the whole period. $\sigma^{*}$ represents the standard deviation of the sample. $\bar{H}$ is the average harvest in million tonnes. $\bar{B}$ is the average stock biomass in million tonnes. $\bar{x}_{0, t}$ is the average recruitment in millions. $\bar{f}$ is the average fishing mortality. Mean values are obtained from $t 15$ to $t 35$.)

| Indicators | Scenario | $H T$ |
| :---: | :---: | :---: |
| $Z$ | 15944 | 28216 |
| $C$ | 47309 | 31067 |
| $\sigma_{Z}^{*}$ | 4080 | 2769 |
| $\sigma_{C}^{*}$ | 396 | 388 |
| $\bar{H}$ | 0.44 | 0.28 |
| $\bar{B}$ | 2.76 | 3.63 |
| $\bar{x}_{0, t}$ | 4369 | 3778 |
| $\bar{f}$ | 0.312 | 0.118 |

As Hannesson (2011) pointed out: 'What pulse fishing means is that a stock of fish is fished down heavily for a short period of time and then left to replenish itself for a longer period. But what does the industry do in the meantime?' The added constraint in this section successfully limits the variations of fishing mortality over time.

It is shown in Table. 1.5 that as the constraint gets tighter, both profit and cost tend to decrease; the shadow cost for having the constraint rises. Still, the scale of the constraint shadow cost is low: within $10 \%$. At a relatively low cost, the constraint has made the proposal more realistic since stable quotas are favoured by fishers as they enable the decision making to be more predictable and correct. When the annual change rate of fishing mortality is within $2 \% ~\left(k_{1}=\right.$ 0.98 and $k_{2}=1.02$ ), it is still valid that in more than $70 \%$ of the time, $O P$ results in higher average net profits than $H T$. It seems possible to obtain better results than $H T$ if the harvest strategy is to apply a constant fishing mortality level. Such a simple strategy can have limited risk, higher mean profits and more straightforward implications for the sector.

Table 1.5 Results for $O P$ under various sets of $k_{1}$ and $k_{2}$
( $Z$ and $C$ are net profit and fishing cost for the whole period. Shadow cost is calculated as the percentage difference of the objective value $Z$ with (set 1-4) and without (set 5) the underlying constraint. $\sigma^{*}$ represents the standard deviation of the sample.)

| Parameter sets | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $k_{1}$ | 0.75 | 0.9 | 0.95 | 0.98 | $+\infty$ |
| $k_{2}$ | 1.25 | 1.1 | 1.05 | 1.02 | $-\infty$ |
| $Z$ | 28216 | 27934 | 27512 | 26894 | 29332 |
| $C$ | 31067 | 30451 | 30547 | 30209 | 31770 |
| Shadow cost | $3.8 \%$ | $4.8 \%$ | $6.2 \%$ | $8.3 \%$ | $/$ |
| $\sigma_{Z}^{*}$ | 2769 | 2748 | 2825 | 2942 | 2903 |

### 1.7 Conclusion

This paper sets up an age-structured bioeconomic model of NEA mackerel and introduces an environmental factor that affects the weight gain process and the yearly recruitment of the population. To deal with such a complex model, certain assumptions are required. All the simplifications are not inventions but based on existing studies. Several methods have been implemented to analyse the problem: simple optimization of nonlinear programming problem;
extracting average results from multiple optimizations; simulations and comparison between historical and optimal policy.

It is found that weight-dependent price, knife-edge selectivity and constant natural mortality promote pulse fishing as the optimal exploitation pattern. Any parameter combination that advocates the older age classes encourages the policy to give time for the population to accumulate biomass before removing a large quantity in a short period. The behaviour of the optimal policy is quite sensitive to the combination of parameters. The challenge for such agestructured models lies not only in estimating the numerous age-dependent parameters, but also in choosing the assumptions that sway the results to the minimum extent.

The mean results from multiple optimizations suggest that a higher volatility of the environmental factor leads to higher net profits on average but with a smaller likelihood of achieving the mean values. When recruitment is constant, the seemingly small stochasticity in the weight gaining process causes huge volatility in net profits and fishing costs. Although the Ricker recruitment function is estimated from the historical data in a mathematically sound manner, still it seems oversimplified for generating reasonable recruitments. In our numerical example, even with a certain amount of randomness involved, the recruitment curve constantly produces the most pessimistic scenarios of all.

When a constant policy is applied for simulations, it is found that a fishing mortality around 0.06 produces the highest net profits. This result promotes a much more conservative policy for NEA mackerel than what ICES reports have examined. The simulations yield at least one $\sigma_{Z}^{*}$ lower net profits than optimizations regardless of the scenario, which is an expected situation. In reality, a stable and predictable fishing pattern without intense harvesting peaks is practical and valuable to the industry and management of the resource.

Comparisons between the historical and the optimal harvest prove that the past exploitation has been economically inefficient and biologically unsustainable. A proper and constant fishing mortality policy has the possibility to exceed the performance of the historical harvest. When conducting the optimization in this section of the paper, an extra constraint on the annual change rate of the decision variable is attached in order to smooth out the policy. The cost of having such a constraint is inexpensive: within $10 \%$ of the total profit. The comparison manifests that in more than $70 \%$ of the time, the optimal harvest offered by the model leads to $43 \%$ higher net profit and $34 \%$ lower fishing cost.

## Appendix

Table A. Notations and parameter values

| Subscripts |  | Definitions |
| :---: | :---: | :---: |
| $i$ |  | Age |
| $t$ |  | Time/Period |
| Variables |  |  |
| $x_{i, t}$ |  | Number of individuals of age $i$ at time $t$ (millions) |
| $f_{t}$ |  | Fishing mortality at time $t$ (decision variable) |
| $s_{i}$ |  | Selectivity at age $i$ |
| $w_{i, t}$ |  | Average individual weight of age $i$ at time $t$ (tons) |
| $h_{i, t}$ |  | Harvest biomass at time $t$ of age $i$ (million tons) |
| $H_{t}$ |  | Harvest biomass at time $t$ (million tons) |
| $S_{t}$ |  | Spawning Stock Biomass (SSB) at time $t$ (million tons) |
| $p_{i, t}$ |  | Unit price of age $i$ at time $t$ (1000NOK/ton) |
| $\rho_{t}$ |  | Zooplankton index |
| Parameters | Values |  |
| $m$ | 0.15 | Natural mortality (same for all age classes) |
| $m_{i}$ | $0.32-0.02 i$ | Natural mortality at age $i$ |
| $n$ | 12 | Maximum age class |
| $T$ | 100 | End period |
| $u_{i}$ | Table 1.2 | Proportion of sexually matured individuals at age $i$ |
| $s_{i}^{\prime}$ | Table 1.2 | Knife-edge selectivity |
| $s_{i}^{*}$ | Table 1.2 | Non-selective selectivity |
| $\alpha, \beta$ | 6.37; 0.52 | Parameters for the Ricker's recruitment function |
| $p$ | 8.46 | Constant unit price (NOK/kilo) |
| $r$ | 0.05 | Discount rate |
| c | 23000 | Calibrated cost parameter for pure schooling fisheries |
| $B_{\text {lim }}$ | 1.84 | Biomass reference limit (million tons) |
| $F$ | [0,10] | Fishing mortality range |

## References

Anderson, C. N. K., Hsieh, C., Sandin, S. A., Hewitt, R., Hollowed, A., Beddington, J., ... Sugihara, G. (2008). Why fishing magnifies fluctuations in fish abundance. Nature, 452(7189), 835-839. https://doi.org/10.1038/nature06851

Baranov, F. I. (1945). On the Question of the Biological Basis of Fisheries: On the Question of the Dynamics of the Fishing Industry. Indiana University.

Benton, T. G., Lapsley, C. T., \& Beckerman, A. P. (2002). The population response to environmental noise: Population size, variance and correlation in an experimental system. Journal of Animal Ecology, 71(2), 320-332. https://doi.org/10.1046/j.13652656.2002.00601.x

Beverton, R. J. H., \& Holt, S. J. (1957). On the Dynamics of Exploited Fish Populations. Ministry of Agriculture, Fisheries and Food.

Bjørndal, T., Gordon, D. V., Lindroos, M., \& Kaitala, V. (2000). International management strategies for a migratory fish stock: A bio-economic simulation model of the Norwegian spring-spawning herring [Working paper]. SNF / Centre for fisheries economics.

Bjørndal, T., Ussif, A.-A. M., \& Sumaila, U. R. (2003). A bioeconomic analysis of the Norwegian spring spawning herring (Nssh) stock [Working paper]. SNF.

Clark, C. W. (2010). Mathematical Bioeconomics: The Mathematics of Conservation (3 edition). Hoboken, N.J: Wiley.

Fiskeridirektoratet. (n.d.). Profitability survey on the Norwegian fishing fleet. Retrieved from 2014.

Getz, W. M., \& Haight, R. G. (1989). Population Harvesting: Demographic Models of Fish, Forest, and Animal Resources. Princeton University Press.

Hannesson, Rognvaldur. (1975). Fishery Dynamics: A North Atlantic Cod Fishery. The Canadian Journal of Economics / Revue Canadienne d'Economique, 8(2), 151-173. https://doi.org/10.2307/134113

Hannesson, Rögnvaldur. (2011). Forty years of fisheries economics. In fisheries economics and management. Future challenges. 100 years after Warming's on rent fishing grounds. Book of Abstracts 35:44. Institute of Food and Resource Economics, University of Copenhagen.

Hilborn, R., \& Walters, C. J. (Eds.). (1991). Quantitative Fisheries Stock Assessment: Choice, Dynamics and Uncertainty (1 edition). New York: Springer.

Horwood, J. W. (1987). A calculation of optimal fishing mortalities. Journal Du Conseil, 43(3), 199-208. https://doi.org/10.1093/icesjms/43.3.199

ICES. (2014). Report of the Report of the Working Group on Widely Distributed Stocks (WGWIDE). ICES Headquarters, Copenhagen, Denmark.
Leslie, P. H. (1945). On the use of matrices in certain population mathematics. Biometrika, 33, 183-212.
Lockwood, S. J. (1988). The mackerel. Its biology, assessment and the management of a fishery.
Major, P. F. (1978). Predator-prey interactions in two schooling fishes, Caranx ignobilis and Stolephorus purpureus. Animal Behaviour, 26, 760-777. https://doi.org/10.1016/0003-3472(78)90142-2
Nicolau, J. (2002). Stationary Processes That Look like Random Walks: The Bounded Random Walk Process in Discrete and Continuous Time. Econometric Theory, 18(1), 99-118.

Pauly, D., Christensen, V., Dalsgaard, J., Froese, R., \& Torres, F. (1998). Fishing Down Marine Food Webs. Science, 279(5352), 860-863. https://doi.org/10.1126/science.279.5352.860
Reid, D. G., Walsh, M., \& Turrell, W. R. (2001). Hydrography and mackerel distribution on the shelf edge west of the Norwegian deeps. Fisheries Research, 50(1-2), 141-150. https://doi.org/10.1016/S0165-7836(00)00247-2

Ricker, W. E. (1954). Stock and Recruitment. Journal of the Fisheries Research Board of Canada, 11(5), 559-623. https://doi.org/10.1139/f54-039

Rocha, J.-M. D., Gutiérrez, M.-J., \& Antelo, L. T. (2012). Selectivity, Pulse Fishing and Endogenous Lifespan in Beverton-Holt Models. Environmental and Resource Economics, 54(1), 139-154. https://doi.org/10.1007/s10640-012-9585-z
Schaefer, M. B. (1954). Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. Inter-American Tropical Tuna Commission Bulletin, 1(2), 23-56.

Simmonds, E. J., Campbell, A., Skagen, D., Roel, B. A., \& Kelly, C. (2011). Development of a stock-recruit model for simulating stock dynamics for uncertain situations: The example of Northeast Atlantic mackerel (Scomber scombrus). ICES Journal of Marine Science: Journal Du Conseil, 68(5), 848-859. https://doi.org/10.1093/icesjms/fsr014
Skagen, D. W. (1989). Growth patterns in the North Sea and western mackerel in Norwegian catches 1960-85 [Working paper]. ICES.

Steinshamn, S. I. (2011). A Conceptional Analysis of Dynamics and Production in Bioeconomic Models. American Journal of Agricultural Economics, 93(3), 803-812. https://doi.org/10.1093/ajae/aar007
Tahvonen, O. (2008). Harvesting an Age Structured Population as Biomass: Does It Work. Natural Resource Modeling, 21(4), 525-550. https://doi.org/10.1111/j.19397445.2008.00022.x

Tahvonen, O. (2009). Economics of harvesting age-structured fish populations. Journal of Environmental Economics and Management, 58(3), 281-299. https://doi.org/10.1016/j.jeem.2009.02.001

Tahvonen, O. (2010). Age structured optimizationmodels in fisheries bioeconomics: A survey. In : Boucekkine R, Hritonenko $N$, Yatsenko $Y$ (eds) Optimal control of age-structured populations in economy, demography, and the environment. Routledge, Abingdon.
Vandromme, P., Stemmann, L., Berline, L., Gasparini, S., Mousseau, L., Prejger, F., ... Gorsky, G. (2011). Inter-annual fluctuations of zooplankton communities in the Bay of Villefranche-sur-mer from 1995 to 2005 (Northern Ligurian Sea, France). Biogeosciences, 8(11), 3143-3158. https://doi.org/10.5194/bg-8-3143-2011

Von Westernhagen, H., Westernhagen, H. V., \& Rosenthal, H. (1976). Predator-prey relationship between Pacific herring, Clupea harengus pallasi, larvae and a predatory hyperiid amphipod, Hyperoche medusarum. Fishery Bulletin, 74, 669-674.

Walters, C. J. (1969). A Generalized Computer Simulation Model for Fish Population Studies. Transactions of the American Fisheries Society, 98(3), 505-512. https://doi.org/10.1577/1548-8659(1969)98[505:AGCSMF]2.0.CO;2

Yodzis, P. (1994). Predator-Prey Theory and Management of Multispecies Fisheries. Ecological Applications, 4(1), 51-58. https://doi.org/10.2307/1942114

Zimmermann, F., \& Heino, M. (2013). Is size-dependent pricing prevalent in fisheries? The case of Norwegian demersal and pelagic fisheries. ICES Journal of Marine Science: Journal Du Conseil, fst121. https://doi.org/10.1093/icesjms/fst121

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[^1]:    ${ }^{1} \rho_{t+1}=0.01\left[100 \rho_{t}+e^{-120}\left(e^{-3\left(100 \rho_{t}-100\right)}-e^{3\left(100 \rho_{t}-100\right)}\right)+\sigma_{\rho} \varepsilon_{t}\right]$ where $\varepsilon_{t}$ is a sequence of independent and identically distributed random variables with $E\left[\varepsilon_{t}\right]=0$ and $\operatorname{Var}\left[\varepsilon_{t}\right]=1$ and the volatility $\sigma_{\rho}=2$.

[^2]:    ${ }^{2} \mathrm{http}: / / \mathrm{www} . i m r . n o / t e m a s i d e r / f i s k / m a k r e l l /$ makrell/en

