Cooperation of customers in traveling salesman problems with profits

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Cooperation of customers in traveling salesman problems with profits*

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1 Introduction

The traveling salesman problem and its variants are among the most studied problems in the literature on transportation and logistics. In one of these variants known as the profitable tour problem [2], a profit-maximizing carrier decides whether to visit a particular customer with respect to the prize the customer offers for being visited and traveling cost associated with the visit, all in the context of other customers. The purpose of this paper is to define the profitable tour game, a cooperative version of the profitable tour problem, and to derive its properties. We are particularly interested in prize allocations that create incentives for the carrier to visit all relevant customers.

Applications of the profitable tour game might include for example situations in shipping where a carrier is able to serve demands of several customers with a single vehicle. Whether it comes to delivery or pickup of goods, the customers might need to induce the carrier to visit them by offering sufficient rewards. Subsequently, negotiation with other customers in the same position could lead to better prizes while the carrier's visit would remain guaranteed. This knowledge could also be utilized by the carrier by offering specifically tailored discounts on multiple orders from the same area or by evaluating and pricing of new customers.

2 Problem definition

The problem under study can be seen as a two-stage conflict of $n+1$ decision makers. First, n customers decide on prizes offered to a carrier for being visited and, after all prizes are revealed, the carrier decides which customers to visit and how to perform the tour. The strategy of a rational

*For the final and complete version of this article published in *Optimization Letters*, see [1].

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and profit-maximizing carrier is to choose the tour with the largest difference between the total prize-based profit and the total traveling cost. With this in mind, the customers want to set the prizes in a least costly manner that still guarantees them to be visited. It might be useful to form coalitions with other customers. Such coalitions would then aim to set prizes such that their sum would be the least possible to guarantee being visited regardless of other customers' offered prizes.

The whole situation is a zero-sum game as everything that is paid out by the customers gets collected by the carrier. This does not offer opportunities for a meaningful cooperation of all players. On the contrary, leaving out the carrier and focusing on the conflict of customers only, the prizes can be set in a way that benefits other customers as well. This is the idea for defining the profitable tour game as a cooperative game of the customers. Formally, a transferable utility game is defined as a tuple of two elements. In this case, this is a pair (N, Cost^{PTP}), where N is the set of all customers and Cost^{PTP} is the so-called characteristic function, i.e., the optimal cost associated with each group of customers $S \subseteq N$. These costs can be obtained by solving an integer programming model for each subset S. As opposed to the traveling salesman game [3] and the routing game with revenues [4], we do not assume these costs to be equal to the costs of the least costly tours. In fact, the total cost incurred by the customers equals the sum of all offered prizes and in the profitable tour problem it is sometimes needed to offer more than the cost of the least costly tour to prevent the carrier from excluding some customers.

3 Results

The main results can be summarized in the following two theorems.

Theorem 3.1 *If the core of the traveling salesman game* (N, Cost^{TSP}) *is non-empty, all allocations from the core, and no other, are optimal prize allocations of the grand coalition in the profitable tour game* (N, Cost^{PTP}).

Theorem 3.2 *If the core of the traveling salesman game* (N, Cost^{TSP}) *is empty, then prizes* Prize_j *stand for an optimal prize allocation of the grand coalition in the profitable tour game* (N, Cost^{PTP}) *if and only if they represent an optimal solution of the problem:*

$$\min \varepsilon \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in N} \mathrm{Prize}_j = \mathrm{Cost}^{TSP}(N) + \varepsilon, \tag{2}$$

$$\sum_{j \in S} \mathrm{Prize}_j \leq \mathrm{Cost}^{TSP}(S) + \varepsilon \qquad \forall S \subset N, \tag{3}$$

$$\mathrm{Prize}_j \geq 0 \qquad \forall j \in N, \tag{4}$$

$$\varepsilon \geq 0, \tag{5}$$

where the variable ε stands for the cost to be allocated in form of prizes in addition to the total cost of a tour visiting all customers.

We prove the two theorems above and illustrate them with numerical examples. In addition, we derive a corollary linking optimal prize allocation of the grand coalition in the profitable tour game to the dual game of the traveling salesman game.

4 Conclusions

We define the profitable tour game and find several properties of the optimal prizes to be offered in the case the coalition of all customers, the grand coalition, is formed. These properties describe a relationship between the prizes and the underlying traveling salesman game to provide another connection with an extensive literature on core allocations in traveling salesman games. We prove that, if the core of the underlying traveling salesman game is non-empty, the set of optimal prizes to be offered coincides with this core. For problems in which this core happens to be empty, we present a model to find the optimal prizes to be offered by the customers.

Overall, we analyze the total cost associated with each coalition of customers and, in form of the prizes, the strategies to achieve it. Our analysis opens further research opportunities for studying cost allocation to divide the costs paid out in form of prizes among the cooperating customers. Analyzing the core of the profitable tour game and the performance of specific allocation methods might be some of the interesting avenues for future research.

References

[1] O. Osicka, M. Guajardo and K. Jörnsten, "Cooperation of customers in traveling salesman problems with profits", *Optim. Lett.* (2019). https://doi.org/10.1007/s11590-019-01429-6

[2] G. Laporte and I.R. Martín, "Locating a cycle in a transportation or a telecommunications network", *Networks* 50, 92-108 (2007).

[3] J.A.M. Potters, I.J. Curiel and S.H. Tijs, "Traveling salesman games", *Math. Program.* 53, 199-211 (1992).

[4] A. Estévez-Fernández, P. Borm, M. Meertens and H. Reijnierse, "On the core of routing games with revenues", *Int. J. Game Theory* 38, 291-304 (2009).

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