Essays on Talent Discovery and Allocation

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Acknowledgments

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1 Introduction
Talent-intensive sectors, such as high-tech industry, finance, professional services and academic institutions, are increasingly important in the growth of advanced economies. In these industries, labor productivity depends crucially on the combination of workers’ innate abilities and on-the-job training, more than on tangible assets such as physical equipment and infrastructure. Analyzing how firms discover talents and allocate them within their organizational structure, and how competition for talent between firms affects these processes is extremely important to understand the efficiency and growth of advanced economies. These issues also bear on other important economic issues, such as the allocation of risk, the degree of economic inequality and the incentives to acquire education.

Workers’ talent is typically scarce and – to some extent – portable from one firm to another (Groysberg et al., 2008, and Groysberg, 2010). Firms do not own these inputs as they do with physical capital: the inalienability of talent and human capital generates contract incompleteness (Becker, 1964; Hart and Moore, 1994), insofar as workers are not bonded to their employers by search costs or other frictions (such as limited geographical mobility). Absent these frictions, competition for talent becomes a key feature affecting its allocation within and between organizations.

The existing literature shows that, in general, competition for talent by firms has both benefits and costs. On the one hand, it increases wages and the average level of education (Garmaise, 2011). On the other hand, competition for talent may generate inefficiencies.

First, in organizations whose technology requires assigning employees to heterogeneous tasks (differing in the talent-sensitivity of their output), labor market competition may lead to inefficient task assignment of workers when outside competitors view such assignment as a signal of workers’ ability. Waldman (1984) and Bern-
hardt (1995) show that the inefficiency in talent allocation is increasing with the competitiveness of the underlying labor market.

Second, when workers’ talent is uncertain, its discovery is a source of risk. Workers may turn out to be less productive than they expect and be laid off. Harris and Holmström (1982) and Acharya, Pagano, and Volpin (2016) show that without labor market competition, this risk is privately insurable: firms might commit to pay schemes that are insensitive to talent discovery, as reflected by their employees’ past performance, or give generous severance pay to laid-off employees, and thus compensate them upon being found untalented. However, firms can provide such insurance only if the labor market is not fully competitive, i.e., one where workers are not free to switch to other employers once their talent is discovered. If they are free to switch, firms cannot provide severance payments to low-talent employees: this would require cross-subsidizing them at the expense of high-talent ones, who would react to such a scheme by switching to a competing employer.

This thesis studies the impact of competition for talent on its discovery and allocation beyond the existing literature in organizational economics and applied microeconomic theory. Specifically, it addresses three distinct questions:

1. Can particular organizational structures, such as partnerships, reduce inefficiencies in talent allocation within firms in the presence of labor market competition?

2. Can labor market competition have a negative impact on workers’ training effort due to inefficiencies in promotion schemes?

3. Does public unemployment insurance favor talent discovery within firms and investment in human capital by employees, in the presence of labor market competition?

This thesis includes three theoretical contributions aimed at exploring the questions posed above, in the context of talent-intensive industries.
Organizational Design with Portable Skills

Competition for talent may imply inefficient allocation of jobs across workers, when the current employer has an informational advantage about employees’ talent relative to other firms in the industry (Waldman, 1984; Bernhardt 1995).

I show that asymmetric information among firms is not a necessary assumption for inefficient task allocation to exist. If workers’ talent is nonverifiable in courts, contracts feature bidimensional incompleteness. On the one hand, workers cannot commit to stay with the current employer; on the other hand, firms cannot commit to task allocation, as it is based on talent. In a setting with symmetric information but nonverifiable workers’ ability, firms allocate workers inefficiently to tasks, to reduce retention costs.

I analyze a simple theoretical model of talent allocation within a competitive firm, producing output by means of two tasks (which can be generalized to more than two) differing in two respects: one is more talent-sensitive than the other, and the skills acquired while dealing with the former are more portable than those acquired while dealing with the less talent-sensitive task. This firm is organized as a corporation, namely one shareholder (or a block of homogeneous shareholders) controls task allocation, deciding job assignments for all the firm’s employees. As a result, profit maximization leads to inefficient task allocation.

Efficient task allocation can be attained by removing one of the sources of contract incompleteness featured in the model. However, the core of the paper focuses on an alternative organizational structure. I analyze whether organizing the firm as an equity partnership (as opposed to a corporation) allows it to achieve efficient task allocation and optimal retention of partners and workers. The key difference between a partnership and a corporation is that, in the former, workers have control and cash flow rights, if they become partners. Once they buy equity in the firm, employees do not earn a fixed salary, but share in the profits generated. In this framework, the firm should attain efficient task allocation, as this enables partners to earn the largest possible profit. As in this form the firm generates more surplus than a corporation, its shareholders should be willing to sell it to the employees.

Waldman (1984) and Bernhardt (1995) are the key reference studies on the inefficiency of talent allocation within firms operating in competitive labor markets. Regarding the role of partnerships, no existing paper analyzes the effects of this or-
ganizational structure on the efficiency of talent allocation. Levin and Tadelis (2005) argue that partnerships abound in human capital-intensive industries because clients cannot perfectly observe the quality of the products supplied. They show that firms organized as partnerships signal the quality of their output as partners share profits equally. Namely, partners maximize the average profit instead of the total one. This implies that they will hire only the best workers on the market (the more productive ones). Alchian and Demsetz (1972), emphasize the incentive to monitor peers in such organizational structure. Farrell and Scotchmer (1988) show that many law firms have few partners because the best workers do not want to share their earnings with less productive ones. Garicano and Santos (2004) show how a firm organized as a partnership can favor the transmission of human capital between partners and associates and between senior and junior partners.

Equity-partnerships are the most common form of partnership in countries featuring the common law system. This paper provides a novel rationale for the widespread existence of partnerships in talent-sensitive industries, where the competition for talent is fierce.

**Promotions and Training: Do Competitive Firms set the Bar too High?**

Promotion-based incentives are widespread in organizations (Baker, Jensen, and Murphy, 1988; Baker, Gibbs, and Holmström, 1994a and Baker, Gibbs, and Holmström, 1994b). Promotions improve firms’ performance by providing workers with incentives and by allocating talent within hierarchical organizations in the most productive way.

This paper analyzes the relationship between promotion standards and workers’ incentives to exert training effort in competitive labor markets. Since labor market competition affects talent allocation between and within firms (Waldman, 1984; Bernhardt, 1995), it can also be expected to impact training through the inefficient definition of promotion standards.

I assume firms to produce by means of two different jobs: a talent-sensitive and a routinary, talent-insensitive one. Workers dealing with the first task are promoted and can be poached by competing firms, while non-promoted workers cannot. In this framework, firms will promote fewer employees than efficient if promotion thresholds are not contractible. This hampers workers’ incentives to exert both firm-specific
and portable training effort. Interestingly, workers’ decisions depend both on the probability for them to receive an outside offer (hence, on the competition for talented workers), and on the distribution of their talent.

If firms can commit to promotions before workers acquire human capital, they would lower the threshold to encourage workers to invest more. Such behavior may lead to equilibria featuring over-promotion and over-investment in firm-specific and portable human capital with respect to the Pareto-efficient benchmark, a result consistent with that in Holmström (1999), where workers over-invest because of their career concerns.

Regarding the relationship between promotion-based incentive schemes and workers’ training, Ben-Porath (1967) is the first paper analyzing workers’ incentives to invest in human capital over their career. Carmichael (1983) studies the impact of workers’ seniority and promotion ladders on firms’ and workers’ investments in specific human capital, assuming adverse selection between the firm and its employees. Prendergast (1993) studies how promotions incentivize workers to acquire firm-specific human capital, in a setting with two-sided moral hazard: workers’ investment is assumed to be non-verifiable, hence neither firms nor workers can commit respectively to wages and investments. If different tasks in the firm are associated with different wages, then this commitment issue can be solved under certain conditions allowing the incentive-compatible promotion schemes incentivizing workers to invest in firm-specific human capital. Gibbons and Waldman (1999) analyze wage and promotion dynamics in a setting with job assignments, training and learning about workers’ talent, providing results consistent with the stylized facts in Baker, Gibbs and Holmström (1994a, 1994b).

Talent Discovery, Layoff Risk and Unemployment Insurance (with Marco Pagano)

In talent-intensive jobs, workers’ performance reveals their quality. This enhances productivity and wages, but also increases layoff risk. As shown by Harris and Holmström (1982) and Acharya, Pagano and Volpin (2016), if workers cannot resign from their jobs, firms can insure them via severance pay without losing the best performing workers. However, in perfectly competitive labor markets, private insurance cannot be provided, as it would entail cross-subsidization from the more talented workers
to the less talented ones. The former would not accept such mechanism and would therefore move to a competing firm offering higher wages. In this framework, more risk-averse workers will choose less informative jobs, thus impairing productive efficiency.

We study a dynamic theoretical model with symmetric learning about worker’s talent in a similar framework to that in Harris and Holmström (1982). We aim at showing that if the government provides public unemployment insurance (“UI”), workers will select themselves on more talent-sensitive jobs (namely, featuring higher layoff risk), regardless of their degree of risk-aversion. We obtain empirical predictions on the extent to which talent-sensitive firms can attract workers in environments with competitive markets, depending on the presence and design of UI. Specifically, we prove theoretically and test empirically the existence of a positive correlation between the generosity of UI benefits and the percentage of workers employed in talent-sensitive industries. Our empirical tests draw upon publicly available datasets for the UI replacement rates (used as a measure of UI generosity) and employment in different industries for OECD countries as well as for U.S. states.

Furthermore, we study the possibility of introducing employment protection laws, and compare them with UI systems, in order to set a ranking in terms of productive efficiency and risk sharing of these different policy interventions and to define conditions under which one dominates the other.

The theoretical and empirical literature shows that both the level and the duration of unemployment benefits tend to reduce the effort of the unemployed in labor search, thus increasing the duration of unemployment spells (Moffitt and Nicholson, 1982; Meyer, 1990, and Katz and Meyer, 1990). However, other papers show that unemployment insurance (UI) also allows workers to search longer so as to identify better matches, thus raising aggregate productivity (Diamond, 1981; Acemoglu, 1997; Marimon and Zilibotti, 1999). Indeed, Nekoei and Weber (2017) document empirically that UI improves the quality of firms where the unemployed find jobs and increases their wages. None of these works have studied unemployment benefits as subsidies to talent discovery within the firm, which therefore results a hitherto neglected positive effect of UI.
References


2 Organizational Design with Portable Skills
Organizational Design with Portable Skills*

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Abstract

Workers can move across firms and take with them portable skills. This affects firms’ strategies as inefficient task allocation reduces retention costs. In the existing literature, asymmetric information about workers’ talents makes this retention strategy profitable. In this paper, workers’ skills are observable but nonverifiable, hence task allocation is noncontractible and inefficient task allocation persists. I show that a firm organized as an equity-partnership allocates tasks efficiently. In this framework, partners get cash flow and control rights on task allocation and are retained in equilibrium. This provides a new rationale for the widespread presence of partnerships in human-capital intensive industries.

JEL classification: D86, J24, J54, M52.

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1 Introduction

Workers’ mobility is a relevant issue for firms operating in human capital-intensive industries: in these sectors employers cannot bind the main input of production to the firm. This generates retention costs that may prevent employers from matching workers to jobs efficiently within organizations. Moving across firms along their career path, workers can use skills in firms differing from the ones that trained them. These skills are referred to as portable (Grosyberg et al. 2008; Groysberg, 2010). Skills’ portability depends inversely on their firm-specificity.

Employers use several tools to retain their best workers, such as wage bonuses, noncompete clauses (Mukherjee and Vasconcelos, 2012) and perks. Another common strategy is to allocate talented workers to tasks that make them less attractive for competitors in the industry (Greenwald, 1986; Waldman, 1984 and more recently, Mukherjee and Vasconcelos, 2018). This reduces their outside option after training.

Anecdotal evidence about talent-intensive industries, shows a constant increase in the number of firms organized as profit-sharing partnerships.

This paper analyzes how competition for talented workers affects the organizational design of human capital-intensive firms. More specifically, I address two questions: first, will a profit maximizing firm efficiently allocate workers across tasks featuring heterogeneous production technology and portability? Second, if the firm is organized as a partnership rather than a corporation, will it allocate tasks to workers more efficiently?

To answer these questions, the model studies firms producing their output by means of two tasks. One has a productivity depending on the talent of workers op-

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2One could also think of portable resources. For instance, a lawyer working for a certain company, by moving to a competitor, or by starting up a spin-out firm, can carry with her a certain fraction of clients from the initial firm’s pool.

3For further implications of workers’ mobility and portability of their human capital, Acharya and Volpin (2010) show how the competition for workers in the labor market affects the quality of corporate governance in a firm. Ellingsen and Kristiansen (2017) describe the impact of portability on experts’ competitive compensation.

4IRS Data on the amount of professional partnerships in the U.S. highlight a significant increase in the last ten years, with an average growth rate of 5.6% per year.
erating it. The other task has a fixed productivity. The first task is assumed to be more portable than the second one.\(^5\) The employer hires a pool of workers offering spot wage contracts.\(^6\) After a training period, employees acquire firm-specific human capital. At this stage, workers’ talents become publicly observable yet nonverifiable in courts. Thus, task allocation is noncontractible, as it depends on workers’ productivity.

An efficient cutoff value of ability is derived, such that workers who (do not) fulfill it, shall be allocated to the more (less) portable task. I then describe two benchmark contracts that allow the attainment of the first-best task allocation. First, I assume firms to be able to commit on task allocation (namely, talent to be verifiable). Second, I assume workers to be able to commit not to leave the first employer once task allocation takes place. Specifically, when one source of contract incompleteness is removed, task allocation is efficient.

When instead, neither firms nor workers can commit credibly to agreements, firms assign the more portable task to fewer workers than in the efficient benchmark. This implies that some workers’ talent is not efficiently used in the production process. The magnitude of this inefficiency depends on the relative portability of the skills acquired while executing the two tasks (namely on workers’ outside options deriving from task allocation). Workers who are inefficiently allocated on the less portable task are not productive enough to justify too high a retention cost (deriving from a better outside option).

I also examine to what extent more elaborate contracts can reduce productive inefficiencies. Specifically, I introduce the possibility for the firm to offer \textit{up-or-out contracts}. These contracts are widely used in human capital intensive industries (Waldman, 1990) and are (extreme) forms of tournaments (Lazear and Rosen, 1981). They state that a worker should either perform so as to be promoted to a better position in the organization, or laid off. I show that such a mechanism can restore the efficient task allocation, yet at a cost in terms of productivity. Specifically, the cutoff ability to be kept in the firm needs to be the efficient one for the task allocation

\(^5\)This assumption can be alternatively interpreted as the case in which one task makes the worker more “visible” than the other, in the spirit of Milgrom and Oster (1987)

\(^6\)This assumption can be relaxed without qualitatively altering results. Allowing the employer to offer long-term contracts would not change the predictions of the model as long as contract incompleteness holds.
rule, and for production to be efficient, the skills acquired on the talent-insensitive task should be fully portable. However, even in the best allocative scenario possible, profit-maximizing firms always prefer offering simple wage contracts rather than up-or-out ones.

The model predicts that a change in firms’ organizational form improves efficiency. If the incumbent employer sells the firm to some employees who then run it as an equity partnership, the efficient task allocation can be attained. It requires prospective partners to buy equity of the firm in advance, and then they will be remunerated with realized dividends. By giving control and cash flow rights to some workers, the partnership organizational form eases the ex-post retention of both partners and salaried workers. I show that a “meritocratic” sharing rule entitling more productive workers to higher shares of the realized profit (namely, to more equity and control rights) incentivizes the best workers to become partners.

Partners choose task allocation so as to maximize the profit to be shared. This shift in control rights makes room for efficiency as partners allocate themselves and all other workers on the task in which they are more productive. Henceforth, I show that if (at least) all the inefficiently allocated workers are made partners, the efficient outcome is attained.

The paper is structured as follows. Section 2 reviews the related literature. Section 3 sets up the basic model. Section 4 derives the efficient task allocation. It is shown that the efficient outcome can be implemented if workers’ mobility can be limited or contracts are complete. Section 5 introduces the allocative inefficiency due to portability of talent and contractual incompleteness. Section 6 modifies the initial model introducing the possibility for the employer to sell the firm out to some workers who run it as a partnership. Section 7 concludes.

2 Related Literature

This paper contributes to two branches of organizational economics: one dealing with optimal allocation of workers within firms, and the other analyzing the design of organizations and the allocation of control rights.

Task allocation across workers has been analyzed in settings characterized by asymmetric information among firms. Greenwald (1986) shows that if the current
employer has an informational advantage about workers’ ability, task allocation can be exploited to prevent poaching raids by rival firms. The latter, in fact, can be refrained from poaching a worker whose ability is uncertain, to avoid paying too much for a “lemon” (winner’s curse). Task allocation may be perceived by the uninformed parties as a signal of workers’ talent. Waldman (1984) considers a framework in which information about workers’ ability is observed only by the incumbent employer. Future potential employers receive a signal from the task assigned to each worker. Henceforth, the current employer may exploit her informational advantage and allocate workers inefficiently in order to send an incorrect signal to the opponents.\footnote{Bernhardt (1995) features a similar argument to justify the existence of the so-called \textit{“Peter principle”}. This principle describes the empirical evidence that some promoted workers turn out to be less productive than before, when they were working on a simpler task.}

In this paper, I show that allocation inefficiencies persist when workers’ abilities are observable in the industry, but task allocation is not contractible.\footnote{This assumption makes the model similar to the matching model presented by Jovanovic (1979) in which workers’ abilities are perfectly observable and they need to be allocated between firms depending on complementarities and technologies so as to attain efficient matches.} I argue that observing workers’ talents is not enough to obtain efficient outcomes if the employer cannot commit to a certain task allocation.

Another branch of the literature on organizational design has focused on the role of asymmetric information between firms and clients. Levin and Tadelis (2005) argue that partnerships abound in human capital-intensive industries because clients cannot perfectly observe the quality of the products supplied.\footnote{For instance, a patient cannot tell whether a diagnosis is correct, or a plaintiff could not perfectly evaluate a lawyer’s technical advice.} The authors show that firms organize as partnerships in order to signal the quality of their output. They assume partners to share the profit equally. Such assumption is fundamental for the signaling purpose: partners maximize the average profit instead of the total one. This implies that they will hire the best workers on the market (the more productive ones).

I develop a different framework with respect to the one in Levin and Tadelis (2005) in several respects. First, I assume the quality of the output produced to be observable. Second, I do not consider a monopolistic firm. Third, in my model the firm hires workers who develop all the possible talents. Indeed, at the beginning of the job relationship, abilities are unobservable. Fourth, I depart from the assumption
that partners share profits equally, as I am not concerned with the signaling problem. The results provided in this paper show that for the retention motive, partners should receive a share of profit proportional to their productivity.

This paper is also related to Rebitzer and Taylor (2006). It focuses on the role of “up-or-out contracts” in law partnerships. In their model there is a continuous turnover of associates, in an overlapping generation framework. Dismissed workers can carry along human capital acquired during the time within the firm. This loss is not featured in Rebitzer and Taylor (2006), whereas the present paper emphasizes that also low-skilled workers' departures cause losses for the incumbent employer. Indeed, the employer bears the cost to train freshly hired workers to substitute the dismissed ones. If these are poached from a competing firm, they cannot produce as well as the dismissed workers because of imperfect portability of skills. Other theoretical contributions on the economics of partnerships focusing on different issues with respect to the impact of workers’ mobility on the design of organizations include: Alchian and Demsetz (1972), emphasizing the incentive to monitor peers in such organization; Farrell and Scotchmer (1988) showing that many law firms have few partners because the best workers do not want to equally share their earnings with weaker ones; Kochan and Rubinstein (2000); Garicano and Santos (2004) showed how a firm organized as a partnership can favor the transmission of human capital between partners and associates and senior and junior partners; Morrison and Whilelm (2004).

Finally, the paper is linked to the classical literature on incomplete contracts and control rights in organizations, dating back to Grossman and Hart (1986), Hart and Moore (1988), Hart and Moore (1990), Aghion and Tirole (1997). In this paper, contracts feature bilateral incompleteness: firms cannot commit to task allocation and workers cannot commit to stay with their first employer. These incompletenesses generate inefficient production and the solution provided is vertical integration as proposed in the above mentioned literature.

3 The Model

A firm takes prices as given and hires a continuum of measure 1 of workers from a perfectly competitive labor market by means of spot wage contracts. Let the output
price be normalized to 1 and workers’ productive effort be costless.\(^\text{10}\) Employer and employees are risk-neutral. The latter get utility from consumption, namely from the wage they earn. Workers’ heterogeneous productivities are denoted as \(y \in [\underline{y}, \bar{y}]\), with \(\underline{y} > 0\). Productivities are distributed according to a generic cumulative distribution function \(F(y)\) with \(\frac{\partial F(y)}{\partial y} = f(y)\). Workers’ productivities are uncertain to everyone at the beginning of the job relationship.\(^\text{11}\)

Initially, employees execute a standard nonproductive task (which can be considered as a training period).\(^\text{12}\) By doing so, their talent becomes observable in the industry, but not verifiable in courts. This last assumption makes contracts contingent on workers’ ability, not enforceable. Since the employer chooses task allocation depending on abilities, it is noncontractible. Once abilities are observed, the employer allocates workers to either of two tasks. This allocation is determined by a new spot contract defining a task and a new wage. Tasks differ in productivity and portability (or specificity) rate of the skills workers acquire by executing them. After task allocation, workers may be poached by competing firms in the industry. Let there be no discounting across the two periods and no financial markets.

### 3.1 Contracts and Tasks

The employer offers spot wage contracts. Let \(w_1\) be the initial wage offer. Let \(w^i_2\), with \(i = \{A, B\}\), denote the wage offered to the worker at the interim stage, namely after her talent becomes observable and she is allocated to task \(i\).

Let \(\theta_i\) define the \textit{portability rate}, of task “\(i\)” (namely, the share of task \(i\) output a leaving worker can produce outside the initial firm). The two tasks are characterized as follows:

\(^{10}\)Picariello (2017) removes this assumption to study the interaction between promotions (or task allocation) and workers’ incentives to acquire more or less firm-specific human capital with competitive labor markets. In such framework, talent allocation has a dual role: on the one hand it can reduce mobility, on the other hand, it serves as an incentive for workers to acquire human capital.

\(^{11}\)This is a common assumption, see for instance Waldman (1984) and Greenwald (1986).

\(^{12}\)The output of this task is normalized to zero for simplicity, but it could be whatever constant value independent of workers’ ability without changing the qualitative results provided throughout the paper.
Assumption 1. Task A produces $\beta y$ with $\beta \in [1; \infty)$ and has a portability rate $\theta_A \in (\theta_B; 1]$.

Task B produces $x \in [y, \bar{y}]$ and has a portability rate $\theta_B \in (0; \theta_A)$.

Notice that assuming $\theta_i \leq 1$ is equivalent to assuming that a worker leaving her current firm may be less productive elsewhere.

To sum up, task A is the more talent-sensitive of the two and the skills it delivers are more portable, whilst task B can be thought of as a routine task. The assumption that the skills deriving from working on task A are more portable than those deriving from working on task B is motivated by the fact that the first yields an output positively correlated with innate talent. Alternatively, one could think of task A as making workers more “visible” (hence, attractive) on the labor market.\footnote{The ranking of portability rates could be changed and all the main results of the paper would hold true, although the inefficiencies shown later are reversed.}

I assume workers’ talent to be unknown to everyone at the beginning of the game. For this reason, workers receive an homogeneous initial wage offer. After talents become observable and task allocation takes place, every worker will have an heterogeneous outside option depending on the skills acquired on the assigned job. Specifically, a worker assigned to task A can produce outside the initial firm

$$\theta_A \beta y.$$ 

A worker assigned to task B, can produce

$$\theta_B x.$$ 

I assume that a worker assigned to task $i$ acquires the skills to execute only that task after poaching. Namely, a workers allocated to task B (respectively, A), cannot be poached to execute task A (respectively B) immediately, as she needs training for the new task.

3.2 Time Line

The time line of the model includes five stages:
• At $t = 1$, firms bid competitively for workers offering $w_1$. Workers who accept will work on a standard nonproductive task.

• At $t = 2$, workers’ productivities become observable to them and to all the firms in the industry. Wages for the standard task are paid.

• At $t = 3$, firms offer a new spot contract specifying task $i$ and wage $w_{i}^2$.

• At $t = 4$, workers can leave the initial firm for a new one.

• At $t = 5$, the production process is completed and wage $w_{i}^2$ is paid.

3.3 Equilibrium Concept

The model features perfect information about workers’ talent in a sequential game. The equilibrium concept is subgame perfect Nash equilibrium. In the simple initial model, workers only decide whether to work for a firm at the beginning of the game, whereas firms choose wage contracts and task allocation. Hence, a subgame perfect Nash equilibrium for this game consists of a vector of wages and a noncontractible task allocation $\{w_1, w_{i}^2, i\}$.

4 Efficient Task Allocation

First, I derive the efficient threshold value for workers’ talent $y^* \in [\underline{y}, \bar{y}]$. It is chosen so that all workers with ability larger or equal (respectively, smaller) than $y^*$ are assigned to task A (respectively, task B). The employer and the employees sign two spot contracts. At the beginning of the job relationship (stage 1), the firm pays a wage $w_1$ to convince workers to join the firm. As described later, this wage is an outcome of Bertrand competition for workers.

After the execution of the standard task, workers’ abilities are revealed and they are allocated one of the two tasks. At this stage, workers are offered a wage depending on task allocation $w_{i}^2$. Let us define the social welfare as

$$W = \int_{\underline{y}}^{\bar{y}} \beta y f(y) dy + F(y)x - w_1 - \int_{\underline{y}}^{\bar{y}} w_{i}^2(y) f(y) dy + w_1 + \int_{\underline{y}}^{\bar{y}} w_{i}^1(y) f(y) dy. \quad (1)$$
Let $\pi$ denote the profit of the firm, whereas the other terms define the sum of wages earned by the employees.

The efficient cutoff value for workers’ productivity is defined as:

$$y^* \in \arg\max \{y\} W.$$ 

The first-order condition delivers the optimal threshold value

$$y^* = \frac{x}{\beta}. \quad (2)$$

This cutoff value maximizes the total surplus. Notice that, ceteris paribus, the higher the production enhancer $\beta$, the lower $y^*$. Hence more workers should be allocated to task A. Instead, when $x$ increases, the threshold value increases. Namely, only very productive workers shall work on task A.

### 4.1 Implementing the Efficient Allocation

The standard model presented in this paper features bilateral contract incompleteness. On the one hand, firms cannot commit to task allocation; on the other hand, workers cannot commit to stay with their employer after task allocation takes place. I will now relax one incompleteness at a time in order to show that when either of the parties can commit to an agreement, efficient task allocation is implemented.

#### 4.1.1 Workers’ Commitment

Assume workers can commit to stay with their employer after task allocation (for instance, because labor contracts feature strict noncompete clauses). In this framework, workers’ ex-post retention is not an issue for the employer. Hence, the latter does not use task allocation strategically to reduce the cost of retention.

Assume the parties can sign unconstrained contracts limiting workers’ mobility. In this environment, the following proposition holds:

**Proposition 1.** If the employer and the employees can sign unconstrained contracts, through which the worker can commit not to leave the firm after task allocation, task allocation is efficient.
The proof of this and all other propositions and lemmas is relegated to the Appendix. Intuitively, if retention is not an issue at the interim stage, the employer pays workers a fixed wage after task allocation, independent of the task they work on. Thus, the firm allocates tasks only considering employees’ marginal productivity on either task: this leads to an efficient outcome. The ability cutoff for a worker to be allocated to task A will be \( y^{**} = y^* \), which maximizes productivity. Workers extract all the surplus generated at \( t = 1 \), as the labor market is perfectly competitive ex-ante.

### 4.1.2 Firms’ Commitment

Suppose workers’ talent is verifiable, so that firms can credibly commit to task allocations ex-ante. In this framework, the firm can offer contracts of the type

\[
\{w(y), i(y)\}.
\]

By means of this contract, the firm can commit to the efficient task allocation.

**Proposition 2.** *If workers’ ability is verifiable, the employer can commit to match workers to tasks efficiently, according to the cutoff value \( y^* = \frac{2}{\beta} \).*

In this case, the firm can attract as many workers as possible in the competitive labor market and offer the highest total expected surplus possible. Since the contract including task allocation is enforceable, the firm cannot holdup at the allocation stage.

### 5 Portability and Inefficiency

Consider now the case in which workers can leave the firm after being matched with a task. In the new firm, workers produce a fraction of what they did in the initial firm, depending on the task they execute. Therefore workers’ outside option depends on task allocation and on their talent. Since talent is nonverifiable, task allocation is noncontractible.

**Proposition 3.** *If workers cannot commit to stay with their initial employer and firms cannot commit to task allocation, it is profit maximizing to assign task A to...*
fewer workers with respect to the efficient equilibrium. In a competitive equilibrium the threshold value is
\[
\hat{y} = \frac{(1 - \theta_B)x}{(1 - \theta_A)\beta} > y^*.
\]

This result shows that if worker can leave the source-firm, \( F(\hat{y}) - F(y^*) \) of them are inefficiently allocated to task B. These workers could potentially be assigned to task A (since \( \beta y > x \) for them), but they are not (see Figure 2). Their productivity is not large enough to compensate the spread between \( \theta_A \) and \( \theta_B \). Namely, the wage necessary to retain them at the interim stage if working on task A, is relatively too high. To reduce retention costs, firms strategically match them with the less portable task.\textsuperscript{14}

![Figure 1: Inefficiency](image)

This is not a surplus maximizing outcome: some workers’ talent is inefficiently used and developed. If a worker is matched with task B, she will not be able to work on task A in another firm, although her talent would potentially allow her to do so.

If \( \theta_A \) increases, ceteris paribus, the threshold value \( \hat{y} \) increases. As in Waldman (1984), the degree of allocative inefficiency is decreasing in the firm-specificity of workers’ human capital. However, in this paper, the result is driven by a different mechanism. I do not consider informational asymmetries across firms, about workers’ talent. I study an informational setting similar to those used in matching models, with symmetric information (Jovanovic, 1979). Suppose workers can send a signal about their ability to the market in the setting presented by Waldman (1984). Such action may reduce the relevance of the signal delivered by task allocation. Workers could do signal jamming (as in Holmström, 1982/1999 and Gibbons, 2005) to convey more precise information about their ability, out of task allocation. The more informative the signal (the more important the signal jamming activity), the less effective is task allocation for firms to retain the best workers. Indeed, if a very talented worker is allocated to a simple routine task, she can signal her actual skills. This would increase her probability of being hired by a competing firm seeking highly productive

\textsuperscript{14}Allowing firms to poach workers before task allocation would not change the result as all firms are identical and solve the same profit maximization problem. Namely, in equilibrium, no firm would bid to poach and allocate to task A a worker with ability \( y \in [y^*, \hat{y}] \).
employees. In this model, task allocation is an effective retention tool. A key role, for this result to exist, is played by contract incompleteness and by firm and task-specificity of the skills acquired by the employees.

5.1 Complete vs Incomplete Contracts

It is now interesting to compare the cases studied so far. It has been shown that bilaterally incomplete contracts yield inefficient production, as a consequence of opportunistic task allocation within firms. However, removing one source of incompleteness allows the implementation of the efficient task allocation. Namely, if either firms or workers are able to commit to agreements, production is efficient. Consider the case where firms can credibly commit to task allocation (for instance because talent is verifiable, or as a result of a reputation building behavior). Note that in this scenario, workers are entitled to a larger expected surplus, as efficient task allocation is implemented. Consider the case in which workers’ interim participation constraints bind in equilibrium.\(^\text{15}\) At \(t = 1\) they earn

\[
w_1(y^*) = (1 - \theta_A) \int_{y^*}^{\hat{y}} \beta y f(y) dy + F(y^*)(1 - \theta_B)x
\]

and they expect

\[
\mathbb{E}[w_2(y^*)] = \theta_A \int_{y^*}^{\hat{y}} \beta y f(y) dy + F(y^*)\theta_B x.
\]

If task allocation is noncontractible, workers earn

\[
w_1(\hat{y}) = (1 - \theta_A) \int_{\hat{y}}^{\hat{y}} \beta y f(y) dy + F(\hat{y})(1 - \theta_B)x
\]

and

\[
\mathbb{E}[w_2(\hat{y})] = \theta_A \int_{\hat{y}}^{\hat{y}} \beta y f(y) dy + F(\hat{y})\theta_B x.
\]

In this case, the following inequalities hold:

\[
w_1(y^*) < w_1(\hat{y}) \tag{3}
\]

\(^{15}\)Since workers are risk-neutral and do not discount future earnings, the interim participation constraints may be slack thus allowing for many possible equilibria.
$$E[w_2(y^*)] > E[w_2(\hat{y})].$$  

(4)

Let

$$w_1(y) + E[w_2(y)] \equiv W(y), \ \forall y \in [y, \tilde{y}]$$  

(5)

thus, in this framework, it is clear that

$$W(y^*) \geq W(\hat{y})$$  

(6)

These inequalities provide a clear picture of the issues generated by firm’s inability to commit to task allocation. Suppose the firm promises a worker that at $t = 3$, task allocation will be efficient. In this case, should the firm be credible, the worker could accept $w_1(y^*)$ smaller than $w_1(\hat{y})$ to be hired. However, if firms cannot actually commit to task allocation, they will have an incentive to allocate tasks inefficiently later on, so as to obtain a positive rent

$$w_1(\hat{y}) - w_1(y^*) = [F(\hat{y}) - F(y^*)](1 - \theta_B)x - (1 - \theta_A) \int_{y^*}^{\hat{y}} \beta y f(y)dy.$$  

(7)

If firms can holdup, they will do it, thus generating less surplus and earning a positive rent with respect to the efficient benchmark case. For this reason, if workers anticipate this, they will not accept a lower wage ex-ante. They will require higher wages to be hired and have a “flatter” wage schedule.

6 Up-or-out Contracts

Thus far, I have considered firms and workers agreeing to simple wage contracts. Now, suppose firms can offer “up-or-out” contracts, which are widespread in human capital-intensive industries. In this case, employers set a certain performance standard and only workers fulfilling it will be kept (promoted, “go up”) in the firm, whereas the others will be laid off. There are two possible ways to define an up-or-out contract: either as a minimum productivity standard denoted as $y_{uo} \in [y, \tilde{y}]$, or as a minimum wage commitment. In the first case, at $t = 2$, when workers’ productivities become observable, the firm will lay off all those producing $y < y_{uo}$ and allocate all the others to task A. In the second case, the firm sets a minimum wage and workers whose productivity is too low to earn that wage are laid off, otherwise the firm would make losses. In this model, workers’ productivity is nonverifiable, therefore
the firm cannot commit to contracts contingent on it. However, firms can commit to wages, thus I study to what extent the second type of up-or-out contracts can improve allocative efficiency.\(^{16}\)

Rebitzer and Taylor (2006) show that, under certain conditions, up-or-out contracts can solve retention issues with no loss of welfare. I show that the firm-specificity of the skills workers acquire within a firm imply a cost of using these contracts.

**Lemma 1.** Suppose an up-or-out contract is in place and after talent revelation, the firm keeps only workers worth earning \(w_2 \geq \theta_A \beta y^*\). In this case:

1. Task allocation is efficient
2. The firm faces a loss \(F(y^*)(1 - \theta_B)x\).

Intuitively, if a firm commits to keep workers who should be paid as much as they earn if efficiently allocated to task A, efficient task allocation is implemented. \(^{17}\)

Workers who do not fulfill the requirement and therefore are laid off, do not acquire firm-specific human capital. Specifically, a share \(1 - \theta_B\) of the human capital they could acquire if staying with the firm would be specific. Replacing laid off workers with poached ones with similar abilities (or with newly hired workers) yields the firms at most zero profit from task B, given labor market perfect competitiveness. For this reason, up-or-out contracts generate a tradeoff between efficient production and losses in terms of human capital.

**Proposition 4.** It is never profitable for firms to use up-or-out contracts instead of simple wage contracts.

The intuition for this result hinges on two factors. First, if all workers who would execute task B are dismissed, the employer will substitute them with workers poached from competing firms. These workers will not be able to produce the same amount as those who were trained inside the firm. There is a fixed cost to be faced.

\(^{16}\)All the results hold even using the first type of up-or-out contracts.

\(^{17}\)I analyze the scenario in which these contracts deliver the most efficient task allocation, and show that it may not suffice to cover replacement costs. Note that the firm would optimally choose a different promotion threshold. See the proofs of Lemma 1 and Proposition 4 in the Appendix for a more detailed analysis.
Second, through these contracts, the firm does not maximize its profit. Profit maximization requires workers with productivity smaller than \( \hat{y} \) to be allocated task B. With up-or-out contracts, these workers will work on task A.

To sum up, even choosing the efficient threshold as up-or-out cutoff (thus attaining the highest productivity), the firm prefers implementing simple wage contracts.

The extant literature has shown the efficacy of up-or-out clauses in providing incentives for workers to exert effort. This is one of the benefits supporting the widespread use of these contracts in talent-sensitive industries. However, this model shows that these contracts impose a loss on firms using technologies requiring the acquisition of specific skills to be operated. Moreover, these contracts may generate efficient, but not optimal talent allocation within organizations.

7 Partnership

In this section, I analyze task allocation in partnerships. Suppose that before task allocation, the employer can decide whether to keep running the firm as a corporation, or to sell it to some workers. In the second case, buyers will run the firm as a partnership, thus changing the structure of the organization. A partnership is an organizational form in which some workers (or partners) have both cash flow and control rights. Most of the firms operating in professional services industries are organized as partnerships (Teece, 2003).

To maximize the sale price of the firm, the employer will select a bounded segment of abilities for prospective partners. The sale price depends on the profit of the partnerships, which in turn depends on who is made partner.

7.1 Equity and Shares

In order to analyze task allocation in an equity partnership, I introduce some notation. Let \( \phi \) denote the price of equity every prospective purchases from the current firm.

\(^{18}\) Up-or-out contracts are an extreme version of tournaments. For further details, see Lazear and Rosen (1981).
owner to buy her stake in the firm.\(^{19}\) Let \(\pi^P\) denote the profit of the firm organized as a partnership. The firm owner defines a segment \(y^P \in [\bar{y}, \bar{y}]\) in which a prospective partner’s ability should lie. Let \(y_1\) and \(y_2\) be respectively the lower and the upper bound of \(y^P\) chosen by the employer.

Every partner is entitled to a share of profit \(s(y) \in [0, 1]\) and for simplicity, assume the firm owner to sell the firm out, so that \(\int_{y_1}^{y_2} s(y)f(y)dy = 1\). The owner contracts \textit{vis-a-vis} with each prospective partner offering a partnership contract \(\{\phi(y), s(y)\} \forall y \in y^P\). This contract includes \(s(y)\) denoting the shares of the firm for a prospective partner with ability \(y\), and the cost of such equity \(\phi(y)\). When offering partnership contracts, the firm owner makes take-it-or-leave-it offers.

### 7.2 New Timing

The baseline timeline is slightly modified. The new timing of the game is the following:

- At \(t = 1\), firms bid competitively for workers offering \(w_1\). Workers who accept will work on a standard nonproductive task.
- At \(t = 2\), workers’ productivities become observable to them and to all the firms in the industry. Wages for the standard task are paid.
- At \(t = 3\), the firm owner selects the length of the segment \(y^P\) and offers a partnership contract \(\{\phi, s\}\).
- At \(t = 4\), potential partners accept or reject.
- At \(t = 5\), partners choose task allocation for themselves and salaried workers.
- At \(t = 6\), partners and salaried workers can leave the firm.
- At \(t = 7\), the production process is completed and wages are paid.

\(^{19}\)This fee may also be considered as a reduction in the ex-ante wage that a prospective partner pays in order to gain a higher wage ex-post. Importantly, this fee entitles the worker with control rights.
7.3 New Constraints

Prospective partners decide whether to buy the firm by accepting the partnership contract. A generic worker accepts the offer if a feasibility condition (defined as “willingness-to-pay” - WTP- constraint) is satisfied. Depending on the task she would be matched with in a corporation, either of two conditions needs to be satisfied for the worker to buy equity:

\[ \phi \leq s\pi^P - \theta_B x \quad \forall y \in [\underline{y}, \hat{y}) \quad (WTP_B) \]

or

\[ \phi \leq s\pi^P - \theta_A \beta y \quad \forall y \in [\hat{y}, \bar{y}] \quad (WTP_A) \]

At \( t = 3 \), the employer selects the boundaries of the segment \( y^P \), in order to maximize \( \int_{y_1}^{y_2} \phi(y)f(y)dy \). The owner is willing to sell the firm if

\[ \int_{y_1}^{y_2} \phi(y)f(y)dy \geq \pi. \quad (8) \]

Partners acquire cash flow and control rights: they earn a share of the realized profit of the firm rather than a fixed wage and decide over task allocation for themselves and all other employees. This affects the employer’s choice on whether to sell the firm, since it changes the profit generated and whereby the surplus to be extracted through the sale of equity \( \phi \).

For a segment \( y_P \) of length \( y_2 - y_1 \), and a certain task allocation, partners and salaried employees earn, respectively, a dividend or a wage. These remunerations should suffice to implement retention at the interim stage (\( t = 4 \)). The “interim” participation constraints for salaried workers are the same as in the maximization program for a corporation in section 4. For partners instead, interim participation constraints depend on the task they are matched with. A partner working on task A will not leave the firm if

\[ s\pi^P(y_1, y_2) \geq \theta_A \beta y. \quad (IPC_A) \]

A partner working on task B, instead, will not leave the firm if

\[ s\pi^P(y_1, y_2) \geq \theta_B x. \quad (IPC_B) \]

Given the interim participation constraints, the following lemma holds:
Lemma 2. The firm owner offers each prospective partner a nondecreasing share of the firm with respect to her ability, so that

\[ \frac{\partial s(y)}{\partial y} \geq 0. \] (9)

For all prospective partners to break even when accepting the partnership contract, the owner needs to offer “meritocratic” contracts. This result is far from obvious and rules out the possibility for the partnership to be an equal-sharing one. Most of the results in the existing literature are based on equal-sharing mechanisms (see, for instance, Levin and Tadelis, 2005). In this paper, workers’ abilities are continuously distributed and this requires the best partners to obtain different rents with respect to the less productive ones in order to break even. Hence, for the retention motive, partners must be entitled to a share of the firm proportional to their productivity.

7.4 The Employer’s Program

I will now analyze the employer’s optimal selection of partners and check whether the efficient task allocation is implemented in this framework.

Lemma 3. Efficiency in task allocation cannot be improved by selling the firm to workers who are efficiently allocated in a corporation.

Partners’ selection is crucial for the implementation of the efficient task allocation. If none of the workers who are inefficiently allocated by the initial firm owner is made partner, running the firm as a corporation or as a partnership makes no difference in terms of surplus generated. Profit maximizing partners would match tasks and workers in the same way as the incumbent owner would. There is no improvement with respect to the corporation case: the same surplus is differently distributed. In this case, the owner herself is indifferent between selling the firm and keeping it as a corporation.

Consider the cases in which the inefficiently allocated workers are offered a partnership contract. It is important to verify that they accept it, and that the dividend they earn will suffice to retain them after task allocation. The following proposition states the result obtained.
Proposition 5. If at least all workers with ability \( y \in [y^*, \hat{y}) \) are made partners, the efficient task allocation is implemented. The partnership generates a higher profit with respect to the corporation.

A necessary condition for the implementation of efficiency is that workers with ability \( y \in [y^*, \hat{y}) \) are made partners. When this is the case, they will have an incentive to accept the partnership contract and not to leave the firm at the interim stage. Since partners’ remuneration is given by a share of the profit realized, they have an incentive to allocate themselves and the other partners to tasks that maximize their productivity, increasing the profit generated. In this scenario, partners are committed to choices made. This allows to circumvent the holdup issue generated by contract incompleteness when only the firm owner has control rights.\(^{20}\)

Efficient task allocation generates more surplus to be split between partners and the incumbent firm owner (through the equity price paid to buy the firm). Moreover, the firm owner is indifferent to how many workers should be made partners on top of those with ability on the \([y^*, \hat{y})\) segment.

If the owner offers contracts such that both the WTP and the interim participation constraints bind, she is able to extract all the surplus generated by the partnership, as she charges positive fees \( \phi = \theta_A \beta y - \theta_B x \) for workers who would be inefficiently allocated in a corporation environment. In this framework partnerships can offer higher wages than corporations at \( t = 1 \). As they allocate talent efficiently, partnerships generate the highest expected surplus possible, which accrues to workers through \( w_1 \).

The model predicts that the firm owner is indifferent about how many workers with productivity larger or equal to \( \hat{y} \) should be made partners, but strictly prefers all workers with ability \( y \in [y^*, \hat{y}) \) to become partners. This is because the latter are efficiently allocated to task A when they are partners, whilst all other workers execute the same task as in a corporation. Thus, not only the best workers should be made partners, but also those who are more productive in task A than in task B and whose talent would not be used efficiently in a corporation. This provides a rationale for a “lower bound” on the ability of workers that should become partners for an organization to produce efficiently.

\(^{20}\)Given linearity of the problem at hand and perfect information, such result is attainable with both majoritarian and proportional voting rule.
8 Conclusions

This paper analyzes the impact of the portability of workers’ skills on task allocation and on the design of organizations in competitive labor markets. First, I study a setting in which the representative firm is organized as a corporation, where shareholders have control rights on task allocation. In the model presented, despite the labor market being perfectly competitive at the beginning of the game, the acquisition of partially portable skills in a firm may generate opportunistic behavior on the employer’s side. In order to reduce retention costs for talented workers, a profit maximizing firm will match workers with tasks inefficiently. This result fits those of a vast branch of the literature predicting inefficient allocation (or promotion) of valuable workers to reduce retention costs (Waldman, 1984; Greenwald, 1986; Bernhardt, 1995). This model’s result differs from the ones in the existing literature as it is driven by contract incompleteness rather than by asymmetric information about employees’ talent across firms.

Organizing the firm as an equity-partnership can implement the optimal task allocation and generate larger profits than when being organized as a corporation. This prediction is driven by the fact that the commitment problem is solved by giving control and cash flow rights to some workers. This solution to holdup issues resembles the one of “vertical integration”, described by Grossman and Hart (1986) and Hart and Moore (1990). These results provide a novel rationale for the widespread existence of partnerships in human capital-intensive industries, which by definition are more exposed to externalities from labor market competition.

I study the choice of a single owner who decides whether to sell the shares of the firm to some employees. The buyers would become partners and turn the corporation into a partnership. The results provided are still valid if the initial firm owner is considered as an individual partner, looking for new partners. Namely, at the beginning of the game, the firm can be assumed to be a partnership with a unique partner who wants to enlarge the pool of partners, thus avoiding the idea of a corporation evolving into a partnership.

I have assumed the skills acquired on the task producing an outcome dependent on workers’ ability to be the more portable ones. This assumption rests on the idea that a worker can carry along more of the output correlated with her talent from one firm to another. However, the ordering of the portability rates of skills acquired
dealing with the two tasks could be reversed, and inefficiency would still hold but in the opposite direction: there would be too many workers dealing with the more talent-sensitive task, so that again efficient production is foregone in favor of profit maximization.

In the model analyzed, workers are risk-neutral and production is deterministic. Relaxing these assumptions will change equilibrium wage contracts and incentives for workers to accept partnership contracts. In fact, salaried workers have limited liability, whereas partners do not (depending on the type of partnership one considers). Such a framework could be analyzed to deepen our understanding of the role of hybrid organizational structures such as limited-liability partnerships, in which some (or all) partners have limited-liability.

Finally, the model provides a partial equilibrium perspective of the partnership equilibrium, in order to focus on the shift of control rights within one organization. However, a general equilibrium analysis could provide broader predictions about the structure of entire industries and the demand for goods would also impact labor market competitiveness.
Appendix

Proof of Proposition 1

Proof. Consider the possibility for employer and employee to sign a contract in which the latter can commit not to leave the firm after task allocation. In this framework, at $t = 3$, workers are locked in and the firm can offer a fixed wage $w_2 = \bar{w}$. Namely, the firm pays workers’ reservation wage regardless of task allocation. Now, by backward induction, consider task allocation. The employer matches workers to tasks to maximize her profit. To do so, she defines an ability threshold $y^{**}$ for a worker to be allocated to task A. Namely, the allocative mechanism $A(y)$ is such that tasks will be assigned as follows:

$$A(y) = \begin{cases} 
\text{Task A} & \forall \ y \in [y^{**}, \bar{y}], \\
\text{Task B} & \forall \ y \in [y, y^{**}). 
\end{cases}$$

The firm’s expected profit is given by

$$\pi = \int_{y}^{\bar{y}} \beta y f(y) dy + F(y)x - \bar{w}. \quad (10)$$

The firm chooses the threshold as:

$$y^{**} \in \arg\max_{\{y\}} \pi \quad (11)$$

The first-order condition for the profit maximization problem is

$$f(y^{**})x - \beta y^{**} f(y^{**}) = 0$$

delivering the optimal threshold value

$$y^{**} = \frac{x}{\beta} = y^*. \quad (12)$$

Therefore the employer allocates all workers with ability $y \geq \frac{x}{\beta}$ to task A, and all the others to task B in a competitive equilibrium without labor market competition after task allocation.
Since the labor market is perfectly competitive at $t = 1$, the worker extracts all the expected surplus through $w_1$. $\blacksquare$

Proof of Proposition 2

Proof. Suppose the employer offers contracts $\{w_1, w_2(y), i(y)\}$. These contracts will be:

$$\{w_1, \theta_A \beta y, A\} \forall y \in [y^*, \bar{y}]$$

(13)

and

$$\{w_1, \theta_B x, B\} \forall y \in [\underline{y}, y^*)$$

(14)

with $y^* = \frac{x}{\beta}$. Namely, the firm commits to allocate tasks efficiently.

In this case, at $t = 1$, the worker expects

$$E[w_2(y^*)] = \theta_A \int_{y^*}^{\bar{y}} \beta y f(y) dy + F(y^*)\theta_B x$$

(15)

if the interim participation constraints bind in equilibrium. The ex-ante individually rational wage is instead

$$w_1(y^*) = (1 - \theta_A) \int_{y^*}^{\bar{y}} \beta y f(y) dy + F(y^*)(1 - \theta_B)x.$$  (16)

Notice that this allocation generates the highest surplus possible, so that, given labor market perfect competitiveness and the possibility for the firm to commit to task allocation upfront, only firms implementing the efficient task allocation will attract workers and be active. $\blacksquare$

Proof of Proposition 3

Proof. As in the proof for Proposition 1, the employer allocates workers across tasks so as to maximize her profit. To do so, she defines an ability threshold $\hat{y}$ for a worker to be allocated to task A. Specifically, the allocative mechanism $A(y)$ is such that tasks will be assigned as follows: 

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\[ A(y) = \begin{cases} 
\text{Task A} & \forall y \in [\hat{y}, \bar{y}], \\
\text{Task B} & \forall y \in [y, \hat{y}). 
\end{cases} \]

Solving the model by backward induction, first consider task allocation at \( t = 3 \). The firm maximizes its expected profit the threshold \( \hat{y} \). It does so by taking into account both the “ex-ante” and the “interim” participation constraints. Therefore the maximization program is:

\[
\begin{aligned}
Max_{\{\hat{y} \in [y; \bar{y}]\}} \pi &= \int_{\hat{y}}^{\bar{y}} \beta y f(y)dy + F(\hat{y})x - w_1 - \int_{\hat{y}}^{\bar{y}} w_2^A(y)f(y)dy - F(\hat{y})w_2^B \\
\text{subject to the “ex-ante” participation constraint}
\end{aligned}
\]

\[
\begin{align*}
 w_1 &\geq \mathbb{E}(\pi) \tag{EAPC} \\
\text{through which workers extract all the expected surplus generated and the “interim” participation constraints depending on task allocation:}
\end{align*}
\]

\[
\begin{align*}
 w_2^A &\geq \theta_A \beta y \tag{IPC_A} \\
 w_2^B &\geq \theta_B x \tag{IPC_B}
\end{align*}
\]

The interim participation constraints bind in equilibrium. By plugging these constraints in the objective function and maximizing with respect to \( \hat{y} \), one gets the first-order condition:

\[
(1 - \theta_A)\beta \hat{y} f(\hat{y}) - f(\hat{y})(1 - \theta_B)x = 0
\]

yielding the equilibrium threshold:

\[
\hat{y} = \frac{(1 - \theta_B)x}{(1 - \theta_A)\beta} \tag{18}
\]
Comparing the profit maximizing threshold (18) with the efficient one (12), since \( \theta_B < \theta_A \), it is immediate to see that \( \hat{y} > y^* \). This result is robust as it persists in the limit values of \( \theta_A \) and \( \theta_B \).

At \( t = 1 \), workers earn the firm’s expected profit, as

\[
w_1 = (1 - \theta_A) \int_{\bar{y}}^{\hat{y}} \beta y f(y) dy + (1 - \theta_B) F(\hat{y})x.
\]

Proof of Lemma 1

Proof. Suppose the employer offers up-or-out contracts and sets the minimum wage to \( w_2 = \theta_A \beta y^* \). Namely, when abilities become observable in the industry, workers whose productivity is lower than \( y^* \) will be laid off. All workers with talent \( y \in [y^*, \bar{y}] \) are kept in the firm. All the employees that are kept in the firm are allocated to task A. Workers who would have been inefficiently allocated in the presence of a simple wage contract, are now paid the amount they produce outside the source-firm if allocated to task A. Furthermore, since the labor market is perfectly competitive, all workers will be paid their marginal productivity outside the source-firm when allocated to task A for the retention motive.

Note that I assume \( w_2 = \theta_A \beta y^* \) as a performance requirement for a worker to be kept in the firm. This is because I study the “best” possible scenario, in which the threshold for the allocation of task A is efficient.

All workers with productivity \( y \in [\underline{y}, y^*) \) are laid off. Hence, the firm needs to replace dismissed workers to also produce by means of task B. Two options are available: either hiring workers with unknown talent from the labor market or poaching workers working on task B in competing firms. Since the first would need a training period before being productive, it is more convenient for the firm to hire workers already trained by competing firms.

However, workers who were assigned to task B in a competing firm produce \( \theta_B x \) outside of it. In order to poach them, the representative firm needs to pay their marginal productivity, so that the profit on task B will be null. Hence, the firm is indifferent between producing or not by means of task B.
Hence, the expected profit at $t = 3$ in the presence of an up-or-out contract is

$$\pi_{UO} = \int_{y^*}^{\hat{y}} (1 - \theta_A) \beta y f(y) dy. \quad (19)$$

With this mechanism, the firm is substituting workers who would have produced $x$ with others that will produce $\theta_B x$. Hence the loss in human-capital due to the up-or-out contract is

$$F(y^*)(1 - \theta_B) x > 0. \quad (20)$$

Since I assume $0 < \theta_B < \theta_A \leq 1$, implementing the efficient task allocation through an up-or-out policy, has a positive cost in terms of human capital.

Moreover, note that I assume $w_2 = \theta_A \beta y^*$, but this is not the optimal wage the firm would set in equilibrium. In fact, in order to maximize $\pi_{UO}$, the firm should set $w_2 = \theta_A \beta y$ so that no worker would be laid off at $t = 2$, and all of them would be allocated task A. This is true if

$$\int_{y^*}^{\hat{y}} (1 - \theta_A) \beta y f(y) dy \leq \int_{y}^{\hat{y}} (1 - \theta_A) \beta y f(y) dy. \quad (21)$$

This condition is satisfied since in the model $\beta y \geq 0$ for any $y \in [y, y^*]$.

\[
\text{Proof of Proposition 4}
\]

\textit{Proof.} The benefit from including an up-or-out clause in the labor contracts is denoted as $\Delta_{UO}$ and is given by

$$\Delta_{UO} = \pi_{UO} - \pi = \int_{y^*}^{\hat{y}} (1 - \theta_A) \beta y f(y) dy - F(\hat{y})(1 - \theta_B) x \quad (22)$$

where, $\pi_{UO}$ is defined in equation (19) and $\pi$ is the profit of the firm when simple wage contracts are in place.

From inequality (21) it is immediate to see that

$$\int_{y^*}^{\hat{y}} (1 - \theta_A) \beta y f(y) dy \leq \int_{y}^{\hat{y}} (1 - \theta_A) \beta y f(y) dy. \quad (23)$$

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and the result of the firm’s profit maximization problem delivers the inequality

$$\int_{y}^{\hat{y}} (1 - \theta_A)\beta y f(y) dy \leq (1 - \theta_B)F(\hat{y})x. \quad (24)$$

These two inequalities imply that $\Delta_{UO} < 0$. Hence, firms prefer offering simple wage contracts instead of up-or-out ones. This is true for any cutoff value chosen for up-or-out contracts.

\section*{Proof of Lemma 2}

\begin{proof}
For the owner to charge non-negative fees, partners need to receive a non-negative rent. This condition is embedded in the interim participation constraints. By backward induction, the owner knows to what task partners will be allocated. For partners operating task B, the share to be offered is

$$s(y) \geq \frac{\theta_B x}{\pi P} \quad (25)$$

which is constant for a given set of prospective partners and profit, such that $\frac{\partial s(y)}{\partial y} = 0$. On the other hand for partners that will be allocated to task A, the owner needs to offer a share

$$s(y) \geq \frac{\theta_A \beta y}{\pi P} \quad (26)$$

which is increasing in $y$, namely, $\frac{\partial s(y)}{\partial y} > 0$.
\end{proof}

\section*{Proof of Lemma 3}

\begin{proof}
I consider two cases:

1. Let $y_1 \in [y, y_2]$ and $y_2 \in [y_1, y^*)$

Proceeding by backward induction, at stage 4 of the game, partners and workers may leave. However the interim participation constraint for partners depends on task allocation which is given for granted at stage 4. Lemma 1 shows that
the firm owner sets the stake $s(y)$ so that the interim participation constraints are satisfied. Given the segment of abilities considered, when choosing over task allocation, partners will keep the same task allocation for salaried workers as in the case in which the firm is organized as a corporation. Hence, the “interim” participation constraints for salaried workers are the same as the ones analyzed in section 4 and bind in equilibrium as partners have all the bargaining power.

When maximizing the firm’s profit, partners face the following choice:

$$
\max \left\{ \left[ F(y_2) - F(y_1) \right] x , \int_{y_1}^{y_2} \beta y f(y) dy \right\}.
$$

Since $y_1$ and $y_2$ are smaller than $y^*$, from the profit maximization problem (both first-best and second-best case), all workers with a lower productivity than $y^*$ would be better off producing on task B, independently of whether they need to be incentivized not to leave the firm ex-post, or not. Moreover, given the linearity of the problem at hand, there is no profitable deviation from allocating all partners to the same task. Thus, the profit of the partnership will be

$$
\pi_P = \left[ F(y_1) + F(y^*) - F(y_2) \right] (1 - \theta_B)x + \int_{y_1}^{y_2} (1 - \theta_A) \beta y f(y) dy + \left[ F(y_2) - F(y_1) \right] x
$$

(27)

Then the employer’s problem will be:

$$
\max_{\{y_1 , y_2\}} \int_{y_1}^{y_2} \phi(y) f(y) dy
$$

subject to the feasibility constraint for prospective partners

$$
\phi(y) \leq s(y) \pi_P (y_1 , y_2) - \theta_B x.
$$

(WTP$_B$)

Their interim participation constraints yield

$$
s(y) \geq \frac{\theta_B x}{\pi_P (y_1 , y_2)} \quad \forall y \in [y_1 , y_2]
$$

(28)

Whenever the employer earns at least zero-profit from running the corporation, the interim participation constraints shall always hold. Should this not be the case, the firm owner would be eager to pay some workers for them to run
the firm. Indeed, should $IPC_B$ not be satisfied, then $WTP_B$ shows that the highest fee that the employer can require from the prospective partners would be negative. Hence, as long as there is a change in the firm management, no partner would leave the firm at the interim stage.

As the employer makes take-it-or-leave-it offers, all the $WTP_B$ constraints will bind in equilibrium, so the equity for any prospective partner in the segment analyzed is $\phi = s(y)\pi^P - \theta_B x$. Summing up all the fees delivers the price at which the owner will sell the firm:

$$
\int_{y_1}^{y_2} \phi(y)f(y)dy = \int_{y_1}^{y_2} s(y)\pi^P f(y)dy - \int_{y_1}^{y_2} \theta_B x f(y)dy.
$$

boiling down to the following maximization program

$$
Max_{\{y_1, y_2\}} \pi^P - [F(y_2) - F(y_1)] \theta_B x = \pi \tag{29}
$$

Namely, the employer again maximizes the profit of the firm as a corporation, which is independent of the bounds of the $y_P$ segment. The owner of the firm is indifferent between selling the firm or keep running it as a corporation, for all $y_1$ and $y_2$ smaller than $y^*$ as the surplus is unchanged.

2. Let $y_1 \in [\hat{y}, y_2]$ and $y_2 \in [y_1, \bar{y}]$

In this case, all prospective partners would be allocated to task A in a corporation. As in the previous case, all the salaried employees are allocated across tasks according to the same mechanism used in a profit maximizing corporation.

The task allocation process for partners, through

$$
max \left\{ \int_{y_1}^{y_2} \beta y f(y)dy , \left[ F(y_2) - F(y_1) \right] x \right\} \tag{30}
$$

leads to a profit
\[ \pi^P = F(\hat{y})(1-\theta_B)x + \int_{\hat{y}}^{y_1} (1-\theta_A)\beta y f(y)dy + \int_{\hat{y}}^{\bar{y}} (1-\theta_A)\beta y f(y)dy + \int_{y_1}^{y_2} \beta y f(y)dy. \]  
(31)

The employer’s maximization problem will be:

\[ \text{Max}_{\{y_1, y_2\}} \int_{y_1}^{y_2} \phi(y) f(y)dy \]

subject to the willingness-to-pay conditions for all the prospective partners

\[ \phi \leq s(y)\pi^P - \theta_A\beta y \forall y \in [y_1, y_2] \]  
(WTP\(_A\))

and the interim participation constraints delivering

\[ s(y) \geq \frac{\theta_A\beta y}{\pi^P(y_1, y_2)} \forall y \in [y_1, y_2]. \]  
(32)

As in the previous case, as long as the firm as a corporation generates at least zero profit, the interim participation constraints shall be satisfied if a change in the management of the firm takes place. Should this not be the case, the employer would pay a positive amount to the prospective partners in order to sell them the firm.

Since the firm owner has all the bargaining power, the WTP-constraints will all bind in equilibrium, hence the price at which the firm can be sold is

\[ \int_{y_1}^{y_2} \phi(y) f(y)dy = \int_{y_1}^{y_2} s(y)\pi^P f(y)dy - \int_{y_1}^{y_2} \theta_A\beta y f(y)dy \]

yielding the objective function

\[ \int_{y_1}^{y_2} \phi(y) f(y)dy = \pi^P - \int_{y_1}^{y_2} \theta_A\beta y f(y)dy = \pi. \]  
(33)

Again, the employer is indifferent about who shall be made partner in the pool of workers with ability larger than \( \hat{y} \). This result hinges on the fact that these workers would not generate any extra surplus through the profit of the firm as a partnership with respect to the case in which it is organized as a corporation. Namely, such a choice of partners would not generate an improvement.
It is straightforward to see that the firm owner would be indifferent between selling the firm or running it as a corporation.

■

Proof of Proposition 6

Proof. To prove this proposition, consider three cases:

1. Let $y_1 \in [y^*, y_2]$ and $y_2 \in [y_1, \hat{y})$

When task allocation is chosen, all salaried employees will be allocated to the same task as in a corporation. Hence, partners are allocated to tasks by choosing

$$\max \left\{ \int_{y_1}^{y_2} \beta y f(y) dy, \left[ F(y_2) - F(y_1) \right] x \right\}.$$

The crucial element here is that if workers with ability included in the segment $[y^*, \hat{y})$ do not need to be paid a fraction of their productivity for retention purposes, they just add their output to the profit of the partnership. For the above mentioned segment of abilities, it will always be the case that

$$\int_{y_1}^{y_2} \beta y f(y) dy > \left[ F(y_2) - F(y_1) \right] x$$

therefore, the profit of the partnership after task allocation will be

$$\pi^P = \left[ F(y_1) + F(\hat{y}) - F(y_2) \right] (1 - \theta_B) x + \int_{y_1}^{y_2} \beta y f(y) dy + \int_{\hat{y}}^{y_2} (1 - \theta_A) \beta y f(y) dy.$$

(34)

And the employer’s maximization program is

$$\max_{\{y_1, y_2\}} \int_{y_1}^{y_2} \phi(y) f(y) dy$$

subject to the willingness-to-pay constraints for prospective partners

$$\phi \leq s(y) \pi^P(y_1, y_2) - \theta_B x \ \forall \ y \in [y_1, y_2] \quad (WTP_B)$$
their interim participation constraints yield
\[ s(y) \geq \frac{\theta_A \beta y}{\pi P(\cdot)} \forall y \in [y_1, y_2]. \tag{35} \]

All the \((WTP_B)\) constraints will bind in equilibrium. The employer’s objective function shall be
\[ \int_{y_1}^{y_2} \phi(y) f(y) dy = \int_{y_1}^{y_2} s(y) \pi P(\cdot) f(y) dy - \int_{y_1}^{y_2} \theta_B x f(y) dy. \]

Boiling down to
\[ \int_{y_1}^{y_2} \phi(y) f(y) dy = \pi P - \left[ F(y_2) - F(y_1) \right] \theta_B x. \tag{36} \]

By further working out (36) the employer’s objective function becomes
\[ \int_{y_1}^{y_2} \phi(y) f(y) dy = \pi + \int_{y_1}^{y_2} \beta y f(y) dy - \left[ F(y_2) - F(y_1) \right] x \tag{37} \]

So the (WTA) constraint for the firm owner will always be satisfied, as workers with ability in the considered segment will produce more when allocated to task A rather than to task B.

The first-order condition with respect to \(y_1\) is
\[ f(y_1) x - \beta y_1 f(y_1) = 0 \tag{38} \]

which is concave with respect to \(y_1\), so that \(y_1 = \frac{\pi}{\beta} = y^*\) is a maximum.

The first-order condition with respect to \(y_2\) boils down to
\[ \beta y_2 f(y_2) - f(y_2) x = 0. \tag{39} \]

In this case, the second derivative with respect to \(y_2\) is positive, hence the objective function is convex with respect to \(y_2\) and the resulting equilibrium level \(y_2 = \frac{\pi}{\beta} = y^* = y_1\) is a minimum. Linearity of the problem, implies that the owner will pick the maximum value achievable \(\hat{y}\) for \(y_2\) in order to maximize the objective function: whenever the employer picks partners in the segment of abilities \([y^*, \hat{y})\), in equilibrium she offers the partnership contract to all of
them.
In equilibrium the profit of the partnership is
\[
\pi^P = F(y^*)(1 - \theta_B)x + \int_{y^*}^{\hat{y}} \beta y f(y) dy + \int_{\hat{y}}^{\bar{y}} (1 - \theta_A)\beta y f(y) dy.
\] (40)

Organizing the firm as a partnership makes room for an increase of surplus at stake as the profit of the firm is increased by
\[
\Delta \pi = \pi^P - \pi = \int_{y^*}^{\hat{y}} \beta y f(y) dy - \left[ F(\hat{y}) - F(y^*) \right] (1 - \theta_B)x > 0 \quad (41)
\]

There is a continuum of possible equilibria, depending on the value of \(s(y)\). It is worth analyzing the case in which all the interim participation constraints bind, so that \(s(y) = \frac{\theta_A y}{\pi^P}\). In this case, the equilibrium fee required of each prospective partner is \(\phi = \theta_A \beta y - \theta_Bx\) so that the owner extracts all the extra surplus generated by the partnership. In this case, the first-order condition with respect to \(y_1\) delivers
\[
y_1 = \frac{\theta_Bx}{\theta_A \beta} < y^*
\]
but since \(y_1\) is bounded between \(y^*\) and \(y_2\), in equilibrium \(y_1 = y^*\) consistent with the general solution provided above. In this case, the owner manages to extract all the extra rent generated by efficient task allocation.

2. \(Let y_1 \in [y^*, \hat{y}) \text{ and } y_2 \in [\hat{y}, \bar{y}]\)

In this case, during the task allocation process, partners take into account the following:
\[
\max \left\{ \int_{y_1}^{\hat{y}} \beta y f(y) dy , \left[ F(\hat{y}) - F(y_1) \right] x \right\} +
\]
and
\[
\max \left\{ \int_{\hat{y}}^{y_2} \beta y f(y) dy , \left[ F(y_2) - F(\hat{y}) \right] x \right\}.
\] (42)

Given the segment of abilities that are considered, it is better if all partners are allocated to task A. The profit thereby generated is

48
\[ \pi^P = F(y_1)(1 - \theta_B)x + \int_{y_1}^{y_2} \beta y f(y) dy + \int_{y_2}^{\tilde{y}} (1 - \theta_A)\beta y f(y) dy. \quad (43) \]

The employer’s maximization program will be

\[ \max_{\{y_1, y_2\}} \int_{y_1}^{y_2} \phi(y) f(y) dy \]

subject to the willingness-to-pay constraints for prospective partners, which differ depending on their abilities:

\[ \phi(y) \leq s(y)\pi^P(\cdot) - \theta_B x \quad \forall y \in [\hat{y}, \tilde{y}] \quad (WTP_B) \]

and

\[ \phi(y) \leq s(y)\pi^P(\cdot) - \theta_A \beta y \quad \forall y \in [\hat{y}, \tilde{y}] \quad (WTP_A) \]

The interim participation constraints are instead equal for all prospective partners and deliver

\[ s(y) \geq \frac{\theta_A \beta y}{\pi^P(\cdot)} \quad \forall y \in [y_1, y_2]. \quad (44) \]

As all the willingness-to-pay constraints will bind in equilibrium, the objective function for the employer will be

\[ \int_{y_1}^{y_2} \phi(y) f(y) dy = \pi^P - \left[ F(\tilde{y}) - F(y_1) \right] \theta_B x - \int_{\hat{y}}^{y_2} \beta y f(y) dy. \quad (45) \]

The first-order condition with respect to \( y_1 \) for this problem is

\[ f(y_1)(1 - \theta_B)x - \beta y_1 f(y_1) + f(y_1)\theta_B x = 0 \quad (46) \]

yielding the maximizer \( y_1 = \frac{x}{\beta} = y^* \).

The first-order condition with respect to \( y_2 \) is

\[ \beta y_2 f(y_2) - (1 - \theta_A)\beta y_2 f(y_2) - \beta y_2 f(y_2) = 0 \quad (47) \]

which implies indifference for the employer.
The firm owner just needs to make all workers with ability on the segment \([y^*, \hat{y})\) partners, and then she is indifferent to the same choice among the most productive workers. This happens because the increase in surplus is deriving from the previously inefficiently allocated workers, whereas workers who are efficiently allocated in a corporation do not increase the surplus at stake in a partnership.

In this case the increase in the realized profit with respect to the case in which the firm is organized as a corporation will be

$$\Delta \pi = \int_{y^*}^{\hat{y}} \beta y f(y)dy + \int_{y^*}^{\hat{y}} \theta_A \beta y f(y)dy - \left[ F(\hat{y}) - F(y^*) \right] (1 - \theta_B)x > 0. \quad (48)$$

It is straightforward to see that the (WTA) for the employer is definitely satisfied in this scenario.

In this case, if the employer offers stakes \(s(y)\) such that the interim participation constraints bind, she charges strictly positive fees only on workers with ability \(y \in [y^*, \hat{y})\).

3. Let \(y_1 \in [y, y^*)\) and \(y_2 \in [y^*, \hat{y})\)

In this case, when choosing upon task allocation, partners select:

$$\max \left\{ \int_{y^*}^{y_2} \beta y f(y)dy , \left[ F(y_2) - F(y^*) \right] x \right\}$$

and

$$\max \left\{ \int_{y_1}^{y^*} \beta y f(y)dy , \left[ F(y^*) - F(y_1) \right] x \right\}.$$

Given the segment of abilities considered, the resulting profit for the partnership is

$$\pi^P = \left[ F(y_1) + F(\hat{y}) - F(y_2) \right] (1 - \theta_B)x + \int_{y^*}^{y_2} \beta y f(y)dy +$$

$$+ \left[ F(y^*) - F(y_1) \right] x + \int_{y}^{\hat{y}} (1 - \theta_A) \beta y f(y)dy. \quad (49)$$
The employer’s maximization problem is
\[
\max_{\{y_1, y_2\}} \int_{y_1}^{y_2} \phi(y) f(y) \, dy
\]
subject to the willingness-to-pay constraints for all partners
\[
\phi \leq s(y) \pi^P(\cdot) - \theta_B x \quad \forall y \in [y_1, y_2]
\]
(WTP_B)
and the interim participation constraints yield
\[
s(y) \geq \frac{\theta_B x}{\pi^P(\cdot)} \quad \forall y \in [y_1, y^*) \tag{50}
\]
and
\[
s(y) \geq \frac{\theta_A \beta y}{\pi^P(\cdot)} \quad \forall y \in [y^*, y_2]. \tag{51}
\]
Given that the (WTP)-constraints will bind in equilibrium, summing them up delivers the objective function for the firm owner:
\[
\pi^P - [F(y_2) - F(y_1)] \theta_B x.
\]
The first-order condition with respect to \(y_1\) is
\[
f(y_1)(1 - \theta_B)x - f(y_1)x + f(y_1)\theta_B x = 0 \tag{52}
\]
so that the owner is indifferent about where to set the lower bound for the segment \(y_P\) in the considered segment of abilities.

The first-order condition with respect to \(y_2\) is
\[
\beta y_2 f(y_2) - f(y_2)(1 - \theta_B) x - f(y_2)\theta_B x = 0 \tag{53}
\]
so the stationary value \(y_2 = \frac{\hat{y}}{\beta} = y^*\) which is a minimum, given that the second derivative with respect to \(y_2\) is positive. For the linearity of the problem, the owner selects \(y_2 = \hat{y}\) as a maximizer.

So the owner is indifferent to how many workers would be made partners in the interval of abilities below \(y^*\), but she wants to make all workers with ability between \(y^*\) and \(\hat{y}\) partners.
Such a result is in line with the one derived for the previous case. The increase in the profit realized is

\[ \Delta \pi = \int_{y^*}^{\hat{y}} \beta y f(y) dy + F(\hat{y}) \theta_B x - \left[ F(\hat{y}) - F(y^*) \right] x > 0 \] (54)

which ensures that the employer will be willing to sell the firm (namely, the WTA-constraint is satisfied).

Even in this case, if the interim participation constraints bind in equilibrium, the owner charges strictly positive fees only on workers with ability \( y \in [y^*, \hat{y}] \).


References


3 Promotions and Training: 
Do Competitive Firms set the Bar too High?
Promotions and Training: Do Competitive Firms Set the Bar too High?∗

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Abstract

Firms use promotions to match workers with jobs that fit their skills, and to provide incentives to exert on-the-job training effort. As promotions make workers more attractive in the labor market, firms will balance productivity and retention costs. I show that if workers exert firm-specific training effort, profit-maximizing firms that cannot commit to promotion rules promote fewer workers than efficient. Differently, if firms can commit to promotion bars, they set the bar efficiently. If workers acquire portable training, this directly increases retention costs. Firms that cannot commit to promotion bars will set them inefficiently high. In this case, workers are discouraged from training when competition for talent is fierce. If firms can commit to promotion bars, they set them lower than without commitment providing strong incentives for workers to acquire portable training. However, in this scenario the promotion bar may be set too low compared with the efficient talent allocation.

JEL classification: D86, M51, M52, M53.

Keywords: Promotions, on-the-job training, poaching, career concerns.

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1 Introduction

In talent-intensive industries, workers’ productivity crucially depends on talent and on-the-job training. Firms often provide workers with financial incentives to exert training effort.\textsuperscript{1} Nonetheless, in many cases, promotion-based incentives are more important than simple wage raises.\textsuperscript{2} Promotions serve two purposes: the provision of incentives for workers to exert training effort and workers’ matching with jobs for which they are best suited. This is especially the case in sectors in which the rigidity of wage schemes (for instance, the public sector), does not allow to reward workers executing the same job with too different salaries. However, the productive benefits of promotions contrast with the drawback that it is more expensive to retain promoted workers, who become more attractive in the labor market.\textsuperscript{3} This generates a tradeoff for firms between productivity and retention costs.

This paper analyzes the relationship between firms’ promotion decisions and workers’ on-the-job specific and portable training effort in competitive labor markets. After having trained and being promoted, workers may receive job offers by competing firms, thus imposing retention costs on the current employer (see for instance, Lazaer, 1986 and Kim, 2014). Henceforth, competition for workers affects talent allocation within organizations (Waldman, 1984; Greenwald, 1986; Bernhardt, 1995; Picariello, 2017; Mukherjee and Vasconcelos, 2018). By affecting talent allocation, firms’ competition for talent may also have an impact on workers’ incentives to exert training effort.

I study a model featuring learning about workers’ talents and unobservable training effort. Firms produce their output by means of two different tasks: one talent-sensitive, the other not. Before being assigned to either task, workers undertake a training stage, whose outcome provides a signal about their talents, based on which firms make their promotion decisions. Specifically, firms infer workers’ talent by observing this signal and promote those whose outcome fulfills a minimum cutoff referred to as promotion bar.

Promoted workers do talent-sensitive tasks and can be poached by competing firms. If profit-maximizing firms cannot commit to promotion bars and compete more fiercely for

\textsuperscript{1}Lazear, 2000; Gaynor, et. al, 2004; Kahn, et. al, 2016; Friebel, et. al, 2017 document the effectiveness of pay-for-performance incentive schemes

\textsuperscript{2}Baker, Jensen and Murphy (1988), Baker, Gibbs and Holmström (1994a) and (1994b) document empirically the prevalence of promotion-based incentive schemes in hierarchical organizations. McCue (1996) shows that promotions account for around 15% of wage growth in organizations.

\textsuperscript{3}For instance because they become more visible as in Milgrom and Oster (1987).
promoted workers than for nonpromoted ones, they set promotion bars inefficiently high. Namely, firms promote fewer workers than is efficient, and the stronger competition for promoted workers, the more severe is this inefficiency.

On-the-job training is either firm-specific or portable (Groysberg, et al., 2008; Groysberg, 2010). The former enhances workers’ productivity only in the current firm, the latter is valuable for any firm in a certain industry, hence it increases workers’ attractiveness on the labor market and raises retention costs. Since firm-specific training effort inflates the signal generated at the training stage, it does not affect firms’ selection of promotion bars. I show that if firms are willing to promote a positive number of workers, effort to acquire specific training monotonically increases with the degree of competition for promoted workers, as this implies higher expected wages conditional on promotion. If instead, firms do not promote workers at all, workers have no incentive to acquire specific training, making effort discontinuous with respect to labor market competition in equilibrium.

Differently, workers’ portable training effort, by raising retention costs, affects firms’ choice of promotion bars. When unable to commit ex-ante, profit-maximizing firms set the bar increasingly high with respect to both labor market competition and workers’ effort, to shield themselves against aggressive poaching raids. This generates two contrasting effects: on the one hand, high promotion bars imply high expected wages for workers, conditional on promotion, so that they are eager to exert training effort to raise the signal sent to the market; on the other hand, workers are less likely to hit too high promotion bars, hence they have weak incentives to exert effort as on-the-job training is valuable only upon promotion and the cost of it may be sunk. For this reason, portable training effort is nonmonotonic with respect to labor market competition in equilibrium. This provides a novel result, as the existing literature (Becker, 1962; Rosen, 1972; Garmaise, 2011) postulates that workers have strong incentives to exert portable training effort in fiercely competitive environments. However, if profit-maximizing firms react to labor market competition by setting the bar very high, workers have weak incentives to exert effort when competition is fierce, as they have low chances of being promoted.

Finally, I show that if firms are able to commit to promotion bars before workers’ training, thus signing long-term agreements with workers, the latter have stronger incentives to exert on-the-job training effort. Inefficiencies in promotions and portable training effort exertion are reduced or reversed with respect to the case without commitment. If workers exert specific training effort, firms commit to promote workers efficiently. If workers exert portable training effort, the promotion bar in equilibrium is nonmonotonic with respect
to the degree of competitiveness for promoted workers. Firms internalize the fact that promotion bars affect workers’ incentives to exert effort and to work for a certain employer at the hiring stage. If labor market competition is low, workers have low incentives to exert training effort, as they secure a small share of the realized surplus, hence their expected wages are too low. In this case, firms strengthen incentives by committing to very low promotion bars, setting them below the efficient benchmark. This raises expected wages by making promotions more affordable in expectation, so that workers exert more effort than without commitment at the cost of productive inefficiency. When instead, competition for workers is sufficiently fierce, a worker secures a large share of surplus, so that the market itself provides strong incentives. But in this case, it is not profitable for firms to provide incentives for workers to exert effort, as retention costs would be extremely high, hence they set the bar higher than the efficient benchmark, to shield themselves against too aggressive poaching raids. In this scenario, workers exert inefficiently too much training effort.

To sum up, portability of training effort and firms’ inability to commit to promotion bars imply a discouragement effect for workers, who exert low training effort in competitive environments as they expect firms to set the bar for promotion too high. However, if firms can commit to promotion bars before workers’ training and labor market competition is not too fierce, promotion bars are set lower than the efficient benchmark. If labor market competition is sufficiently fierce, firms commit to inefficiently high promotion bars, to induce workers to exert less effort, namely, to reduce retention costs. Workers exert more portable training effort when firms commit to promotion bars than when they do not, for any degree of competitiveness.

The structure of the paper is as follows. Section 2 compares its contributions to those in the existing literature. Section 3 lays out the model’s features and assumptions. Section 4 derives the efficient promotion rules and training effort to provide a benchmark for the following analyses. Section 4.1 shows a setting in which efficient promotions and effort are attainable. Section 5 studies workers’ incentives to exert firm-specific training effort, respectively when promotions rules are set after and before training takes place. Section 6 focuses on the incentives to exert portable training effort, in settings in which promotion rules are set ex-post and ex-ante with respect to workers’ training. Section 7 concludes.
2 Related Literature

This paper contributes to the literature in organizational and personnel economics studying the effects of promotion-based incentives in organizations. Specifically, it focuses on the role of promotions in competitive labor markets, both as an incentive for workers to exert training effort and as a device to allocate talent across heterogeneous tasks.

I develop a model featuring uncertain talent and learning, as in career concerns models like Fama (1980), Harris and Holmström (1982), Gibbons and Murphy (1992), Holmström (1999), and Bonatti and Hörner (2017). In my model, workers have career concerns insofar as they exert effort to improve firms’ perceptions of their talent, hence their likelihood to obtain promotions. This paper focuses on promotions as a device to provide workers with incentives to exert effort, while in Holmström (1999) (and the successive literature), competitive wages serve this scope. Having promotions instead of wages allows for the analysis of inefficiencies generated by firms and to study workers’ reaction to them. In Holmström (1999), the labor market is perfectly competitive at any stage of the game, while in this paper, it is only at the hiring stage and then firms compete differently for promoted and nonpromoted workers. Differently with respect to Holmström (1999), in this paper, when firms are unable to commit to promotion bars, workers’ effort results in being nonmonotonic with respect to labor market competition, because the firm takes into account the impact of competitiveness by raising promotion bars. When workers exert portable training effort and competition for promoted workers is fierce, the model I present converges to the one in Holmström (1999) and in both frameworks, workers exert too much effort with respect to the efficient benchmark.

This paper is also related to the literature on job assignment within organizations, including Waldman (1984), Bernhardt (1995) and more recently, Mukherjee and Vasconcelos (2018). These papers share results of inefficient job assignment within organizations with asymmetric learning between firms. They show that if a firm has an informational advantage about the ability of its employees, it may exploit it and allocate them inefficiently across tasks. This is because competing firms perceive task allocation as a signal of workers’ talent. Hence, misallocation serves to discourage poaching. In this paper this is also the case, but I show that labor market competition not only impairs efficient talent allocation, but it also affects workers’ incentives to exert training effort. Hence, I show an hitherto neglected inefficiency from competition for talent. Becker (1962), Rosen (1972) and Garmaise (2011), show that workers are eager to train in competitive environments.
Yet, they did not consider the impact that such competition has on firms’ decisions, which in turn affect workers’ incentives.

The first paper analyzing workers’ incentives to train over their career is Ben-Porath (1967). Carmichael (1983), studies the impact of workers’ seniority and promotion ladders on firms’ and workers’ investments in specific human capital. Both papers study models featuring asymmetric information (adverse selection) between the firm and its employees. I analyze a model in which both firms and workers are uncertain about the latter’s talent. More recently, Ashraf, et. al (2014), study the impact of promotion incentives on workers’ job selection. Karachiwalla and Park (2017) provide empirical evidence of how promotions affect teachers’ performances in Chinese schools. Brogaard et., al. (2017) show the impact of tenure on finance professors’ research performance and risk-taking in terms of projects undergone.

Prendergast (1993) studies how promotions incentivize workers to acquire firm-specific human capital, in a setting featuring moral hazard. As in this paper, Prendergast (1993) assumes workers’ investment to be unobservable, hence neither firms nor workers can commit respectively to wages and investments. If different tasks in the firm are associated to different wages, then this commitment issue can be solved under certain conditions. However, my model and predictions differ from Prendergast’s (1993): I analyze the possibility for workers to exert firm-specific and portable training effort and show differences in the optimal strategies adopted.

Gibbons and Waldman (1999) study wage and promotion dynamics in a setting with job assignments, specific human capital acquisitions and learning about workers’ talent. The model in this paper differs from Gibbons and Waldman’s (1999) in several respects and in the predictions provided. First, they consider training effort to be costless and not strategically chosen by workers; as Prendergast (1993), they focus their attention on firm-specific human capital, foregoing considerations relative to portable training; third there is completely asymmetric learning about workers’ talent, namely it is revealed only to the current employer, while I consider a framework in which promoted workers’ talent is revealed to all firms in the industry.

Finally, Ferreira and Nikolowa (2018), study the optimal design of long-term labor contracts in a framework in which workers have preferences over job attributes (such as perks, status, visibility). They study the optimal design of contracts to hire workers at the lowest cost possible. The authors find that the optimal contract is a long term one, in which at the beginning of their careers, workers take less desirable jobs, and later on have
a chance of being promoted to more desirable ones. In this paper, I also show that long term contracts including promotion rules provide stronger incentives for workers to exert on-the-job training effort.

3 The Model

Homogeneous price-taking firms hire a continuum of measure 1 of workers with unknown talent each from a perfectly competitive labor market. Firms’ production technology is labor-intensive. Namely, firms produce a positive output only if they are able to hire (at least) a worker. Let the output price be normalized to 1. All agents in the model are risk-neutral. In the first stage workers train. For simplicity, I assume that workers do not produce at the training stage. As is explained below in greater detail, the training outcome provides a signal about workers’ talent. This signal is observed by everyone in the industry, but may be not verifiable.

After training, workers are either promoted and produce via a talent-sensitive technology, or not, and produce by means of a talent-insensitive one.

Finally, assume no discounting across periods, neither for firms nor for workers.

3.1 Tasks, Training and Productivity

Workers’ talent is a continuous real-valued random variable to everyone in the economy. Talent \( \eta \) is uniformly distributed over the support \([0, \tilde{\eta}]\), with \( \tilde{\eta} \geq \frac{3}{2} \) and finite.\(^6\)

At the first stage of the game, workers execute a training task, yielding a signal

\[
y_1 = \eta + a
\]

\(^4\)Assuming productive training would not change the qualitative results presented throughout the paper, as long as the output is constant and does not provide further information about workers’ talent.

\(^5\)Assuming the lower bound of the support of \( \eta \) to be negative, would make the quantitative analysis of the model more tedious, without adding any qualitative results to the ones presented throughout the paper.

\(^6\)This assumption makes all the maximization programs concave.
where \( a \in \mathbb{R}^+ \) is training effort and is finite. Effort cost is given by

\[
\psi(a) = \frac{a^2}{2}.
\]

Training can be either firm-specific or portable and effort is unobservable. In the first case, training effort is productive only in the worker’s current firm; in the second case, training effort is productive for any firm in the industry.

Firms cannot observe training effort, but based on the signal received, they form conjectures about it and workers’ talent:

\[
\mathbb{E}(\eta|y_1) = \hat{\eta} = y_1 - \hat{a}, \quad \forall \hat{\eta} \in [0, \bar{\eta}],
\]

where the firm expects the workers to exert training effort \( \hat{a} \).

However, a worker may choose \( a \neq \hat{a} \) (namely, deviate from the equilibrium expectation), in order to change the firm’s expectation about her talent. If \( \hat{\eta} < 0 \) or \( \hat{\eta} > \bar{\eta} \), equation (2) yields beliefs about talent outside of the feasible range. In these cases, I assume that if \( \hat{\eta} < 0 \), the firm believes that the worker is not worthy of promotion, whilst if \( \hat{\eta} > \bar{\eta} \), the firm believes that the worker is worthy of promotion with probability \( \frac{1}{2} \), otherwise, with same probability, the worker is not promoted.\(^8\)

There exists a cutoff value of \( \eta \) for which even an infinitesimally larger \( a \) with respect to \( \hat{a} \) yields a signal \( \hat{\eta} > \bar{\eta} \), thus generating uncertainty about promotion for the worker. This is given by \( E \equiv \bar{\eta} - a + \hat{a} < \bar{\eta} \).

Based on the signal observed (namely on the conjecture \( \hat{\eta} \)), firms decide upon promotions. Workers who are promoted to the talent-sensitive task \( A \) produce

\[
y_{2A} = \eta + a
\]

\(^7\)The outcome \( y_1 \) resembles the one in Holmström (1999). The main difference is the absence of a white noise error term. Differently with respect to Holmström (1999), the model in this paper consists of two periods and effort is exerted only in the first of these, when workers’ talent is still uncertain. Hence, adding a white noise error term in \( y_1 \) would not change the qualitative results provided throughout the paper.

\(^8\)The assumption on beliefs is made for the sake of algebraic simplicity. However, changing this belief with any other possible one, would not change the qualitative results provided throughout the paper. To better understand this assumption, suppose that upon observing a signal \( \hat{\eta} > \bar{\eta} \), the firm performs an investigation on the ability of the worker, and that this investigation does not provide results at \( t = 2 \) with probability \( \frac{1}{2} \).
whereas, nonpromoted workers execute the routinary task $B$ and produce

$$y_{2B} = x + a, \quad \text{with} \quad x = \mathbb{E}(\eta) = \frac{\bar{\eta}}{2}. \quad (4)$$

A worker with average talent is equally productive in doing both tasks, so that for her $y_{2A} = y_{2B}$.\footnote{Allowing $x$ to be any value in the interval $[0, \bar{\eta}]$ would not change the qualitative results provided throughout the paper.} Notice that training effort is productive both if workers are promoted and execute task $A$ and if they are not promoted and execute task $B$.\footnote{This assumption does not change the qualitative insights of the model, although it allows to rule out less interesting corner results.} This assumption is motivated by the fact that training is not intended to be task-specific, but is either firm or industry-specific.

Let $\theta_i \in [0, 1]$ denote the probability that a competing firm tries to poach a worker executing task $i$. Let $\theta_A > \theta_B$, and to simplify notation, let $\theta_B = 0$ and define $\theta_A$ as just $\theta$.\footnote{Alternatively, one can think of this assumption as stating that nonpromoted workers are invisible in the sense of Milgrom and Oster’s (1987) assumption.} As is explained later in greater detail, the assumption that firms compete only for promoted workers affects the results of the model. However, such assumption can be relaxed by allowing firms to compete also for nonpromoted workers. If firms compete more aggressively for promoted workers than for nonpromoted ones (so that $\theta_A > \theta_B$), all qualitative results in this paper are preserved. If instead, competition for nonpromoted workers is fiercer than for promoted ones (so that $\theta_A < \theta_B$), the inefficiencies shown throughout the model analysis are reversed.

### 3.2 Labor Contracts

The firm offers spot wage contracts to workers at each stage. Let $w_1$ denote the wage offered at the hiring stage, when the labor market is assumed to be perfectly competitive, henceforth, firms compete à la Bertrand to hire employees. Throughout the paper, I consider two possible frameworks: one in which firms can commit to promotion bars (hence, these are set at the hiring stage, before workers’ on-the-job training takes place) and one in which this is not the case (hence, they are set after training). Furthermore, workers’ talent is assumed to be nonverifiable, so that firms cannot commit to wage contracts contingent on talent.
After promotions, firms pay wages $w_{2i}$ to workers assigned to task $i$, in order to retain them at the interim stage. Workers cannot commit not to leave the current employer for a competitor. Hence, ex-ante, firms earn expected profit

$$\pi = \mathbb{E} \left[ \sum_{i=A}^{B} (y_{2i} - w_{2i}) \right] - w_{1}. \quad (5)$$

Note that, given their production function, the two tasks are substitutes for firms, so that they do not need to promote workers if it is not profitable doing so.

Workers’ lifetime expected utility is

$$U(w, a) = w_{1} + \mathbb{E}(w_{2i}) - \psi(a) \text{ for } i = \{A, B\}. \quad (6)$$

As a result of fierce competition among firms to attract workers, $w_{1}$ is set so that workers extract all the expected surplus and firms themselves earn zero profit in expectation (namely, $w_{1}$ is driven by the firm’s zero-profit condition). At the interim stage, the current employer can match possible external bids, thus expected wage conditional on promotion is

$$\mathbb{E}(w_{2A}) = \theta(\eta + \mathbb{I}a) \quad (7)$$

where $\mathbb{I}$ is an indicator function equal to 1 when the training effort exerted is portable and zero otherwise.

Since it is assumed that there is no competition for nonpromoted workers, firms extract all the surplus produced in task B and set $w_{2B} = 0$.

### 3.3 Time Line

The time line of the model includes five stages:

- $t = 0$ (hiring stage), firms offer $w_{1}$ to hire workers. If the promotion bar is contractible, it is set.
- $t = 1$ (training stage), if workers accept the wage offer, they execute a training task and exert training effort $a$.
- $t = 2$, training yields a signal $y_{1}$. Based on this, firms form a conjecture $\hat{\eta}$ on workers’ talent.
- $t = 3$ (promotion stage), if the promotion bar is noncontractible, firms set it. Workers who fulfill it are promoted. Nonpromoted workers earn their reservation wage.

- $t = 4$ (interim poaching stage), promoted workers may leave the current firm for a competitor if an external bid occurs. The current employer can match outside offers with $w_{2A}$.

- $t = 5$, production is completed and revenues $y_{2i}$ are produced, for $i = \{A, B\}$.

### 3.4 Equilibrium Concept

The time line of the model features sequential actions and workers have private knowledge of the training effort they exert. The equilibrium concept studied throughout the model is *Perfect Bayesian Nash Equilibrium (PBNE)*.

A PBNE is defined by wages $w_1$ and $w_{2i}$, a promotion bar $\eta^* \in [0, \infty)$, training effort $a$ and a conjecture about such effort given by $\hat{a}$. Along the equilibrium path, despite the presence of hidden information (about training effort), information will be symmetric, as firms’ conjecture $\hat{a}$ will be correct.

Moreover, if the equilibrium promotion bar $\eta^*$ exceeds the upper bound of the support of workers’ talents $\bar{\eta}$, it is equivalent to saying that the firm does not promote any worker.

### 4 Benchmark

Consider the promotion rule $\eta^o$ and training effort $a^o$ that maximize expected surplus

$$W = \int_{\eta^o}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^o)\bar{\eta}}{2} + a^o - \frac{a^{o2}}{2}. \quad (8)$$

A social planner solves:

$$\max_{\{\eta^o, a^o\}} W$$

**Proposition 1.** *The total surplus is maximized by a promotion bar $\eta^o = \frac{\bar{\eta}}{2}$ and training effort $a^o = 1$*

The proof for this and all other lemmas and propositions are relegated to the Appendix.
This proposition provides a benchmark to be compared with promotion bars and effort exerted in a competitive equilibrium. Notice that for the maximization of total surplus, it makes no difference whether training effort is portable or not.

Intuitively, this result points out that firms’ productivity is maximized if all workers with a talent \( \eta \geq \bar{\eta}^2 \) are promoted to task A. Namely, all workers producing “above average” should be promoted.

### 4.1 Implementing the Efficient Allocation

Suppose firms can commit to wage contracts contingent on the signal \( y_1 \) at the hiring stage. In this case, I assume \( y_1 \) to be not only observable in the whole industry, but also verifiable in courts. Suppose at \( t = 0 \) the firm offers a contract \( \{ w_1, w_2 \} \), with \( w_1 \) defined by the firm’s zero-profit condition given perfect labor market competition at the hiring stage, and \( w_2 = y_1 = \eta + a \) irrespective of whether the worker is promoted or not. Notice that the wage \( w_2 \) is contingent on training effort, although this may be firm-specific. In this case, at the training stage, the worker exerts the training effort that maximizes her expected utility, since \( w_1 \) is paid at \( t = 0 \), so that her maximization program is

\[
\alpha^* \in \arg\max \frac{\bar{\eta}}{2} + a - \frac{a^2}{2} \quad \text{subject to} \quad \psi(a).
\]

**Proposition 2.** Assume \( y_1 \) is verifiable, then the efficient allocation \( \{ \alpha^*, \eta^* \} \) can be implemented. If the firm offers a contract

\[
w_1 = E \left[ \sum_{i=1}^{R} (y_{2i} - w_{2i}) \right], \quad w_2 = y_1, \tag{9}
\]

in equilibrium workers will exert effort \( \alpha^* = 1 \) and firms will promote workers fulfilling the threshold \( \eta^* = \bar{\eta}/2 \). This wage schedule is such that all workers are retained for any \( \theta \).

Intuitively, if firms offer a wage \( w_2 \) equal to the signal irrespective of whether the worker is promoted or not, the latter has an incentive to acquire training efficiently. Since the wage at \( t = 2 \) is paid even if the worker is not promoted, when choosing the promotion bar, the firm sets it efficiently, as this will increase its profit for any level of effort of the worker. Specifically, promotion decisions do not affect \( w_2 \). Since the firm offers \( w_1 \) such that \( E(\pi) = 0 \), this result holds both with and without commitment to promotion bars at
the hiring stage. Specifically, the assumption that the profit-maximizing firms are able to commit to wages depending on the signal $y_1$ is sufficient to convey this outcome.

Note that the contract proposed in this section is one of many possible ones to implement the efficient allocation in equilibrium. The only condition needed to attain the efficient allocation is that $y_1$ is verifiable in courts, so that firms can offer contingent contracts on the signal. In the following sections, I assume $y_1$ to be observable by firms and workers but nonverifiable in courts, so that firms can only use promotion-based incentives.

### 5 Firm-Specific Training

Before production, workers undergo a training stage. Training effort is unobservable, hence firms cannot design a contract conditional on a certain level of it. This section studies how promotion decisions affect workers’ incentives to exert firm-specific training effort.\(^\text{12}\)

In this framework, effort increases $y_1$ used by firms as a signal of workers’ talent to decide whether to promote them or not. Namely, workers exert firm-specific training effort because they are concerned about being promoted, although this will not directly affect expected wages.

#### 5.1 No Commitment to Promotion Bars

Suppose neither promotion bars nor wages contingent on talent are contractible. In this case, the firm sets the promotion bar after workers’ training. Recall that workers’ training delivers a signal $y_1 = \eta + a$, observed by everyone in the industry. After observing $y_1$, firms make a conjecture about workers’ talent:

$$E(\eta|y_1) = \hat{\eta} = y_1 - \hat{a}.$$  

Recall that if the firm observes an off-equilibrium signal $\hat{\eta} > \bar{\eta}$, it will promote the worker with probability $\frac{1}{2}$.

---

\(^{12}\)This case is similar to a standard moral hazard problem.
Solving the model by backward induction, at \( t = 3 \) firms set a promotion bar \( \eta^*_s \) such that workers are promoted if \( \hat{\eta} \geq \eta^*_s \). In order to choose the bar, firms solve the following maximization program

\[
\eta^*_s \in \arg \max_{\eta} \pi = (1 - \theta) \int_{\eta}^{\hat{\eta}} \hat{\eta} f(\hat{\eta}) d\hat{\eta} + \frac{F(\eta) \hat{\eta}}{2} + a
\]

delivering the optimal promotion bar without commitment.

**Lemma 1.** If firms are unable to commit to promotion bars before workers exert specific training effort, the optimal bar is \( \eta^*_s = \frac{\hat{\eta}}{2(1 - \theta)} \geq \eta^o \). For any \( \theta > \frac{1}{2} \), firms do not promote any worker as \( \eta^*_s > \hat{\eta} \).

The optimal bar \( \eta^*_s \) has two interesting features: first, it is independent of workers’ firm-specific training effort. Since training is only valuable for the current employer, it does not affect directly workers’ equilibrium wages.

Second, it is increasing in \( \theta \), the degree of competitiveness for promoted workers. Intuitively, firms choose the promotion bar opportunistically, to reduce retention costs. This behavior results in inefficient underpromotion, thus fewer workers are promoted with respect to the efficient benchmark, for the firm to extract as much rent as possible. Moreover, as competition for promoted workers exceeds the threshold \( \frac{1}{2} \), firms find it profitable to produce only via the routine task, namely, they promote no worker. To see this, consider two polar cases: let \( \theta = 0 \), so that promoted workers never receive an outside offer. In this case, the firm cares only about workers’ productivity and pays them a fixed wage, thus in equilibrium \( \eta^*_s = \eta^o \). Then consider the case in which \( \theta = 1 \). In this scenario, the optimal promotion bar is \( \eta^*_s \to \infty \). Intuitively, in this case, promoted workers would extract all the surplus generated, thus the firm has no incentive to promote. This result provides an important prediction: absent commitment to promotion bars, competition for talent implies inefficient production (namely, inefficient promotions), since firms trade off efficient productivity with higher profits.

At \( t = 1 \), workers rationally anticipate the firm’s optimal promotion rule \( \eta^*_s \) and decide how much firm-specific training effort to exert in order to maximize their expected utility.

\[^{13}\text{Waldman (1984) and Greenwald (1986) feature similar results in a model featuring asymmetric information about workers’ productivity. Picariello (2017) features a similar structure to the model presented in this paper with similar results.}\]
Recall that workers’ lifetime expected utility is

\[ U(w, a) = w_1 + \mathbb{E}(w_{2i}) - \psi(a) \quad \text{for } i = \{A, B\}. \]

However, \( w_1 \) is paid at \( t = 0 \), so that it is a constant in the worker’s maximization program. Moreover, since \( \mathbb{E}(w_{2B}) = 0 \), the worker solves

\[ a^*_p \in \arg\max_{\{a \geq 0\}} \frac{\theta}{2} \left[ \int_{\eta - a + \hat{a}}^{\bar{\eta} - a + \hat{a}} (\eta + a - \hat{a}) f(\eta) d\eta + \int_{\eta - a + \hat{a}}^{\bar{\eta}} (\eta + a - \hat{a}) f(\eta) d\eta \right] - \frac{a^2}{2}. \]

Note that workers care about the expected wage they could earn conditional on the signal \( y_1 \) and on promotion. This is because every firm in the industry observes \( y_1 \), hence workers have an incentive to exert specific training effort in order to increase the value of the signal they send to the market. In this way, they hope to earn higher wages upon promotion (although, along the equilibrium path, every firm will correctly conjecture workers’ effort and be able to tell what their actual talent is). This maximization program yields the following result:

**Proposition 3.** The optimal firm-specific training effort when firms cannot commit to promotion bars is:

\[ a^*_p = \begin{cases} \frac{\theta}{2} & \text{if } \theta \in [0, \frac{1}{2}], \\ 0 & \text{if } \theta \in (\frac{1}{2}, 1]. \end{cases} \]

This proposition shows that optimal specific training effort is discontinuous. Workers have an incentive to exert positive effort as long as firms have an incentive to promote some employees and there is some labor market competition for them (namely, if \( 0 < \theta \leq \frac{1}{2} \)). For values of \( \theta \leq \frac{1}{2} \), workers have an increasing incentive to exert effort as labor market competition increases, since this results in higher expected wages conditional on promotion. Specifically, since specific effort does not affect these wages (hence, the promotion bar), workers try to increase their likelihood of promotion by inflating \( y_1 \).

However, if \( \theta > \frac{1}{2} \), as stated in lemma 1, firms find it profitable not to promote any worker. In this case, workers have no reason to exert specific training effort, as they anticipate that the cost of their investment will be sunk. The same outcome realizes if \( \theta = 0 \), yet the driving mechanism is different: although firms promote workers efficiently in this scenario, the latter are locked in as no firm would bid an offer for them upon
promotion. Hence, workers are indifferent between being promoted and not, thus they exert no training effort.  

5.2 Commitment to Promotion Bars

Consider now the case in which firms can commit to promotion bars before workers’ training. At \( t = 0 \), firms offer contracts \( \{w_1, \eta^*_s\} \) to hire workers. Let \( \eta^*_s \) denote the contractible promotion bar and \( a^*_s \) the training effort workers exert in equilibrium. Since the firm produces via a labor-intensive technology, it needs necessarily to hire some workers in order to produce a positive output. Should the firm fail to hire any worker, it would produce no output and earn no profit.

Solving the model by backward induction, the firm anticipates that workers’ effort decision is independent of the promotion bar:

\[
a^*_s = \frac{\theta}{2}.
\]

(10)

Promotion bars change the wages firms offer and workers accept the offer providing them the highest lifetime expected utility as of \( t = 0 \)

\[
U(w, a) = w_1 + \mathbb{E}(w_2) - \psi(a).
\]

(11)

Since the labor market is perfectly competitive, \( w_1 \) equates the firm’s expected profit. The latter chooses the promotion bar by solving

\[
\text{Max}_{\{\eta^*_s\}} U = \int_{\eta^*_s}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^*_s)\bar{\eta}}{2} + a^*_s \left(1 - \frac{a^*_s}{2}\right).
\]

Proposition 4. When workers exert firm-specific training effort, and firms can commit to promotion bars at the hiring stage, the optimal effort is

\[
a^*_s = \frac{\theta}{2}
\]

(12)

\(^{14}\)Effort discontinuity results from the fact that I assume talent to be uniformly distributed. Such assumption allows to rule out the role of the distribution of talent itself in workers’ choice of training effort, thus implying that firm-specific effort only depends on the competition for promoted workers. This also allows to stress uniquely the impact of portability of the acquired training on the results shown later on.
firms, instead, commit to the efficient promotion bar

\[ \eta^{**} = \bar{\eta} = \bar{\eta}^{o}. \]  

(13)

Intuitively, at \( t = 0 \) firms know that promotion bars affect both retention costs (namely, \( w_2 \)) and wages they can offer to attract workers (namely, \( w_1 \)). Profit maximizing firms prefer being productive and to do so, they need to hire workers, so they commit to the promotion bar that maximizes workers’ lifetime expected utility as of \( t = 0 \). As a result, in equilibrium, firms commit to the surplus maximizing promotion bar, although workers’ effort decision is independent of it.

Since specific training effort does not affect promotion bars, but only the signal \( y_1 \), workers’ optimal effort only depends on the degree of competitiveness for promoted workers, as it raises expected wages upon promotion.

Differing from the case without commitment, firms commit to promote a constant positive share of workers for any level of labor market competition. Hence, workers have an incentive to exert positive training effort in equilibrium, for any \( \theta > 0 \).

Note that, for any value of \( \theta \), effort is inefficiently low, yet for any \( \theta > \frac{1}{2} \), firms’ ability to commit to promotion bars, allows for increasingly more efficient training effort with respect to the case without commitment, since workers have an incentive to exert strictly positive training effort.

6 Portable Training

Consider now the case in which workers exert portable training effort, increasing their productivity in any firm in the industry. In this framework, workers’ effort affects firms’ choice of the promotion bar, as it directly increases retention costs for any degree of labor market competition.

6.1 No Commitment to Promotion Bars

If firms cannot commit either to wages contingent on talent, or to promotion bars, the latter are set after workers’ training. After having observed the signal \( y_1 \), firms promote
workers whose conjectured talent fulfills
\[ \hat{\eta} = y_1 - \hat{a} \geq \eta^*_p \]
where \( \eta^*_p \) denotes the noncontractible optimal promotion bar. At \( t = 3 \), firms set a bar for every possible level of training effort in order to maximize their expected profit:
\[ \eta_p(a_p) \in \arg\max_{\{\eta\}} \pi = (1 - \theta) \int_{\eta}^{\hat{\eta}} (\eta + a_p) f(\eta) d\eta + F(\eta) \left( \frac{\hat{\eta}}{2} + a_p \right). \]

**Lemma 2.** If firms cannot commit to promotion bars and workers exert portable training effort, the profit-maximizing promotion bar is
\[ \eta_p(a_p) = \frac{\hat{\eta}}{2(1 - \theta)} + \frac{\theta a_p}{1 - \theta} \geq \eta^*_s \geq \eta^0 \]
for any effort \( a_p \geq 0 \) and \( \theta > 0 \).

Firms’ best reaction is a promotion bar \( \eta_p(a_p) \), increasing with workers’ portable training effort and with the probability for promoted workers to receive an offer from a competing firm, \( \theta \).

As in Section 5.1, firms cope with a trade off between productivity and high retention costs. For this reason, when unable to commit to bars, they promote fewer workers with respect to the efficient benchmark, in order to reduce retention costs. Both competition for promoted workers and portable training effort increase such costs, hence, promotion bars. Intuitively, if \( \theta \) increases, promoted workers are more likely to receive an offer from a competing firm, hence their expected wage increases. Furthermore, since workers’ training effort is portable, it also increases their expected wage upon promotion, for any \( \theta \), as it makes them more productive within the whole industry. These two effects, combined together, imply a stronger inefficiency in promotions without commitment to bars. Furthermore, if competition for promoted workers is too fierce, firms may find it profitable not to promote workers at all.

At \( t = 1 \), workers take into account the firm’s reaction to their training and exert the portable training effort that maximizes their expected utility
\[ U(w, a) = w_1 + \mathbb{E}(w_2) - \psi(a) \text{ for } i = \{A, B\}. \]
At the training stage, \( w_1 \) has already been paid, hence is a constant for the worker, whilst \( E(w_{2B}) = 0 \). So the worker’s maximization program is

\[
a^*_p \in \arg\max_{\{a_p \geq 0\}} \theta \left\{ \int_{\bar{\eta}}^{\tilde{\eta} - a_p + \hat{a}} (\eta + a_p - \hat{a}) f(\eta) d\eta + \left[ F(\tilde{\eta} - a_p + \hat{a}) - F(\eta_p(a_p) - a_p + \hat{a}) \right] a_p + \right. \\
+ \int_{\eta_p(a_p) - a_p + \hat{a}}^{\tilde{\eta}} (\eta + a_p - \hat{a}) f(\eta) d\eta + \left[ 1 - F(\eta_p(a_p) - a_p + \hat{a}) \right] a_p \left. \right\} - \frac{a^2_p}{2}
\]

and provides the following results:

**Proposition 5.** If firms cannot commit to promotion bars, in equilibrium:

1. workers’ portable training effort is
   
   \[
a^*_p(\theta) = \frac{\tilde{\eta} \theta \left[ 3(1 - \theta)^2 - 1 \right]}{(2\tilde{\eta} - \theta)(1 - \theta)^2 + 2(2 - \theta)\theta^2}
   \]
   which is nonmonotonic with respect to \( \theta \) and zero for \( \theta = 0 \) or \( \theta \) sufficiently large.

2. The profit-maximizing promotion bar is
   
   \[
   \eta^*_p(\theta) = \frac{\tilde{\eta}}{1 - \theta} \left[ \frac{1}{2} + \frac{\theta^2 \left[ 3(1 - \theta)^2 - 1 \right]}{(2\tilde{\eta} - \theta)(1 - \theta)^2 + (2 - \theta)2\theta^2} \right]
   \]
   which is larger than \( \eta^0 \) for any \( \theta \) and larger than \( \bar{\eta} \) for any \( \theta > \frac{1}{2} \).

When workers exert portable training effort, firms raise promotion bars as a response to such effort. This happens because training effort and competition for promoted workers raise retention costs at the interim stage. If firms compete too fiercely for promoted workers (namely, if \( \theta \geq \frac{1}{2} \)), the former will not promote any worker in equilibrium, thus preferring to produce only by means of routine jobs.\(^\text{15}\)

Workers rationally anticipate that in equilibrium, firms will increase the promotion bar when they exert more effort. This implies that portable training effort is driven by two contrasting forces. On the one hand, as \( \theta \) increases, the expected wage conditional on promotion increases. Thus, workers want to increase the signal \( y_1 \) to improve their

\(^\text{15}\)As in Section 5.1, if \( \theta \to 1 \), firms set \( \eta^*_p \to \infty \).
likelihood of promotion. I refer to this as the “expected wage effect”. On the other hand, firms react more fiercely to workers’ effort when $\theta$ is high, since fierce competition imposes a strong externality on firms promoting workers, as it implies high retention costs. Thus, the firm shields itself against competition by setting the promotion bar so high that workers are discouraged from exerting effort, as their chances of obtaining a promotion are low and the cost of effort will most likely be sunk. I refer to this as the “discouragement effect”.

![Figure 1: Portable training effort (\(\bar{\eta} = 4\)).](image)

This implies that optimal effort is nonmonotonic with respect to the degree of competition for promoted workers: for low values of $\theta$, the expected wage effect dominates and effort is increasing up to a cutoff value $\hat{\theta}$; as $\theta$ exceeds $\hat{\theta}$, the discouragement effect dominates and effort decreases towards zero, as shown in Figure 1. The figure also shows that portable training effort is always below the efficient benchmark ($a^o = 1$).

Figure 1, compares portable and specific training effort in the absence of commitment to promotion bars. When competition for promoted workers is low, workers exert more portable effort than the specific one, because the former not only inflates the signal $y_1$, but also raises expected wages conditional on promotion. However, as competition becomes fiercer, the discouragement effect implies that portable training effort falls toward zero, whilst specific training effort increases up to when firms do not promote any worker. When $\theta \geq \frac{1}{2}$, workers do not exert either firm-specific or portable training effort.
This result provides a novel prediction on portable training effort in competitive labor markets. The existing literature (Becker, 1962; Rosen, 1972; Garmaise, 2011) postulates that workers are eager to exert portable training effort in more competitive environments, as it increases their expected wage. The model presented hereby, instead, predicts that in highly competitive labor markets, workers may not be willing to exert portable training effort due to the impact of competition on firms’ promotion decisions. The innovation in this model with respect to the existing literature is that not only workers, but also firms react to labor market competition. This implies that in highly competitive environments, firms set promotion bars very high in order to lower retention costs, thus making promotions less likely for workers who would then have no incentive to exert training effort, as its costs will most likely be sunk.

6.2 Commitment to Promotion Bars

Consider now the case in which firms commit to promotion bars at the hiring stage. At \( t = 0 \), firms offer contract \( \{ w_1, \eta^*_p \} \) to hire workers. Let \( \eta^*_p \) denote the optimal promotion bar and \( a^*_p \) optimal training effort in this framework.

Solving the problem by backward induction, first consider workers’ choice of effort \( a_p(\eta_p) \) that maximizes their expected utility at \( t = 1 \). Since \( w_1 \) is constant at this stage and \( \mathbb{E}(w_{2B}) = 0 \), workers’ maximization program is

\[
a_p(\eta_p) \in \arg \max_{a \geq 0} \frac{\theta}{2} \left\{ \int_{\eta_p - a + \hat{a}}^{\eta - a + \hat{a}} (\eta + a - \hat{a}) f(\eta) d\eta + \left[ F(\eta - a + \hat{a}) - F(\eta_{p} - a + \hat{a}) \right] a + \int_{\eta_p - a + \hat{a}}^{\eta} (\eta + a - \hat{a}) f(\eta) d\eta + \left[ 1 - F(\eta_{p} - a + \hat{a}) \right] a \right\} - \frac{a^2}{2}
\]

**Lemma 3.** If firms commit to promotion bars at \( t = 0 \), workers’ optimal effort is

\[
a_p(\eta_p) = \frac{\theta(3\bar{\eta} - 2\eta_p)}{2\bar{\eta} - \theta}
\]

for any promotion bar \( \eta_p \).

The higher the promotion bar \( \eta_p \), the lower is workers’ effort, as a consequence of the discouragement effect.
Given perfect labor market competition at the hiring stage, firms earn zero expected profit and commit to the promotion bar that maximizes workers’ lifetime expected utility:

\[ \max_{\{\eta_p^*\}} U = \int_{\eta_p^*}^{\tilde{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_p^*)\tilde{\eta}}{2} + a_p^*(\eta_p^*) \left( 1 - \frac{a_p^*(\eta_p^*)}{2} \right). \]

The following proposition describes firms and workers’ equilibrium behavior.

**Proposition 6.** If firms can commit to promotion bars at \( t = 0 \) and workers exert portable training effort, in equilibrium:

1. the promotion bar is

\[ \eta_p^{**}(\theta) = \frac{\tilde{\eta}}{2} \left[ \frac{4\tilde{\eta}^2 - 12\tilde{\eta}\theta(1 - \theta) + 5\theta^2}{\theta^2 + 4\tilde{\eta}(\tilde{\eta} - \theta(1 - \theta))} \right], \quad (16) \]

nonmonotonic with respect to \( \theta \): there exists \( \theta^o \equiv \frac{\tilde{\eta}}{2\tilde{\eta} + 1} > 0 \) such that \( \eta_p^{**} < \eta^o \) for any \( \theta \in (0, \theta^o) \); \( \eta_p^{**} > \eta^o \) for any \( \theta \in (\theta^o, 1) \) and \( \eta_p^{**} = \eta^o \) if \( \theta = 0 \) or \( \theta = \theta^o \);

2. workers exert effort

\[ a_p^{**}(\theta) = \frac{3\theta\tilde{\eta}}{2\tilde{\eta} - \theta} - \frac{2\tilde{\eta} \left[ \tilde{\eta}(4\tilde{\eta} - 12\theta(1 - \theta) + 5\theta^2) \right]}{(2\tilde{\eta} - \theta) \left[ 2\theta^2 + 8\tilde{\eta}(\tilde{\eta} - \theta(1 - \theta)) \right]]. \quad (17) \]

Compare now the equilibrium outcomes with and without commitment to promotion bars.

First, consider the optimal training effort exerted in the two scenarios. Figure 2 depicts levels of effort \( a^o, a_p^{**}, a^*_p \) and \( a_p^{**} \) as functions of \( \theta \). When the firm commits to promotion bars, workers exert more portable training effort than the firm-specific equivalent (namely, \( a_p^{**} \geq a^*_p \)). This happens because portable training not only increases the signal \( y_1 \), but it also directly increases workers’ expected wages upon promotion. Namely, portability itself provides incentives to exert training effort, for any degree of competiveness at the interim stage.

Moreover, the ability of firms to commit to promotion bars, makes workers’ portable training effort increasing with \( \theta \), and always larger or equal to the one exerted absent commitment to promotion bars. Namely, \( a_p^{**} \geq a^*_p \) for any \( \theta \in [0, 1] \).
Interestingly, there exists a strictly positive degree of competitiveness for promoted workers $\theta^*$, at which, if firms commit to promotion bars at $t = 0$, workers exert efficient portable training effort. For any $\theta$ smaller than $\theta^*$, $a_{p}^{**}$ is inefficiently low, whilst for any $\theta$ larger than $\theta^*$ workers exert too much portable effort with respect to the efficient benchmark. As $\theta$ increases, workers expect to secure a larger share of the surplus produced, henceforth they have stronger incentives to exert effort: it enhances their likelihood to be promoted, but also the wage they could be offered by competing firms. However, when poaching raids happen more frequently (namely, when the competition for promoted workers is very fierce), workers have an incentive to exert too much effort. This result resembles the one in Holmström (1999), showing that career concerns provide too strong incentives for workers to exert effort.$^{16}$ In this paper, promotion bars and competition for talent provide these incentives. However, I show this result in a two-period framework, whereas Holmström (1999) studies an infinitely repeated game.

$^{16}$If workers exert portable training effort and the labor market becomes increasingly competitive, the framework shown in this paper becomes similar to the one in Holmström (1999), where workers are entitled to higher wages, the higher the effort they exert one period before compensation, for any talent. The fact that in Holmström (1999), effort raises upwards workers’ expected productivity, translates into higher wages, similar to the model I present, although the driving mechanism is different.
To sum up, firm’s commitment to promotion bars increases efficiency in effort exertion up to a certain cutoff $\theta^*$. When firms compete more aggressively for promoted workers, the latter have too strong incentives to exert effort which in equilibrium exceeds the efficient benchmark. As compared to the case without commitment, when firms commit to promotion bars, workers exert more efficient effort when $\theta$ is sufficiently low. Otherwise, when $\theta$ is too high, the inefficiency persists but is reversed, as workers exert too much effort. This result makes room for policy discussion. In talent-intensive industries, many labor contracts feature noncompete clauses, preventing workers from accepting job offers from firms competing with the initial one (Samila and Sorenson, 2011; Prescott, et., al., 2016). These clauses lock workers in by endogenously reducing competitiveness. In the model shown hereby, firms could use such covenants in order to reduce workers’ portable training effort, leading it to the efficient level, even in highly competitive environments. Clearly, such clauses should not be too binding, otherwise workers would have no incentive to train. The firm should fine tune them in order to achieve efficiency.

Regarding promotion bars, Figure 3 shows $\eta_p^*$, $\eta_p^{**}$ and $\eta^o$ as functions of $\theta$. Notice that $\eta_p^*$ is larger than or equal to $\eta_p^{**}$ for any $\theta$. Specifically, when committing to a promotion bar at the hiring stage, the firm sets it always below the one it would choose after the
training stage. This is because at $t = 0$ the firm internalizes the impact its choice has on workers’ effort and on the ability to attract the latter.

The promotion bar $\eta_p^{**}$ is nonmonotonic with respect to $\theta$. There exists a cutoff value of $\theta$ denoted $\theta^o \equiv \frac{\bar{\eta}}{2\bar{\eta} + 1}$ such that for any $\theta \in (0, \theta^o)$, the firm commits to promotion bars below the efficient benchmark (namely, $\eta_p^{**} < \eta^o$). For any $\theta \in (\theta^o, 1]$, instead the firm commits to promotion bars above the efficient benchmark, yet smaller than the one it would choose without commitment (namely, $\eta_p^* > \eta_p^{**} > \eta^o$). Finally, the firm commits to the efficient promotion bar for $\theta = 0$ or $\theta = \theta^o$.

Intuitively, when competition for promoted workers is low ($\theta < \theta^o$), workers secure a small share of the produced surplus and therefore have little incentive to exert training effort in poorly competitive environments. This is because they know that their effort may increase promotion bars for any degree of competition and make promotions less likely, thus further reducing expected wages. Firms, instead, earn a large share of surplus and would like workers to exert effort, so that they commit to low and more affordable promotion bars. By doing so, firms lower the bar below the efficient benchmark.

However, as competition for promoted workers increases ($\theta > \theta^o$), the labor market itself provides incentives for workers to exert portable training effort. As seen also in Figure 2, workers secure a larger share of the surplus produced, thus they have a strong incentive to exert portable training effort. This makes promoted workers’ retention more expensive, hence the firm adjusts the promotion bar upwards, even setting it above the efficient benchmark when competition becomes too fierce, in order to shield itself against aggressive poaching raids.

Notice that firms commit to promote a positive share of workers for any $\theta$, differently with respect to the case without commitment (namely, $\eta_p^{**} < \bar{\eta}$). This is because firms anticipate that not promoting workers implies that they will not exert training effort. This, in turn generates low surplus and the inability for firms to attract workers at $t = 0$.

To sum up, when workers exert portable training effort and firms commit to promotion bars at the hiring stage, inefficiencies may persist with respect to the case without commitment. More specifically, when $\theta$ is low, firms set the promotion bar below the efficient benchmark, so that the inefficiency in production persists, but is reversed, as for the same region of parameters, without commitment they would set the bar too high with respect to the efficient benchmark. If competition for promoted workers is sufficiently fierce, instead, firms set the promotion bar higher than the efficient one. However, the inefficiency
is smaller with respect to the case without commitment, as, by committing to promotion bars at $t = 0$, firms take into account the impact their choice has on workers’ effort and on their ability to hire them.

7 Conclusions

Workers’ talent and on-the-job training are key inputs in talent-intensive industries, like the financial, the legal and the medical sectors. To provide incentives to exert training effort, firms may reimburse workers’ cost of such training. However, if effort is unobservable, its cost cannot be reimbursed. Promotion-based incentive schemes are widespread in organizations (Baker, Jensen and Murphy, 1988), especially in those sectors in which wages are rigid and firms are unable to commit to pay-for-performance schedules. When promotions make workers more attractive in the labor market, firms face a tradeoff between productivity and high expected retention cost for promoted workers.

This paper shows the linkage between promotion-based incentive schemes and training effort in competitive labor markets when workers’ talent is uncertain at the training stage. Profit-maximizing firms may use promotions to reduce retention costs. Indeed, if firms cannot commit to set promotion bars when hiring workers, they set them higher than the efficient benchmark. This inefficiency becomes stronger as competitiveness for promoted workers increases. In fact, in sufficiently competitive environments, firms prefer not to promote workers at all.

If workers exert firm-specific training effort, their decision does not affect promotion bars, hence, such effort is increasing with the degree of competitiveness for promoted workers, provided that firms promote workers in equilibrium. This happens because specific training effort only serves to improve the signal about workers’ talent on which firms base their promotion decisions. In this scenario, when able to commit to promotion bars, firms choose the efficient one since it allows them to offer the highest possible flow of wages to hire workers and be productive. If instead workers exert portable training effort, the latter, together with labor market competition, affect firms’ decisions and these will raise promotion bars above the efficient benchmark. Although higher promotion bars raise expected wages conditional on promotion, they also generate a discouragement effect, as workers are less likely to be promoted, hence the optimal effort is nonmonotonic concave with respect to labor market competition. This result differs from those in the existing
literature (Becker, 1962; Rosen, 1972; Garmaise, 2011) postulating that workers are eager to exert more training effort in highly competitive environments. In this paper, not only workers, but also firms react to labor market competition. Specifically, they shield themselves against aggressive poaching raids by setting the bar very high. This in turn has an impact on workers’ incentives, so that competition affects their choice through two channels: directly raising expected wages, and indirectly lowering their chances of being promoted.

If firms commit to promotion bars before workers’ training, when competition for promoted workers is not too high, they reduce the promotion bar below the efficient benchmark, making promotions easier to obtain with respect to the case without commitment at the cost of inefficient production. When competition increases, the firm raises the promotion bar above the efficient benchmark, since in this case, workers have strong market incentives to exert portable training effort, thus increasing retention costs. With commitment to promotion bars, workers exert more effort with respect to the case without commitment, for any level of competition for promoted workers. However, if firms compete too fiercely for promoted workers, the latter exert inefficiently too much effort. This implies that firms’ ability to commit to promotion bars before workers exert training effort, does not necessarily implement efficiency (as in the case with firm-specific effort), but it actually reverses such inefficiencies.

In this paper, I assume that firms are homogeneous in their production technology, so that even though the signal workers produce at the training stage is observable by all firms in the market, a worker who is not promoted by her first employer cannot move to a competing firm and be promoted thereby. As a robustness remark, notice that if firms were heterogeneous this would not be the case. However, if only the first employer observes the signal about workers’ talent and all other firms observe only promoted workers’ productivity, then the qualitative results I have shown throughout the paper hold true.

Furthermore, I study frameworks in which workers exert either firm-specific or portable training effort. However, it would be worth analyzing workers’ behavior in a framework in which they can choose a mix of the two. This would allow to understand how do the weights associated with each type of training effort change in frameworks with noncontractible and contractible promotion rules. To do so, one should also change the firms’ production function for this analysis to be meaningful. More specifically, the returns to each type of training effort should be modeled in a more structured way.
Another possible venue for future research would be to allow for heterogeneity across firms and workers. Including technological heterogeneities could make workers employed by a certain firm more or less attractive to different competitors. One core assumption of the paper is that the talent-sensitive and routine tasks are substitutes for the productive scope. However, allowing for complementarity between the two tasks, the firm would need always to promote a positive share of workers. If firms are all technologically identical, then the results are qualitatively unchanged with respect to those presented in this paper (although there would not be corner solutions in the equilibria). If instead firms were heterogeneous in the degree of complementarity between the two tasks, then only the firms producing via technologies featuring more complementarity between tasks would be active and able to attract workers.

Workers, instead, may differ for the distribution of their talent. In this scenario, the probability of poaching raids may vary across firms, thus implying different promotion rules across different potential employers. If this is the case, workers may select themselves across employers, depending on their subjective talent distribution. Such result would be even more interesting if workers are risk-averse, as this would also introduce a hedging aspects in job selection. Such approach could generate further testable predictions.
Appendix

Proof of Proposition 1

The proof of this proposition is straightforwardly provided by the first-order conditions of the maximization program

$$\max_{\{\eta^o, a^o\}} W = \int_{\hat{\eta}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^o)\bar{\eta}}{2} + a^o - \frac{a^{o2}}{2}.$$ 

The first-order condition with respect to effort $a$ yields

$$a^o = 1 \quad (18)$$

whilst the first-order condition with respect to the promotion bar $\eta$ gives

$$\eta^o = \frac{\bar{\eta}}{2} \quad (19)$$

Proof of Proposition 2

Proof. Throughout this proof, it is assumed that at $t = 0$ firms are able to commit to wages contingent on the signal $y_1$. At the hiring stage, firms offer wage contracts $\{w_1, w_2\}$. By the zero-profit condition, the hiring wage is set as

$$w_1 = \mathbb{E}\left[\sum_{i=A}^{B} (y_{2i} - w_{2i})\right],$$

whereas $w_2 = y_1 = \eta + a$.

In this case, at the training stage, the worker exerts the training effort that maximizes her expected utility at the next stage:

$$a^* \in \arg\max_{\{a\}} \mathbb{E}(w_2) = \frac{\bar{\eta}}{2} + a - \frac{a^2}{2}$$

The first-order condition for an interior optimum of the above maximization program yields

$$a^* = a^o = 1. \quad (20)$$
Since $w_2$ is paid irrespective of whether the worker is promoted or not, the firm’s expected profit is
\[
\pi = \int_{\eta^*}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^*)\bar{\eta}}{2} - w_1 - w_2
\]
which is maximized for $\eta^* = \frac{\bar{\eta}}{2} = \eta^o$.

This outcome holds true both if the firm cannot commit to promotion bars at the hiring stage, so that the promotion bar $\eta^*$ is set after the training effort and if the firm can commit to promotion bars at $t = 0$. In the second case, the firm offers a contract $\{w_1, w_2, \eta^*\}$ at $t = 0$, thus $\eta^*$ is chosen so as to convince workers to accept the contract ex-ante. Namely, firms maximize workers’ lifetime expected utility. Again, since $w_2$ is independent of the promotion decision, the firm tries to maximize $w_1 = \mathbb{E}\left[\sum_{i=1}^{B} (y_{2i} - w_{2i})\right]$. This procedure yields again the same result as the one shown above.

For which concerns promoted workers’ retention at the interim stage, note that $w_2 = \eta + a$ is exactly as much as a worker would be offered by a competing firm with portable effort and $\theta = 1$, as shown in equation (7). Hence, for any $\theta \in [0, 1]$, workers have no incentive to move to another employer. Trivially, the same result holds if workers acquire specific training.

\[\Box\]

**Proof of Lemma 1**

Proof. The proof for this lemma comes from the firm’s profit maximization problem. To choose the promotion bar, the firm solves:
\[
\eta^*_s \in \arg\max_{\{\eta\}} (1 - \theta) \int_{\eta}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta)\bar{\eta}}{2} + a
\]
delivering the first-order condition for an interior solution
\[
\frac{f(\eta)\bar{\eta}}{2} - (1 - \theta)\eta f(\eta) = 0 \Rightarrow \eta^*_s = \frac{\bar{\eta}}{2(1 - \theta)}. \quad (21)
\]

Now, $\eta^*_s \leq \bar{\eta}$ for any $\theta \leq \frac{1}{2}$, otherwise it exceeds the upper bound of the support of talent, so that the firm does not promote workers at all.

\[\Box\]
Proof of Proposition 3

Proof. To prove this proposition, recall from Lemma 1 that the firm sets a promotion rule so that a worker producing \( y_1 \) is promoted if \( y_1 - \hat{a} \geq y^* = \frac{x}{1-\theta} \). This condition can be rewritten as \( \eta \geq y^*-a+\hat{a} \). Recall that when observing \( \hat{\eta} < 0 \), the firm does not promote the worker who produced such signal, who then earns zero. If instead the firm observes too high a signal \( \hat{\eta} > \tilde{\eta} \), the worker who produced such signal is promoted (respectively, not promoted) with probability \( \frac{1}{2} \).

Workers exert firm-specific training effort in order to maximize their expected utility, as depicted in equation (5).

Since every firm in the industry observes \( y_1 \), a worker exerts firm-specific effort so that

\[
a^*_s \in \arg\max_{\{a\geq 0\}} \frac{\theta}{2} \left[ \int_{y^*-a+\hat{a}}^{\tilde{\eta}+\hat{a}} (\eta + a - \hat{a}) f(\eta) d\eta + \int_{y^*-a+\hat{a}}^{\hat{\eta}} (\eta + a - \hat{a}) f(\eta) d\eta \right] - \frac{a^2}{2}.
\]

However, Lemma 1 shows that firms may not be willing to promote workers if there is too much competition for talent. Thus, I take into account two possible scenarios:

1. If \( \theta \leq \frac{1}{2} \), firms promote a positive fraction of workers, so the first-order condition for an internal solution is

\[
\frac{\theta}{2} \left( 1 + \frac{a - \hat{a}}{\tilde{\eta}} \right) - a = 0 \tag{22}
\]

The second-order condition is satisfied if

\[
\frac{\theta}{2\tilde{\eta}} < 1 \tag{23}
\]

Note that since \( \tilde{\eta} \) (the upper bound of the support of talent \( \eta \)) is strictly larger than 1, the second-order condition for this program is fulfilled.

Along the equilibrium path, the firm perfectly anticipates workers’ effort, hence \( a = \hat{a} \) and the first order condition yields

\[
a^*_s = \frac{\theta}{2} \tag{24}
\]
2. If $\theta > \frac{1}{2}$, the firm does not promote any workers, so that the first term of workers’ maximization program is zero. Given the nonnegativity constraint imposed on effort, in equilibrium the worker exerts no training effort, namely $a_s^* = 0$.

Notice that such solution also emerges in the case in which $\theta = 0$, for a different reason. In this case, the firm promotes workers efficiently, yet there is no competition in the labor market, hence, workers are indifferent between being promoted and not, so that they have no incentive to exert training effort.

\[ \boxed{\text{Proof of Proposition 4}} \]

Proof. By backward induction workers’ effort is the same as in the case without commitment, since it does not depend on the promotion bar. This implies that the optimal effort is the same as the one derived in the proof of Proposition 3:

\[ a_s^* = a_s^{**} = \frac{\theta}{2} \]  \hspace{2cm} (25)

provided that the firm promotes some workers in equilibrium.

Moving backwards to $t = 0$, the firm offers the pair \{ $w_1$, $\eta_s^{**}$ \} that maximizes workers’ lifetime expected utility

\[ U(w, a) = w_1 + \mathbb{E}(w_2) - \psi(a) \]

to ensure activity.

At $t = 0$ the firm has expected profit

\[ \mathbb{E}_0(\pi) = \int_{\eta_s^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a_s^* - \mathbb{E}(w_2) - w_1. \]  \hspace{2cm} (26)

Since the labor market is perfectly competitive at the hiring stage and firms are unable to commit directly to wages, $w_1$ is driven by the zero-profit condition hence

\[ w_1 = \int_{\eta_s^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a_s^* - \mathbb{E}(w_2). \]
Given these wages, the firm chooses the promotion bar solving:

$$\max_{\eta^*} U = \int_{\eta^*}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^*)\bar{\eta}}{\theta} + a^* \left( 1 - \frac{a^*}{2} \right)$$

This delivers the first-order condition

$$\eta^* = \frac{\bar{\eta}}{2} = \eta^o.$$  \hspace{1cm} (27)

Since promotion bars do not provide incentives for workers to exert firm-specific training effort, the firm ends up choosing the bar that maximizes productivity.

Proof of Lemma 2

Proof. The proof for this lemma comes from the firm’s profit maximization problem:

$$\eta_p(a) \in \arg\max_{\eta} \pi = (1 - \theta) \int_{\eta}^{\bar{\eta}} (\eta + a) f(\eta) d\eta + F(\eta) \left( \frac{\bar{\eta}}{2} + a \right).$$

This yields the first-order condition for an interior solution:

$$f(\eta) \left[ \frac{\bar{\eta}}{2} + a - a(1 - \theta) - (1 - \theta)\eta \right] = 0$$

which can be simplified and deliver the best reaction function

$$\eta_p(a) = \frac{\bar{\eta}}{2(1 - \theta)} + \frac{\theta a_p}{1 - \theta}$$ \hspace{1cm} (28)

for any training effort $a_p$ exerted by the worker.

Proof of Proposition 5

Proof. I prove this proposition in two steps: first, I derive the optimal effort and promotion bar, showing that the latter is larger than the efficient benchmark and exceeds $\bar{\eta}$ if $\theta \geq \frac{1}{2}$; second, I prove the nonmonotonicity of optimal portable training effort with respect to $\theta$. 

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1. By backward induction, workers know that the firms’ best reaction function $\eta_p(a)$ is increasing in their effort. For this reason, at $t = 1$, their maximization program is:

$$a_p^\ast \in \arg\max_{\{a_p \geq 0\}} \left\{ \int_{\eta_p(a_p)-a_p+\hat{a}}^{\bar{\eta}-a_p+\hat{a}} (\eta + a_p - \hat{a}) f(\eta) d\eta + \int_{\eta_p(a_p)-a_p+\hat{a}}^{\bar{\eta}} (\eta + a_p - \hat{a}) f(\eta) d\eta \right\} - \frac{a_p^2}{2}$$

The first-order condition for an interior solution, taking into account the fact that the firm correctly conjectures $a$ along the equilibrium path (namely, $a = \hat{a}$), is

$$\frac{\theta}{2\bar{\eta}} \left[ 3\bar{\eta} - 2\eta_p(a_p) \right] + a_p \left( 1 - \frac{2\theta}{1 - \theta} \right) = 0 \quad (29)$$

Plugging the firm’s reaction function (28) into (29), one gets

$$a_p^\ast(\theta) = \frac{\bar{\eta}\theta[(1-\theta)^2 - 1]}{(2\bar{\eta} - \theta)(1-\theta)^2 + 2(2-\theta)\theta^2} \quad (30)$$

Plugging the optimal effort (30) into the firm’s best reaction function (28), delivers the equilibrium promotion bar without commitment:

$$\eta_p^\ast(\theta) = \frac{\bar{\eta}}{1 - \theta} \left[ 1 + \frac{\theta^2[(1-\theta)^2 - 1]}{(2\bar{\eta} - \theta)(1-\theta)^2 + (2 - \theta)2\theta^2} \right] \quad (31)$$

Compare now $\eta_p^\ast$ with the efficient benchmark $\eta^o = \frac{\bar{\eta}}{2}$. Notice that $\eta_p^\ast$ can be written as

$$\eta_p^\ast = \frac{\bar{\eta}}{2(1-\theta)} + \frac{\theta a_p^\ast}{1 - \theta} \quad (32)$$

Since $\theta \in [0, 1]$, it is sure that $\frac{\bar{\eta}}{2(1-\theta)} \geq \frac{\bar{\eta}}{2}$, so that the first term in (32) is larger than the efficient promotion bar. The second term is the optimal effort multiplied by $\frac{\theta}{1 - \theta}$. These two elements multiplied are by definition larger or equal to zero, thus implying that $\eta_p^{**} \geq \eta^o$, for any $\theta \in [0, 1]$. The promotion bar equates the efficient benchmark only if $\theta = 0$, namely in the absence of labor market competition for promoted workers.
Moreover, note that if \( \theta \geq \frac{1}{2} \), \( \eta^*_p \geq \bar{\eta} \), as in the case in which workers exert firm-specific training effort. If \( \theta = \frac{1}{2} \), the first term of equation (32) is exactly equal to \( \bar{\eta} \) and it is summed to something at least equal to zero.

2. To prove that \( a^*_p \) is nonmonotonic concave with respect to \( \theta \), first notice that \( a^*_p \) is continuous, as its denominator is always different from zero. Moreover, one can show that \( a^*_p \) has zeros at \( \theta = 0 \) and \( \theta \approx 0.43 \).

Second, take the total differentiation of \( a^*_p \)

\[
\frac{d(a^*_p)}{d\theta} = \frac{2\bar{\eta}[3\bar{\eta}\theta^4 + (3 - 12\bar{\eta})\theta^3 + (19\bar{\eta} - 3)\theta^2 - 12\bar{\eta}\theta + 2\bar{\eta}]}{[3\theta^3 - (2\bar{\eta} + 6)\theta^2 + (4\bar{\eta} + 1)\theta - 2\bar{\eta}]^2}.
\]

(33)

To determine the sign of (33), note that the denominator is always positive. This implies that

\[
sign\{\frac{d(a^*_p)}{d\theta}\} = -sign\{2\bar{\eta}[3\bar{\eta}\theta^4 + (3 - 12\bar{\eta})\theta^3 + (19\bar{\eta} - 3)\theta^2 - 12\bar{\eta}\theta + 2\bar{\eta}]\}.
\]

One can see that

\[
\lim_{\theta \to 0} \frac{d(a^*_p)}{d\theta} > 0 \text{ and } \lim_{\theta \to 1} \frac{d(a^*_p)}{d\theta} = 0.
\]

By continuity of \( a^*_p \) with respect to \( \theta \), there exists a positive cutoff value \( \hat{\theta} \), such that

\[
\frac{da^*_p}{d\theta} \geq 0 \text{ for any } \theta \leq \hat{\theta} \text{ and } \frac{da^*_p}{d\theta} < 0 \text{ for any } \theta > \hat{\theta},
\]

so that optimal effort starts falling up to zero (for \( \theta \approx 0.43 \)).

Finally, tedious algebra shows that the second derivative of \( a^*_p \) with respect to \( \theta \) is negative for any \( \theta \in [0, 1] \), which proves concavity of optimal effort and promotion bar with respect to competition for promoted workers.

\[ \blacksquare \]

**Proof of Lemma 3**

**Proof.** To prove this lemma, consider workers’ best reaction function for any promotion bar \( \eta_p \). This is derived by solving

\[
a_p(\eta_p) \in \arg\max_{a \geq 0} \frac{\theta}{2} \left\{ \int^{\eta-a+\hat{\eta}}_{\eta-a+\hat{\eta}} (\eta + a - \hat{\eta}) f(\eta) d\eta + \int^{\eta}_{\eta-a+\hat{\eta}} (\eta + a - \hat{\eta}) f(\eta) d\eta + \right.
\]

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The first-order condition for this problem is
\[
\frac{\theta}{2} \left[ 3 - \frac{2(\eta_p - a + \hat{a})}{\bar{\eta}} + \frac{a}{\eta} \right] - a = 0
\] (34)
and the second-order condition is
\[
\frac{3\theta}{2\bar{\eta}} < 1
\]
which is fulfilled, since it is assumed that \( \bar{\eta} \geq \frac{3}{2} \).

Along the equilibrium path, the firm holds consistent beliefs, so that \( a = \hat{a} \). Thus, worker’s best reaction function is
\[
a_p(\eta_p) = \frac{\theta(3\bar{\eta} - 2\eta_p)}{2\bar{\eta} - \theta}.
\] (35)
for any promotion bar \( \eta_p \in [0, \bar{\eta}] \).  

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**Proof of Proposition 6**

*Proof.* I prove this proposition in three steps. First, I derive the optimal promotion bar. Second, I show that in equilibrium, the promotion bar is smaller or equal to the efficient benchmark in a certain interval of values of \( \theta \), more specifically, it is nonmonotonic with respect to labor market competition for promoted workers. Third, I derive the optimal portable training effort.

1. The firm rationally anticipates workers’ reaction specified in equation (35). As in the proof of Proposition 4, at \( t = 0 \) the firm offers a contract \( \{w_1, \eta_p^{**}\} \) to hire workers. Workers are willing to work for the firm offering the contract that maximizes their lifetime expected utility
\[
U(w, a) = w_1 + \mathbb{E}(w_2) - \psi(a).
\]

The firm’s expected profit as of \( t = 0 \) is
\[
\mathbb{E}_0(\pi) = \int_{\eta_p^{**}}^{\bar{\eta}} \left( \eta + a(\eta_p^{**}) \right) f(\eta) d\eta + F(\eta_p^{**}) \left( \frac{\eta}{2} + a(\eta_p^{**}) \right) - w_1 - \mathbb{E}(w_2).
\]
Since the labor market is perfectly competitive at $t = 0$, $w_1$ is driven by the zero-profit condition, yielding $w_1 = E_0(\pi)$.

Given these wages, at the hiring stage, the firm commits to the promotion bar solving

$$Max_{(\eta_p^{**})} U = \int_{\eta_p^*}^{\bar{\eta}} \eta f(\eta)d\eta + \frac{F(\eta_p^{**})\bar{\eta}}{2} + a(\eta_p^{**})\left(1 - \frac{a(\eta_p^{**})}{2}\right).$$

The first-order condition for this maximization program yields

$$\eta_p^{**}(\theta) = \frac{\bar{\eta}}{2} \left[4\bar{\eta}^2 - 12\bar{\eta}\theta(1 - \theta) + 5\theta^2\right].$$

(36)

Note that $\eta_p^{**}$ is continuous with respect to $\theta$ as it can be easily shown that its denominator is never equal to zero.

2. Equating $\eta_p^{**}$ with $\eta^o = \frac{\bar{\eta}}{2}$, one obtains:

$$\theta[(2\bar{\eta} + 1)\theta - \bar{\eta}] = 0$$

which has solution roots $\theta_1 = 0$ and $\theta_2 = \frac{\bar{\eta}}{2\bar{\eta} - 1} \equiv \theta^o$. As both roots are real, one can conclude that $\eta_p^{**} < \eta^o \forall \theta \in (0, \theta^o)$, whilst $\eta_p^{**} > \eta^o \forall \theta \in (\theta^o, 1]$.

Note that the total differentiation of the optimal promotion bar with respect to $\theta$ yields

$$\frac{d\eta_p^{**}}{d\theta} = \frac{-4\bar{\eta}[\theta^2 - (8\bar{\eta}^2 + 4\bar{\eta})\theta + 4\bar{\eta}^2]}{(4\bar{\eta} + 1)\theta^2 - 4\bar{\eta}\theta + 4\bar{\eta}^2}.\quad (37)$$

The sign of (37) is such that

$$\lim_{\theta \to 0} \frac{d\eta_p^{**}}{d\theta} < 0 \text{ and } \lim_{\theta \to 1} \frac{d\eta_p^{**}}{d\theta} > 0.$$

Hence, by continuity of $\eta_p^{**}$, there exists a value of $\theta$ such that the sign of the derivative switches from negative to positive, hence the promotion bar is nonmonotonic with respect to $\theta$. 
3. The optimal effort $a_{p}^{**}$ is straightforwardly derived by plugging equation (36) into the worker’s best reaction function (35). This delivers

$$a_{p}^{**}(x, \theta) = \frac{\bar{\eta}\theta}{2(\bar{\eta} - \theta)} \left\{ 4 - \left[ \frac{4\bar{\eta}\theta^2 + (\bar{\eta} - \theta)(\bar{\eta} - 3\theta)}{\bar{\eta}\theta^2 + (\bar{\eta} - \theta)^2} \right] \right\} \geq 0.$$
References


4 Talent Discovery, Layoff Risk and Unemployment Insurance
Talent Discovery, Layoff Risk and Unemployment Insurance

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Abstract

In talent-intensive jobs, workers’ performance reveals their quality. This enhances productivity and wages, but also increases layoff risk. If workers cannot resign from their jobs, firms can insure them via severance pay. If instead workers can resign, private insurance cannot be provided, and more risk-averse workers will choose less informative jobs. This lowers expected productivity and wages. Public unemployment insurance corrects this inefficiency, enhancing employment in talent-sensitive industries and investment in education by employees. The prediction that the generosity of unemployment insurance is positively correlated with the share of workers in talent-sensitive industries is consistent with international and U.S. evidence.

JEL: D61, D62, D83, I26, J24, J65.

Keywords: talent, learning, layoff risk, unemployment insurance.

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Discovering workers’ talent is increasingly important in the knowledge economy, whose ability to innovate – e.g., by introducing a new app or investment strategy – crucially depends on the quality of employees’ human capital (Kaplan and Rauh, 2013). The hallmark of talent-intensive industries is that their technology does not rely on the ability to perform routine tasks efficiently, but rather on employees’ qualities such as imagination and intelligence, as well as their education. In this setting, corporate success often hinges on identifying the most talented workers and assigning them to the task they are best at.

If the labor market is competitive, talented workers share in the productivity gains they generate, in the form of high salaries or bonuses. However, *ex ante* talent discovery is a source of risk for workers, if they are not fully aware of their own quality: *ex post*, they may turn out to be worse than expected, and if so they may be laid off and forced to seek a more suitable job. Such risk imposes considerable welfare losses on workers (Low, Meghir and Pistaferri, 2010): laid off workers experience earnings losses, not only while unemployed but also upon reentry (Jacobson, LaLonde and Sullivan, 1993), and typically cut back on their expenditures (Gruber, 1997; Browning and Crossley, 2001).

In principle, this risk is privately insurable: firms might commit to give generous severance pay to laid-off employees, and thus compensate them upon being found untalented. But firms can provide such insurance only if the labor market is not fully competitive, in that workers are not free to switch to other employers once their talent is discovered. If they are, firms cannot provide severance payments to low-talent employees: this would require cross-subsidizing them at the expense of high-talent ones, who would react to such a scheme by switching to a competing employer (Harris and Holmström, 1982).

Hence, in the presence of *ex-post* competition for talent, workers are left to bear the layoff risk arising from the talent discovery process, absent any public unemployment insurance. We show that in this situation, risk-averse workers have the incentive to mitigate or eliminate such risk by choosing to work for firms and industries whose projects convey little information about employees’ quality. These firms and industries naturally feature less efficient talent allocation than those where employers can learn more about their employees’ quality. Therefore, they also feature
no layoff risk, but pay lower average wages than the latter. As a result, industries with talent-sensitive technologies (where on-the-job performance reveals a worker’s ability) will find it difficult to hire workers and develop: only the least risk-averse workers – if any – will want to work in such industries.

As pointed out by Hirshleifer (1971), information revelation brings benefits in terms of productive efficiency, but also has costs due to forgone insurance opportunities. In this paper, we show that this destruction of insurance opportunities can impair the development of talent-sensitive industries or technologies. By the same token, this market failure highlights a hitherto neglected efficiency rationale for public unemployment insurance (UI), whereby society – rather than firms – supports laid-off workers, funding their benefits with payroll taxes on employees who retain their jobs. Being buffered against layoff risk, even risk-averse workers will prefer jobs in talent-sensitive industries, which pay high wages. The prediction is that such industries should be able to flourish in one of two alternative settings: either in economies with little labor market competition (because of employee loyalty, switching costs or regulatory frictions) or in economies where competition for workers’ talent is associated with a generous public safety net against layoff risk.

Compared with public UI, trying to protect workers against job loss by limiting firms’ ability to fire them is socially inefficient. Employment protection legislation (EPL) effectively forces firms to keep also low-quality workers on board: this will induce firms in more talent-intensive industries to refrain from hiring in the first place, in order to break even. This is because, due to limited liability, workers can share in the firm’s surplus but are protected from the losses that they generate. Thus EPL leads to an inefficiently low level of learning about workers’ talent, and results in lower average wages, not just reduced layoff risk. Hence, in our framework it is dominated by UI.

We also investigate the impact of talent discovery and layoff risk on workers’ accumulation of human capital. To this purpose, we expand the baseline model to allow for an initial stage where workers can invest in education, and find that the introduction of UI spurs such investment by workers, as it decreases the risk of the return to human capital. Insofar as it encourages employment in talent-sensitive industries, it also increases the total number of workers who acquire education. Hence, UI acts both on the intensive and the extensive margin of education acquisition – a channel that in turn compounds the impact of UI on talent discovery.
Our model produces several testable predictions. One of these is a positive correlation between the generosity of UI and the share of workers employed in talent-sensitive firms. We show that such correlation is broadly consistent both with OECD country-level data and with U.S. state-level data from the Bureau of Labor Statistics (BLS), using in both cases the income replacement rate (i.e. the ratio between unemployment benefits and the last wage) as a measure of the generosity of the UI system.

The structure of the paper is as follows. Section 2 sets our contribution against the backdrop of the relevant literature. Section 3 lays out the model’s assumptions. Section 4 derives the evolution of beliefs about employees’ talent and firms’ resulting optimal layoff rule. Sections 5.1 and 5.2 characterize equilibrium in the absence and in the presence of labor market competition, and compare them. Section 6 shows how public UI affects the equilibrium. Section 7 investigates the effects of employment protection legislation, and compares them with those of UI. Section 8 extends the model to a setting where workers can invest in education before entering the labor market. Section 9 summarizes the empirical predictions of the model and provides evidence for some of them. Section 10 concludes.

2 Related Literature

This paper lies at the intersection of two strands of research: the literature on learning about the quality of workers, and that on the insurance that they receive by employers and public institutions. What naturally joins these two strands of research is the simple fact that learning about one’s talent is a source of risk.

Learning about talent can occur either within the firm (from one’s work performance with a given employer) or in the market (from sequential matching with different employers). In our model, learning occurs within the firm, as in career concerns models dating back to Fama (1980) and Holmström (1999). Since however such learning spills over to other potential employers, as in Harris and Holmström (1982) competition prevents firms from being able to insure workers against talent uncertainty. In our setting, the non-insurability of human capital risk leads not only to inefficient risk-sharing within firms, but also to low average productivity: efficient talent discovery within firms takes place only if employees are insured against the implied risk, and this cannot occur in a competitive labor market, as in Acharya,
Pagano and Volpin (2016). This result differs sharply from that by Jovanovic (1979) and other search models of the labor market, where learning about workers’ quality occurs in the marketplace: in those models, mobility allows employees and firms to attain efficient matches.

In our setting, workers bear the cost of talent discovery in the form of layoff risk. In reality, also firms bear costs in such a learning process, since experimenting with novices requires forgoing employment of senior employees with a proven track record. Terviö (2009) shows that, in a search model with uncertain worker quality, this implicit screening cost deters efficient talent discovery: firms pay inefficiently high wages to mediocre incumbent workers rather than testing promising novices. Also in Terviö’s model labor market competition leads to inefficiently low talent discovery, but in our framework this inefficiency arises from uninsurable layoff risk rather than screening costs.

Far from being inessential, however, this feature of our model is at the root of its main prediction: that public provision of UI can restore efficiency in talent discovery even in the presence of labor market competition. Interestingly, a substitutability relationship between firm-level insurance provision and public UI is documented empirically by Ellul, Pagano and Schivardi (2017).

By highlighting that UI enhances productive efficiency, our paper contributes to the literature on the costs and benefits of UI. This literature has recognized that UI stabilizes workers’ consumption (Gruber, 1997) and avoids mortgage defaults by the unemployed (Hsu, Matsa and Meltzer, 2017), but also stressed its disincentive effects on labor search by the unemployed, and the resulting increase in the duration of unemployment spells (Moffitt and Nicholson, 1982; Meyer, 1990, and Katz and Meyer, 1990). But other papers show that UI also allows workers to search longer so as to identify better matches, thus raising aggregate productivity (Diamond 1981; Acemoglu 1997; Marimon and Zilibotti 1999). Indeed, Nekoei and Weber (2017) document empirically that UI improves the quality of the firms where the unemployed find jobs and raises their wages. While in these papers UI raises productivity by subsidizing talent discovery in the marketplace, in our setting it acts as a subsidy to

\[1\] Moffit and Nicholson (1982) find that a 26-week extension of the benefit duration lengthens the average unemployment spell by about 2.5 weeks. Meyer (1990) shows that the probability of leaving unemployment is negatively affected by the level of benefits, and increases just before the entitlement period expires.
talent discovery within the firm.

The only search-theoretic model of UI with risk-averse workers is Acemoglu and Shimer (1999, 2000). In their general equilibrium setting, if firms choose a labor-intensive technology, they create many job vacancies and can fill them offering low wages: risk-averse workers will accept such low wages because they have a high probability of filling a vacancy, and thus avoiding unemployment. If instead firms choose a capital-intensive technology, they create few vacancies: even if they offer high wages, few workers will bid for them for fear that the job will be filled by a competing applicant. This creates vacancy risk for firms, which will deter them from opting for such a technology. UI changes this result, as it makes even risk-averse workers willing to bear the unemployment risk associated with a capital-intensive technology.

Hence, also in Acemoglu and Shimer UI implies higher productivity of employed workers, as well as higher level and risk of unemployment, as in our model. But the two models differ in two important respects. First, in ours unemployment risk arises from the danger of being laid off, not from the risk of the job being filled by a competing applicant. Second, in our model the productivity-enhancing effect of UI comes from better talent discovery, whereas in Acemoglu and Shimer it stems from firms choosing a more capital intensive technology. This translates into different predictions about the effects of UI: according to our model, UI reallocates employment towards talent-intensive industries, while according to Acemoglu and Shimer UI induces all firms to adopt a more capital-intensive technology.

3 The Model

We study a two-period model with Bayesian learning about workers’ talent. The economy is populated by competitive firms owned by risk-neutral shareholders and a continuum of measure $N$ of workers. Each worker can operate at most one project. Each project lasts for two periods and in both it must be operated by the same worker: if the worker leaves the firm at the end of the first period, the project is terminated prematurely.

Firms belong to one of two industries, $j = \{1, 2\}$, whose respective technologies feature a specific sensitivity to employees’ talent $\lambda_j \in [0, 1]$, as will be explained
below in greater detail. Industry \( j \) is populated by \( F_j \) firms, so that \( F = F_1 + F_2 \) is the total number of firms in the economy. Each industry \( j \) is endowed with a continuum of measure \( G_j > N \) of homogeneous projects. As a result, in each industry there is at least one project per worker: workers – not projects – are the scarce factor of production. The model easily generalizes to any number of industries.

Workers are risk-averse: their instantaneous utility \( u(w_t) \) is increasing and concave in their time-\( t \) wage \( w_t \). Furthermore, they have no time-discounting, no initial wealth, and no access to financial markets.\(^2\) Hence, their lifetime utility is \( U = u(w_1) + u(w_2) \).

### 3.1 Types and Productivity of Workers

Workers differ in their talent: worker \( i \)'s quality is \( q_i = \{G, B\} \) (either “good” or “bad”) and initially it is unknown to everyone in the economy, including workers themselves. The common prior belief about workers’ quality is \( \Pr(q = G) = p \in [0, 1] \). The revenue produced by a worker in each period is observable by all firms, so that also Bayesian posterior beliefs about worker’s quality are common. This assumption is without loss of generality, provided previous work experience (as opposed to performance) is common knowledge.\(^3\)

All workers have a reservation wage \( w_0 > 0 \) per period, whose utility is standardized to zero, for simplicity: \( u(w_0) = 0 \). In each of the two periods of its life, a project produces revenue \( y_t \). The revenue \( y_t \) can take either a high value \( \overline{y} > w_0 \) or a low value \( \overline{y} - c \), which does not cover the worker’s reservation wage \( w_0 \), leading to a negative surplus: \( \overline{y} - c - w_0 < 0 \). The revenue produced by a project depends on a combination of technological risk and talent of the worker in charge of it, as illustrated by Figure 1. With probability \( 1 - \lambda \), the payoff depends only on technological

\(^2\)This assumption allows us to focus on the firm and the labor market as the only sources of insurance against human capital risk. Otherwise, laid-off workers would be able to buffer their consumption by borrowing or decumulating their wealth.

\(^3\)Intuitively, suppose that workers’ first-period performance were observed only by their current employer, but that those leaving their firm after the first production period can be told apart from others. Then, potential employers would infer that such employees were laid off, and therefore performed poorly in the first period, since it is optimal to fire only such employees, as we will prove. Employees who performed well in the first period will have no incentive to resign from their current employer, as otherwise they would be mistaken for low-quality workers. Hence, other employers’ belief about the low quality of workers leaving their firm after the first period is rational.
risk: the project’s revenue is $\bar{y}$ with probability $p$ and $\bar{y} - c$ with probability $1 - p$. Instead, with probability $\lambda$ the project’s revenue reflects the worker’s talent: if she is good, the project delivers revenue $\bar{y}$; if she is bad, it yields $\bar{y} - c$.

Hence, $\lambda$ can be seen as the project’s sensitivity to workers’ talent: the higher $\lambda$, the lower the “noise” in the project’s payoff, and the sharper the “signal” that it conveys about the talent of the project’s executor.4 For example, in the extreme case where $\lambda = 1$, the project always succeeds if executed by a good worker and fails otherwise, and therefore it is perfectly informative about the worker’s talent. In the polar opposite case $\lambda = 0$, the project succeeds with the unconditional probability $p$, and therefore is totally uninformative about its executor’s talent.

Notice that $\lambda$ does not affect a project’s unconditional probability of success and thus its expected revenue, $\bar{y} - (1 - p) c$, as well as its variance $p(1 - p) c^2$. As we shall see, in this model a project’s sensitivity to talent, $\lambda$, raises its expected return and

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4By the same token, we shall see that $\lambda$ also determines workers’ returns to talent.
its risk only because it sharpens the firm’s learning and thus increases its propensity to liquidate bad-performing projects before completion: the relationship between $\lambda$ and payoff moments is driven by the firm’s behavioral response, not by technology.

To make the problem interesting, we impose the following parameter restrictions:

$$\bar{y} - (1 - p)c \geq w_0 > \bar{y} - c > 0.$$  \hspace{1cm} (1)

The left-hand-side inequality implies that initially it is efficient to hire any worker, since her unconditional expected revenue is positive. The right-hand-side inequality implies that the productivity of a bad worker is low enough that the employer does not wish to retain her. Condition (1) can be rewritten as

$$p \geq \frac{c - \bar{y} + w_0}{c} > 0,$$  \hspace{1cm} (2)

so that in what follows we restrict our attention to the interval $p \in \left[ 1 - \frac{\bar{y} - w_0}{c}, 1 \right]$.

### 3.2 Labor Contracts

Firms are assumed to compete for workers when initially hiring them. After the first production period, workers can be fired by their employer or not. Regarding workers’ mobility, we consider two labor market regimes: a non-competitive regime, where workers cannot resign from their current employer, because of loyalty or market frictions (such as search costs or regulation), and a competitive one, where workers are free to resign and switch to a new employer. Otherwise stated, in the first regime workers can commit to stay with their initial employer, whereas in the second they have no such commitment ability.

Instead, firms are assumed to be able to commit to long-term contingent contracts: when hiring workers, they offer wage contracts for both production periods, $\{w_t\}_{t=1}^2$, conditional on retaining the employee in the second period. In other words, workers’ performance in each period is assumed to be not only observable but also verifiable.\(^5\)

Firms condition the wage to be paid in the first production period on their prior belief $\theta_0 = p$ about workers’ quality, and the wage to be paid in the second production

\(^5\)Failing this, firms would not be able to offer any insurance, even in the non-competitive regime: see footnote 5 below.
period on the revenue $y_1$ previously generated by the worker, and therefore on their posterior belief, $\theta_1 = Pr(q = G|y_1)$. Hence, the second-period wage is effectively a function of their belief, $w_2(\theta_1)$. Wages can never be negative, as employees are protected by limited liability.

Once a worker is hired and assigned to a project, she generates revenue $y_1$. Based on this initial payoff, the firm decides whether to keep the worker running the project or not: if the expected “continuation revenue” produced by the worker, denoted by $y_2$, is negative, then the firm will want to liquidate the project and fire the worker. This decision is captured by an indicator variable $\gamma = 1$ if the worker is retained within the firm and her project is continued, and $\gamma = 0$ if she is laid off and the project is liquidated. Once an employee’s project is liquidated, the firm is better off firing her and producing nothing rather than keeping her idle and paying her reservation wage $w_0$.

Recalling that the firm’s revenue at time $t$ equals $y_t$, its profit equals $y_t - w_t$. Assuming no discounting, the firm maximizes expected profits:

$$\mathbb{E}_0 [y_1 - w_1 + \gamma(y_2 - w_2)],$$

and workers maximize expected utility as of the beginning of the game:

$$\mathbb{E}_0 [u(w_1) + \gamma u(w_2)],$$

with $u(\cdot)$ increasing and concave.

### 3.3 Time Line

The time line of the model is composed of four stages, as shown in Figure 2.

At $t = 0$, firms compete for workers by offering two-period contingent wage contracts $\{w_t\}_{t=1}^2$, and workers choose which firm to work for.

At $t = 1$, each worker initiates a project in the chosen firm, produces revenue $y_1$, and earns wage $w_1$.

At $t = 2$, beliefs about each employee’s quality are updated, and based on such beliefs firms decide whether to retain or fire workers; even if retained, workers can resign if the labor market features ex post competition.
At $t = 3$, retained employees continue to operate the project, produce revenue $y_2$ and receive wage $w_2$; otherwise, their project is liquidated and they earn the reservation wage $w_0$ absent any insurance, a severance pay if pledged by the firm, or a unemployment benefit in the presence of public insurance.

### 4 Profits, Beliefs and Layoffs

The expected revenue that projects produce at $t = 1$ is the same for all firms, irrespective of $\lambda$:

$$\mathbb{E}_0(y_1) = \bar{y} - (1 - p)c. \quad (5)$$

However, the actual value of the revenue $y_1$ will generally differ depending on the employee operating the project. Based on its realization, the belief about the quality of the employee in charge of the project is updated from the prior $\theta_0 = p$ to the posterior $\theta_1$, which can take one of two values: $\Pr(q = G | y_1 = \bar{y}) \equiv \theta_H$ for workers that generated a profit at $t = 1$ or $\Pr(q = G | y_1 = \bar{y} - c) \equiv \theta_L$ for those that produced a loss.
This Bayesian updating depends on the informativeness \( \lambda \) of the firm’s technology:

\[
\theta_H = \lambda + (1 - \lambda)p \geq p
\]

and

\[
\theta_L = (1 - \lambda)p \leq p.
\]

Hence, the expected second-period revenue of the project upon good performance, \( y_{2H} \equiv \mathbb{E}_1(y_2 | y_1 = \bar{y}) \) is

\[
y_{2H} = \bar{y} - (1 - \theta_H(\lambda))c,
\]

while the corresponding expression upon bad performance, \( y_{2L} \equiv \mathbb{E}_1(y_2 | y_1 = \bar{y} - c) \), is

\[
y_{2L} = \bar{y} - (1 - \theta_L(\lambda))c.
\]

These two expressions bracket the first-period average revenue:

\[
y_{2H} \geq \mathbb{E}_0(y_1) \geq y_{2L}, \quad \forall \lambda: \text{the revenue from the project is expected to increase upon good performance, and to drop upon bad.}
\]

Based on such updated beliefs, firms will choose different optimal firing policies depending on the informativeness of their technology, \( \lambda \):

**Lemma 1** If the revenue is \( y_1 = \bar{y} \), the worker is retained and the project is continued, irrespective of the firm’s talent-sensitivity \( \lambda \). If \( y_1 = \bar{y} - c \), the worker is laid off only by firms with talent-sensitivity \( \lambda \geq \hat{\lambda} = \frac{\bar{y} - (1-p)c - w_0}{pc} \) and the corresponding project is liquidated.

This lemma, proved in the Appendix (as all subsequent results), is illustrated by Figure 3. The informativeness of the firm’s technology, \( \lambda \), ranges between 0 and 1. If \( \lambda \) exceeds the threshold value \( \hat{\lambda} \), it becomes optimal for the firm to fire low-performing workers. Such policy raises the firm’s productive efficiency, as measured by its ex-ante expected surplus \( \mathbb{E}_0(y_2) - w_0 \), because, when \( \lambda \) exceeds \( \hat{\lambda} \), the firm’s screening ability is sufficiently high to determine the liquidation of unpromising projects, and retain only those that are likely to be profitable, and thus able to pay a higher average wage. Such layoff policy is equivalent to an “up-or-out” mechanism, by which employees that prove successful at \( t = 1 \) receive a wage increase and the others are laid off. Indeed, “up-or-out” contracts are typical of talent-sensitive industries, such as academia, professional services and high-tech.
However, this gain in productive efficiency is obtained at the cost of unemployment risk, as workers that happen to perform poorly at $t = 1$ are fired. This can be seen in Figure 3, where the $y_{2L} - w_0$ line flattens to the right of $\lambda = \hat{\lambda}$: highly talent-sensitive firms produce nothing when the project’s expected surplus is negative, conditional on $y_1$, as they would make losses even if they paid workers just their reservation wage $w_0$. By firing workers upon bad performance at $t = 1$, such firms raise their unconditional expected revenue at $t = 2$:

$$E_0(y_2) = \begin{cases} py_{2H}(\lambda) + (1-p)y_{2L}(\lambda) = \bar{y} - (1-p)c = E_0(y_1) & \text{if } \lambda < \hat{\lambda}, \\ py_{2H} = p[\bar{y} - (1-p)(1-\lambda)c] > E_0(y_1) & \text{if } \lambda \geq \hat{\lambda}, \end{cases}$$

(10)

where $\theta_H$ is replaced by expression (6) in the expressions (8) and (9) for $y_{2H}$ and $y_{2L}$.

5 Labor Market Equilibrium

We now turn to the analysis of the labor market equilibrium. First, we consider the benchmark case of the noncompetitive regime, where workers cannot be poached by other firms at $t = 2$, after projects have generated their first payoff. Second, we study the regime where such poaching is possible, so that there is competition for workers.
also at $t = 2$. Finally, we contrast the allocation of risk and workers across firms in these two labor market regimes.

5.1 Benchmark: Noncompetitive Labor Market

We start with a labor market regime without ex-post competition for workers, because – for instance – prohibitive switching costs or regulatory constraints prevent workers from resigning after $t = 1$. In this regime, when firms bid for workers’ services at $t = 0$, they commit to pay workers a lifetime wage equal to the revenue that they are expected to generate during their whole career: ex-ante competition will lead each firm to bid wages up to the point where this is the case, so that total expected profits (3) are zero.

As a consequence, in this regime workers’ lifetime compensation does not depend on the first-period payoff of their project: they are perfectly insured against human capital risk. Notice that firms with highly informative technology (i.e. such that $\lambda \geq \hat{\lambda}$) will still optimally use the information about their employees’ quality inferred from their first-period performance and terminate the projects that make losses in the first period. But even these firms will pay the same lifetime compensation to the workers in charge of loss-making projects as to those in charge of profit-making ones: upon liquidation of their projects, the former will receive a severance pay that equals the salary paid to the latter – or alternatively are kept idle within the firm and paid the same salary as other workers.

Formally, the lifetime compensation that each employee of the same firm receives is

$$w_1 + w_2 = E_0(y_1) + E_0(y_2) = \begin{cases} 2E_0(y_1) & \text{if } \lambda < \hat{\lambda}, \\ E_0(y_1) + py_{2H} & \text{if } \lambda \geq \hat{\lambda}, \end{cases}$$

drawing on expressions (5) and (10) above.

Notice that in firms where $\lambda \geq \hat{\lambda}$ employees earn a strictly larger amount than in those where $\lambda < \hat{\lambda}$, since $py_{2H} > E_0(y_1)$. Moreover, since in these firms the expected second-period payoff $py_{2H}$ is increasing in $\lambda$, the employees of the most informative firms will receive the highest possible compensation, without bearing any risk. Therefore, in this labor market regime the firms with the highest value of $\lambda$ – namely, those with the most informative technology and the highest expected
productivity – will be able to attract all the employees, while other firms will not be able to operate. This is summarized in the following:

**Proposition 1** If the labor market is noncompetitive at $t = 2$, efficiency in production and risk sharing is attained in equilibrium, as the most talent-sensitive firms employ the entire workforce, and insure their employees fully.

As we shall see in Section 5.2, this result breaks down if the labor market is competitive at $t = 2$.

### 5.2 Competitive Labor Market

If the labor market is competitive both at $t = 0$ and $t = 2$, the workers whose projects are profitable at $t = 1$ can be poached by other firms at $t = 2$, who can attract them by offering a wage higher than the unconditional expectation of their revenue (i.e. the highest wage consistent with zero profits and full insurance by their current employer). Hence, their former employer would be left just with overpaid low-quality workers, as in Harris and Holmström (1982) and Acharya, Pagano and Volpin (2016).

In this labor market regime, competition allows workers to extract all the surplus that they generate in each period, so that the wage at time $t$ is

$$ w_t = \max\{E_{t-1}(y_t), 0\}, $$

which guarantees also that the worker’s participation constraints are satisfied: the expected wage of an employee in a firm with $\lambda < \widehat{\lambda}$ is $E_{t-1}(y_t) > w_0$ for both periods $t = 1$ and $t = 2$ (as shown by Lemma 1) and therefore her expected utility is

$$ E_{0}(U) = u(\bar{y} - (1 - p)c) + pu(y_{2H}) + (1 - p)u(y_{2L}). $$

---

$^{6}$It is worth noticing that for this outcome to obtain in equilibrium, it is necessary not only that workers commit not to resign from their job, but also that firms commit to the payments envisaged in their contracts, conditional on workers’ performance. Thus, commitment is required on both sides: otherwise, firms could hold up their employees and earn higher profits by paying less than the agreed wages. Clearly, this would prevent efficient risk-sharing.
Instead, employees in a firm with $\lambda \geq \hat{\lambda}$ have unconditional expected utility

$$E_0(U) = u(\bar{y} - (1-p)c) + pu(y_2H),$$ (14)

as in these firms a worker producing $y_1 = \bar{y} - c$ yields a conditional expected revenue $y_{2L} < w_0$ and is laid off at $t = 2$.

We now analyze how workers choose among employment opportunities. The most interesting case is that in which workers can choose between “safe” jobs offered by firms with talent intensity $\lambda_S < \hat{\lambda}$, and “risky” jobs in firms with talent intensity $\lambda_R \geq \hat{\lambda}$. In this case, workers will self-select into the two sets of firms depending on their risk aversion $\rho$, the more risk-averse opting for safe jobs, and the less risk-averse for risky ones:

**Proposition 2** Workers prefer offers from firms with $\lambda_S < \hat{\lambda}$ over those from firms with $\lambda_R \geq \hat{\lambda}$ if and only if their risk aversion $\rho$ exceeds $\widehat{\rho} \equiv \frac{[1-(1-\lambda_R)p|c-\bar{y}+w_0]}{y_{2L}^e - w_0} \geq 0$, which is increasing in $\lambda_R$.

The proof of this proposition relies on the fact that the expected benefit of a safe job compared to a risky one increases with workers’ risk aversion. Hence, workers with risk aversion below the threshold $\widehat{\rho}$ are willing to give up job security in order to earn higher expected wages, while the opposite applies to more prudent ones. The threshold risk aversion $\widehat{\rho}$ is monotonically increasing from 0 to a maximal value as the talent-sensitivity $\lambda_R$ of the risky industry rises from $\hat{\lambda}$ to 1: intuitively, as the informativeness of technology increases, jobs become more productive, hence pay higher wages, which induces even more risk-averse workers to accept the implied higher layoff risk. This prediction is far from obvious, because a more informative technology raises both risk and expected return to human capital; however, the implied increase in expected return dominates the increase in risk, resulting in a larger number of workers being attracted to the talent-sensitive industry.

If instead all the available jobs are either of the safe or of the risky variety, workers’ choices polarize:

**Proposition 3** (i) If all firms have $\lambda < \hat{\lambda}$, risk-averse workers choose to work for those with the lowest $\lambda$.

(ii) If all firms have $\lambda \geq \hat{\lambda}$, all workers choose to work for those with the highest $\lambda$, irrespective of their risk attitudes.
The intuition for the first part of the proposition is that firms with talent-intensity below $\hat{\lambda}$ effectively offer wage lotteries that are mean-preserving spreads of those offered by firms with $\lambda = 0$, whose technology is completely insensitive to talent. Since all the wage lotteries at $t = 2$ have the same unconditional expected payoff, but variance that increases in $\lambda$, at $t = 0$ risk-averse workers will prefer the least informative firm (i.e. choose the lowest-risk lottery in the sense of Rothschild and Stiglitz, 1970). If instead only firms with high talent-sensitivity are present on the market, workers cannot insure themselves against layoff risk by picking a safer but less lucrative job. Absent the possibility to limit downside risk, workers will want to maximize upside risk, and thus work for the most informative firm on the market, recalling that the expected wage is linearly increasing in $\lambda$.

Taken together, the results of the last two propositions enable us to address the more general case in which talent-sensitivity $\lambda$ of the firms potentially active in the economy is distributed on a continuum that includes $\hat{\lambda}$. In this more general case, the model predicts that relatively risk-averse employees (those with $\rho \geq \hat{\rho}$) will only accept offers from firms featuring the lowest level of talent-sensitivity; conversely, employees with risk-aversion $\rho < \hat{\rho}$ will accept labor contracts only from the most talent-sensitive firms in the economy.

### 5.3 Inefficiency of Labor Market Competition

Section 5.2 shows that labor market competition at $t = 2$ prevents firms from insuring their employees against layoff risk, and thus induces the more risk-averse workers to insure themselves by choosing less talent-sensitive jobs. In contrast, in the non-competitive labor market analyzed in Section 5.1, where workers cannot resign from their employer at $t = 2$, firms offer severance payments that implement efficient risk-sharing, so that all workers accept to be employed in the most talent-sensitive firms.

Hence, labor market competition destroys risk-sharing opportunities while leading to a less efficient allocation of the workforce. The model predicts that, if workers are sufficiently risk-averse (namely, at least some of them have risk aversion larger than $\hat{\rho}$), labor market competition will lead to fewer workers choosing to be employed in talent-sensitive firms. In the limit, no such firm may be viable. Thus, the economy will feature less talent discovery, less layoff risk (hence, a lower unemployment rate),
as well as lower average productivity (and consequently, wages) than if firms were able to provide severance payments to laid off employees.

If instead all workers have low risk aversion \( (\rho < \hat{\rho}) \), they will choose jobs in highly talent-sensitive firms (those with \( \lambda > \hat{\lambda} \)) even in a competitive labor market, but such efficiency in production will be attained at the cost of less efficient risk-sharing. In principle, in this economy layoff risk is insurable, being idiosyncratic; however, firms cannot insure employees against it, being unable to cross-subsidize laid-off workers via severance payments funded by lower wage to retained, high-quality workers.

This suggests that, under labor market competition, public intervention can raise efficiency by providing the risk sharing that firms cannot provide. In the next two sections we will consider two alternative government interventions in this economy, and explore to what extent they can increase efficiency.

6 Public Unemployment Insurance

The government can intervene by introducing a public UI scheme to protect laid-off employees of talent-intensive industries. We assume the social security system to run UI on a balanced budget: the unemployment benefits \( b \) paid to laid-off workers are funded by taxing the income of employees in the same firms at rate \( \tau \in [0, 1] \). Moreover, the insurance system is assumed to have no deadweight costs: the taxes levied to fund it require no collection costs and impose no distortion of labor supply decisions.\(^7\) We discuss below what are the implications of relaxing the latter assumption.

The introduction of the UI system affects both firms’ and workers’ optimal strategies:

**Lemma 2** With a public UI system, the employees of firms with \( \lambda_R \geq \lambda^* = \frac{\bar{y}-(1-p)c-(w_0+b^*)}{pc} < \hat{\lambda} \) pay payroll taxes at the rate \( \tau^* = 1-p \) and receive unemployment benefits \( b^* = py^R_{2H} \), and therefore are fully insured against layoff risk.

Intuitively, the UI system has two effects. First, the availability of the unemployment benefit raises the outside option of workers: when contracting with firms, their

\(^7\)Thus, it is irrelevant whether the taxes that fund the system are lump-sum or payroll-based.
outside option is \( w_0 + b \) rather than the reservation wage \( w_0 \). As this raises retention costs, firms become more demanding in their layoff policy than they would be in the absence of UI: not only firms with talent sensitivity \( \lambda \geq \hat{\lambda} \), but also those with \( \lambda \in [\lambda^*, \hat{\lambda}] \) will lay off workers upon bad performance at \( t = 1 \). Second, UI eliminates all layoff risk by insuring workers against it.

Hence, UI implies that workers in risky firms have the same income whether employed or not. This affects the choice between risky and safe jobs:

**Proposition 4** If offered labor contracts by firms with different talent sensitivity, in the presence of public UI workers accept the offer from the most talent-sensitive firm, regardless of their risk aversion.

A key difference between firms’ provision of severance pay and a public UI scheme is that the latter are universal in their coverage. As seen in Section 5.1, if a firm were to commit to provide severance pay in a competitive labor market regime, it would lose its best workers to its competitors, and be left with a pool of overpaid employees. Hence no firm can commit to insure laid-off workers via severance pay. In contrast, a public UI scheme effectively forces all firms to fund unemployment benefits via their payroll tax. Hence, when the government provides workers with insurance against layoff risk, labor market competition is no longer an issue.

Public UI will induce all workers – irrespective of their risk aversion – to accept jobs from the most talent-sensitive firms at \( t = 0 \), since these will be able to offer the highest possible expected salaries. This implies that the economy achieves efficient production, on top of efficient risk sharing.

This is clearly an extreme prediction, resulting from the assumption that the government designs UI to provide complete coverage against layoff risk: it is straightforward to show that, if coverage were less than complete, the most risk-averse workers may still prefer to take a job in a talent-insensitive firm. Hence, the empirical prediction is that the fraction of employees working in talent-sensitive firms is positively correlated with the coverage of layoff risk offered by public UI.

In fact, incomplete coverage of layoff risk may be an optimal feature of UI if there are deadweight costs in the redistribution from employed to unemployed workers, in the form of either costly tax enforcement or labor supply distortions. Cross-country differences in such costs may indeed explain why in practice public UI systems feature
different coverage in terms of benefits relative to pre-layoff wages – i.e. different “replacement rates”.

In the model as laid out so far, workers are the only agents who respond to the introduction of public UI and can generate a reallocation of employment by accepting job offers from riskier firms. However, it is also possible to envisage a variant of the model where firms themselves may increase the talent-sensitivity of their production technology at a cost, by investing in costly R&D. In this case, the introduction of UI may trigger an increase in such investment. To see this, consider an economy where initially all firms have talent-sensitivity $\lambda_S < \hat{\lambda}$ and all workers have risk-aversion $\rho \geq \hat{\rho}$. In this case, even if firms could increase their talent-sensitivity to $\lambda_R \geq \hat{\lambda}$ by investing in R&D, none of them would have an incentive to do so, because it would no longer be able to hire workers. However, if a UI system offering perfect insurance is set up in this economy, firms would have an incentive to invest in R&D and transform their technology in a talent-sensitive one, provided the cost of R&D investment is not prohibitively high: in fact, firms who were not to invest in R&D could no longer attract workers and thus would shut down.

7 Employment Protection Laws

An alternative public intervention that is often thought to reduce employment risk is to restrict the freedom of firms in their firing decisions, via “employment protection legislation” (EPL). Such restrictions can take several forms: (i) prohibition of layoffs, (ii) requirement that terminations be motivated by a “just cause”, or (iii) requirement of a pre-set payment to laid-off workers. When EPL takes the last of these three forms, it effectively amounts to a universal mandatory severance pay, and therefore it plays a function that is akin to that of a public insurance system. We will therefore focus on EPL restricting layoffs – in fact, for the sake of clarity, we shall focus on the case where it forbids them altogether.

Our main result is that, in a competitive labor market, the effects of such a restriction to layoffs are quite different from those of UI:

Lemma 3 If EPL forbids layoffs, firms with $\lambda_R \geq \hat{\lambda}$ are not viable.

If firms are forced to keep workers upon bad performance at $t = 1$, the more
talent-sensitive ones will refrain from hiring them at $t = 0$, expecting not to break even otherwise. This result hinges on two key assumptions of the model: labor market competition and workers’ limited liability. Competition implies that workers appropriate all the surplus that they generate, when this is positive. Yet, limited liability shields them from the losses that they generate at $t = 2$ when the firm forcibly retains them despite a poor performance at $t = 1$. As a result, talent-sensitive firms will not break even in expectation: only firms with $\lambda_S < \hat{\lambda}$ will be active in the market.

This result plays an important role in the effects of EPL, both when benchmarked against no government intervention and when compared with public UI:

**Proposition 5** (i) When labor markets are competitive and EPL forbids layoffs, production is (weakly) less efficient than without government intervention.

(ii) Compared with public UI, EPL implies less efficient production, and (weakly) lower insurance against layoff risk.

This proposition points out that government intervention via EPL weakly decreases welfare, because it eliminates the more talent-sensitive firms, whose jobs may appeal to the least risk-averse workers. Hence, the introduction of EPL decreases expected revenue and wages below the no-intervention level: the elimination of layoff risk occurs at the cost of lower production efficiency. This result is consistent with the finding by Bartelsman et al (2016) that, in countries with restrictive EPL, risky industries contributing to aggregate productivity growth are small or exhibit relatively low productivity.

The comparison with public UI contained in the second statement of the proposition is even starker, because with UI all workers prefer jobs in firms with high talent sensitivity and productivity, while with EPL all of them will have to take jobs in firms with low talent sensitivity and productivity. Moreover, this loss in productive efficiency does not imply better insurance of workers, since UI eliminates all layoff risk, while with EPL workers remain exposed to wage risk in firms with low talent sensitivity.
8 Education

So far in our model workers choose only which job to accept. In reality, career choices are preceded by educational ones. Insofar as education impacts on-the-job performance, it contributes to determine expected wages as well as layoff risk. In this section we show that the introduction of UI encourages investment in education by lowering the riskiness of their human capital, and via this channel it further enhances workers’ expected productivity (compared to the baseline model where workers make no educational choices).

In the context of this model, it is natural to posit that education reduces the importance of noise (“errors”) in production and thus increases the dependence of payoffs on the intrinsic quality of workers – that is, it raises the parameter $\lambda$ for any given technology of the firm. In other words, the talent-sensitivity parameter will be dictated not only by technology, but also by workers’ educational level.

To capture this idea with the smallest possible change to our setting, suppose that the economy is populated by identical (safe) firms with $\lambda < \hat{\lambda}$, and suppose that the process of education allows workers to increase the informativeness of the revenue they generate at $t = 1$ to some $\lambda' > \hat{\lambda}$. For simplicity, initially assume education to be costless (below we relax this assumption). Then, by Proposition 3 only workers with sufficiently low risk aversion ($\rho < \hat{\rho}$) will become educated: those with high risk aversion ($\rho \geq \hat{\rho}$) would be damaged if they were to increase the informativeness of the revenue they generate. By getting educated, workers with low risk aversion will increase both the mean and the variance of their compensation, exposing themselves to layoff risk. Other workers will avoid such risk by not getting educated.

Now, assume that UI were to be introduced in this economy. Based on Proposition 4, being insured against layoff risk, also workers with high risk aversion ($\rho \geq \hat{\rho}$) will become educated, and upon doing so they will increase their expected compensation, bringing it in line with that of less risk-averse workers. Hence, the introduction of UI enhances the investment in human capital, and also via this channel increases the expected productivity of firms and the expected income of workers, at the same time as it increases the unemployment rate.

In the setting considered above, workers’ educational choice is assumed to be binary, so that UI increases the number of people who acquire education, i.e. the extensive margin, but not the amount of education that they acquire, i.e. the in-
tensive margin. A simple way of capturing also the effect of UI on the intensive margin is to consider a variant where, beside costless basic education, workers can invest more in their human capital at a cost $\psi$. This investment further increases the informativeness of their performance to $\lambda'' > \lambda'$.

As a benchmark, consider the educational choice made by a risk-neutral worker: she will get costly education if and only if the cost $\psi$ does not exceed the threshold value $\overline{\psi} \equiv p(1 - p)(\lambda'' - \lambda')c$, which measures the implied increase in the expected wage (i.e., the expected incremental return to education).

Instead, a risk-averse worker may wish not to invest in costly education even if $\psi \leq \overline{\psi}$, because – unlike a risk-neutral worker – she must consider the incremental layoff risk associated with higher education, not only its expected net benefit. Since however in the presence of UI also these workers effectively behave as if they were risk-neutral, its introduction will induce all of them to invest in costly education. More precisely:

**Proposition 6** If $\psi \leq \overline{\psi}$, in the absence of UI workers with risk-aversion $\rho < \hat{\rho}$ acquire costly education if and only if $\rho \leq \rho_E \equiv \overline{\psi} - \overline{\psi} (1 - p)\overline{\psi} > 0$. In the presence of UI, all workers invest in education regardless of their risk aversion.

Hence, if $\psi \leq \overline{\psi}$, educational choices in the absence of UI differ across three groups of workers, depending on their risk aversion:

- those with $\rho \geq \hat{\rho}$ do not acquire any education;
- those with $\rho \in [\rho_E, \hat{\rho})$ acquire only costless education;
- those with $\rho \in [0, \rho_E)$ acquire both costless and costly education.

In the presence of UI, instead, all three types of workers will acquire both types of education. Hence, UI has both an effect on the extensive margin, by inducing workers with high risk-aversion ($\rho \geq \hat{\rho}$) to become educated (and indeed to invest also in costly education), but also on the intensive margin, by encouraging workers with low risk-aversion ($\rho \in [\rho_E, \hat{\rho})$) to acquire also costly education, which they would not have done in the absence of UI.

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8 The value of $\overline{\psi}$ is derived from the incentive constraint for a risk-neutral worker to invest in costly education:

$$p[\overline{\gamma} - (1 - \theta H(\lambda'))c] - \psi \geq p[\overline{\gamma} - (1 - \theta H(\lambda'))c]$$

This inequality implies that a risk-neutral worker will invest in further education for any $\psi \leq p(1 - p)(\lambda'' - \lambda')c \equiv \overline{\psi}$. 
9 Empirical Predictions and Some Evidence

Despite its simplicity, our model provides a rich set of empirical predictions:

1. Competition for talent in the labor market weakens the protection against layoff risk offered by firms, for instance in the form of severance pay.

2. In a competitive labor market and absent public UI, more talent-sensitive industries feature greater layoff risk, higher average wages and steeper career profiles.

3. The fraction of employees in talent-sensitive industries and firm investment in R&D are positively correlated with the generosity of public UI, other things being equal.

4. The introduction of UI increases the expected compensation of workers, as well as the unemployment rate.

5. In talent-sensitive industries, the returns to education are higher but riskier than in other industries, and the level of education of employees is increasing in the generosity of public UI.

To the best of our knowledge, most of these predictions have not been tested by empirical work. Here, we start to explore whether the evidence is consistent with the third prediction, namely that the generosity of UI systems is positively correlated with the fraction of employees in talent-sensitive industries, as well as with firm investment in R&D. We focus on these predictions in light of their great policy relevance: in spite of the vast literature on UI, there is no research on the correlation between its design and industrial structure, in terms of both employment allocation across industries and firms’ technological choices.

In probing the evidence, we do not aim to pin down the direction of causality between UI generosity – as measured by the level and duration of UI benefits – and industrial structure. In principle, causality might go in either direction. On one hand, a more generous UI should make employees more inclined to work in talent-sensitive industries, and allow these to attract a larger fraction of the total workforce. On the other hand, if most employees work in talent-sensitive industries – for instance, because they have low risk aversion or are highly educated – there will be a strong
constituency in support of a generous UI system, while the opposite will happen if most workers are employed in industries with low talent-sensitivity. Both of these lines of argument are consistent with our theoretical framework, and accordingly we investigate correlations rather than causal relationships.

Mapping the prediction of interest to the data requires finding an empirical counterpart for the talent-sensitivity of industries. We gauge it by the knowledge intensity of the sector’s technology: we consider professional, scientific and technological services, as well as the production and dissemination of knowledge as more talent-sensitive sectors than manufacturing, and accordingly we expect them to employ a larger fraction of workers in jurisdictions with more generous UI systems.

We analyze the relationship between sectoral employment and UI generosity using alternatively two panel data sets: yearly country-level data for 17 developed countries in 1995-2013, and yearly state-level data for the U.S. in 1990-2013. We measure the ratio of employees in the sector of interest to total employment (excluding self-employed workers) drawing country-level data from the OECD database, and U.S. state-level data from the Bureau of Labor Statistics (BLS).

In both data sets, the measure of the generosity of public UI is the income “replacement rate”, i.e. the ratio of unemployment benefits to the average salary, and varies both across countries (or states) and over time. The country-level replacement rate is the ratio of the UI benefits received by a worker in the first two years of unemployment to the worker’s last gross wage in the corresponding country, so as to capture both the level and the duration of unemployment benefits. These data are based on Aleksynska and Schindler (2011), as extended by Ellul, Pagano and Schivardi (2016) from 2005 to 2013. The replacement rate averages 0.35 for the whole sample, but features significant differences across countries: in France, the Netherlands, Norway, Portugal and Spain, its average exceeds 0.40; in contrast, in the Czech Republic, Greece, Israel and the U.K. it is below 0.20. Moreover, in some countries UI replacement rates vary significantly over time – within the same country: this is the case of Denmark, Italy, Norway and Portugal. In other countries they are quite stable: for example, in the Czech Republic it did not change throughout the whole period, and in Austria, Belgium, Spain and the UK it changed little over time.

The estimates of panel regressions based on country-level data are shown in Table 1, separately for two relatively talent-intensive sectors in columns 1 and 2 (professional, scientific and technological services, and information and communication,
respectively), and for manufacturing in column 3. All regressions include country fixed effects to control for unobserved heterogeneity due to time-invariant differences in countries' industrial specialization, and calendar year effects to absorb common trends in the relative employment shares of these three sectors, arising for instance from global changes in technology or in product variety. Standard errors are reported in parenthesis below the respective coefficients.

Table 1. Country-Level Sectoral Employment Regressions
(OECD Yearly Data, 1995-2013)

<table>
<thead>
<tr>
<th>Dep. var.: % Employees</th>
<th>Professional, Scientific &amp; Technological Services (1)</th>
<th>Information &amp; Communication (2)</th>
<th>Manufacturing (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replacement Rate (UI)</td>
<td>0.013**</td>
<td>0.005**</td>
<td>−0.044***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>State FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.975</td>
<td>0.923</td>
<td>0.972</td>
</tr>
<tr>
<td>N. obs.</td>
<td>314</td>
<td>314</td>
<td>295</td>
</tr>
</tbody>
</table>

The results show that the fraction of employees in the two more talent-intensive sectors is positively and significantly correlated with the replacement rate, while the corresponding fraction in manufacturing is negatively and significantly correlated with it. To have an idea of the economic significance of the estimates, consider that increasing the replacement rate from its average level in the Czech Republic (0.06, the lowest in the sample) to that of Portugal (0.65, the highest in the sample) is associated with an increase of 0.8 percentage points in the fraction of employees in professional, scientific and technological services, and a decrease in the fraction of manufacturing employment of 2.6 percentage points, to be respectively compared with overall sample means of 12 and 18 percent for these two sectors.
We apply this approach also to U.S. data, exploiting variation in state-level replacement rates. These are defined as the product of the maximal UI benefits and the respective maximal duration, measured in 2002 constant dollars using the Consumer Price Index (as done by Agrawal and Matsa, 2013), standardized by the average wage in the relevant sector, state and year. The data for UI benefits and duration are drawn from the “Significant Provision of State UI Laws” of the U.S. Department of Labor, and the data for average wage by sector, state and year are based on BLS data. The replacement rate averages 0.22 for the whole sample, but differs substantially across states: its mean ranges from 0.42 in Massachusetts, 0.32 in Rhode Island, and 0.30 in Pennsylvania, to 0.14 in Alabama, Arizona and District of Columbia. Moreover, in some states – such as Minnesota and Pennsylvania – it varies appreciably over time.

Also for U.S. state-level data, we estimate panel regressions – shown in Table 2 – for two relatively talent-intensive sectors and for the manufacturing sector. The former sectors differ from those in Table 1, because BLS statistics define sectors differently from the OECD: for the U.S. we consider health and education services (columns 1 and 2) and professional and business services (columns 3 and 4) as more talent-intensive than manufacturing (column 5 and 6).

<table>
<thead>
<tr>
<th>Table 2. State-Level Sectoral Employment Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U.S. Yearly Data, 1990-2013)</td>
</tr>
<tr>
<td>Dep. var.: % Employees</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Replacement Rate (UI)</td>
</tr>
<tr>
<td></td>
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<tr>
<td>State FE</td>
</tr>
<tr>
<td>Year FE</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
</tr>
<tr>
<td>N. obs.</td>
</tr>
</tbody>
</table>
The regressions in the odd columns include only country fixed effects, while those in even columns include also calendar year effects. The results are broadly in line with those of Table 1 based on country-level data: the coefficient of the replacement rate is estimated to be positive for the two more talent-sensitive sectors, and negative for manufacturing. All estimates are significantly different from zero, except for the coefficient in column (4), which refers to professional and business services in the specification that includes year effects.

The regressions shown in Tables 1 and 2 are based on aggregate data. An additional piece of evidence can be gleaned from firm-level data on R&D investment by U.S. companies: recall that, according to our model, higher UI replacement rates may induce firms to become more talent-sensitive by investing in R&D. Evidence on this point is provided by Ellul, Wang and Zhang (2016), who find that firms in states with more generous UI insurance tend to feature greater risk-taking behavior along various dimensions, including R&D investment. In Table 12 of their study, they regress the ratio of R&D investment to total assets on the replacement rate in the state where the company is headquartered, and on lagged company level controls (total assets, leverage, ROA, market-to-book ratio, asset tangibility and sales growth), and find that the coefficient of the replacement rate is positive and significant. While their R&D evidence is based on a subsample of firms for which they observe managerial compensation data, the same result is obtained using a comprehensive sample of 139,210 firm-year observations between 1992 and 2013, drawn from Compustat.\footnote{We are very grateful to Kuo Zhang for kindly agreeing to re-estimate the R&D regressions on this larger sample.}

On the whole, the evidence presented in this section is broadly consistent with the prediction that the generosity of UI is positively correlated with the development of talent-sensitive industries. It is a task for future research to investigate whether this prediction is also upheld in the context of quasi-natural experiments associated with reforms of the social security system.

10 Conclusions

Talent discovery is crucial in human capital-intensive industries (such as high-tech, professional services and health), as it allows for efficient matching of workers to tasks,
translating into higher production and wages. However, it also generates risks for workers who are uncertain about their own skills, as after some work experience they may turn out to be either more or less talented than expected, and if insufficiently talented they may be laid off.

When the labor market is non-competitive, firms can insure their employees against the resulting human capital risk, compensating them with severance pay in case of layoff. However, labor market competition prevents firms from insuring workers against layoff risk, as it comes at the expense of more talented workers: the cross-subsidy given to low-performing employees would induce high-performing ones to switch to a competitor, leaving their initial employer only with overpaid and untalented employees. Absent any insurance, risk-averse workers will select themselves into less talent-sensitive occupations, which discover less precise information about their skills and thus generate less or no layoff risk.

The core policy implication of our model is that in competitive labor markets, public UI induces workers to seek employment in more talent-sensitive industries, irrespective of their risk aversion, as they prefer to test their own skills in jobs that reveal sharper information about their talent. This allows for more efficient job-talent matches, hence higher average wages, than in the absence of such intervention. The resulting increase in layoff risk (and consequently in the unemployment rate) generates no welfare losses because of the safety net provided by UI. The higher layoff risk reflects the more frequent firings of workers upon bad performance: the availability of unemployment benefits increases workers’ reservation wage, so that firms are less likely to break even, and more demanding in their criterion to retain employees.

We also show that UI dominates another possible policy intervention, namely employment protection legislation (EPL) that constrains firms’ ability to lay off workers. In fact, if the labor market is competitive and workers are protected by limited liability, EPL will prevent highly talent-sensitive firms to break even, and therefore will distort employment towards firms with less talent-sensitive technologies and therefore with lower expected productivity. Hence, in order to foster the discovery and efficient allocation of talent, policy should prefer insurance of employees against layoffs over norms that penalize layoffs. Another interesting policy implication of the analysis is that UI encourages workers to acquire education, irrespective of their degree of risk-aversion, which in turn further enhances talent discovery.
Admittedly, these strong results would be to some extent mitigated or modified in a richer model that were to allow for the potential efficiency costs of UI. For instance, if workers’ labor supply were modelled as resulting from a leisure-consumption trade-off, the payroll taxes required to fund unemployment benefits would distort labor supply.

Moreover, in the model we rule out the workers’ ability to insure themselves via financial markets, for instance by borrowing after being laid off: we do so to focus on firms and on the social security system as the only possible sources of insurance against workers’ human capital risk. This assumption is not unrealistic, as workers are often credit constrained (Jacobson, LaLonde and Sullivan, 1993). However, clearly self-insurance of workers via precautionary saving would reduce the benefits from the presence of UI.

Finally, our analysis abstracts from general equilibrium effects of the allocation of workers across industries, such as its effect on the relative prices of goods produced by industries featuring different talent sensitivity – an assumption that is appropriate in a small open economy where the relative prices of tradeable are dictated by the international market. For instance, the model predicts that upon the introduction of UI all workers will switch to the most talent-sensitive industries. If instead the relative price of these industries’ output were determined endogenously in the economy, it would decline in response to the increase in their output of these industries, and this would limit the extent of the reallocation process. However, the result that more labor would be employed in talent-sensitive industries would still hold true qualitatively.
Appendix: Proofs

Proof of Lemma 1

Proof. Since $\theta_H \geq p$, by condition (2) we have $1 - \theta_H < \frac{\bar{y} - w_0}{c}$. Therefore, if at $t = 1$ the project yields a positive surplus, the employee is retained and the project continued. If instead the project delivers a loss in $t = 1$, the belief that the worker is good is updated to $\theta_L \leq p$. We need to distinguish two possible cases for the conditional expected revenue:

1. $1 - \theta_L < \frac{\bar{y} - w_0}{c}$: the worker is retained for any realization of $y_1$, as she is expected to produce a positive surplus;

2. $1 - \theta_L \geq \frac{\bar{y} - w_0}{c}$: the worker is laid off, as she is expected to generate a loss for any wage higher than (or equal to) $w_0$.

Whether a firm ends up in case 1 or 2 depends on the talent-sensitivity of its production technology $\lambda$. By continuity of $\theta_L$, $\exists \hat{\lambda} : \bar{y} - (1 - \theta_L)c = w_0$ given by

$$\hat{\lambda} \equiv \frac{\bar{y} - (1 - p)c - w_0}{pc}.$$ (15)

If the project’s informativeness is $\hat{\lambda}$, the firm is indifferent between laying off and keeping a worker who failed in the previous period as in expectation it will always break even. If $\lambda < \hat{\lambda}$, the firm’s optimally keeps all its employees (case 1). If $\lambda \geq \hat{\lambda}$, instead, the firm lays off workers who generate a loss at $t = 1$, and retain those who generate a positive surplus (case 2), as the former would not make it break even. □

Proof of Proposition 1

Proof. We prove this proposition in two steps: we show that

1. if all firms offer contracts with severance pay, workers choose to work for firms with $\lambda \geq \hat{\lambda}$.

2. given 1), workers choose to work for the most talent-sensitive firm on the market.
1. Any firm with \( \lambda < \hat{\lambda} \) pays all workers a constant wage across the two periods, irrespective of their performance, and therefore provides the same unconditional expected utility for both periods:

\[
E_0 \left[ U(\lambda < \hat{\lambda}) \right] = 2u[\bar{y} - (1 - p)c].
\]

Instead, firms with \( \lambda \geq \hat{\lambda} \) offer different wages across the two periods and fire workers who had a bad performance at \( t = 1 \). Hence they provide the following unconditional expected utility:

\[
E_0 \left[ U(\lambda \geq \hat{\lambda}) \right] = u[\bar{y} - (1 - p)c] + u[p(\bar{y} - (1 - \theta_H)c)].
\]

Since \( u(w) \) is an increasing function, and \( \bar{y} - (1 - p)c < p(\bar{y} - (1 - \theta_H)c) \) for any \( \lambda \geq \hat{\lambda} \), any worker prefers to work for firms with \( \lambda \geq \hat{\lambda} \).

2. To show that workers prefer the firm with the highest \( \lambda \) among those with \( \lambda \geq \hat{\lambda} \), notice that \( w_1 \) is independent of \( \lambda \), while \( w_2 = p(\bar{y} - (1 - \theta_H)c) \) is increasing in \( \lambda \):

\[
\frac{\partial w_2}{\partial \lambda} = \frac{\partial w_2}{\partial \theta_H} \cdot \frac{\partial \theta_H}{\partial \lambda} = p(1 - p)c > 0.
\]

Hence, they will pick the most talent-sensitive firm in the market.

\[\blacksquare\]

**Proof of Proposition 2**

**Proof.** Let \( \Delta U_S \) denote the expected benefit from choosing the safe job rather than the risky one, so that

\[
\Delta U_S = pu[\bar{y} - (1 - \theta_{2H}^R)c] + (1 - p)u[\bar{y} - (1 - \theta_{2L}^S)c] - \left\{ pu[\bar{y} - (1 - \theta_{2H}^R)c] + (1 - p)u(w_0) \right\}
\]

\[
= pu[\bar{y} - (1 - \theta_{2H}^R)c] + (1 - p)u[\bar{y} - (1 - \theta_{2L}^S)c] - pu[\bar{y} - (1 - \theta_{2H}^R)c], \quad (16)
\]

where in the second step we have used the assumption \( u(w_0) = 0 \). To simplify notation, let us define:

\[
y_{2H}^S \equiv \bar{y} - (1 - \theta_{2H}^S)c, \quad y_{2L}^S \equiv \bar{y} - (1 - \theta_{2L}^S)c, \quad y_{2H}^R \equiv \bar{y} - (1 - \theta_{2H}^R)c, \quad (17)
\]
which allows us to rewrite (16) as follows:

$$\Delta U_S = (1 - p)u(y_{2L}^S) - p[u(y_{2H}^R) - u(y_{2H}^S)].$$  \hspace{1cm} (18)$$

Consider any two expected revenues \(x_1\) and \(x_2\) such that \(x_1 \in (w_0, y_{2H}^S)\) and \(x_2 \in (y_{2H}^S, y_{2H}^R)\). By the mean value theorem, we can write (18) as

$$\Delta U_S = (1 - p)u'(x_1)(y_{2L}^S - w_0) - pu'(x_2)(y_{2H}^R - y_{2H}^S),$$  \hspace{1cm} (19)$$

where by concavity of the utility function \(u'(x_1) > u'(x_2)\) since \(y_{2L}^S < y_{2H}^S < y_{2H}^R\). Using (17) in equation (19) yields

$$\Delta U_S = (1 - p)[u'(x_1)y_{2L}^S - u'(x_2)(\lambda_R - \lambda_S)pc].$$  \hspace{1cm} (20)$$

By adding and subtracting \((1 - p)u'(x_2)(y_{2L}^S - w_0)\) on the right-hand side of (20), dividing and multiplying it by \(u'(x_2)\) and simplifying, we obtain:

$$\Delta U_S = (1 - p)u'(x_2) \left[ \rho(y_{2L}^S - w_0) + \bar{y} - c + (1 - \lambda_R)pc - w_0 \right],$$  \hspace{1cm} (21)$$

where \(\rho \equiv \frac{u'(x_1) - u'(x_2)}{u'(x_2)}\) is a measure of worker’s risk aversion: for fixed values of \(x_1\) and \(x_2\), the greater the curvature of the utility function, the larger the numerator and the smaller the denominator. By the continuity of \(\Delta U_S\) in \(\rho\), there exists a critical risk aversion level:

$$\hat{\rho} \equiv \frac{[1 - (1 - \lambda_R)p]c - \bar{y} + w_0}{y_{2L}^S - w_0} \geq 0$$  \hspace{1cm} (22)$$

such that, for any \(\rho \geq \hat{\rho}\), workers will prefer the safe job, and for any \(\rho < \hat{\rho}\) they will prefer the risky one. Clearly, the threshold risk aversion \(\hat{\rho}\) is increasing in \(\lambda_R\): indeed, it equals 0 for \(\lambda_R = \hat{\lambda}\) and \((c - \bar{y})/y_{2L}^S > 0\) for \(\lambda_R = 1\). \(\blacksquare\)

**Proof of Proposition 3**

**Proof.** (i) In firms with \(\lambda < \hat{\lambda}\), the unconditional expected wage at \(t = 2\) equals the worker’s expected productivity:

$$E_0(y_2) = p[\bar{y} - (1 - \theta_H)c] + (1 - p)[\bar{y} - (1 - \theta_L)c].$$  \hspace{1cm} (23)$$
Upon substituting for $\theta_H$ and $\theta_L$, this expression becomes

$$
\mathbb{E}_0(y_2) = p [\bar{y} - (1 - p)(1 - \lambda)c] + (1 - p) \{\bar{y} - [1 - (1 - \lambda)p]c\}
= \bar{y} - (1 - p)c = \mathbb{E}_0(y_1) \forall \lambda < \tilde{\lambda},
$$

which is independent of $\lambda$. Instead, the unconditional variance of the wage is increasing in $\lambda$:

$$
\sigma^2 = p \{y_{2H} - [\bar{y} - (1 - p)c]\}^2 + (1 - p) \{y_{2L} - [\bar{y} - (1 - p)c]\}^2 = p(1 - p)\lambda^2 c^2.
$$

hence, the wage paid by firms with informativeness $\lambda < \tilde{\lambda}$ is a mean-preserving spread of the distribution of the wage that would be paid by a firm with $\lambda = 0$, which does not update its beliefs. Thus, a risk-averse worker will always choose the least informative project available.

(ii) In firms with $\lambda \geq \tilde{\lambda}$, a worker that produces $y_1 = \bar{y} - c$ at $t = 2$ is laid off and gets zero utility. If instead $y_1 = \bar{y}$ at $t = 2$, the worker’s wage is increasing in $\lambda$ (as shown in the proof of Proposition 1). Thus, all workers prefer to work for the firm featuring the highest $\lambda$. ■

**Proof of Lemma 2**

**Proof.** First, we derive the new condition that defines the firms that lay off underperforming workers at $t = 1$ under UI. As in the proof of Lemma 1, we distinguish two possible cases for the conditional expected revenue:

1) $1 - \theta_L < \frac{y - w_0 - b}{c}$: the worker is retained for any realization of $y_1$, as she is expected to produce a positive surplus;

2) $1 - \theta_L \geq \frac{y - w_0 - b}{c}$: the worker is laid off, as she is expected to generate a loss for any wage higher than (or equal to) $w_0 + b$.

Whether a firm ends up in case 1 or 2 depends on the talent-sensitivity of its production technology $\lambda$. By continuity of $\theta_L$, $\exists \lambda^* : \bar{y} - (1 - \theta_L)c = w_0 + b$ given by

$$
\lambda^* = \frac{\bar{y} - (1 - p)c - (w_0 + b)}{pc}. \quad (24)
$$
The government chooses the optimal tax rate \( \tau \) and transfer to unemployed workers \( b \) in order to maximize the social welfare function subject to the binding budget constraint and the non-negativity constraint for the tax rate \( \tau \):

\[
\max_{\{\tau, b\}} pu[y_2^R (1 - \tau)] + (1 - p)u(b),
\]

subject to

\[
py_2^R \tau = (1 - p)b,
\]

\[
\tau \in [0, 1],
\]

which is equivalent to:

\[
\max_{\{\tau\}} pu[y_2^R (1 - \tau)] + (1 - p)u \left( \frac{py_2^R \tau}{1 - p} \right)
\]

Working out the first-order condition for an interior solution to this problem delivers the optimal level of \( \tau \):

\[
\tau^* = 1 - p.
\] (25)

Substituting \( \tau^* \) in the budget constraint yields the optimal UI benefit:

\[
b^* = py_2^R,
\] (26)

so that employees in firms with \( \lambda_R \geq \lambda^* \) obtain full insurance. Replacing the unemployment benefit \( b \) with its optimal value \( b^* \) in (26) yields the value of \( \lambda^* \). Since \( b^* > 0 \), it is immediate that \( \lambda^* < \hat{\lambda} \). \( \blacksquare \)

**Proof of Proposition 4**

**Proof.** In the presence of public UI, a worker employed by a firm featuring \( \lambda_R \geq \hat{\lambda} \) earns utility

\[
u [\bar{y} - (1 - p)c] + u(py_2^R)
\] (27)

whereas a worker employed by a firm exhibiting \( \lambda_S \in [0, \hat{\lambda}) \) has unconditional expected utility

\[
u [\bar{y} - (1 - p)c] + pu(y_2^S) + (1 - p)u(y_2^L).
\] (28)
We know that
\[ y_{2H}^R > y_{2H}^S \geq y_{2L}^S \Rightarrow u(p y_{2H}^R) > p u(y_{2H}^S) + (1 - p) u(y_{2L}^S) \]
and this holds for every concave utility function and for every \( \lambda_R \geq \lambda > \lambda_S \).

Hence, any worker would choose the more talent-sensitive job over the less informative one. ■

Proof of Lemma 3

**Proof.** If firms cannot fire workers in a competitive labor market, those featuring talent-sensitivity \( \lambda < \hat{\lambda} \) earn zero unconditional expected profit. On the other hand, if workers are not laid off after a bad outcome in \( t = 1 \), the unconditional expected profit for a firm with \( \lambda \geq \hat{\lambda} \) is:
\[
E_0(\pi) = (1 - p)[\bar{y} - (1 - \theta_L)p]c] \leq 0. \tag{29}
\]
Note that firms with \( \lambda \geq \hat{\lambda} \) will not want to keep badly performing employees idle, as this would generate an expected loss equal to their reservation wage:
\[
E_0(\pi) = -w_0 < 0. \tag{30}
\]
Hence, if highly talent-intensive firms do not fire workers after a bad outcome in \( t = 1 \), they would make losses. Anticipating this at \( t = 0 \), in an EPL regime such firms have no incentive to hire workers, and will be inactive. This is an equilibrium, since there are no profitable deviations from a situation in which all such firms are inactive: if any single one of them were to start production and enter the labor market, other firms would have an incentive to poach the employees tested by this firm: any other firm with \( \lambda \geq \hat{\lambda} \) has an incentive to free ride on the others, so that in equilibrium none of them would be active at \( t = 0 \). ■

Proof of Proposition 5

**Proof.** (i) By Proposition 2, in a competitive labor market without government intervention, workers with risk-aversion \( \rho < \hat{\rho} \) choose to work for firms with \( \lambda \geq \hat{\lambda} \).
By Lemma 3, when EPL is in place, these jobs are no longer available, so that the expected revenue and wages in the economy is lower than without EPL. If instead all workers have risk-aversion $\rho \geq \hat{\rho}$, then they will all work for firms with $\lambda < \hat{\lambda}$ that feature no layoff risk, so that the introduction of EPL is inconsequential.

(ii) By Proposition 4, in a competitive labor market with public UI, all workers choose the most talent-sensitive (highest-$\lambda$) job available, which generates the highest feasible production while keeping risk-sharing efficient. By Lemma 3, when EPL is in place, only jobs in firms with $\lambda_S < \hat{\lambda}$ are available, so that the expected revenue and wages in the economy is strictly lower than with public UI. Moreover, with EPL all workers will have to take jobs in firms with $\lambda_S < \hat{\lambda}$, which feature wage risk (unless $\lambda_S = 0$), whereas in the presence of UI they would have chosen jobs in firms with $\lambda_R \geq \hat{\lambda}$, yet bear no layoff risk. Hence, EPL also implies less efficient risk sharing than UI.

10.1 Proof of Proposition 6

Proof. Let $\Delta U_E$ denote the benefit for a risk-averse worker with $\rho < \hat{\rho}$ from investing in costly education. Moreover, let $y_{2H}(\lambda')$ and $y_{2H}(\lambda''$) denote the expected revenue respectively generated by workers with and without costly education, conditional on observing $y_1 = \overline{y}$. Since $\lambda'' > \lambda$, $y_{2H}(\lambda'') > y_{2H}(\lambda')$. We assume that $\psi < \overline{\psi}$, so that at least risk-neutral workers invest in costly education. This condition implies $\psi < y_{2H}(\lambda'') - y_{2H}(\lambda') = (1 - p)(\lambda'' - \lambda')c$. The net utility gain from costly education is

$$
\Delta U_E = pu(y_{2H}(\lambda'') - \psi) + (1 - p)u(w_0 - \psi) - pu(y_{2H}(\lambda')) - (1 - p)u(w_0).
$$

Consider any two expected revenues $x_1$ and $x_2$ such that $x_1 \in (w_0 - \psi, w_0)$ and $x_2 \in (y_{2H}(\lambda'), y_{2H}(\lambda'') - \psi)$. By the mean value theorem, we can write (31) as

$$
\Delta U_E = p[u(y_{2H}(\lambda'') - \psi) - u(y_{2H}(\lambda'))] - (1 - p)[u(w_0) - u(w_0 - \psi)].
$$

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where, by concavity of the utility function, \( u'(x_1) > u'(x_2) \) since \( w_0 < y_2H(\lambda') < y_2H(\lambda'') - \psi \). Hence, \( \Delta U_E \geq 0 \) if and only if

\[
(1 - p)u'(x_1)\psi \leq u'(x_2)p[(1 - p)(\lambda'' - \lambda')c - \psi].
\] (32)

By adding and subtracting \((1 - p)u'(x_2)\psi\) on its left-hand side, (32) can be rewritten as

\[
\rho \leq \rho_E \equiv \frac{p(1 - p)(\lambda'' - \lambda')c - \psi}{(1 - p)\psi} = \frac{\psi - \psi}{(1 - p)\psi},
\]

where \( \rho \equiv \frac{u'(x_1) - u'(x_2)}{u'(x_2)} \) is a measure of worker’s risk aversion, as it denotes the slope of marginal utility: workers whose risk aversion is below the threshold \( \rho_E \) will invest in private education, while those with risk aversion above \( \rho_E \) will not. The threshold risk aversion \( \rho_E \) is decreasing in the cost \( \psi \) of additional education. It is immediate that costly education implies a net benefit \( \Delta U_E > 0 \) in the presence of public UI, since effectively all workers behave as risk-neutral. \( \blacksquare \)
References


