



# Taylor Rule Estimation with the Presence of a ZLB-Period

*How the inclusion of shadow rate affect the precision of Taylor rule  
estimation on the federal funds rate.*

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This thesis was written as a part of the Master of Science in Economics and Business Administration at NHH. Please note that neither the institution nor the examiners are responsible – through the approval of this thesis – for the theories and methods used, or results and conclusions drawn in this work.



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# Abstract

This thesis estimates monetary policy reaction functions for the United States' economy from 1987 until 2015 using a Taylor rule. The period 2009-2015 was characterized by the federal funds rate being bounded below by zero, commonly known as the zero lower bound (ZLB). To measure the effect of monetary policy during this period, a shadow rate was proposed. A shadow rate is an estimated, theoretical interest rate not bounded by the ZLB. We compare estimations of the Taylor rule on out-of-sample data using both the federal funds rate bounded below by zero and the federal funds rate with a shadow rate. We find that the Taylor rule estimations conducted with the shadow rate fits closer to the out-of-sample effective federal funds rate, than standard estimations that includes the data bounded below by zero. Our thesis suggests that shadow rate can be used as a tool to analyze monetary policy in a Taylor rule setting, using data with the presence of a ZLB-period. Furthermore, our results suggests that there is value in including shadow rate in macroeconomic models for monetary policy.

**Keywords** – Taylor rule, shadow rate, zero lower bound, monetary policy

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Background</b>	<b>3</b>
2.1	The Federal Reserve and its mandate . . . . .	3
2.2	Policy tools . . . . .	4
2.3	The Federal Reserve and the Taylor rule . . . . .	5
2.4	Zero lower bound and shadow rate . . . . .	6
<b>3</b>	<b>Taylor rule</b>	<b>8</b>
3.1	The original Taylor rule . . . . .	8
3.2	Forward-looking Taylor rule . . . . .	9
3.2.1	Interest rate target . . . . .	9
3.2.2	Interest rate smoothing . . . . .	10
3.2.3	Our chosen model . . . . .	11
<b>4</b>	<b>Data</b>	<b>12</b>
4.1	Time series . . . . .	12
4.1.1	The federal funds rate . . . . .	12
4.1.2	Shadow rate . . . . .	13
4.1.3	Expected inflation . . . . .	14
4.1.4	The output gap . . . . .	15
4.1.5	Instrumental variables . . . . .	16
4.2	Structural breaks and sample period . . . . .	21
<b>5</b>	<b>Methodology</b>	<b>23</b>
5.1	Econometric approach . . . . .	23
5.2	GMM . . . . .	23
5.3	GMM and Taylor rule . . . . .	24
5.3.1	Out-of-sample performance . . . . .	26
5.3.2	Overidentification . . . . .	26
5.3.3	Stationarity . . . . .	27
5.3.4	Robustness . . . . .	27
5.3.5	Misspecification . . . . .	28
<b>6</b>	<b>Analysis</b>	<b>29</b>
6.1	Forward-looking Taylor rule . . . . .	29
6.1.1	Weak instruments test with OLS . . . . .	34
6.1.2	ADF-test . . . . .	35
6.2	Robustness . . . . .	35
6.2.1	1-step iteration . . . . .	36
6.2.2	Model with unemployment gap . . . . .	36
6.2.3	Models with different horizons . . . . .	39
6.2.3.1	ZLB sub-period . . . . .	39
6.2.3.2	Adjusted data samples . . . . .	39
6.3	Limitations . . . . .	42

<b>7 Conclusion</b>	<b>44</b>
<b>References</b>	<b>46</b>
<b>Appendix</b>	<b>50</b>
A1 Data appendix . . . . .	50
A2 Shadow rate estimation . . . . .	51
A3 Figures . . . . .	53
A4 Tables . . . . .	53

## List of Figures

4.1	Effective federal funds rate and shadow rate . . . . .	13
4.2	Expected inflation . . . . .	15
4.3	The output gap . . . . .	16
4.4	Commodity price inflation . . . . .	18
4.5	House price inflation . . . . .	18
4.6	Long-short spread . . . . .	19
4.7	Change in return on equity for U.S banks . . . . .	19
4.8	Change in total public debt . . . . .	20
4.9	Change in US/UK foreign exchange rate . . . . .	20
4.10	Unemployment gap . . . . .	21
6.1	Extended model . . . . .	33
6.2	1-step iteration model . . . . .	36
6.3	Robustness: unemployment gap . . . . .	38
6.4	Robustness: ZLB sub-period . . . . .	39
6.5	Robustness: adjusted sample 1991Q2 - 2015Q4 . . . . .	41
6.6	Robustness: adjusted sample 2001Q2 - 2015Q4 . . . . .	41
6.7	Robustness: adjusted sample 1988Q2 - 2016Q4 . . . . .	42
A3.1	Baseline model . . . . .	53

## List of Tables

6.1	Baseline model . . . . .	30
6.2	Baseline and extended model . . . . .	31
6.3	RMSE and MAE results . . . . .	33
6.4	Extended and OLS model . . . . .	34
6.5	Extended and UEG . . . . .	37
6.6	RMSE and MAE results: unemployment gap . . . . .	38
6.7	Adjusted samples . . . . .	40
A4.1	Augmented Dickey-Fuller (ADF) test on regressors . . . . .	53
A4.2	Extended and 1-step . . . . .	54
A4.3	Extended model and Pre-ZLB period . . . . .	54
A4.4	RMSE and MAE results: 1991Q2-2015Q4 . . . . .	55
A4.5	RMSE and MAE results: 2001Q4-2015Q4 . . . . .	55
A4.6	RMSE and MAE results: 1988Q2-2016Q4 . . . . .	55
A4.7	Results from GMM estimations. . . . .	56



# 1 Introduction

Monetary policy is critical for a well-balanced and well-functioning economy. The Federal Reserve is the United States' monetary policy authority. Their goal is to maximize sustainable output and employment, while managing stable prices through low and stable inflation. It cannot control inflation, output or employment directly, but affects these indirectly, mainly through adjusting the federal funds rate. From December 2008, until December 2015, the effective federal funds rate was in the 0 to 0.25 percent range. During this time the Federal Reserve could not lower the rate further to provide more economic stimulus, as it had officially entered a zero lower bound. The zero lower bound (*ZLB*), is the condition in which the interest rate cannot become negative. However, some central banks (e.g European Central Bank, Sweden's Riksbank and the Bank of Japan) have at some point adopted negative short-term interest rates. A negative interest rate could lead to market participants hoarding cash, as it becomes an arbitrage opportunity (Altavilla et al., 2019). Hence, in this thesis we will treat zero as a strict lower limit for the effective federal funds rate. In the presence of a *ZLB* regime, the effects of monetary policies will not be visible by the change in the interest rate. To measure such effects, Wu and Xia (2016) proposed an estimated interest rate named the shadow rate. The shadow rate (*SR*) can be defined as an implied, negative rate capable of measuring the effects of monetary policies in the case of a *ZLB* regime. Given the likelihood of entering such a regime later on, the rate can be useful in macroeconomic models that utilize the federal funds rate, for example the Taylor rule.

John Taylor introduced what later became known as the Taylor rule in 1993 to illustrate how the Federal Reserve conducts monetary policy. The Taylor rule suggested that the interest rate should be set to minimize the difference between inflation and the inflation target, and real output and potential output (Taylor, 1993). Following the introduction in 1993, several additions have been made to the literature, a prominent contribution was made by Clarida, Gali and Gertler (1998). They presented an updated policy rule, using a forward-looking model named the baseline model, which will be the initial basis for our analyses.

In this thesis we want to answer the research questions of whether employing the shadow

rate in Taylor rule estimation produces superior out-of-sample results, compared to using data containing measurements bounded below by zero. We will confirm through relevant literature if it is possible to estimate a Taylor rule using measures of the shadow rate as a substitute for the federal funds rate when the ZLB is binding. Our estimated Taylor rules will be based on an in-sample period of 111 observations, extending from 1988Q2 until 2015Q4.

Our findings presented in this thesis confirms that employing a shadow rate in a Taylor rule estimation yields superior out-of-sample performance. We have verified our results, both in terms of RMSE and MAE, by comparing our two estimated models (with and without shadow rate) to out-of-sample federal funds rate data, from 2016Q1 to 2019Q3. The out-of-sample period is the latest available data without the presence of a ZLB. Relevant literature also confirms that the shadow rate can be used in macroeconomic models as a substitute for the federal funds rate (e.g. Bullard et al. (2012) and Krippner (2012, 2013)). Hence, we find it reasonable that it can be used as a substitute in a Taylor rule estimation as well.

Answering our research question may prove valuable for several reasons. Firstly, entering a ZLB regime in the future is increasingly likely, making the use of a shadow rate advantageous in a Taylor rule interpretation of a normalized federal funds rate. Secondly, advocating the performance of the shadow rate implies that further use of the shadow rate, or alternative estimates of a negative interest rate, can improve other macroeconomic models.

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## 2 Background

In this section, we will first introduce the Federal Reserve and their primary objectives, which tools they have at their disposal to reach these objectives, before we consider their use of the Taylor rule. Lastly we will present the necessary context of zero lower bound and shadow rate.

### 2.1 The Federal Reserve and its mandate

Functioning as the central bank of the United States, the Federal Reserve has four main objectives. These are to (1) conduct monetary policy, (2) provide emergency liquidity, (3) to supervise certain banks and financial firms to secure safety and soundness, and lastly, (4) to operate key payment system services for the government and financial firms (Labonte and Makinen, 2019). These responsibilities are delegated by the Congress through the Federal Reserve Act with the intention of ensuring or maintaining “maximum employment, stable prices, and moderate long-term interest rates” (Labonte and Makinen, 2019). The necessary tasks that supports this statutory mandate is divided between three core entities within the Federal Reserve system (Labonte, 2019). First, it is the Board of Governors who are in charge of overseeing the operations of the regional federal banks, as well as directing and guiding the reserve banks when they are conducting financial services to depository institutions or the government (Board of Governors’ Publications Committee, 2016). Second, there is twelve regional Reserve banks responsible for carrying out the main objectives of the Federal Reserve within its region/district. This implies that the regional banks are responsible for supervising and examining both financial and non-financial institutions, lending to depository institutions, providing and maintaining key financial services as the distributor of the nation’s currency, clearing checks, serving as a bank for the U.S. Treasury, and finally, to examine certain financial institutions to ensure compliance with federal consumer protection (Ryan et al., 2009). Lastly, the Federal Open Market Committee (*FOMC*) is the decision making entity of the Federal Reserve system in regards to monetary policy. They meet every six weeks, in addition to unscheduled meetings, to announce their stance regarding monetary policy (Labonte, 2019). In 2012 the FOMC was responsible for adopting the 2% inflation target which is

the Federal Reserve's interpretation of "stable prices" as of their mandate (Lavigne et al., 2012).

## 2.2 Policy tools

To carry out monetary policy the Federal Reserve targets the federal funds rate. The FOMC determines the federal funds target rate, while the lending and borrowing between depository institutions reflects the market equilibrium of the same rate, constituting the *effective* federal funds rate. There is several ways for the Federal Reserve to keep the federal funds rate close to its target. By imposing a reserve requirement held at the federal bank for every depository institution, the Federal Reserve creates a stable demand for reserves. Traditionally, they have used a method of open market operations to adjust the supply of reserves. This method implies selling or buying U.S Treasury notes in the secondhand market. Doing so alters the money supply and credit conditions, which in turn affects the interest rate that depository institutions charge each other for overnight loans (i.e. the effective federal funds rate) in order to meet the reserve requirement. In addition to open market operations, the Federal Reserve can initiate discount window lending. This implies that the depository institutions borrows directly from the Federal Reserve Bank at a rate determined by the Board of Governors, rather than from other depository institutions. The rate of the discount window have been placed above the federal funds rate since 2003, implying that depository institutions only utilized the discount window when the market behaved in a manner that pushed the effective federal funds rate above the discount window rate. (Board of Governors' Publications Committee, 2016)

Recessions can alter the way a central bank conducts its monetary policy. At the start of the financial crisis the Federal Reserve increased its lending through the discount window and initiated several programs stated in the Federal Reserve Act to help stabilize the shortage of liquidity in the market. In addition, the FOMC also cut the federal funds target rate from 5.25% to about 0% between 2007 and the end of 2008. Although this response somewhat stabilized the financial markets to a degree that it could function normally, the impact of the financial crisis was so severe and long-lasting that these actions where not sufficient. Given that the federal funds rate was closing in on its zero lower bound, the Federal Reserve could not utilize the federal funds rate to support the economy.

Instead, the FOMC chose to use two different kinds of monetary policy, namely large-scale asset purchases (i.e. quantitative easing) and forward guidance. (Board of Governors' Publications Committee, 2016)

Large-scale asset purchase, often termed as quantitative easing (*QE*), is a monetary policy tool that the Federal Reserve used during the end of 2008 when they purchased large amounts of long-term securities to put a downward pressure on the long-term interest rate (Board of Governors' Publications Committee, 2016). This is done to move investors and financial institutions away from investing in long-term securities and over to real investments that drive employment and growth. During the financial crisis the Federal Reserve bought U.S Treasury notes, agency debt and agency mortgage backed securities for a total of \$2.5 trillion (Labonte and Makinen, 2019). When the federal funds rate hit zero in late 2008, the Federal Reserve initiated forward guidance. This implies that the FOMC informs the public of its future intention for the federal funds rate in its post meeting statements to affect expectations. (Board of Governors' Publications Committee, 2016)

In 2015 the FOMC increased the federal funds target rate to 0.25%, its first change since December 2008. This also marked the start of normalising their stance of monetary policy, as the employment was consistent with what can be seen as "maximum employment" (Board of Governors' Publications Committee, 2016). Normalisation in this case implies rising short-term interest rates to more normal levels and also decreasing the size of the Federal Reserve balance sheet. As of 2019 the Federal Reserve is well underway towards normalisation as the federal funds target rate has hit 1.25% and the balance sheet is steadily shrinking as securities reach maturity.

## 2.3 The Federal Reserve and the Taylor rule

When deciding upon the direction of the monetary policy the FOMC "regularly consults the policy prescriptions from several monetary policy rules along with other information that is relevant to the economy and the economic outlook". (Board of Governors' Publications Committee, 2018b)

Following existing literature published by the Federal Reserve (e.g Knotek II et al. (2016), Kliesen (2019)) the Taylor rule, first proposed by Taylor (1993), is acknowledged as a

benchmark for assessing the stance on monetary policy. The Taylor rule states that the monetary authority, in our case the Federal Reserve, should set its policy rate in accordance with the change in inflation and the output gap. In critique to such policy rules, one can argue that the simple policy prescriptions do not take into account the many complexities of the real economy (Board of Governors' Publications Committee, 2018a). However, models like the Taylor rule should simply serve as a guidance when deciding on monetary policy (Taylor, 1993).

## 2.4 Zero lower bound and shadow rate

As mentioned above, the FOMC lowered the federal funds target rate to approximately zero between 2007 and December 2008. Thus, reaching its zero lower bound. Since currency is available as an alternative asset, negative nominal interest rates would lead to riskless arbitrage opportunities, thus rates are bounded below by zero (Bauer and Rudebusch, 2016). The ZLB affects not only the way the central bank conducts its monetary policy, but as pointed out by Lindé et al. (2017), it also affects the behaviour of economic models.

To explain the effects of monetary policy with the federal funds rate at the lower bound some research has focused on the ZLB sub-period (Wu and Xia, 2016). This approach leads to the exclusion of six years of valuable macroeconomic data. Furthermore, it poses an issue in summarizing monetary policy after the economy exits the ZLB. The shadow rate is a possible solution to aid economists in analyzing monetary policy with the presence of a ZLB-period.

An initial model for implied interest rates was created by Black (1995), namely the shadow rate term structure model (*SRTSM*). Wu and Xia (2016) used the shadow rate from the SRTSM to construct a new measure for the monetary policy stance when the effective federal funds rate is bounded below by zero, and employed this measure to study unconventional monetary policy's (e.g. QE) impact on the real economy. The shadow rate term structure model is able to capture a scenario of the yield curve going into negative territory, rather than being restrained to a zero lower bound, like other term structure models (Lemke and Vladu, 2015). We explain how the shadow rate is estimated in section 4.1.2 and appendix A2.

Wu and Xia (2016) approached the question of shadow rate validity using a formal hypothesis test, which tested whether the parameters related to the shadow rate were different from the parameters related to the federal funds rate. They used a 3-factors Factor-Augmented Vector Autoregressive (*FAVAR*) model to study the effects of monetary policy interventions. The method allowed them to summarize information from a large set of economic variables. From the analysis, Wu and Xia found that the shadow rate showed similar dynamic correlations with macroeconomic variables during the ZLB-period, as the federal funds rate did prior to the financial crisis. Hereby confirming the validity of using the shadow rate in place of the federal funds rate. The results substantiated the findings of Bullard et al. (2012) and Krippner (2012), who advocated the value of shadow rate to describe monetary policy stance.

Given the fact that the federal funds target rate is well underway to more normal short-term levels we find it highly interesting to study the possible insights that the shadow rate may provide on out-of-sample performance. More explicitly, we will study the implications of shadow rate versus the actual effective federal funds rate in estimating a Taylor rule for the U.S economy with a ZLB regime.

## 3 Taylor rule

In the following section, we will present the original Taylor rule along with a version that can be characterized as forward-looking, and lastly, a smoothed version of the forward-looking policy rule.

### 3.1 The original Taylor rule

The original Taylor rule was proposed by Taylor (1993) as a simple policy rule that could effectively "call for changes in the federal funds rate in response to changes in the price level or changes in real income". This resulted in the following equation:

$$r = \bar{\pi} + 0.5y + 0.5(\bar{\pi} - \bar{\pi}^*) + 2 \quad (3.1)$$

where  $r$  is the federal funds rate,  $\bar{\pi}$  is the rate of inflation over the previous four quarters and  $y$  is the percent deviation of real GDP from a target. Taylor chose to set the inflation target ( $\bar{\pi}^*$ ) at two percent, which as mentioned in section 2.1, the FOMC also did in 2012. This yields the original version that Taylor proposed:

$$r = \bar{\pi} + 0.5y + 0.5(\bar{\pi} - 2) + 2 \quad (3.2)$$

This can be simplified to the following equation:

$$r = 1 + 1.5\bar{\pi} + 0.5y \quad (3.3)$$

Equation 3.3 implies that the "equilibrium" nominal rate is four percent when the inflation is at its target of two percent and the real GDP is equal to potential GDP, giving an output gap ( $y$ ) of zero. Further, Taylor (1993) states that his policy rule "cannot and should not be mechanically followed by policymakers", but rather function as an indicator for what needs to be done. Following this, the "Taylor principle" provides clarification. It states that a one percentage change in inflation should lead to a greater than one percentage change in the nominal interest rate. The reason for this is to make sure that



the rise in real interest rate is above the rise in inflation, so that the interest rate do not accommodate for any shocks that initially caused the rise in inflation (Woodford, 2001). Taylor (1993) found the original policy rule to fit quite well to the federal funds rate between 1987 and 1992, with respect to its simplicity. However, there have been countless attempts to estimate different versions of the Taylor rule since the paper of John Taylor was released.

## 3.2 Forward-looking Taylor rule

The original Taylor rule proposed by Taylor (1993) is a backward-looking model. This entails that, for instance, the FOMC sets the federal funds target rate  $r$  for period  $t$  based on both the inflation and output gap for the period  $t-n$  until time  $t$ . Researchers point out that the Federal Reserve also look at expected inflation when they are to decide the federal funds target rate (Clarida et al., 2000, 1998). Hence, we have been inspired by the work of Clarida et al. (1998) and Clarida et al. (2000) which estimate a Taylor rule using data for expected inflation and output gap, categorizing it as a forward-looking model. Formally, the model is a hybrid with both expected values and lagged values of variables as they use a lagged interest rate as a regressor.

### 3.2.1 Interest rate target

A forward-looking Taylor rule with an interest rate target, e.g. the model by Clarida et al. (2000), assumes that the central bank chooses a target for the nominal interest rate that they wish to reach in a given period. Hence, the target rate in period  $t$  is

$$i_t^* = i^* + \beta(E[\pi_{t,k}|\Omega_t] - \pi^*) + \gamma[x_{t,q}|\Omega_t] \quad (3.4)$$

where  $i^*$  is the "equilibrium" nominal interest rate, as mentioned above, the nominal interest rate when both inflation and output gap is equal to their targets.  $\pi_{t,k}$  is the quarterly expected inflation between period  $t$  and period  $t+k$ .  $x_{t,q}$  is the quarterly output gap between period  $t$  and period  $t+q$  in the forward-looking model. However, in our version we will use expected four quarters ahead data for inflation, substituting the  $t+k$  term with  $t+4$ . In addition, we will use current data for the output gap, therefore

substituting the  $t+k$  term with  $t$ . The output variable is nonetheless still presented as an expected variable, given that the potential GDP is a long-term estimated value. Further,  $\beta$  and  $\gamma$  are the coefficients of expected inflation and the output gap, respectively, while  $\pi^*$  is the inflation target rate. The model can be simplified by introducing a constant term  $\alpha = i^* + \beta\pi^*$ . This yields the following equation

$$i_t^* = \alpha + \beta E[\pi_{t+4}|\Omega_t] + \gamma E[x_t|\Omega_t] \quad (3.5)$$

$\beta$  and  $\gamma$  determines the responsiveness of policy to the change in expected inflation and output gap, respectively.  $\Omega$  is the information set at time  $t$ , which is further discussed in section 5.3.

### 3.2.2 Interest rate smoothing

The output of the above model displays the target rate as the interest rate the central bank pursues to close the deviations from its inflation and output targets. However, the model does not consider the fact that central banks tend to smooth interest rates. There are various reasons for the central bank to do this. Goodfriend (1991) states that the federal reserve smooth interest rates to cushion the banking system against interest rate shocks. He also says that the Federal Reserve dislike changing the interest rate in different directions close to each other, called "whipsawing the market".

By adding interest smoothing into equation 3.5 we can create an expression for the *actual* nominal interest rate. This implies that the central bank will choose the weight between  $i^*$ , the target interest rate, and  $\rho$ , the smoothing of the interest rate. Equation 3.6 displays this relationship as

$$i_t = (1 - \rho)i_t^* + \rho i_{t-1} + v_{1t} \quad (3.6)$$

where  $\rho$  is the smoothing coefficient and  $v_{1t}$  is an exogenous interest rate shock term with a zero mean. Fully introducing equation 3.5 into equation 3.6 will yield a Taylor rule for the actual nominal interest rate, presented as

$$i_t = (1 - \rho)(\alpha + \beta E[\pi_{t+4}] + \gamma E[x_t]) + \rho i_{t-1} + v_{1t} \quad (3.7)$$

The model of equation 3.7 is no longer linear in its parameters, and clearly displays the effects of interest rate smoothing. For example, if you have a smoothing parameter  $\rho$  of 0.85, then the interest rate target of equation 3.5 is only approached by 15% of what it would have been if there was no smoothing present. Further, given an output gap of zero and a  $\beta$  of 1, a positive 1% inflation shock would imply that the central bank only adjusted the interest rate up by 0.15%. By using a lagged dependent variable we soak up serial correlation, implying that we do not test our model for serial correlation.

### 3.2.3 Our chosen model

Inspired by the work of Clarida et al. (1998) and Clarida et al. (2000), our interpretation of a forward-looking Taylor rule is displayed below. The model will be derived with regards to GMM in section 5.3.

$$i_t = (1 - \rho)(\alpha + \beta E[\pi_{t+4}] + \gamma E[x_t]) + \rho i_{t-1} + v_{1t} \quad (3.8)$$

## 4 Data

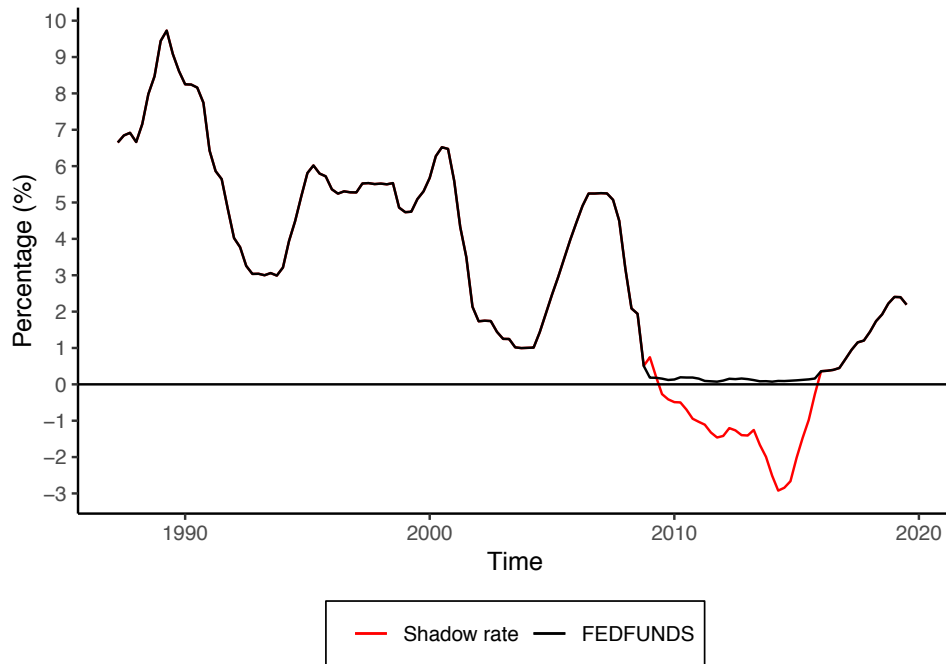
The following section presents the data used in our analysis. In subsection 4.1 we present the time series used in our Taylor rule estimation, along with separate figures of each variable. In subsection 4.2 we present and explain the structural break that supports our chosen sample period.

### 4.1 Time series

The variables used for our Taylor rule estimation is presented along with their respective figures. Our in-sample-period mainly stretches from 1987Q2 until 2015Q4, but as we conduct a robustness check with a sample ending in 2016Q4, we display the variables graphically with the period 1987Q2-2016Q4. The federal funds rate is used in the out-of-sample comparisons and are therefore displayed from 1987Q2 to 2019Q3. Our sources of data have been the FRED® data service provided by the Federal Reserve Bank of St.Louis and the Federal Reserve Bank of Atlanta. The mnemonics shown in parentheses are the labels used to identify each series by FRED®. The figures presented in this section have all been created using RStudio and the "ggplot2"-package.

#### 4.1.1 The federal funds rate

As our measure of the short-term nominal interest rate we use the effective federal funds rate (FEDFUNDS). As mentioned in section 2.2, the rate is calculated as an weighted-average of the agreed rate for an overnight loan between depository institutions. The Federal Open Market Committee (FOMC) determines the federal funds target rate, while the lending and borrowing between depository institutions reflects the market equilibrium of the same rate, constituting the effective federal funds rate. The rate applied in our analysis is calculated as daily averages converted to quarterly averages. Figure 4.1 illustrates the development of the effective federal funds rate.

**Figure 4.1:** Effective federal funds rate and shadow rate

### 4.1.2 Shadow rate

We chose the estimated rate by Wu and Xia (2016) as our shadow rate throughout our thesis. The rate is extracted from the Federal Reserve bank of Atlanta. The input data for the shadow rate are one-month forward rates beginning  $n$  years hence. Wu and Xia use forward rates corresponding to  $n = 1/4, 1/2, 1, 2, 5, 7,$  and 10 years. The forward rates are from estimated Svensson (1995) and Nelson and Siegel (1987) model parameters. The parameters are obtained by independently fitting observations of coupon-paying yield curve data from the Gürkaynak et al. (2007) data set. In short, the shadow rate is assumed to be a linear function of three latent variables called factors, which follow a VAR(1) process. The latent factors and the shadow rate are estimated with the extended Kalman filter, by adding it to equation 4.1. The Kalman filter takes a linear approximation for the yield data, as the factors are non-linear. The forward rate in the SRTSM described in appendix A2 with equations A.1 - A.5 can be approximated to equation 4.1:

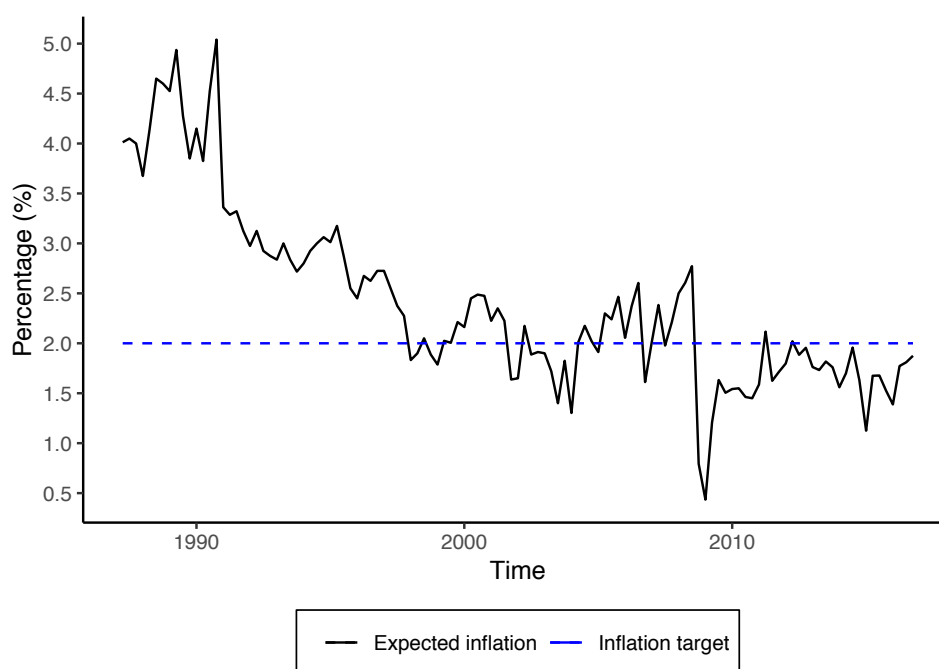
$$f_{n,n+1,t}^{SRTSM} = \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right) \quad (4.1)$$

where  $(\sigma_n^{\mathbb{Q}})^2 \equiv \text{Var}_t^{\mathbb{Q}}(s_{t+1})$ . The function  $g(z)$  consists of a normal cumulative distribution

function and a normal probability density function. The approximation for the equation can be found in appendix A2. The computations is derived from the paper of Wu and Xia. For further insight we refer to "Measuring the macroeconomic impact of monetary policy at the zero lower bound" by Wu and Xia (2016). Their computed shadow rate is displayed in figure 4.1. In this thesis we assume that we can treat the estimated shadow rate like observations. However, this will imply larger standard errors when accounting for estimation uncertainty, but this is regarded beyond the scope of our thesis.

### 4.1.3 Expected inflation

For expected inflation we chose the forecasted value of four-quarter PCE inflation three quarters ahead provided by the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. It is a measure of the PCE inflation of the current quarter together with the median forecast of the subsequent three quarters. Prior to 2007 this forecast was not collected. To collect a long-term time series for this specific data the Survey of Professional Forecasters subtracts the consumer price index by 0.3 percentage points to obtain the pre-2007 values. The choice of PCE inflation is regarded as favorable in contrast to the Consumer Price Index (CPI) (Mehra and Sawhney, 2010). Figure 4.2 displays the expected inflation along with the 2% inflation target. As mentioned in section 2.1 the inflation target was officially adopted by the FOMC in 2012. Taylor (1993) also suggested 2% as a measure of inflation target in his original policy rule. Hence, we use 2% as our inflation target for our entire sample period.

**Figure 4.2:** Expected inflation

#### 4.1.4 The output gap

As our measure of the output gap we use the difference between real potential gross domestic product (GDPPOT) and real gross domestic product (GDPC1). The former is the level of real GDP that is consistent with steady growth and a stable rate of inflation. Potential GDP is estimated on the basis of fundamental determinants of supply, like labor, capital and unemployment. The estimates are conducted on a sectorial level comprising of six sectors being non-farm business, farm, the federal government, state and local governments, households, and nonprofit institutions (Shackleton, 2018). Both the measurements of GDP are provided with a quarterly frequency and is reported in chained 2012 dollars. The output gap is illustrated in figure 4.3.

**Figure 4.3:** The output gap

### 4.1.5 Instrumental variables

As a solution to endogenous regressors we include instrumental variables. This method provides a way to obtain consistent parameter estimates. Further, as our thesis is inspired by the works of Clarida, Gali & Gertler, we apply the instruments included by them, i.e. commodity price inflation and the long-short spread. To reach a model with all parameters being significant we decide to extend the information set. When deciding which instrumental variables to include in our estimation, we tried to approximate the true information set, hereby obtaining consistent and significant parameters. The financial markets are known for their efficiency of applying all available information, accounting for both past, present and expected information. Hence, we pursued to include instrumental variables that communicate data related to financial markets.

We use seven instrumental variables in our extended information set when carrying out our empirical analysis. We have chosen to include commodity price inflation (PPIACO), the spread between the 10-year Treasury note and the 3-month Treasury bill (T10Y3M), house price inflation (CSUSHPINSA), return on average equity for all U.S banks (USROE), total public debt (GFDEBTN) and the US/UK exchange rate (DEXUSUK). The data have all been collected from FRED<sup>®</sup>, as seen from the mnemonics in the parentheses.

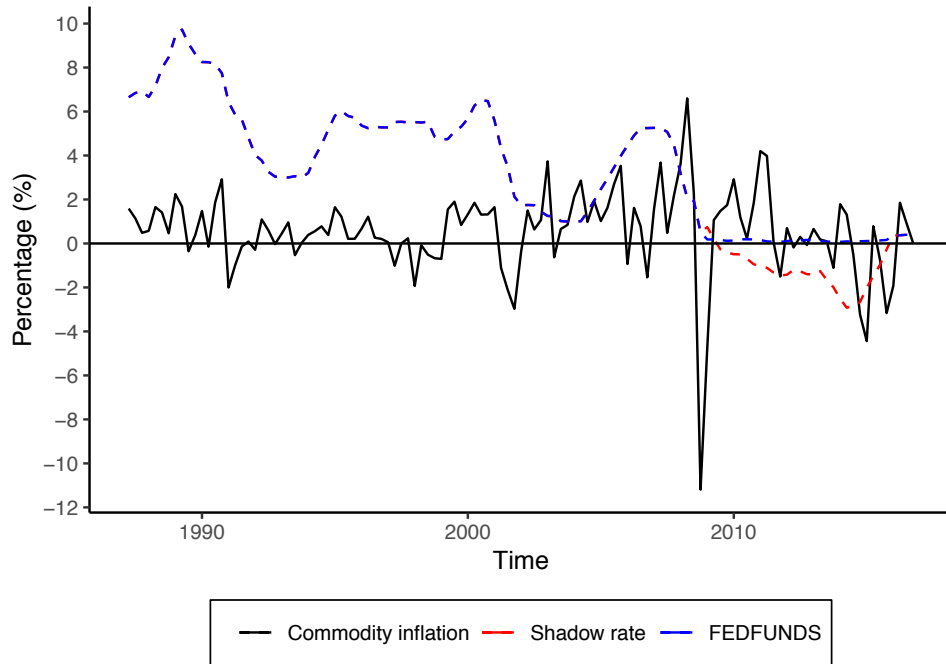


Additionally, we have also chosen to use a measure of unemployment gap as a regressor when conducting a robustness check. In addition it will be added as an instrumental variable along with the data mentioned above. The unemployment gap is measured as the difference between the actual unemployment rate and the Congressional Budget Office's estimate of the natural rate of unemployment. This natural rate is a nonaccelerating inflation rate of unemployment, often referred to as NAIRU, which is consistent with a stable rate of inflation (Arnold, 2001). The data is sourced from the website of the Atlanta Federal Reserve. The instrumental variables data is presented more thoroughly in the appendix section A1.

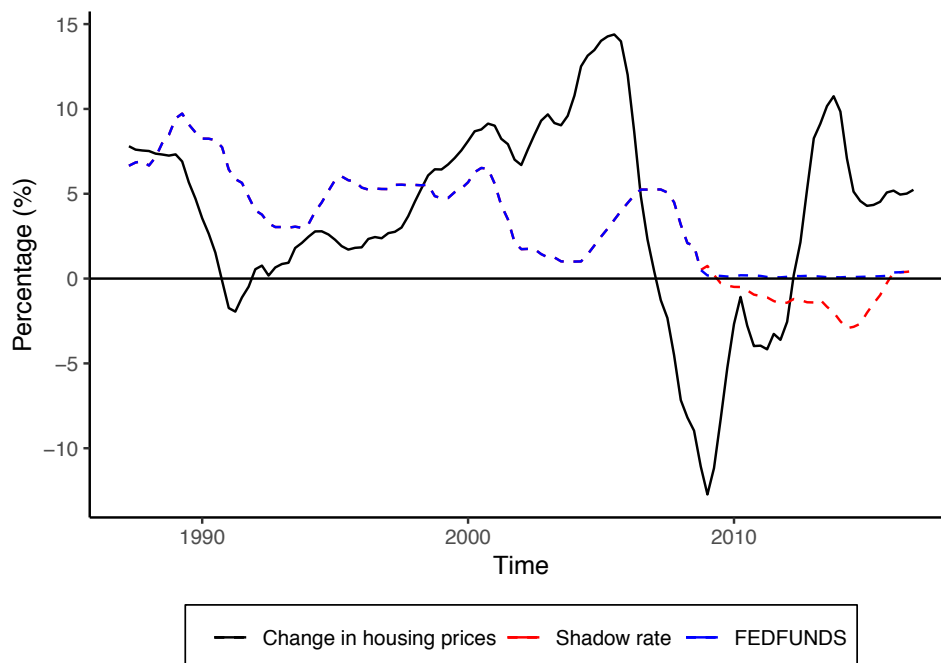
As mentioned we included the variables since we believe they reflect the stance of the financial markets. The variable that displays the change in housing prices is included due to its ability to show the expectations of future economic conditions, perfectly aligning with our forward-looking model. It does so by exhibiting the households expectations for both their personal economy as well as the entire macro economy, which was proven for Norwegian data (Anundsen and Jansen, 2013). Further, exchange rate have shown to alter the expectations of the market agents, again providing the analysis with a useful expectations term (Kallianiotis, 2016). The US/UK exchange rate is used in favor of the US/EU exchange rate due to its time of existence. Also, the return on average equity for all U.S banks can measure different risk characteristics important in pricing the market (He and Ng, 1994). Followingly, it provides relevant information for the stance of the financial markets and is therefore included in our extended information set. Lastly, the change in public debt has an effect on real per capita GDP (Kumar and Woo, 2010). This will in turn indicate that an increase in the public debt will have a negative impact on the financial markets and vice versa. These instrumental variables have also been used for Taylor rule estimation on Norwegian data by Skumsnes (2013), serving as an additional guidance in the choice of instrumental variables. All of the instrumental variables in our extended model have been differenced. Furthermore, we tested the variables for seasonality and found that the data for house price inflation needed to be seasonally differenced. To confirm whether our additional instrumental variables supports our model, we compare the J-stats and parameters of the baseline and extended model in section 6.1. The extended model yields a higher J-stat and all significant parameters, indicating that the extended instrument set improves the model. Figure 4.4 - 4.10 displays the instrumental variables,

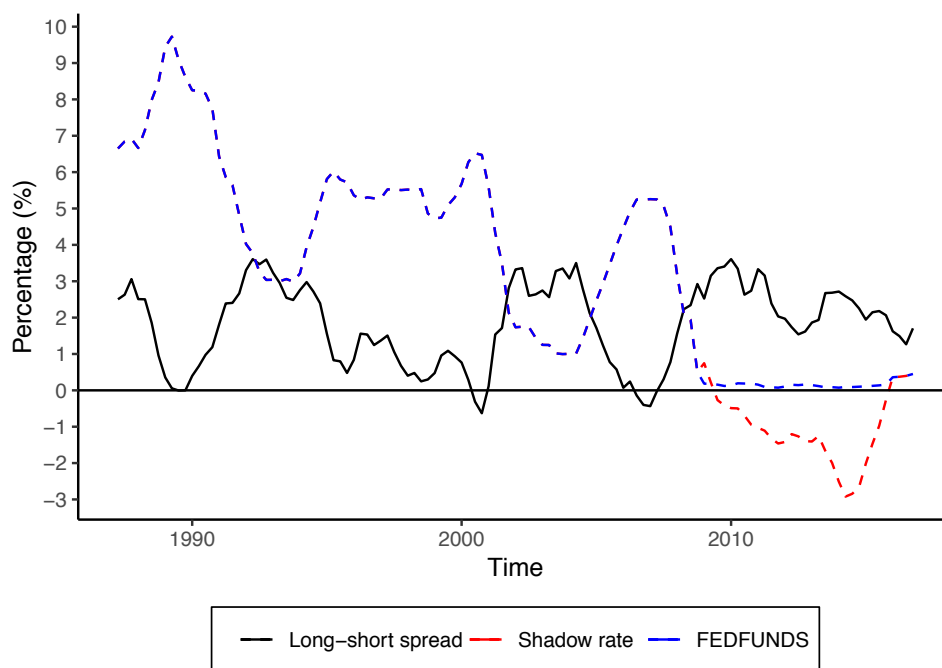
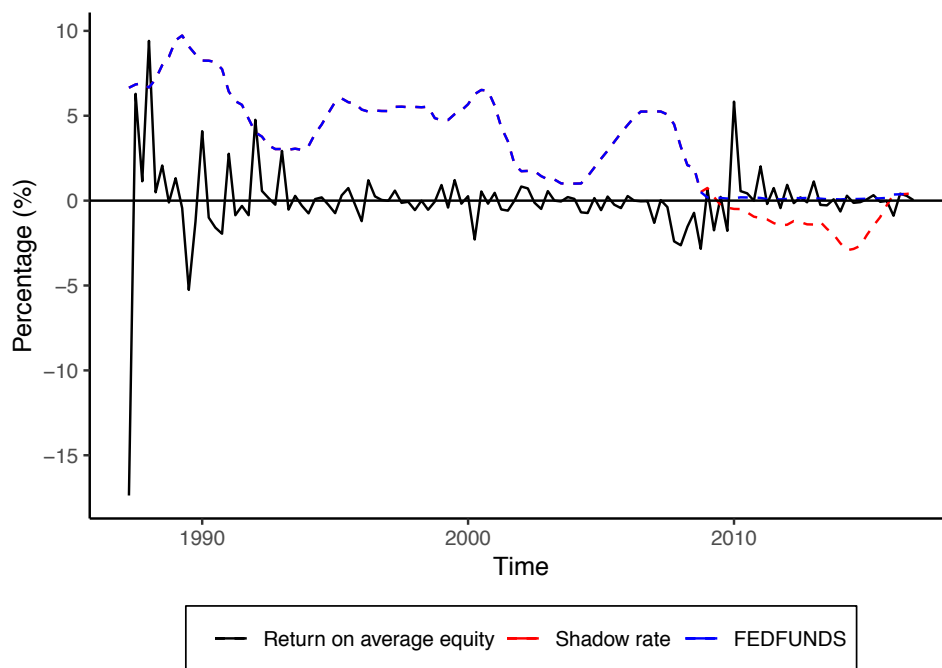
including the unemployment gap.

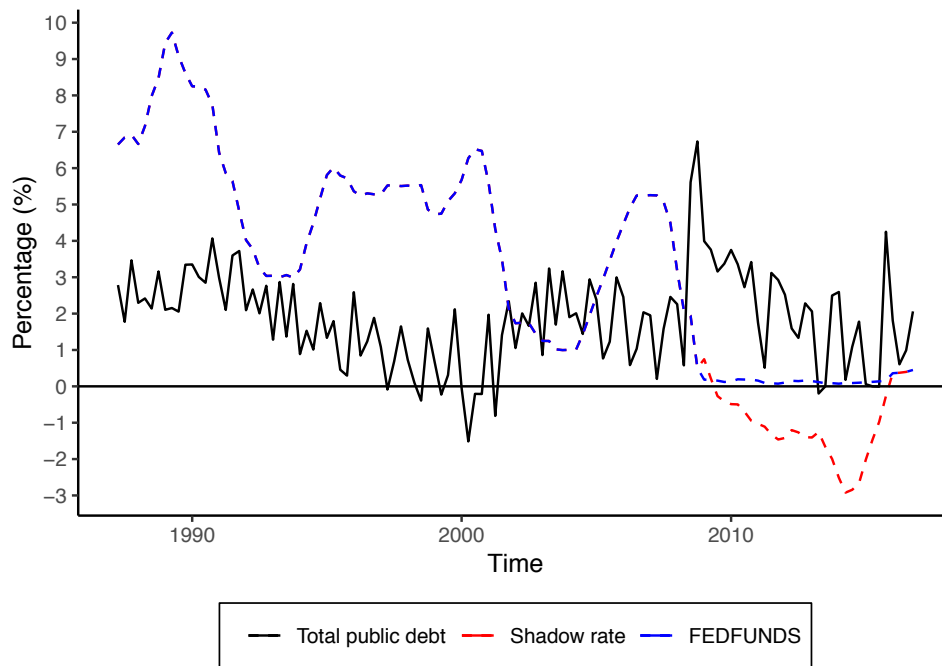
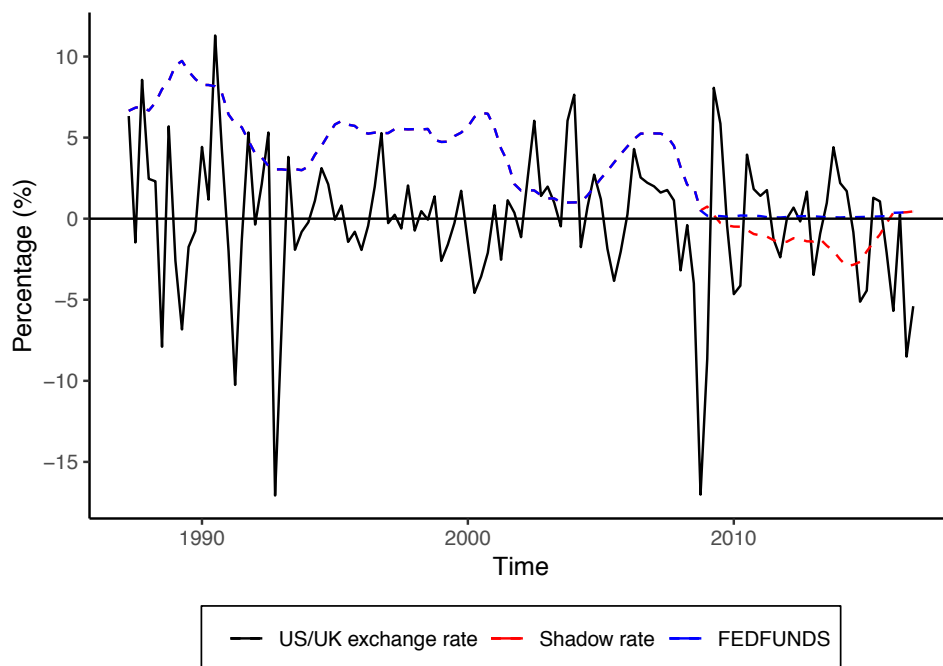
**Figure 4.4:** Commodity price inflation

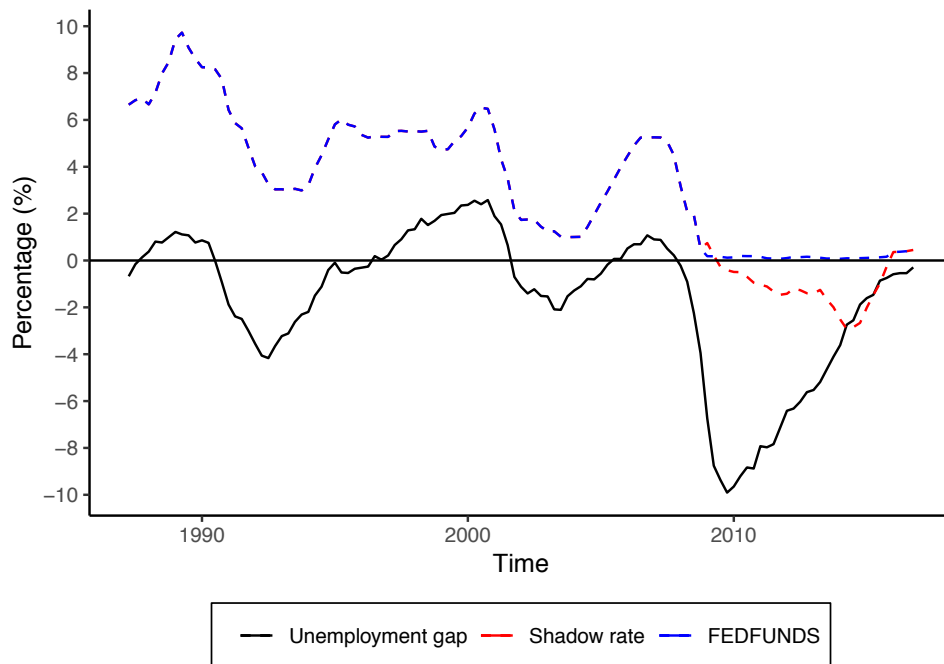


**Figure 4.5:** House price inflation



**Figure 4.6:** Long-short spread**Figure 4.7:** Change in return on equity for U.S. banks

**Figure 4.8:** Change in total public debt**Figure 4.9:** Change in US/UK foreign exchange rate

**Figure 4.10:** Unemployment gap

## 4.2 Structural breaks and sample period

Clarida et al. (2000) found significant differences in their estimated Taylor rules across different chairs of the Federal Reserve, serving as natural choices of structural breaks. Especially the period served by Paul Volcker has been influential on the U.S economy as he marked the start of a different monetary policy with his work on disinflation. The later period of Volcker, followed by his successor Alan Greenspan, have been regarded as a period with focus on fighting inflation (Goodfriend and King, 2005). Empirical evidence also show that there have been a drop in the persistence of the inflation, dropping the measurement to significantly lower levels in the Volcker-Greenspan era in contrast to Arthur Burns and George William Miller's chairmanships (Beechey and Österholm, 2007; Clarida et al., 2000). Hence, we find it reasonable to use a sample period from 1987Q2 with quarterly data, as this marks the period when Greenspan was appointed chair of the Federal Reserve. Belke and Klose (2010) have estimated a Taylor rule for U.S data and argued that quarterly data do not properly catch the dynamics as well as monthly data. On the other hand, Islam (2011) estimated both a forward- and backward-looking Taylor rule using U.S data, finding no evidence that the frequency of the data affects the results. In addition, as our Taylor rule estimation is inspired by Clarida et al. (2000) who uses

quarterly data, our analysis will employ the same frequency.

## 5 Methodology

The following sections presents the econometric methods we use in analyzing our data. Firstly, we explain why we have chosen the generalized method of moments (*GMM*) for Taylor rule estimation. Then we discuss how our estimations are performed, and lastly we present limitations with the econometric framework.

### 5.1 Econometric approach

From our discussion of the preferable Taylor rule we presented a policy rule using a smoothing parameter. The original Taylor rule presented by Taylor (1993) does not include a smoothing parameter, which implies linearity in the parameters. Following the findings from the presented literature by Clarida et al. (1998) we see that it is determined that using a smoothing parameter results in more precise estimations. When we apply a smoothing parameter the parameters are no longer linear. Non-linearity is a violation of the OLS assumptions of consistent and unbiased estimators. Hence, we must apply a method that can account for non-linearity. Furthermore, since we are dealing with a forward-looking model we have to apply a method that is robust to issues with endogeneity. Since we use an expected value as a regressor - see section 4.1.3 - the explanatory variables will be correlated with the error term at time  $t$ . Thus, in addition to non-linearity, we have to use a method that accounts for endogeneity.

### 5.2 GMM

Generalized method of moments (GMM) estimation is one of the most used methods for estimating models in economics (Hall, 2005). Through the use of instrumental variables the method handles non-linear estimations with endogenous explanatory variables. Furthermore, GMM brings the advantage of dealing with heteroskedasticity and autocorrelation in the residuals. The method uses a set of moment conditions to solve for the parameters of the model. Inspired by Drukker (2010), Monsrud and Mjelde (2018) formulates the moment conditions as follows.

$$E[m(y_t, x_t, z_t, \theta) = 0] \quad (5.1)$$

$m$  is a  $q \times 1$  vector of functions, where the expected values are zero. The left-hand side variable is defined by  $y_t$  and  $x_t$  is the explanatory variable vector. Additionally,  $z_t$  is a  $q \times 1$  vector of instrumental variables and  $\theta$  is a  $k \times 1$  vector of parameters, where  $k \leq q$ . The sample moments from the population moments are:

$$\bar{m}(\theta) = \frac{1}{T} \sum_{t=1}^T Tm(y_t, x_t, z_t, \theta) \quad (5.2)$$

The goal is to solve the over-identified system of moment conditions. When  $k < q$ , the GMM chooses the parameters that minimize the following objective function with respect to the parameter vector :

$$\theta_{GMM} \equiv \sum_{t=1}^T \operatorname{argmin}_{\theta} \bar{m}(\theta)' W \bar{m}(\theta) \quad (5.3)$$

Only, when  $k = q$  can we get an explicit formula where the moment conditions are exactly satisfied. Then the GMM estimator solves  $\bar{m}(\theta)$  so that  $\bar{m}(\theta)' W \bar{m}(\theta) = 0$ . When the system is overidentified, we minimize with the use of numerical optimization methods. This is because a large number of moment conditions require a numerical minimization.

### 5.3 GMM and Taylor rule

In this section we will use the Taylor rule presented in section 3.2.2, and show how we apply the generalized method of moments. Initially, we present equation 3.8.

$$i_t = (1 - \rho)(\alpha + \beta E[\pi_{t+4} | \Omega] + \gamma E[x_t | \Omega_t]) + \rho i_{t-1} + v_{1t}$$

Furthermore, we eliminate the unobserved forecast variables by introducing an auxiliary variable. (Clarida et al., 1998)

$$\epsilon_{1t} = -(1 - \rho)(\beta(\pi_{t+4} - E[\pi_{t+4} | \Omega_t]) + \gamma(x_t - E[x_t | \Omega_t])) + v_{1t} \quad (5.4)$$



This equation is a combination of the forecast errors and the exogenous error term. Consequently, it is orthogonal to variables in the information set. Next, we solve the equation for  $v_{1t}$ .

$$v_{1t} = -(1 - \rho)(\beta(\pi_{t+4} - E[\pi_{t+4}|\Omega_t]) + \gamma(x_t - E[x_t|\Omega_t])) + \epsilon_{1t} \quad (5.5)$$

To remove the expectation term we insert 5.5 into 5.3. Resulting in the following equation:

$$\begin{aligned} i_t = & (1 - \rho)(\alpha + \beta E[\pi_{t+4}|\Omega_t] + \gamma E[x_t|\Omega_t]) + \rho i_{t-1} + \\ & (1 - \rho)(\beta(\pi_{t+4} - E[\pi_{t+4}|\Omega_t]) + \gamma(x_t - E[x_t|\Omega_t])) + \epsilon_{1t} \end{aligned} \quad (5.6)$$

Lastly, we rewrite the policy rule in terms of realized variables, hereby eliminating the unobserved forecast variables. The result is the policy reaction function:

$$i_t = (1 - \rho)(\alpha + \beta\pi_{t+4} + \gamma x_t) + \rho i_{t-1} + \epsilon_{1t} \quad (5.7)$$

To reach the parameter vector for estimation, we use an instrument set  $z_t$ , which is a vector of variables within the information set  $\Omega_t$ . The instrument set can consist of any lagged and current variables that are uncorrelated with  $\epsilon_{1t}$ , and hereby orthogonal to the error term. Thus, the condition is  $E[\epsilon_{1t}|Z_t] = 0$ , which we write as:

$$E[i_t - (1 - \rho)[\alpha + \beta\pi_{t,k} + \gamma x_{t,q}] - \rho i_{t-1} | Z_t] = 0 \quad (5.8)$$

Therefore, what we want to estimate is the parameter vector  $[\rho, \alpha, \beta, \gamma]$ . To perform this estimation we use GMM. In the initial Taylor rule model we use the instrumental variables proposed by Clarida et al. (1998). This implies four lags of the regressors, commodity inflation and long-short spread. Hence, the number of instruments surpasses the number of parameters in our vector, implying that our model is overidentified. The handling of overidentification will be presented in the next section.

The Federal Reserve might include other factors when determining the funds rate, which implies that adding more variables could contribute to a better fitting model. However,

the goal of our thesis is to compare the inclusion of shadow rate to a standard rule. Thus, focusing on fitting the most precise model is out of our scope, and we will not present a model with additional variables beyond the given parameter vector.

### 5.3.1 Out-of-sample performance

To answer our question of whether the Taylor rule estimated with shadow rate outperforms the regular Taylor rule, we apply the RMSE- and the MAE-criterion. RMSE estimates the root-mean-squared-error and gives a measure of the differences between values. We compare the RMSE of our Taylor rule constructed with shadow rate and the standard Taylor rule, to the federal funds rate of our “out-of-sample”-period. The series that returns the lowest RMSE is the one that has the highest precision of estimating the funds rate. The mean absolute error (MAE) measures the average of the absolute differences between our estimated and the realized values. The reason for including this measure in addition to RMSE is that the RMSE squares the residuals, hereby penalizing large errors more. Thus, which is the preferred method depends on the relative weight one wish to place on larger errors. To gain a more comprehensive insight into the difference between our estimations we include both measures in the analysis section.

### 5.3.2 Overidentification

To determine whether our estimations provides valid results, we must test for overidentification (Guay et al., 2004). This is done by the use of Hansen’s J-statistic, where we test the null hypothesis that the values for our parameter vector  $[\rho, \alpha, \beta, \gamma]$  exist. Not rejecting the null hypothesis implies that the residual is orthogonal to the variables in the information set. On the other hand, rejecting the null implies that the orthogonality condition is violated, and we have omitted relevant explanatory variables. Rejection of the null can additionally imply that our model is misspecified, as the J-test function as an overall test for misspecification. In section 6 we include the results from the J-test inn our GMM-estimations.

### 5.3.3 Stationarity

Our method of estimating Taylor rules using GMM relies on the assumption that all variables are stationary. This implies that the statistical properties (e.g. mean, variance and autocorrelation) are all constant over time. A stationary time series does not contain a unit root. Thus, we test for stationarity in our regressors using the augmented Dickey-Fuller (ADF) test,

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{p=1}^P \delta_p \Delta Y_{t-p} + \epsilon_t$$

in which  $\alpha$  is a constant. Our null hypothesis is that  $|\gamma| = 0$ , i.e., meaning that the time series is not stationary, while our alternative hypothesis is that  $|\gamma| \neq 0$ , meaning that the time series  $Y_t$  converges to a stationary time series. The results from our ADF-test are presented in section 6.1.2 and appendix A4.1.

### 5.3.4 Robustness

To ensure robustness in our estimations we use different gap measurements, as well as using data samples from different time periods. This is done to ensure that we have robust estimates that are representative for a larger scope than the initially applied data and timeframe.

When using GMM to estimate our models we set the weight updating to N-step iterative. Contrary to 1-step iteration, the N-step re-estimates the covariance matrix until it obtains converging estimates. This process gives different weightings to different moment conditions. The observations with higher variance get a lower weighting, while the lower variance observations get a higher weighting. The covariance matrix gets reiterated as many times as needed until it obtains a numerical convergence. This implies that the estimations of  $\hat{\beta}$  and  $\hat{\Omega}$  have numerical converged. Thus, the converged estimates present a more robust model with a more stable  $\beta$ -coefficient. We include a 1-step iteration estimation for robustness in section 6.2.1.

Further options chosen for our GMM-estimations include HAC (newey-west), which is applied to overcome autocorrelation and heteroscedasticity, as well as applying the

Marquardt optimization algorithm, which minimizes the sum of the weighted residuals in a non-linear regression.

### 5.3.5 Misspecification

There is no clear literary consensus regarding the validity of a smoothing parameter in the policy reaction function. Rudebusch (2002) argues that there is misspecification in the model proposed by (Clarida et al., 1998). The argument is that the coefficient does not reflect smoothing, but rather other factors that cause a deviation from the policy rule. The findings from Rudebusch has gained literary support, see for instance English et al. (2003) and Gerlach-Kristen (2004). They find that adding more regressors in the model leads to a drop in the smoothing parameter. However, they do not support that there are no monetary policy inertia. Hence, they argue that the estimations of large smoothing coefficients reflects an over-cautiousness in central banks, which they believe to be inaccurate, but they still argue the validity of the smoothing coefficient. (Carare and Tchaidze, 2005)

## 6 Analysis

To estimate our Taylor rules we use the preferred Taylor rule presented by Clarida et al. (1998). There appears to be a literary consensus about the validity of their Taylor rule, see for example Orphanides (2007) and Ascari and Ropele (2009). In their paper they argue that the forward-looking Taylor rule gives the most precise estimations. To determine whether inclusion of shadow rate data yields a more precise model we will use the RMSE and MAE-criterion, testing out-of-sample performance, in addition to a graphical comparison. For robustness we also present an estimation where we use 1-step iteration, a forward-looking model using employment gap instead of output gap and estimations conducted with different time horizons.

### 6.1 Forward-looking Taylor rule

Initially, we fit the forward-looking Taylor rule, presented in section 3.2.3. As we use a smoothing parameter, we are dealing with a non-linear estimation. Hence, we use the General Method of Moments (GMM) in EViews10. See section 5.2 for a description of GMM. The model assumes that the Federal reserve uses four quarters ahead expected inflation and output gap when determining the federal funds rate, substantiated by research from Federal Reserve Bank of Cleveland (2019). The variables used are inflation, output gap, a smoothing parameter and a constant, all of which are presented in section 4. As our model is based on the best fitting model from Clarida et al. (1998), we apply the same instruments as presented in their paper. This implies four lags of the regressors as well as commodity inflation and long-short spread, both presented in section 4.1.5. Including our constant, we have a total of 21 instruments. The estimated models span from 1987Q2 until 2015Q4. Since we use four lagged terms as instrumental variables the adjusted sample starts at 1988Q2. Clarida et al. referred to the model as their baseline model, for readability purposes we use the same name. The estimated Taylor rule model - previously presented as equation 3.8 - is as follows:

$$i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma[x_t]) + \rho i_{t-1} + \epsilon_t$$

Our goal is to measure how the estimations differ when we include data for shadow rate in place of the federal funds rate during the ZLB-period. Firstly, we present the coefficients resulting from the GMM estimations. The results are displayed in table 6.1 below.

**Table 6.1:** Baseline model

<b>Baseline</b>		
	(1)	(2)
	Standard	SR
$\rho$	0,80*** (0,03)	0,87*** (0,02)
$\alpha$	-0,88 (0,55)	-0,67 (1,00)
$\beta$	2,29*** (0,20)	2,20*** (0,39)
$\gamma$	0,93*** (0,10)	1,28*** (0,16)
$N$	111	111
J-stat	13,88	13,27
P-value	0,68	0,72

Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Estimated model:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ .

Model (1) includes four lags of the effective federal funds rate, while model (2) includes four lags of the effective federal funds rate and shadow rate for the period it persisted. Both models include four lags of inflation, output gap, commodity inflation(PPIACO) and the spread between the 10-year Treasury note and the 3-month Treasury bill(T10Y3M) as instrumental variables.

As opposed to the original Taylor rule presented in section 3.1, the output displays larger coefficients for inflation and output gap. This is reasonable as we introduce the smoothing parameter. Most of the coefficients are highly significant, except for the constant terms. The meaning of this coefficient is presented in section 3.2. The p-value for the Hansen j-statistic is 0.68, see section 5.3.2. To approximate a true information set and reach a model where all coefficients are significant, we introduce more instrumental variables (see section 4.1.5). When including more instrumental variables we must be thoughtful in that the instruments should correlate with the regressors but not with the error term. Following existing literature, see for example Siklos et al. (2004) and Skumsnes (2013), we decide to include data for US house price inflation, return on average equity, total public debt, unemployment gap and the exchange rate between US dollars and British pounds to represent the information of the financial markets (i.e. approximate the true information set). See section 4.1.5 and appendix A1 for more insight into our chosen

instruments. By adding these we have a total of 41 instrumental variables. The result is that all coefficients are significant at a five percent level. Furthermore, the resulting p-value from the J-statistic is 0.98, implying that we reject the null of overidentification. The results achieved from adding the additional instruments are shown in Table 6.2 below, compared to the results from the baseline model.

**Table 6.2:** Baseline and extended model

	Baseline		Extended	
	(1) Standard	(2) SR	(3) Standard	(4) SR
$\rho$	0.80*** (0.03)	0.87*** (0.02)	0.80*** (0.02)	0.86*** (0.01)
$\alpha$	-0.88 (0.55)	-0.67 (1.00)	-1.11*** (0.35)	-1.60*** (0.52)
$\beta$	2.29*** (0.20)	2.20*** (0.39)	2.37*** (0.12)	2.58*** (0.18)
$\gamma$	0.93*** (0,10)	1.28*** (0,16)	0.90*** (0,06)	1.24*** (0,08)
$N$	111	111	111	111
J-stat	13,88	13,27	17,79	18,77
P-value	0,68	0,72	1,00	0,99

Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Estimated model:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ .

Model (1) and (3) includes four lags of the effective federal funds rate, while model (2) and (4) includes four lags of the effective federal funds rate and shadow rate for the period it persisted. All the models include inflation, output gap, commodity inflation(PPIACO) and the spread between the 10-year Treasury note and the 3-month Treasury bill(T10Y3M) as instrumental variables. Model (3) and (4) also includes change in housing prices(CSUSHPINSA), return on average equity for all U.S banks(USROE), total public debt(GFDEBTN) and also, the US/UK exchange rate(DEXUSUK).

Model (4), i.e. the extended model including shadow rate, will be our leading model throughout the thesis. Next, we discuss the coefficients and the economic intuition behind their values. A prominent difference between our two secondary estimations is noticed when viewing the coefficients relating to the smoothing parameters. The coefficients are relatively large, which could imply that the Federal Reserve reacts slowly with monetary policy when there are changes in inflation and output gap. However, we cannot deduce anything about the short-term difference by looking at the smoothing parameters isolated, because the real effect is determined by combining coefficients. To compare the implied one-period reaction to changes in inflation and output gap we view the coefficients less the value deducted from the smoothing parameter. Thus,  $(1-\rho)*\pi$  equals the one period

response to a one percent change in inflation.

$$(1 - 0.80) * 2.37 = 0.47 \tag{6.1}$$

$$(1 - 0.86) * 2.58 = 0.36 \tag{6.2}$$

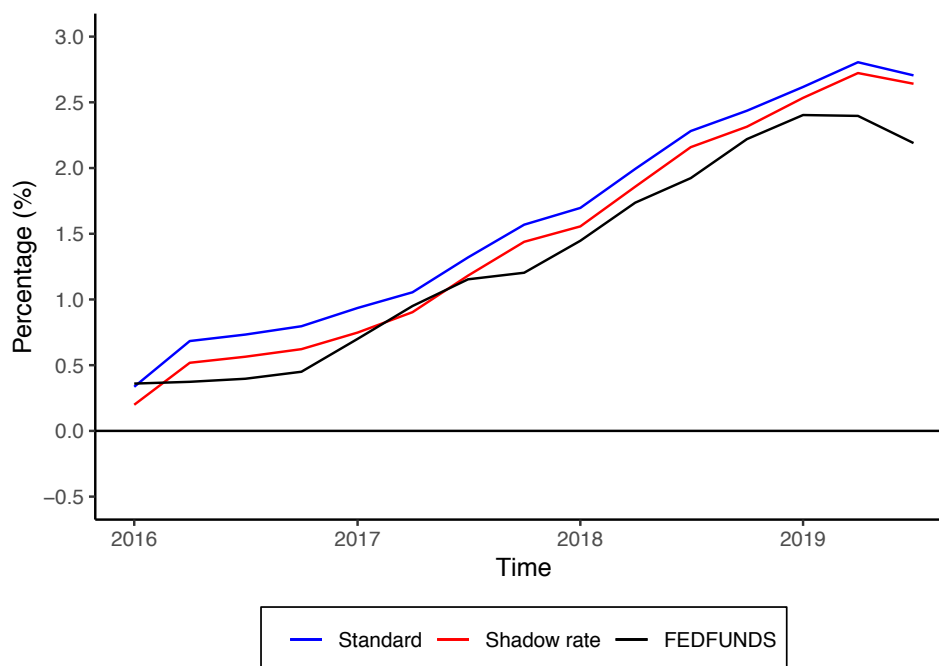
From equations 6.1 and 6.2 we can see that the estimated response to a one percent change in inflation at time  $t$  is a 0.47% in the effective federal funds rate for the regular estimation and 0.36% from the estimation with shadow rate. Implying that the inclusion of shadow rate yields a less responsive policy towards inflation targeting. Following the logic from inflation,  $(1-\rho)*\gamma$  equals the response to a one percent change in output gap.

$$(1 - 0.80) * 0.90 = 0.18 \tag{6.3}$$

$$(1 - 0.86) * 1.24 = 0.17 \tag{6.4}$$

From the estimated responses to output gap changes, equations 6.3 and 6.4 we see that we get similar results from the two estimations, 0.18% versus 0.17%. Thus, there are negligible deviations in how the estimations react to changes in output gap at time  $t$ . Furthermore, we see that the constant from both estimations are negative. The term is defined as  $\alpha = (i^* - 2*\beta)$ , see section 3.2. Substituting our coefficients, we get  $i^*=3.63$  for our regular estimation, and  $i^*=3.57$  for the shadow rate estimation. The long run real equilibrium rates are therefore 1.63 and 1.57, respectively. Compared to values obtained from similar research in the existing literature we can see that the results appear reasonable (Clarida et al., 2000). Our estimated smoothing coefficient (see model (4) in table 6.2) coincides with existing literature (e.g. Bullard et al. (2018) and Kliesen (2019)), who assigns the parameter  $\rho$  a value of 0.85. To further compare our two estimates, we initially show them graphically, while also displaying the effective federal funds rate.



**Figure 6.1:** Extended model

As we can see from the graphs the estimations yield similar results. However, the estimation that accounts for the shadow rate appears to be a better fit. To confirm which estimate is the most precise we apply the RMSE and MAE criteria, presented in section 5.3.4. The resulting outputs are presented in the following table.

**Table 6.3:** RMSE and MAE results

	Standard	SR
RMSE	0.299	0.197
MAE	0.274	0.165

From the measures we see that the Taylor rule estimated with shadow rate is a better fit for the federal funds rate, substantiating our graphical findings. The output from the estimation with shadow rate shows 0.197 and 0.165 for RMSE and MAE, respectively. From the regular estimation without shadow rate we get results of 0.299 and 0.274, respectively. However, the two estimations display similar properties, both graphically and by the difference between the RMSE- and MAE-measures.

### 6.1.1 Weak instruments test with OLS

In our model with the extended information set, the choice of instrumental variables is largely based on comparable literature and trial and error, to reach a significant model. Thus, the degree of relevance of the instruments is uncertain. Weak instruments implies that they are weakly correlated with the endogenous regressors. In this case, GMM estimations can perform poorly in finite samples. Following literature from Bun and Windmeijer (2010), we see that, in the case of weak instruments, the GMM estimations is biased in the direction of the OLS estimations. Therefore, to assess the strength of our instruments we compare the results from our extended estimations to OLS estimation. It is worth noticing that the OLS estimation suffers from endogeneity (i.e. violation of the zero conditional mean for the error term), and its inclusion is solely to help in determining whether we are suffering from weak instruments. The parameters are displayed in table 6.4 below.

**Table 6.4:** Extended and OLS model

	Extended		OLS	
	(3)	(4)	(5)	(6)
	Standard	SR	Standard	SR
$\rho$	0.80*** (0.02)	0.86*** (0.01)	0.82*** (0.03)	0.87*** (0.02)
$\alpha$	-1.11*** (0.35)	-1.60*** (0.52)	-0.99 (0.61)	-1.28 (0.92)
$\beta$	2.37*** (0.12)	2.58*** (0.18)	2.19*** (0.22)	2.31*** (0.34)
$\gamma$	0.90*** (0.06)	1.24*** (0.08)	0.81*** (0.11)	1.07*** (0.19)
$N$	111	111	111	111

Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Estimated models:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ .

Model (3) and (4) is estimated using GMM, applying the extended information set.

Model (5) and (6) is estimated using OLS.

From comparing the related parameters (i.e. model (3) with (5), and model (4) with (6)) we see that there is no evident equality between the parameters. Thus, it appears that we can reject that we have biased GMM estimations with irrelevant instruments. Additionally, Bun and Windmeijer (2010) presents further tests that can be more precise in quantifying a potential relative bias. However, this is defined to be beyond the scope of

our thesis.

### 6.1.2 ADF-test

As mentioned in section 5.3.3, stationarity is critical for robust and valid results. In appendix table A4.1 the values of the test are presented. We have checked the main variables of our model, being the federal funds rate with and without the inclusion of shadow rate, inflation, the output gap and the unemployment gap. The output gap is included as it will be used as a regressor for robustness check in the forthcoming section. In the ADF-test we have added lags to ensure that autocorrelation is not creating biased standard errors, where we have chosen four lags due to our four quarter perspective on inflation.

We can reject the null hypothesis of non-stationarity on our output gap and unemployment gap variable. The variables also show robustness at different lags with both being significant at the 5% level on the first to fourth lag. The test shows that the inflation variable and the variable for effective federal funds rate with and without shadow rate is non-stationary. However, using economic intuition, we believe that the interest rate and inflation is stationary in the long run, as it will converge towards the long-term equilibrium interest rate and inflation target, respectively.

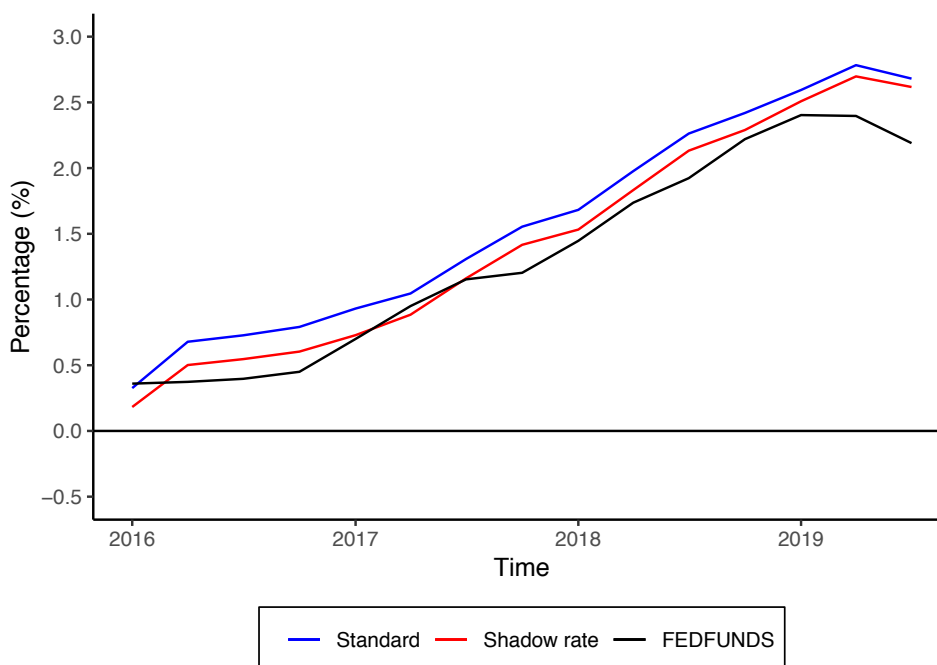
## 6.2 Robustness

The results presented in the previous section relies on assumptions presented throughout our thesis. This implies validity in the use of GMM, the chosen data and the time horizon, see sections 4 and 5.2. To confirm that our results are representative in answering our research question we therefore have to conduct robustness checks. Firstly, we challenge the use of GMM by performing a 1-step iteration estimation. Then, we do estimations where we replace the output gap data with data for unemployment gap. Lastly, we change the time horizon and do additional estimations with adjusted samples. If the results from our robustness checks coincide with the presented results (section 6.1), we can assume robustness.

### 6.2.1 1-step iteration

For an initial robustness check we include the results from a 1-step iteration. Contrary to the n-step iteration this option gives one estimation of  $\hat{\beta}$  and  $\hat{\Omega}$ . It does not converge, thus providing less good finite sample properties, see section 5.3.4 for a more comprehensive description. When comparing the results of the different weighting options we see that the estimations yields comparable results, with minor deviations in the coefficients. The coefficients are presented in appendix A4.2. To see whether changing the updating of the weighting matrix yields different results we display the models graphically.

**Figure 6.2:** 1-step iteration model



As we can see from the graphs, the results are highly comparable to our previous findings when we allowed the coefficients to be re-estimated to numerical convergence. The model that includes the shadow rate fits closest to the federal funds rate on our out-of-sample data.

### 6.2.2 Model with unemployment gap

For robustness purposes we include estimations from a Taylor rule using unemployment gap instead of the output gap data. This data is used by the Federal reserve bank of Atlanta, and existing literature, see for example Taylor (1999) and Orphanides and

Williams (2007), argues its value in interest rate setting. We include this estimation to ensure that our results are valid to different gap measures. We apply the same estimation method and use the same instrumental variables as with our extended model. The results from the GMM estimation are presented in Table 6.5 below, compared to the results from our extended model.

**Table 6.5:** Extended and UEG

	Extended		UEG <sup>1</sup>	
	(3) Standard	(4) SR	(7) Standard	(8) SR
$\rho$	0.80*** (0.02)	0.86*** (0.01)	0.85*** (0.02)	0.87*** (0.01)
$\alpha$	-1.11*** (0.35)	-1.60*** (0.52)	-1.33* (0.73)	-1.11 (0.72)
$\beta$	2.37*** (0.12)	2.58*** (0.18)	2.26*** (0.23)	2.15*** (0.26)
$\gamma$	0.90*** (0.06)	1.24*** (0.08)	0.44*** (0.08)	0.68*** (0.07)
$N$	111	111	111	111
J-stat	17.79	18.77	18.23	19.90
P-value	1.00	0.99	1.00	0.99

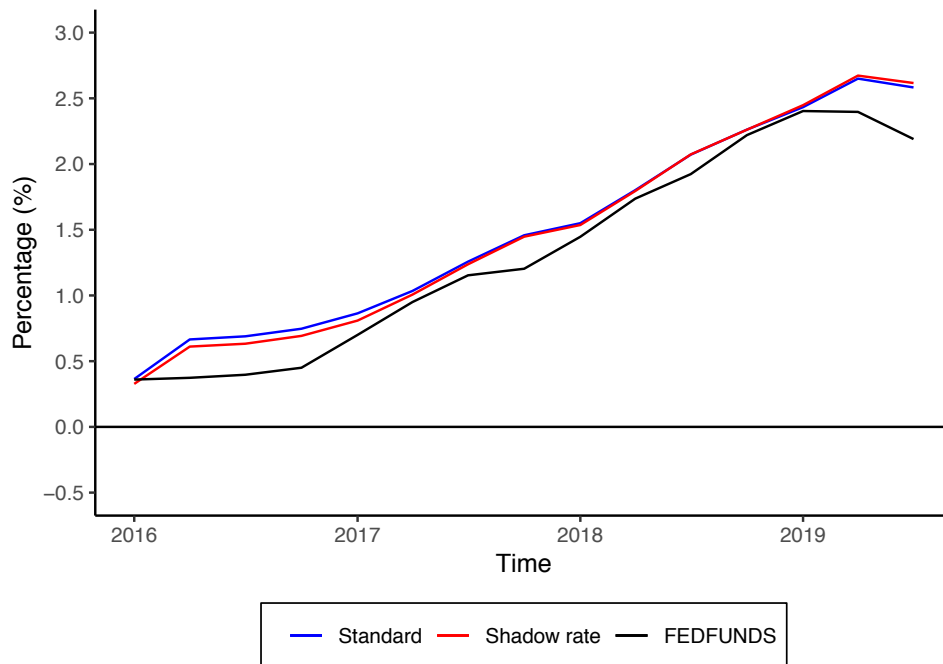
<sup>1</sup> UEG = Unemployment gap

Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Estimated model:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ . For model (3) and (4)  $x =$  output gap, while  $x =$  unemployment gap as a regressor in model (7) and (8).

Model (3) and (7) includes four lags of the effective federal funds rate, while model (4) and (8) includes four lags of the effective federal funds rate and shadow rate for the period it persisted. All models include four lags of inflation, output gap, commodity inflation(PPIACO), the spread between the 10-year Treasury note and the 3-month Treasury bill(T10Y3M), change in housing prices(CSUSHPINSA), return on average equity for all U.S banks(USROE), total public debt(GFDEBTN) and also, the US/UK exchange rate(DEXUSUK)

Firstly, we see that both estimations have a P-value for the J-statistic of about 0.99. Except for the constant, all coefficients are significantly different from zero at a 1-percent level. The constant is significant at the 10-percent level for the standard estimation, while the constant from the estimation with shadow rate is almost significant with a p-value of 0.1276. To compare the models to the out-of-sample effective federal funds rate we view them graphically.

**Figure 6.3:** Robustness: unemployment gap

From the visualization there is no clear indication on which estimation is the better fit. Due to the graphical similarities we include accuracy metrics to determine which yields the best out-of-sample performance.

**Table 6.6:** RMSE and MAE results: unemployment gap

	Standard	SR
RMSE	0.204	0.191
MAE	0.168	0.155

From the accuracy metrics we see that the estimation conducted by including shadow rate yields a slightly better fit. It seems that changing the gap measure leads to inconclusive results regarding which model is the most precise. This poses an interesting point regarding the choice of gap measures by the central bank. If the Federal Reserve uses unemployment gap as their measure, our findings suggests that including shadow rate yields minor out-of-sample performance improvements. On the contrary, it is worth noticing that the parameters displays higher standard errors, and neither  $\alpha$ -parameters are significant at the five percent level. Hence, the out-of-sample comparisons yields less conclusive findings than our previous finding using output gap. Furthermore, from our accuracy metrics we see that the estimation using shadow rate yields a better out-of-sample fit. This implies

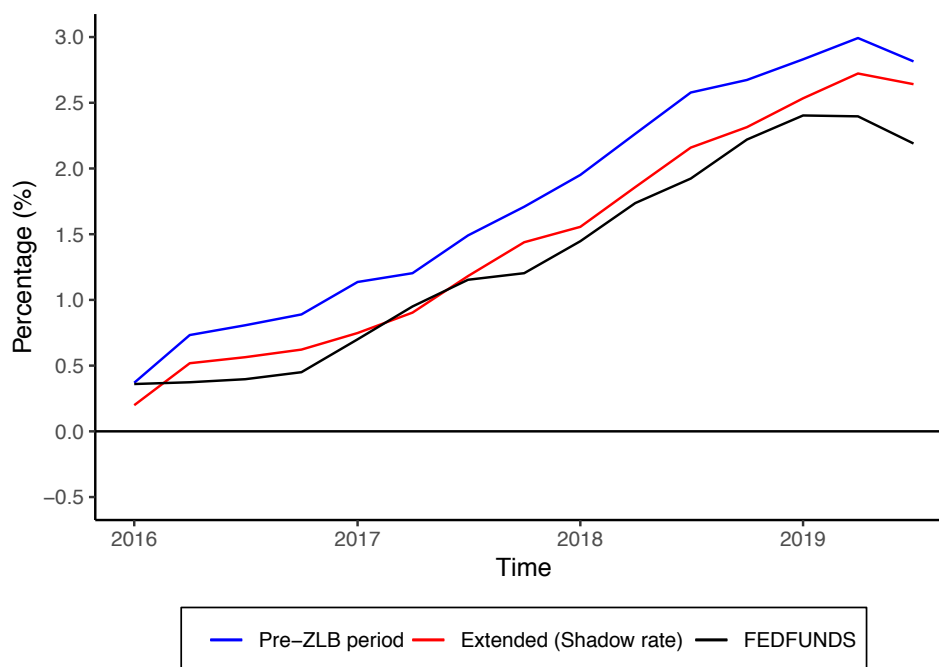
that the results are comparable to the estimation from section 6.1.

## 6.2.3 Models with different horizons

### 6.2.3.1 ZLB sub-period

In section 2.4 we claimed that research has focused on the pre ZLB-period to summarize the effects of monetary policy. This implies that we should view the findings from our extended model compared to a pre ZLB-estimation. Thus, we estimate an adjusted sample ending in 2008Q4, i.e. at the start of the zero lower bound. For robustness, we present the resulting output graphically, compared to our extended model with shadow rate.

**Figure 6.4:** Robustness: ZLB sub-period



From the figure we see that our extended model yields a more precise estimation. Thus, we can confirm that our findings are robust to analyses excluding the ZLB-period. The related parameters are presented in appendix A4.3.

### 6.2.3.2 Adjusted data samples

When deciding the period to start our estimation we used the structural break following the policies of the newly elected chairman of the federal reserve in 1987. Judd et al. (1998) and Hamalainen et al. (2004) estimates policy rules for different sample periods. They

argue that  $\alpha$  and  $\beta$  may be sensitive to the current policy regime. They find that the Federal Reserve has reacted differently over time to inflation and output gap. To ensure that our results are robust to different time horizons we adjust the start and end point of our estimation sample. We use the recession indicators from the federal funds rate data to set starting points for robustness estimates, indicating alternative structural breaks. From the data we view recessions ending in 1991Q2 and 2001Q4. Thus, we use these as starting points for our adjusted samples. Furthermore, we wish to include an estimation that accounts for more data surpassing the ZLB-period. Hence, we include an estimation where we set the end of the sample to 2016Q4, one year later than our original estimation. The results from the GMM estimations is presented in table 6.7 below.

**Table 6.7:** Adjusted samples

	(1991Q2-2015Q4)		(2001Q4-2015Q4)		(1988Q2-2016Q4)	
	(9)	(10)	(11)	(12)	(13)	(14)
	Standard	SR	Standard	SR	Standard	SR
$\rho$	0.83*** (0.01)	0.88*** (0.01)	0.87*** (0.01)	0.91*** (0.01)	0.81*** (0.02)	0.86*** (0.01)
$\alpha$	-1.51*** (0.44)	-1.62* (0.67)	-1.53*** (0.51)	3.43*** (0.92)	-1.26*** (0.36)	-1.50*** (0.56)
$\beta$	2.48*** (0.18)	2.57*** (0.26)	2.18*** (0.23)	0.12 (0.43)	2.41*** (0.12)	2.55*** (0.20)
$\gamma$	0.88*** (0.07)	1.27*** (0.09)	0.68*** (0.05)	1.66*** (0.09)	0.91*** (0.06)	1.24*** (0.08)
$N$	99	99	57	57	115	115
J-stat	19.23	20.85	13.48	14.32	18.57	18.77
P-value	0.99	0.99	1.00	1.00	1.00	0.99

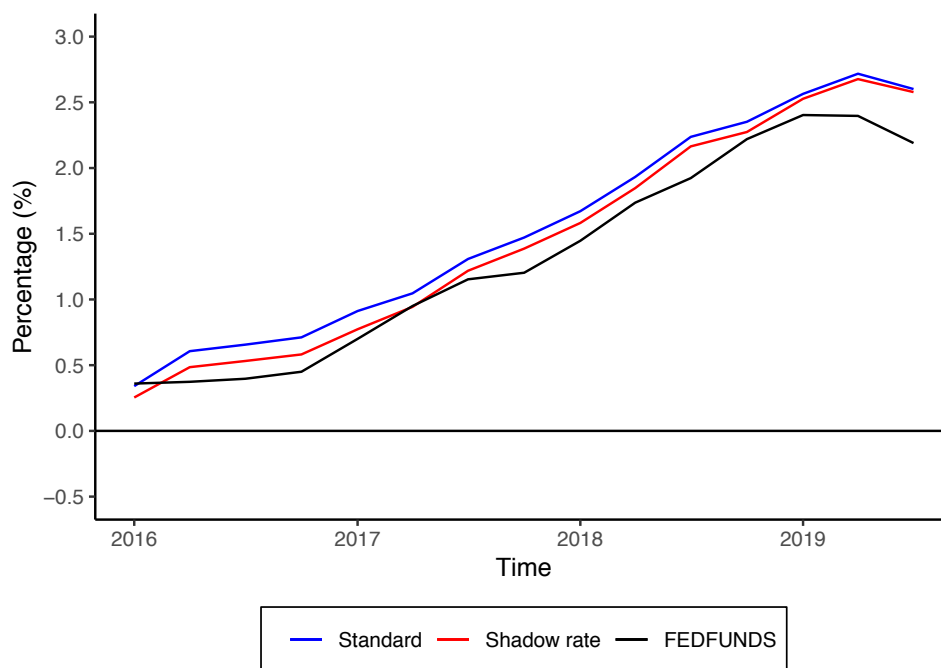
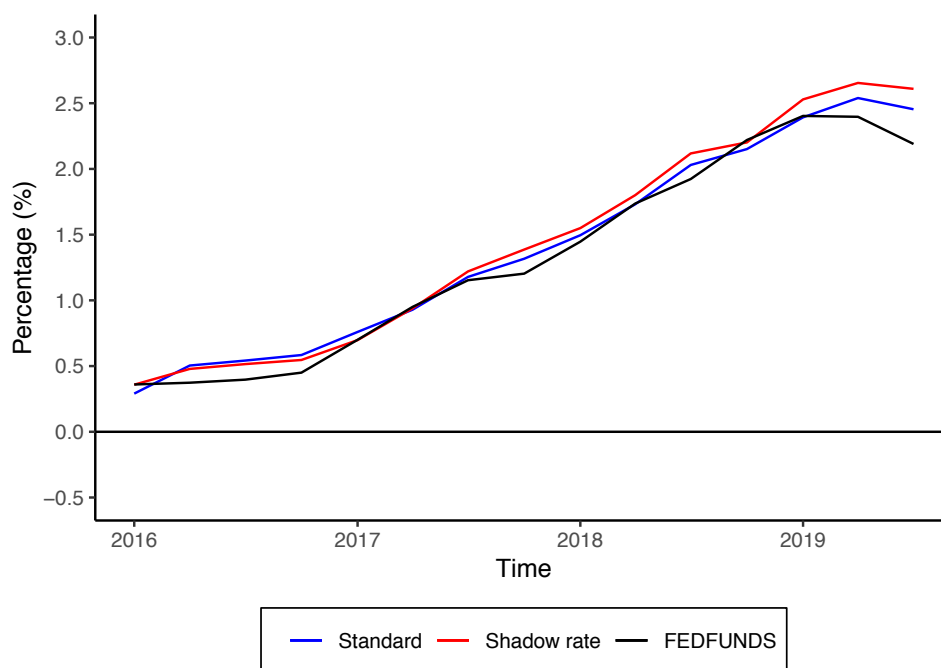
Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

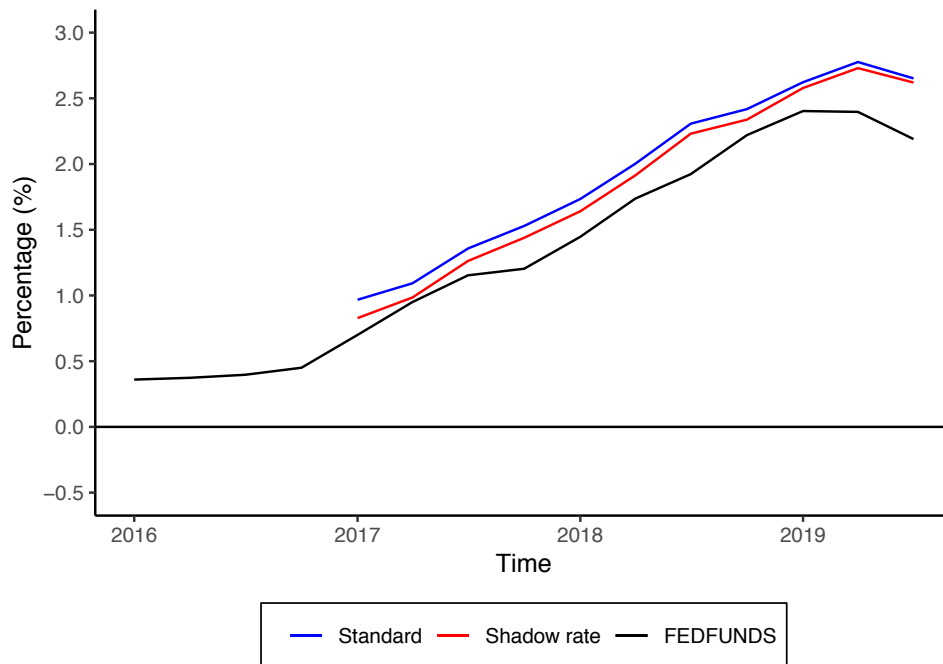
Estimated model:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ .

Model (9), (11) and (13) includes four lags of the effective federal funds rate, while model (10), (12) and (14) includes four lags of the effective federal funds rate and shadow rate for the period it persisted. All models include four lags of the effective federal funds rate, inflation, output gap, commodity inflation(PPIACO) and the spread between the 10-year Treasury note and the 3-month Treasury bill(T10Y3M), change in housing prices(CSUSHPINSA), return on average equity for all U.S banks(USROE), total public debt(GFDEBTN) and also, the US/UK exchange rate(DEXUSUK) as instrumental variables.

All estimations appear to be valid and the coefficients are of reasonable values. All coefficients are significant except for the one relating to inflation from the estimation starting in 2001Q4. To determine whether our results are robust to changes in the sample horizon we display the estimations graphically. The graphs are displayed below.



**Figure 6.5:** Robustness: adjusted sample 1991Q2 - 2015Q4**Figure 6.6:** Robustness: adjusted sample 2001Q2 - 2015Q4

**Figure 6.7:** Robustness: adjusted sample 1988Q2 - 2016Q4

From the graphical visualizations we can see that in two of the three estimations, including the shadow rate results in more precise estimations. In our adjusted sample starting in 2001Q2 we see that the regular estimation fits better. However, as we saw in table 6.7, the  $\beta$ -coefficient is insignificantly different from zero. Additionally, the estimation is conducted using only 57 observations, which could imply ambiguous results. Thus, it is difficult to draw conclusions from the deviations between those two estimations. The accuracy metrics for the adjusted samples are presented in appendix A4.4, A4.5 and A4.6. Generally, our results implies that the findings in section 6.1 are representative for a broader spectrum of sample horizons than initially presented.

## 6.3 Limitations

Before presenting our concluding remarks it is beneficial to include critique and possible limitations to our research. Our findings is based on assumptions regarding the shadow rate, and its validity in describing the stance of monetary policy, presented in sections 2.4 and 4.1.2. This implies that our conclusions will be limited by the efficacy of the shadow rate. The fact that the employed shadow rate is estimated, and not observed like the effective federal funds rate, poses a potential limitation regarding the correctness of the

rate and presented standard errors. Our estimations are based on the assumption that the estimated shadow rate are highly comparable to the observed federal funds rate, which is questionable. Hence, more research on the efficacy of the shadow rate by Wu and Xia would imply more reliable results.

The estimations conducted using GMM relies on valid and relevant instruments. Our choice of instrumental variables in the extended information set is based on comparable literature and a process of trial and error. Furthermore, our model validity is largely based on the tests of the orthogonality restrictions. However, literature suggests that we could be suffering from weak, but many instruments, see section 6.1.1. We test our results against an OLS estimation, but further quantitative testing may be necessary to confirm strong instruments. Thus, our instruments may be weakly correlated with the endogenous variables, i.e. low instrument relevance. This would impose challenges to inference from our GMM estimations.

Next, our robustness check using unemployment gap yielded inconclusive results. Further research may be necessary to conclude whether employing shadow rate in a Taylor rule with unemployment gap is beneficial. There are also critiques to the general validity of Taylor rule estimations on monetary policy (e.g. Siklos and Wohar (2006)). But, due to the scope of our thesis, we do not view this as limitations to our findings.

## 7 Conclusion

In this thesis we have estimated Taylor rules using different datasets to determine whether the inclusion of shadow rate for federal funds data improves the estimations. We have based our model selection on the work of Clarida, Gali & Gertler (1998, 2000). To ensure robustness we changed the horizon of our estimation, included a 1-step iteration and changed the gap parameter. Due to the structural break following the replacement of the chairman of the federal reserve, we started our sample period in 1987Q2. We used the period from 2016Q1-2019Q3 to test the accuracy of our estimations.

From the comparison of the baseline and extended models we saw that including shadow rate yielded a lower effective coefficient for inflation, while the coefficients for output gap were fairly similar. This was also the case when we changed the model input for robustness purposes. Most of the robustness checks implied validity in our extended results, as they showed improvements from including the shadow rate. The estimation with the shortest adjusted sample did not contribute to our extended results, but insignificance in the parameter makes the estimation more negligible.

To determine which estimations that most accurately described the federal funds rate we used the RMSE- and MAE-criterion. The results were that the estimation conducted using shadow rate gave the most accurate results. However, the two estimation pairs displayed similar properties, as one would expect considering the majority of the data is the same, meaning that the results were somewhat inconclusive. The resulting coefficients were significant, and we could reject the J-test for overidentifying restrictions for both estimations. In addition, the conducted ADF-test showed that both our inflation and federal funds rate data was non-stationary. However, we believe that both these measures converge towards their long-term equilibrium targets.

Furthermore, our results contributes to the findings of Wu and Xia (2016), who advocated the value of the shadow rate in long-term macroeconomic models. Our analyses suggests that shadow rate can be used as a tool to view monetary policy in a Taylor rule setting, using data with the presence of a ZLB-period.

Our goal was to figure out whether there was valuable information in including the implied interest rate during the ZLB when estimating the Taylor rule. By this standard, we can

conclude that there is value in inclusion of the shadow rate.

As mentioned in section 3.1, Taylor stated that the policy rule should not be followed mechanically, but rather function as an indicator. With this in mind, the findings in this thesis suggests that including shadow rate data could bring a more comprehensive indication of monetary policy.

The findings from this thesis validates the usefulness of shadow rate in Taylor rule estimation. Few models have incorporated shadow rate in macroeconomic analyses. Including shadow rate to account for monetary policy during a ZLB-period in further macroeconomic models are left for future research.

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# Appendix

## A1 Data appendix

***FEDFUNDS*** - Quarterly averages of daily figures of the effective federal funds rate. The data is provided by the Board of Governors of the Federal Reserve System.

***Shadow rate*** - Quarterly averages of monthly figures of the shadow rate calculated by Wu and Xia (2016). The data is provided by the Federal Reserve Bank of Atlanta.

***Expected inflation*** - For expected inflation the forecasted value of four-quarter PCE inflation three quarters ahead is used. It is a measure of the PCE inflation of the current quarter together with the median forecast of the subsequent three quarters. It is provided by the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters.

***(GDPC1) & (GDPPOT)*** - The output gap measured as the percentage difference between potential gross domestic product and real gross domestic product. Potential GDP is an estimate of the output the economy would produce with a high rate of use of its capital and labor resources, adjusted for inflation. Real GDP is the inflation adjusted value of the goods and services produced by labor and property located in the United States. The data for real GDP and potential GDP is provided by the U.S. Bureau of Economic Analysis and the U.S. Congressional Budget Office, respectively.

***PPIACO*** - Commodity price inflation measured as average change over time in the selling prices received by domestic producers of goods and services. The sample goods and services included in the index is weighted by their size and importance. The U.S. Bureau of Labor Statistics provides the index.

***CSUSHPINSA*** - The SP/Case-Shiller Home Price Indices is an quarterly average of monthly data on the value of single-family housing within the United States. Given a constant level of quality, the indices measure percentage changes in housing market prices.

***T10Y3M*** - Quarterly averages of daily figures of the spread between 10-Year Treasury Constant Maturity and 3-Month Treasury Constant Maturity. The primary data is obtained from the U.S. Treasury Department.

***USROE***- Quarterly frequency of the change in ratio between net income and average of total equity capital of all US banks. This constitutes the return on average equity. The raw data is collected by the Federal Financial Institutions Examination Council and structured by the Federal Reserve Bank of St. Louis.

***GFDEBTN*** - Quarterly(end of period) frequency of change in total public debt for the U.S. The data is collected by U.S. Department of the Treasury.

***DEXUSUK*** - The US/UK foreign exchange rate displays the change in number of U.S dollars for one British Pound. The data exhibits the quarterly average of daily figures, and is provided by the Board of Governors of the Federal Reserve System.

## A2 Shadow rate estimation

Following the research by Wu and Xia (2016) we derive the shadow rate. They initially assume that the short term interest rate is the maximum of the shadow rate  $s_t$  and a lower bound  $\underline{r}$ :

$$r_t = \max(\underline{r}, s_t) \quad (\text{A.1})$$

Next, they assume that the shadow rate is an affine function of some state variables  $X_t$ ,

$$s_t = \delta_0 + \delta_1' X_t \quad (\text{A.2})$$

which follows a first order vector autoregressive process:

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I) \quad (\text{A.3})$$

Following existing literature they view the log stochastic discount factor as affine

$$M_{t+1} = \exp \left( -r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right) \quad (\text{A.4})$$

where the price of risk  $\lambda_t$  being linear in the factors

$$\lambda_t = \lambda_0 + \lambda_1 X_t$$

Implying the risk neutral measure  $\mathbb{Q}$  dynamics for the factors are also first order vector autoregressive:

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}, \quad \varepsilon_{t+1}^{\mathbb{Q}} \stackrel{\mathbb{Q}}{\sim} N(0, I) \quad (\text{A.5})$$

The parameters are related as follows:

$$\mu - \mu^{\mathbb{Q}} = \Sigma \lambda_0,$$

$$\rho - \rho^{\mathbb{Q}} = \Sigma \lambda_1.$$

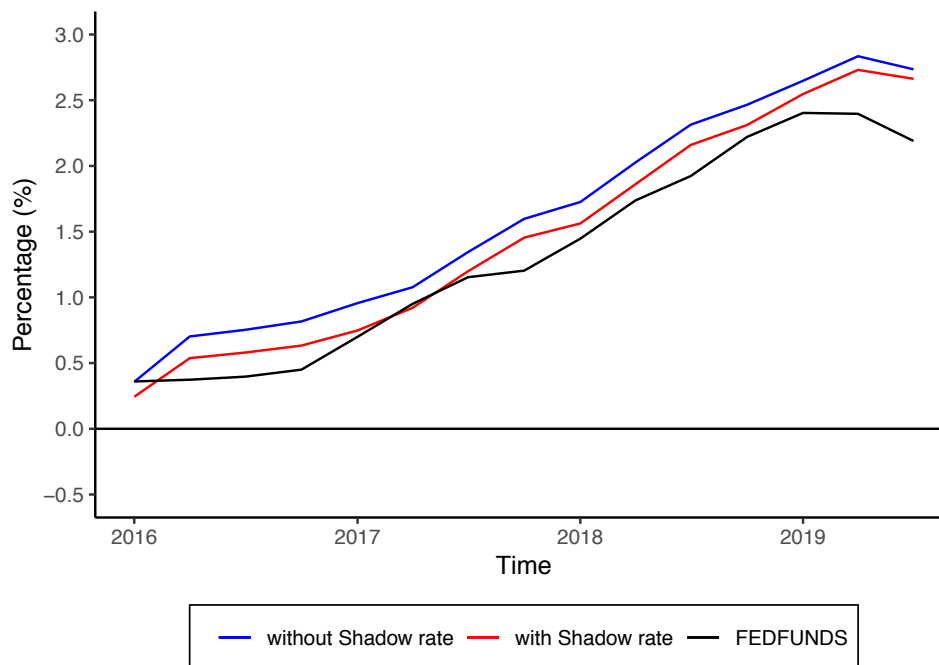
Lastly, they propose an analytical approximation for the forward rate in the SRTSM. Followingly, they define  $f_{n,n+1,t}$  as the forward rate at time  $t$  for a loan starting at  $t+n$  maturing at  $t+n+1$ ,

$$f_{n,n+1,t} = (n+1)y_{n+1,t} - ny_{nt} \quad (\text{A.6})$$

Final approximation is shown as equation 4.1 in section 4.1.2. Lastly, they use the extended Kalman filter, as the observation equation is non-linear in the factors. The filter linearize the non-linear function  $g(\cdot)$  around the current estimates. See Wu and Xia (2016) for more details.

## A3 Figures

Figure A3.1: Baseline model



## A4 Tables

Table A4.1: Augmented Dickey-Fuller (ADF) test on regressors

Lags	(0)	(1)	(2)	(3)	(4)	Constant
FEDFUNDS	-0.77	-2.03	-2.09	-2.14	-1.94	Yes
Shadow rate	-0.75	-1.89	-2.17	-2.08	-1.84	Yes
Expected inflation	-2.52	-2.32	-1.85	-1.71	-1.77	Yes
Output gap	-1.33	-1.77*	-2.07**	-2.03**	-1.97**	No
Unemployment gap	-0.8519	-2.07**	-2.43**	-2.41**	-2.07**	No

\* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Value of test statistics are presented in the table.

**Table A4.2:** Extended and 1-step

	Extended		1-step	
	(3)	(4)	(15)	(16)
	Standard	SR	Standard	SR
$\rho$	0.80*** (0.02)	0.86*** (0.01)	0.80*** (0.03)	0.86*** (0.02)
$\alpha$	-1.11*** (0.35)	-1.60*** (0.52)	-1.27** (0.58)	-1.78* (0.91)
$\beta$	2.37*** (0.12)	2.58*** (0.18)	2.39*** (0.21)	2.61*** (0.33)
$\gamma$	0.90*** (0.06)	1.24*** (0.08)	0.85*** (0.10)	1.22*** (0.18)
$N$	111	111	111	111

Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Estimated model:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ .

Model (3) and (15) includes four lags of the effective federal funds rate, while model (4) and (16) includes four lags of the effective federal funds rate and shadow rate for the period it persisted. All the models include inflation, output gap, commodity inflation(PPIACO) and the spread between the 10-year Treasury note and the 3-month Treasury bill(T10Y3M) as instrumental variables. Model (3) and (4) also includes change in housing prices(CSUSHPINSA), return on average equity for all U.S banks(USROE), total public debt(GFDEBTN) and also, the US/UK exchange rate(DEXUSUK).

**Table A4.3:** Extended model and Pre-ZLB period

	Extended (SR)	Pre-ZLB period
	(4)	(17)
$\rho$	0.86*** (0.02)	0.73*** (0.01)
$\alpha$	-1.60*** (0.52)	-1.47*** (0.15)
$\beta$	2.58*** (0.18)	2.54*** (0.04)
$\gamma$	1.24*** (0.08)	1.07*** (0.03)
$N$	111	83
J-stat	18.77	19.70
P-value	0.99	0.99

Standard errors in parentheses. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

Estimated model:  $i_t = (1 - \rho)(\alpha + \beta E[\pi_{t,k}] + \gamma [x_t]) + \rho i_{t-1} + \epsilon_t$ .

Models are estimated using GMM, applying the extended information set.

**Table A4.4:** RMSE and MAE results: 1991Q2-2015Q4

	Standard	SR
RMSE	0.237	0.171
MAE	0.217	0.143

**Table A4.5:** RMSE and MAE results: 2001Q4-2015Q4

	Standard	SR
RMSE	0.112	0.160
MAE	0.090	0.118

**Table A4.6:** RMSE and MAE results: 1988Q2-2016Q4

	Standard	SR
RMSE	0.299	0.231
MAE	0.285	0.204

Table A4.7: Results from GMM estimations.

	Baseline		Extended		UEG <sup>1</sup>		(1991Q2-2015Q4)		(2001Q4-2015Q4)		(1988Q2-2016Q4)	
	(1) Standard	(2) SR	(3) Standard	(4) SR	(7) Standard	(8) SR	(9) Standard	(10) SR	(11) Standard	(12) SR	(13) Standard	(14) SR
$\rho$	0.80*** (0.03)	0.87*** (0.02)	0.80*** (0.02)	0.86*** (0.01)	0.85*** (0.02)	0.87*** (0.01)	0.83*** (0.01)	0.88*** (0.01)	0.87*** (0.01)	0.91*** (0.01)	0.81*** (0.02)	0.86*** (0.01)
$\alpha$	-0.88 (0.55)	-0.67 (1.00)	-1.11*** (0.35)	-1.60*** (0.52)	-1.33* (0.73)	-1.11 (0.72)	-1.51*** (0.44)	-1.62* (0.67)	-1.53*** (0.51)	3.43*** (0.92)	-1.26*** (0.36)	-1.50*** (0.56)
$\beta$	2.29*** (0.20)	2.20*** (0.39)	2.37*** (0.12)	2.58*** (0.18)	2.26*** (0.23)	2.15*** (0.26)	2.48*** (0.18)	2.57*** (0.26)	2.18*** (0.23)	0.12 (0.43)	2.41*** (0.12)	2.55*** (0.20)
$\gamma$	0.93*** (0.10)	1.28*** (0.16)	0.90*** (0.06)	1.24*** (0.08)	0.44*** (0.08)	0.68*** (0.07)	0.88*** (0.07)	1.27*** (0.09)	0.68*** (0.05)	1.66*** (0.09)	0.91*** (0.06)	1.24*** (0.08)
$N$	111	111	111	111	111	111	99	99	57	57	115	115
J-stat	13.88	13.27	17.79	18.77	18.23	19.90	19.23	20.85	13.48	14.32	18.57	18.77
P-value	0.68	0.72	1.00	0.99	1.00	0.99	0.99	0.99	1.00	1.00	1.00	0.99

<sup>1</sup> UEG = Unemployment gap

Standard errors in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

Estimated model:  $i_t = (1 - \rho)\alpha + \beta E[\pi_{t,k}] + \gamma [x_t] + \rho i_{t-1} + \epsilon_t$ . For model (1)-(11)  $x =$  output gap, while  $x =$  unemploymentgap as a regressor in model (7) and (8). Model (1), (3), (7), (9), (11) and (13) includes four lags of the effective federal funds rate, while model (2), (4), (8), (10), (12) and (14) includes four lags of the effective federal funds rate and shadow rate for the period it persisted. Model (1) and (2) include four lags of inflation, output gap, commodity inflation(PPIACO) and the spread between the 10-year Treasury note and the 3-month Treasury bill(T10Y3M) as instrumental variables. Model (3)-(12) include the mentioned instrumental variables as well as change in housing prices(CSUSHPINSA), return on average equity for all U.S banks(USROE), total public debt(GFDEBTN) and also, the US/UK exchange rate(DEXUSUK) as instrumental variables.