# Essays on game theory and fisheries management

**Evangelos Toumasatos**

Dissertation submitted for the degree of Philosophiae Doctor in Economics
Department of Business and Management Science, Norwegian School of Economics
Bergen, Norway, March 2020

No man ever steps in the same river twice, for it is not the same river and he is not the same man.

Heraclitus of Ephesus, ca. 540-480 BCE

# Contents

# List of Figures

# List of Tables

# Acknowledgements

I would have never embarked on that journey had it not been for one of my supervisors, Stein Ivar Steinshamn. I am truly grateful to him for introducing me to the academic world and for his constant support, advice and guidance throughout the PhD program. They say that it is the journey that matters, and not the destination. And this journey would have never been the same without the presence of my co-supervisor Leif Kristoffer Sandal. I am thankful for all the times he challenged me and for always being patient with my ignorance and mistakes. I would also like to express my gratitude to my external supervisor Marko Lindroos of the University of Helsinki. His work on game theory and fisheries management has been a source of inspiration and motivation for this dissertation. I can not imagine how I would have managed to achieve my goal without all of you showing me the way, and for that I thank you from the bottom of my heart!

I would also like to thank all of my colleagues both at SNF and NHH for making this journey such a great experience. Special thanks to Gunnar Eskeland, Sturla Kvamsdal, Nils-Arne Ekerhovd, Frode Skjeret, Alexander Jakubanecs and Mario Guajardo for many interesting and stimulating discussions on several topics. I am thankful to Svenn-Åge Dahl for welcoming me and making me feel part of SNF. Tusen takk for det! Many thanks to Kristin, Natalia, Charlotte, Turid, Lis, Anne-Guri, Cathrine and the rest of the administration for their help.

Journeys are a great setting for making new and strengthen old friendships. I owe a special thank you to Evangelos, Ondřej, Henrik, Atle, Gabriel, Yuanming, Somayeh, Vit, Rezvan, Nahid, Ritvana, Yewen, and Kyriaki, among others. It is a privilege to have become acquainted with you.

Finally, I would like to express my deepest gratitude to my wife, family and friends back home for their continuous support and encouragement, although they still do not have a clue what I am doing! This dissertation is dedicated to them.

Evangelos Toumasatos

Bergen, Norway, March 2020

# Introduction

The purpose of this thesis is to investigate various game theoretic aspects of fisheries management. Game theory is the study of strategic interaction among rational decision makers. It employs mathematics to describe, explain and predict outcomes in situations where a conflict of interest exists. A game theoretic model consists of a set of interacting players, a set of strategies available to those players, and a specification of a reward function, also known as payoff, for each player and all combinations of strategies. Game theory dates back to 1944, the year when John von Neumann and Oskar Morgenstern published the classic book titled: *Theory of Games and Economic Behavior*. However, it was the fundamental work of John Nash on non-cooperative (Nash, 1951) and cooperative (Nash, 1953) games that made game theory more popular and acceptable among economists.[1] Since then, it has become a standard tool of economic analysis in many subfields, including fisheries economics and management.

The integration of game theory in the economics of the management of fisheries resources became more apparent with the advent of the 1982 United Nations Convention on the Law of the Sea (UNCLOS) and the establishment of the exclusive economic zone (EEZ) regime, despite the fact that fisheries economics have received attention almost thirty years earlier with the publication of H. Scott Gordon's seminal article in 1954, "The economic theory of a common property resource: the fishery". As Grønbæk et al. (2020, p.2) put it: "The evolution of the relevance and application of game theory to the economics of capture fisheries management follows the evolution, although not precisely, of the economic management of capture fisheries." And it was not until the latter part of the last century that game theory became an indispensable part of fisheries research.

Until the early twentieth-century, there was the belief that the best course of action regarding fisheries management was unrestricted management as resources were thought to be inexhaustible. This behaviour can be attributed to the state of fishing technology at that time, which made it very costly to significantly deplete the resources. As fishing technology improved, costs declined and fishing increased, and by the late 1930s what was once seen as inexhaustible it turned out to be exhaustible. The early literature on fisheries economics (Gordon, 1954; Schaefer, 1957; Smith, 1969; Clark 1973; Clark and Munro, 1975) explored the economic consequences of fisheries exploitation characterised by perfectly competitive fishing agents, also known as open-access, and the exact opposite where the fishery is managed by a single agent, referred as a sole-owner.

The collapse of many commercial fisheries, e.g., the North Atlantic herring fisheries in the 1960s and 1970s, has signalled the need to regulate fishing activity. It took three UN conventions in 1956-58, 1960, and 1973-82 before fundamentally changing the management of world marine captured fisheries by recognizing property rights through the establishment of EEZs (Hannesson, 2004). This regime change has placed overnight al-

[1] John Nash was awarded the Nobel Prize in economics in 1994 for his work on game theory.

most 90% of the marine resources worldwide in the control of coastal states (Bjørndal and Munro, 2012).

It was during the last UN convention, when the first article unifying fisheries management and game theory was written by Gordon R. Munro in 1979. As Bailey et al. (2010, p.2) write: "The author was motivated to write his seminal paper by the increasing acceptance of extended fisheries jurisdiction which he believed would, and in fact did, lead to increased management of fisheries by individual coastal states." In the article titled "The optimal management of transboundary renewable resources", Munro (1979) investigated the optimal outcome in a fishery jointly owned by two coastal states with different preferences and fishing costs. The purpose of his study was to address the requirements needed for a cooperative fisheries agreement to be stable in time. A year after, Levhari and Mirman (1980) and Clark (1980) published two more influential game theoretic papers on fisheries management. Both papers delved on the consequences of sharing a fishery resource without cooperating in its management. The four authors have paved the way for two major strands of the literature, one dealing with cooperative solutions and the other with competitive, Nash-Cournot outcomes (Hannesson, 2011).

It has been four decades since then, and the literature on game theory and fisheries management has seen a considerable growth. Many types and variations of fishery games have been explored both on a theoretical and applied basis (see Bailey et al., 2010 and Hannesson, 2011 for comprehensive reviews). From the early two-player fishery games (Sumaila, 1999 and references therein) to multi-player coalitional fishery games (Kaitala and Lindroos, 2007 and references therein). From games with a single stage structure to multi-stage and sequential games (Hannesson, 1995; Kronbak and Lindroos, 2006).

This thesis is organised into three self contained chapters that fit well under the research umbrella of game theory and fisheries management. A fishery game with unique characteristics and structure is presented in each chapter. In chapter 1, coalition formation in the mackerel fishery is investigated. In chapter 2, a dynamic multi-stage game with two types of players where the stock dynamics follow a seasonal pattern is analysed. In chapter 3, a framework for quantifying the basis upon which fisheries agreements are being drawn up is proposed based on a static three-stage game with four interacting agents. Although the setting and structure of the three games differ and are not directly comparable, the underlying bioeconomic models develop progressively mainly in terms of the market structure. In the first chapter, the selling price of the resource is exogenous. In the second chapter, an endogenous and non-linear price specification is assumed. Both models assume the existence of a single resource market. This assumption is relaxed in the last chapter, where players have the option to choose between two markets. The assumption of endogenous prices is retained but the functional relationship is assumed linear. A brief description of each chapter follows.

In the first chapter, we draw from the literature on coalitional games and in particular on the ones with externalities also known as partition function games (Thrall and Lucas, 1963; Yi, 1997; Pintassilgo, 2003). This class of games is based on the notion of coalition structure, i.e., the partition of players in coalitions where the economic performance of a coalition is affected by the collective behaviour of all other coalitions. This means that the payoff of a coalition depends on the coalition structures, which gives rise to free-riding incentives.

The partition function approach is applied to the Northeast Atlantic mackerel fishery. The motivation is to study the degree of cooperation before and after Brexit between the European Union (EU), the United Kingdom (UK), Norway, the Faroe Islands and Iceland.

We find that in the absence of Brexit, the current management regime at that time is a stable outcome in all scenarios tested, whereas after Brexit, only in one. This implies that it is very highly, post-Brexit, that the UK will set its mackerel quota unilaterally, in the same spirit as Iceland has been doing. This will further increase the pressure on the mackerel stock. However, it will most likely not go unpunished, since both the EU and Norway can respond harshly by introducing trade sanctions, as they have already done to Icelandic and Faroese catches in 2013.

The bioeconomic model applied in the first chapter is a generalisation of the annual stock-recruitment model introduced by Clark (1973). The model is in discrete time between periods but continuous within them. In addition, it is linear in the control variable, i.e., harvest, which follows from the assumption that the demand for fish is infinitely elastic, i.e., price is fixed, and the specification of fishing costs.

In the second chapter, we apply a more detailed bioeconomic model to address the consequences of non-cooperation in fisheries that exhibit periodic or seasonal variations, like Arctic Cod, Atlantic Mackerel, Norwegian spring-spawning Herring, etc. Seasonality is an important feature of many commercial fisheries since both biological processes and human activities occur on a seasonal instead of an annual basis, as is often assumed. Besides the inclusion of multiple seasons of differing length and biomass dynamics, demand functions in each season are endogenous and non-linear.

Our approach expands the seasonal model of Ni and Sandal (2019) by allowing for non-cooperative behaviour between two types of players: i) an incumbent leader, and ii) multiple asymmetric potential entrants (followers). The game is dynamic and sequential in the sense that the leader acts first. The feedback Nash equilibrium for the $n$-follower game is derived analytically and used as input into the optimisation process of the leader. A numerical scheme based on recursion is then applied to derive the dynamic feedback policies of the leader. The results are compared to the benchmark case without strategic interaction. In presence of multiple followers, the leader adopts a more aggressive fishing strategy in all seasons. As a consequence, entry for some followers is delayed or not even realised. This increases the pressure on the stock and therefore the long-term biomass is reduced. In addition, there is an almost 50% value reduction for the leader along the state space, implying rent dissipation.

In the third and last chapter, an attempt to better understand and quantify the basis upon which fisheries agreements are being drawn up is made. Since the establishment of the EEZ regime, a number of nations have entered into bilateral agreements over access to fishing stocks that occurred beyond their sovereignty. Today the most known, perhaps, agreements of such type are the so-called sustainable fisheries partnership agreements (SFPAs) between the EU and non-EU coastal states, like Mauritania, Maroco, etc. SFPAs, which were introduced during the latest common fisheries policy (CFP) reform in 2013, allow EU vessels to fish in the signatory countries' EEZs, and in exchange, the EU provides both financial and sectoral support towards the partner countries.

A game theoretic model is proposed where a country with some sort of property right over a fishing resource is faced with the following dilemma: freely grant fishing quotas to a domestic firm or sell them to a foreign agent in return for an endogenously determined price. All purchased quotas are granted to the foreign firm. Both firms exploit the resource according to their quotas and have the option to sell their harvest in two markets, one at home and one abroad. To focus on the strategic interaction between the players, we disregard the problem of optimal fishing, and assume that for any fixed period of time the total allowable catch (TAC) is exogenous. This means that the problems of how much to fish and who should fish can be dealt and analysed separately. Once all strategic outcomes are identified, it is possible to determine the optimal fishing policy by optimising over them. This is illustrated at the end of the chapter by allowing the TAC to be endogenous. Besides the sequential structure of the game, which consists of three stages when the TAC is exogenous and four otherwise, the complexity of the model stems from the inclusion of a second market where the resource can be sold and the fact that prices in both markets are endogenously determined.

# References


Bailey, M., Sumaila, U. R. and Lindroos M. (2010). Application of game theory to fisheries over three decades, *Fisheries Research*, Vol. 102, 1-8.

Bjørndal, T. and Munro, G. (2012). *The economics and management of world fisheries*, Oxford: Oxford University Press.

Clark, C. W. (1973). Profit Maximization and the Extinction of Animal Species, *The Journal of Political Economy*, Vol. 13, 149-164.

Clark, C. W. and Munro, G. R. (1975). The economics of fishing and modern capital theory: a simplified approach, *Journal of environmental economics and management*, Vol. 2, 92-106.

Clark, C. W. (1980). Restricted access to common-property fishery resources: a game-theoretic analysis. In Liu, P.-T. (Ed.), *Dynamic Optimization and Mathematical Economics*. Plenum Press, pp. 117–132 (Chapter 7).

FAO (2003). *Code of Conduct for Responsible Fisheries*, Rome.

Gordon, H. S. (1954). The economic theory of a common-property resource: the fishery, *Journal of Political Economy*, Vol. 62, 124-42.

Grønbæk, L., Lindroos, M., Munro, G. and Pintassilgo, P. (2020). *Game Theory and Fisheries Management*, Springer Books.

Hannesson, R. (1995). Sequential fishing: cooperative and non-cooperative equilibria. *Natural Resource Modeling*, Vol. 9, 51-59.

Hannesson, R. (2004). *The Privatization of the Oceans*, The MIT Press.

Hannesson, R. (2011). *Game theory and fisheries*, *Annu. Rev. Resour. Econ.*, Vol. 3, 181-202.

Kaitala, V. and Lindroos, M. (2007). Game theoretic applications to fisheries, In Weintraub, A., Romero, C., Bjørndal, T. and Epstein, R. (Eds.) *Handbook of operations research in natural resources*, Springer.

Kronbak, L. G. and Lindroos, M. (2006). An enforcement-coalition model: fishermen and authorities forming coalitions, *Environmental and Resource Economics*, Vol. 35, 169-194.

Levhari, D. and Mirman, L. J. (1980). The great fish war: an example using a dynamic Cournot-Nash solution, *The Bell Journal of Economics*, Vol. 11, 322-344.

Morgenstern, O. and Von Neumann, J. (1953). *Theory of Games and Economic Behavior*, Princeton university press.

Munro, G. R. (1979). The optimal management of transboundary renewable resources, *Canadian Journal of Economics*, Vol. 12, 355-376.

Nash, J. (1951). Non-cooperative games, *Annals of mathematics*, Vol. 54, 286-295.

Nash, J. (1953). Two-person cooperative games, *Econometrica*, Vol. 21, 128-140.

Ni, Y. and Sandal, L. K. (2019). Seasonality matters: A multi-season, multi-state dynamic optimization in fisheries, *European Journal of Operational Research*, Vol. 275, 648-658.

Pintassilgo, P. (2003). A coaltion approach to the management of high seas fisheries in the presence of externalities, *Natural Resource Modeling*, Vol. 16, 175-197.

Schaefer, M. B. (1957). "Some considerations of population dynamics in economics in relation to the management of marine fisheries", *Journal of Economic Dynamics and Control*, Vol. 28, 1781-1799.

Smith, V. L. (1969). On models of commercial fishing, *Journal of political economy*, Vol. 77, 181-198.

Thrall, R. M. and Lucas, W. F. (1963). N-person games in partition function form, *Naval Research Logistics*, Vol. 10, 281-298.

United Nations (1982). *United Nations Convention on the Law of the Sea*, UN Doc. A/Conf.62/122.

Yi, S. S. (1997). Stable coalition structures with externalities, *Games and economic behavior*, Vol. 20, 201-237.

# Chapter 1

# Coalition formation with externalities: The case of the Northeast Atlantic mackerel fishery in a pre and post Brexit context

Evangelos Toumasatos[a,b] and Stein Ivar Steinshamn[b]

[a]SNF - Centre for Applied Research, Norwegian School of Economics
[b]Department of Business and Management Science, Norwegian School of Economics

Published in International Game Theory Review

## Abstract

The partition function approach is applied to study coalition formation in the Northeast Atlantic mackerel fishery in the presence of externalities. Atlantic mackerel is mainly exploited by the European Union (EU), the United Kingdom (UK), Norway, the Faroe Islands and Iceland. Two games are considered. First, a four-player game where the UK is still a member of the EU. Second, a five-player game where the UK is no longer a member of the union. Each game is modelled in two stages. In the first stage, players form coalitions following a predefined set of rules. In the second stage, given the coalition structure that has been formed, each coalition chooses the economic strategy that maximises its own net present value of the fishery given the behaviour of the other coalitions. The game is solved using backward induction to obtain the set of Nash equilibria coalition structures in pure strategies, if any. We find that the current management regime is among the stable coalition structures in all eight scenarios of the four-player game but in only one case of the five-player game. In addition, stability in the five-player game is sensitive to the growth function applied and the magnitude of the stock elasticity parameter.

*Keywords:* Mackerel dispute; straddling fish stock; brexit; game theory; externalities; coalition formation; coalition structure stability.

Subject Classification: C71, C72, Q22, Q57.

## 1.1 Introduction

The 1982 United Nations Convention on the Law of the Sea (UNCLOS) recognized a 200 nautical-mile Exclusive Economic Zone (EEZ) stretching from the baseline of a coastal state (United Nations, 1982). The establishment of the EEZ has fundamentally changed the management of world marine captured fisheries by recognizing property rights. Thus, allowing coastal states to manage their stocks for their own benefit. However, such regime has inadequately addressed issues arising from internationally shared fishery resources,[1] e.g., unregulated fishing, over-capitalization, excessive fleet size and etc. (United Nations, 1995; Munro, 2008). Therefore, if the harvesting activities of one coastal state have a significant negative effect on the harvesting opportunities of the other coastal state(s), a coordinated plan for sustainable management from all parties is required.

This need for cooperation has led to the adoption of the 1995 United Nations Fish Stocks Agreement (UNFSA), which supplements and strengthens the 1982 UNCLOS by addressing the problems related to the conservation and management of internationally shared fishery resources (United Nations, 1995). According to UNFSA, exploitation of a shared fish stock within its spatial distribution should be coordinated by a coalition of all interest parties through a UN sanctioned Regional Fisheries Management Organisation (RFMO), e.g., the Northeast Atlantic Fisheries Commission (NEAFC). Membership into an RFMO is open both to nations in the region, i.e., coastal states, and distant nations with interest in the fisheries concerned, as long as they agree to abide by the RFMO's conservation and management measures.

Although UNFSA has established robust international principles and standards for the conservation and management of shared fish stocks (Balton and Koehler, 2006), the fact that RFMOs lack the necessary coercive enforcement power, either to exclude non-members from harvesting or to set the terms of entry for new members, has caused doubts over the long-term viability of such regional management mechanisms (McKelvey et al., 2002). These two inter-related problems, namely the "interloper problem" (Bjørndal and Munro, 2003) and the "new member problem" (Kaitala and Munro, 1993), merge when a nation with no past interest in a particular shared fishery starts exploiting the resource. In this case, the interests of the traditional fishing nations (incumbents) and the new entrant(s) are strongly opposed. On the one hand, incumbents face the prospect of having to give up a share of their quotas to the new entrant(s) in order to join their coalition and exploit the resource sustainably; whereas on the other hand, it might be more profitable for the new entrant(s) not to join and therefore harvest without having to abide by the coalition's conservation measures.

The aforementioned situation gives rise to the free-rider problem due to stock externalities, i.e., the effect of this period's harvest on next period's stock level (Bjørndal, 1987). Stock externalities, which occur when the cost of fishing changes as the population of fish is altered, are negative externalities (Smith, 1969; Agnello and Donnelley, 1976). That is, a nation's harvesting activities lead to less fishing opportunities for another nation and therefore increase the other's nation fishing cost. As nations start cooperating, the externality is internalised and thus the external cost is reduced. The externality disappears if all nations cooperate together. Because the reduction of the negative externality leads to higher benefits for all nations, not only the ones cooperating, some authors within the fishery literature refer to it as positive.

The intuition is as follows. Assume that a cooperative agreement, which aims to

[1] See FAO (2003) and Gulland (1980) for a categorization of shared fish stocks.

preserve a fish stock by limiting the number of catches and thus increasing its population, is signed by a group of nations. A nation who is not part of such agreement can still enjoy the positive effects that the agreement has on the fish stock level without having to reduce its fishing activities. Therefore, a free-rider (non-cooperating nation or coalition of nations)[2] can enjoy a lower cost of fishing without having to mitigate its fishing strategy. Because of the free-rider problem, cooperative agreements among all interest parties in a fishery have not always been possible to achieve.

The importance of externalities emanating from coalition formation where the economic performance of a coalition, including singletons,[3] is affected by the structure of other distinct coalitions has been studied both within game theoretic and fisheries literature. Bloch (1996), Yi (1997), and Ray and Vohra (1999), among others, have established the theoretical framework to analyse coalition formation in the presence of externalities, also referred as endogenous coalition formation, using the partition function approach introduced by Thrall and Lucas (1963). The advantage of those models to the ones using the traditional characteristic function approach is that they consider all possible coalition structures and compute coalition values for every one of them, instead of fixating on some. Thus, stability of different coalition structures, i.e., partial cooperation, can be tested and externalities across coalitions can be captured.

Within the fisheries literature, Pintassilgo (2003), and Pham Do and Folmer (2003) have introduced the partition function approach to fishery games. Pintassilgo (2003) applies this method to the Northern Atlantic bluefin tuna. Pham Do and Folmer (2003) study feasibility of coalitions smaller than the grand coalition. Kronbak and Lindroos (2007) apply different sharing rules to study the stability of a cooperative agreement for the Baltic cod in the presence of externalities. They state that even though the benefit from cooperation is high enough for a cooperative agreement to be reached, its stability is very sensitive to the sharing rule applied due to free-riding effects. For more comprehensive reviews on coalition games and fisheries, as well as game theory and fisheries, see Kaitala and Lindroos (2007), Lindroos et al. (207), Bailey et al. (2010) and Hannesson (2011).

In this article, we implement the partition function approach to study coalition formation in the Northeast Atlantic mackerel fishery. Atlantic mackerel is a highly migratory and straddling stock making extensive annual migrations in the Northeast Atlantic. The stock consist of three spawning components, namely, the southern, the western and the North Sea component, which mix together during its annual migration pattern. As a result, exploitation of mackerel in different areas cannot be separated. Thus, all three spawning components are evaluated as one stock by the International Council for the Exploration of the Sea (ICES) since 1995 (ICES CM, 1996).

Because of the wide geographic range that mackerel is distributed, it is exploited by several nations both in their EEZs and the high seas. Traditionally, mackerel has been

[2]It is possible, although not usual, that a shared fishery is managed by more than one cooperative agreements, where the signatories of one agreement differ from the signatories of the other agreement. An example presented in Munro (2003) consists of the fourteen independent Pacific Island Nations, which were coalesced into two sub-coalitions. If this is the case, then a coalition of nations can free-ride on another coalition.

[3]A coalition consisting of one member.

cooperatively exploited by the European Union[4] (EU), Norway and the Faroe Islands, with the latter taking only a small proportion of the overall catch until 2010 (2%[5] on average). Also, the NEAFC, of which the three nations are members, allocates a share of the mackerel quota to Russia (7% on average), which can fish mackerel in the high seas. In the last decade, however, mackerel has extended its distribution and migration pattern starting to appear in the Icelandic and Greenlandic economic zones. Although the causes of such northward expansion are not fully understood, increased sea surface temperatures in the northeast Atlantic (Pavlov et al., 2013) and high population size of the mackerel stock (Hannesson, 2012) are mostly referred in the literature.

Due to mackerel's distributional shifting, Iceland, which in the past had requested and been denied to be recognised as a coastal state for the management of mackerel, has begun fishing mackerel at increasingly large quantities in 2008 (approximately 18% of the total catch). In 2009, the Faroese, having observed the quantities that Iceland was harvesting, withdrew from the cooperative agreement with the EU and Norway on the grounds that their quota was very low. A bilateral agreement between the EU and Norway was not reached until 2010. Since then, and despite many rounds of consultations, no consensus agreement by all four nations has been reached. However, in 2014, the Faroe Islands together with Norway and the EU signed a 5-year arrangement, which is still in place, determining the total allowable catch (TAC) and the relative share for each participant.

In the past, several authors have closely examined the so-called mackerel dispute between the EU, Norway, Iceland and the Faroe Islands. Ellefsen (2013) applied the partition function approach to study the effects of Iceland's entry into the fishery. He considered two games, a three-player game between the EU, Norway and the Faroe Islands, and a four-player game where he included Iceland. His results indicated that the grand coalition is potentially stable, i.e., it is stable for some but not all sharing rules, in the three-player but not in the four-player game. Hannesson (2012, 2013) studied the outcome of cooperation assuming different migratory scenarios of the mackerel stock. He found out that if the migrations are stock dependent, then minor players, like Iceland and the Faroe Islands, are in a weak position to bargain. The opposite is true if the migrations are purely random or fixed. Jensen et al. (2015) tried to empirically explain the outcome of the mackerel crisis after Iceland's entry into the fishery. They considered two strategies for all nations, namely, cooperation and non-cooperation. They concluded that non-cooperation is a dominant strategy for each player.

The purpose of this article is to investigate how the UK's decision to withdraw from the EU is likely to affect the current management regime in the mackerel fishery. The UK, which has been a member of the EU since 1973, voted on 26 June 2016 to leave the Union. Nine months later, on 29 March 2017, the British government officially initiated Brexit by invoking Article 50 of the European Union's Lisbon Treaty. This will lead to the conclusion of an international agreement between the two parties by the 29th of March 2019 unless the European Council extends this period. Such agreement will define the terms of the UK's disengagement from the European legal system, internal market and

[4]It is assumed that the European Union acts as a nation in this context due to the fact that all of its members abide by the Common Fisheries Policy (CFP). The CFP gives the EU exclusive competence when it comes to negotiating and signing fisheries agreements with non-EU nations. Therefore, EU member states are no longer able to negotiate fisheries agreements by themselves. This is a common assumption when analysing fishery games that include the EU as a player, see Kennedy (2003), Hannesson (2012), Ellefsen (2013) and Jensen et al. (2015).

[5]Unless otherwise stated, all computations in this article are based on ICES (2016a) advice report 9.3.39, tables 9.3.39.12 and 9.3.39.14.

other policies, including the Common Fisheries Policy (Sobrino Heredia, 2017). Being a member state of the EU, the UK has not been directly involved in the negotiations for the mackerel quota but represented by the EU, which allocates fishing opportunities to member states based on the principle of relative stability, i.e., a fixed percentage of the quota based on historical catch levels. Thus, after Brexit is concluded, the UK will have to negotiate on its behalf with the remaining coastal states regarding its share of the mackerel quota, which will most likely be based on the principle of zonal attachment, i.e., each party's share of the quota should be proportional to the catchable stock found in its EEZ (Churchill and Owen, 2010).

In what follows, we focus on two games: (i) a four-player game where the UK is still part of the EU, and (ii) a five-player game where the UK is allowed to make its own decisions. The remaining players/nations considered are Norway, Iceland and the Faroe Islands. Both games are analysed using the partition function approach. That is, we investigate how players are likely to organise themselves in coalitions, which result in the formation of a coalition structure. The objective of a coalition is to maximise its own net present value of the fishery given the behaviour of the other coalitions in the coalition structure. The optimal strategies and payoffs of the games are derived as pure Nash equilibria between coalitions in a coalition structure. Finally, stability of a coalition structure is tested and the set of the Nash equilibria coalition structures is obtained.

The article is structured as follows. In sections 1.2 and 1.3 we lay out the bioeconomic and game theoretic models employed in the article. The empirical model specification is presented in section 1.4. In section 1.5, we report the solution of both games, evaluate the stability of the coalition structures and discuss the results. Finally, section 1.6 summarises our main findings and concludes the article.

## 1.2 Bioeconomic Model

The bioeconomic model we expand on is a deterministic stock-recruitment model introduced by Clark (1973).[6] The model is in discrete time between seasons but continuous within them. Also, it is linear in the control variable, i.e., harvest.

The spawning stock biomass of a fishery at the beginning of a period $t$, for $t = 0, 1, 2, \ldots, \infty$, is referred to as the recruitment $R_t$. The harvested biomass in a period $t$ is denoted by $H_t$ and must be between zero and the recruitment, $0 \leq H_t \leq R_t$. The spawning stock biomass at the end of a period is the difference between the recruitment and the harvest and is called the escapement $S_t$, $S_t = R_t - H_t$. The spawning stock biomass at the beginning of the next period $R_{t+1}$ is a function of the spawning stock biomass at the end of the current period $S_t$, $R_{t+1} = F(S_t)$. The schema below illustrates the stock dynamics between time periods.

$$R_t \longrightarrow H_t \longrightarrow S_t \longrightarrow R_{t+1} = F(S_t) \ldots .$$

The function $F(S)$, which is usually referred to as the stock-recruitment relationship, is assumed to be continuous, increasing, concave and differentiable in $[0, K]$ with $F(0) = 0$ and $F(K) = K$, where $K > 0$ is the carrying capacity of the fishery.

[6]Important contributors towards the development of stock-recruitment models have also been Reed (1974) and Jaquette (1974) who analysed stochastic stock-recruitment models in discrete time.

Note that only harvest mortality occurs during a period $t$. Natural mortality is accounted for within the stock-recruitment relationship, which can be viewed as the net recruitment function or the "natural" production function (Clark and Munro, 1975).

### 1.2.1 Cooperative management

Suppose now that a shared fishery, like the Northeast Atlantic mackerel, is cooperatively managed by a coalition whose members are all the relevant coastal states, also referred to as grand coalition. The goal of the grand coalition is to maximise the net present value of the fishery over an infinite horizon subject to the biological constraint. The maximisation problem can be expressed as follows:

$$\begin{aligned} \underset{S_t}{\text{maximise}} \quad & \sum_{t=0}^{\infty} \gamma^t \Pi(R_t, S_t) \\ \text{subject to} \quad & R_{t+1} = F(S_t), \\ & 0 \leq S_t \leq R_t, \end{aligned}$$

where $\Pi(R_t, S_t)$ is the joint profit from the fishery for each period, which is defined as the difference between gross revenue and total cost. Two assumptions are made when specifying the net revenue function. First, the demand curve is assumed to be infinitely elastic, i.e., each harvested unit of fish can be sold at a fixed price $p$. Thereafter, the gross revenue from the fishery is expressed as $TR(R_t, S_t) = p(R_t - S_t)$. Second, the unit cost of harvest is assumed to be density dependent, i.e., it increases as the size of the stock decreases. Thus, for a given stock size $x$ the unit cost of harvest is equal to $c(x)$, which is a continuous and decreasing function. Consequently, the total cost of harvest within one period is defined as $TC(R_t, S_t) = \int_{S_t}^{R_t} c(x)dx$. To sum up, the joint profit in period $t$ can be written as:

$$\Pi(R_t, S_t) = p(R_t - S_t) - \int_{S_t}^{R_t} c(x)dx.$$

Clark (1973) showed that, if the profit function is specified as above, then the optimal harvest strategy that maximises the net present value of the fishery is given by a "bang-bang" strategy with equilibrium escapement $S^*$

$$H_t = \begin{cases} R_0 - S^*, & t = 0, \\ F(S^*) - S^*, & t \geq 1, \end{cases}$$

i.e., for the initial period the stock should be depleted to the equilibrium escapement level and then harvest the difference between optimal recruitment and escapement. The optimal escapement level is independent of $t$ and must satisfy the so-called "golden rule"

$$\pi(S^*) = \gamma F'(S^*) \pi[F(S^*)], \tag{1.1}$$

where $\pi(x)$ is the marginal profit defined as $\pi(x) = p - c(x)$. The interpretation of the "golden rule" is straightforward, a cooperatively managed fishery is exploited until the marginal profit of harvesting the last unit of the stock is equivalent to the marginal profit of letting that unit grow and be harvested in the next period.

### 1.2.2 Non-cooperative management

Although cooperative management is the desired outcome from the perspective of stock conservation, it is often the case that shared fisheries are non-cooperatively managed. In this subsection, we generalise the above model in order to allow for non-cooperative behaviour among nations. First, we describe how the mackerel stock is exploited in the presence of two or more distinct coalitions. Then, we specify coalition's $i$ maximization problem and derive the non-cooperative "golden rule".

If the mackerel fishery is non-cooperatively managed, then a number of coalitions[7] interacting with each other must exist. Each coalition acts on its own, aiming to maximise its own net present value of the fishery, which is potentially detrimental to other coalitions. Coalitions are assumed to harvest mackerel in the EEZs of their members. Furthermore, we ignore mackerel exploitation on international waters for the following reasons. First, the size of the high seas territory where mackerel potentially exists is relatively small and remote, compared to the rest of its habitat. Second, mackerel is mainly exploited on the high seas by Russia, which receives a small proportion of the total quota and is not directly involved in the management of the stock.

Let $\theta_l$ be the share of the mackerel stock that only appears in the EEZ of nation $l$ for a whole year. The share of the mackerel stock that coalition $i$ enjoys is simply the sum of its members' shares, i.e., $\theta_i = \sum_{l \in i} \theta_l$. For example, if EU and NO form a coalition, then $\theta_{(EU,NO)} = \theta_{EU} + \theta_{NO}$. Parameter $\theta$ is assumed to be stationary, i.e., constant through all time periods. For details on the specification of the share parameter see section 1.4.

Although each coalition exploits mackerel in its own zone, the stock-recruitment relationship specified in the beginning of this section still holds for the aggregated population level, i.e., $R_{t+1} = F(S_t)$. Let $m$ be the number of coalitions that non-cooperatively manage the mackerel fishery. The share parameter $\theta_i$, where $i = 1, 2, \ldots, m$, enables us to work out the share of recruitment $R_{it}$ for each coalition in a time period, i.e., $R_{it} = \theta_i R_t$. After mackerel harvesting activities $H_{it}$ are performed by all coalitions, the escapement from the zone of each coalition is $S_{it} = R_{it} - H_{it}$. The total recruitment for the next time period is determined by the total escapement of the current period through the stock-recruitment relationship on the aggregated escapement level $S_t$, where $S_t = \sum_{i=1}^{m} S_{it}$. The schema below illustrates such process when three coalitions exist, $m = 3$.

$$
\begin{array}{ccccc}
 & \nearrow R_{1t} = \theta_1 R_t \longrightarrow H_{1t} \longrightarrow S_{1t} \searrow & & \\
R_t & \longrightarrow R_{2t} = \theta_2 R_t \longrightarrow H_{2t} \longrightarrow S_{2t} \longrightarrow & S_t = \sum_{i=1}^{3} S_{it} \longrightarrow R_{t+1} = F(S_t) \ldots . \\
 & \searrow R_{3t} = \theta_3 R_t \longrightarrow H_{3t} \longrightarrow S_{3t} \nearrow & & 
\end{array}
$$

Based on the above setting, a coalition $i$ maximises its own net present value of the fishery subject to its recruitment share $R_i$, the escapement strategies of the other coalitions $S_j$ and the stock-recruitment relationship. Such maximisation problem can be

[7] The term coalition is typically used to refer to situations where two or more entities, e.g., companies, political parties, nations etc., cooperate together to achieve a goal. However, within the game theory literature the term is used as follows: given a set of players, any subset of the given set can be a coalition. Thus, according to game theorists, an individual player acting on its behalf can be a coalition. Coalitions consisting of only one player are usually referred to as singletons.

expressed as follows:

$$
\begin{aligned}
\underset{S_{it}}{\text{maximise}} \quad & \sum_{t=0}^{\infty} \gamma^t \Pi_i(R_{it}, S_{it}) \\
\text{subject to} \quad & R_{it} = \theta_i R_t, \\
& R_{t+1} = F(S_t), \\
& S_t = S_{it} + \sum_{j=1}^{m-1} S_{jt} \quad i \neq j, \\
& 0 \leq S_{it} \leq R_{it}.
\end{aligned}
\tag{1.2}
$$

$\Pi_i(R_{it}, S_{it})$ is the profit for coalition $i$ for each period and is specified as in the cooperative case, i.e.,

$$
\Pi_i(R_{it}, S_{it}) = p(R_{it} - S_{it}) - \int_{S_{it}}^{R_{it}} c_i(x)dx.
$$

The optimal harvest strategy that maximises the net present value for coalition $i$ is given by a target escapement strategy with equilibrium escapement $S_i^*$

$$
H_{it} = \begin{cases} R_{i0} - S_i^* = \theta_i R_0 - S_i^*, & t = 0, \\ R_i - S_i^* = \theta_i F\left( S_i^* + \sum_{j=1}^{m-1} S_j \right) - S_i^*, & t \geq 1, \end{cases}
$$

i.e., for the first period the initial recruitment of coalition $i$ should be depleted to its equilibrium escapement level, and then harvest the difference between its recruitment share and its optimal escapement. The recruitment share of coalition $i$ is determined by its share and the stock-recruitment relationship, which depends on the optimal escapement of coalition $i$ and the escapement strategies of the other coalitions $j$. The optimal escapement level is independent of $t$ and must satisfy the following "golden-rule" (see appendix A.1 for the proof):

$$
\pi_i(S_i^*) = \gamma \theta_i F'(S) \pi_i[\theta_i(F(S)],
\tag{1.3}
$$

where $\pi_i(x)$ is the marginal profit for coalition $i$ defined as $\pi_i(x) = p - c_i(x)$ and $S$ is the aggregated escapement defined as $S = S_i^* + \sum_{j=1}^{m-1} S_j$.

It is evident from the non-cooperative golden rule (1.3) that the optimal escapement strategy $S_i^*$ of coalition $i$ depends on the escapement strategies of the other coalitions $j$. Therefore, in order for coalition $i$ to be able to determine its optimal escapement strategy $S_i^*$, it has to have some information regarding the escapement strategies of the remaining coalitions $j$.

Suppose that coalition $i$ makes an educated guess about the escapement strategies of all the remaining coalitions $j$ based on the information it possesses. Coalition $i$ is now able to compute its optimal escapement strategy $S_i^*$ by substituting its educated guess in (1.3). If all coalitions act in the same manner, i.e., they make an educated guess for the strategies of their counterparts, substitute in (1.3), and compute their escapement strategies, then all educated guesses that have been made will probably differ from the escapement strategies that have been computed. Suppose now that some sort of updating based on the newly computed escapement strategies takes place and updates the information set of the coalitions allowing them to adjust their escapement strategies on the new information. Then, all coalitions will have to recompute their escapement strategies based on the new information. This process will keep repeating until no coalition can further gain by adjusting its escapement strategy, then the Nash equilibrium is reached.

Since this article intention is to compute the Nash equilibrium escapement strategies for the coalitions formed and not to derive the optimal escapement paths for these coalitions, there is no need to make any further specification upon the information coalitions have and how this information is updated. The Nash equilibrium escapement strategies can be obtained by solving a system of equations as will be shown in the next section.

Finally, the non-cooperative "golden-rule" is a generalisation of the cooperative one. To see this, assume that all nations cooperate and the grand coalition is formed. The stock share of the grand coalition is equal to one, $\theta_i = 1$, and since no other coalition exist the aggregated escapement is equivalent to the optimal escapement of the grand coalition, $S = S_i^*$. Thus, the two rules are equivalent under full cooperation.

## 1.3 Game Theoretic Model

A coalition game with externalities is modelled in two stages. In the first stage, players, i.e., nations, form coalitions following a predefined set of rules. For our fishery game, we adopt the simultaneous-move "Open Membership" game described in Yi and Shin (1995). According to this rule, players can freely form coalitions as long as no player is excluded from joining a coalition. This type of coalition game is in line with how membership is established within an RFMO according to Article 8(3) of the UNFSA. Also, it is the de facto framework used so far to analyse coalition games in fisheries.

Let $N = \{1, 2, \ldots, n\}$ be the set of players. A coalition $C$ is a subset of $N$, i.e., $C \subseteq N$, with $2^n$ being the number of coalitions that can be formed, including the empty set. The coalition(s) formed in the first stage lead to a coalition structure $CS = \{C_1, C_2, \ldots, C_m\}$, where $1 \leq m \leq n$. A coalition structure has at least one coalition, i.e., full cooperation, and at most $n$ coalitions, i.e., full non-cooperation. The formal definition of a coalition structure as provided in Yi (1997) states that a coalition structure is a partition of the players $N$ into disjoint, non-empty and exhaustive coalitions, i.e., $C_i \cap C_j = \varnothing$ for all $i, j = 1, 2, \ldots, m$ and $i \neq j$, and $\bigcup_{i=1}^{m} C_i = N$. This means that within a coalition structure each player belongs only to one coalition and some players may be alone in their coalitions.

Given the coalition structure that has been formed in the first stage, in the second stage, each coalition chooses the economic strategy that maximises its own net present value of the fishery given the behaviour of the other coalitions. If the grand coalition is formed then the total net present value of the fishery is maximised. The economic strategies in the second stage game, as well as the respective payoffs, are pure strategy Nash equilibria[8]. Given the optimal strategies in the second stage of the game, the Nash equilibria coalition structures in pure strategies are the ones that satisfy the stability criteria.

The game is solved using backward induction to obtain the set of stable coalition structures, if any. First, we fix all coalition structures. Then, we compute optimal strategies and payoffs for all coalitions in every coalition structure. Finally, we check which coalition structures satisfy the stability criteria.

[8] No mixed strategies are considered when solving this game.

### 1.3.1 Second stage of coalition formation

Let $K = \{CS_1, CS_2, \ldots, CS_\kappa\}$ be the set of coalition structures and $\kappa$ the number of coalition structures that can be formed.[9] From the $\kappa$ coalition structures, the $\kappa-1$ consist of two or more coalitions, which non-cooperatively manage the fishing resource. The $\kappa$-th coalition structure contains only one coalition the grand coalition that cooperatively manages the stock.

For a given coalition structure $CS_k = \{C_1, C_2, \ldots, C_m\}$, where $k = 1, 2, \ldots, \kappa$, we denote the payoff of coalition $C_i$, where $i = 1, 2, \ldots, m$, as $v_i(S_i, S)$. The coalitional payoff depends on the escapement strategy of the coalition, $S_i$, and the overall escapement strategy profile of the coalition structure, $S = S_i + \sum_{j=1}^{m-1} S_j$.[10] Also, the set of feasible escapement strategies for any coalition $i$ is between zero, i.e., harvest everything, and its recruitment, i.e., harvest nothing, $S_i \in [0, R_i]$.

The equilibrium escapement strategies $S_i^*$ for all coalitions $C_i$ in a coalition structure $CS_k$ are derived as a Nash equilibrium between coalition $C_i$ and coalitions $C_j$ where $j = 1, 2 \ldots, m-1$, $i \neq j$ and $C_i \cup C_j = CS_k$, and must satisfy the following $m$ inequalities:

$$
\begin{gathered}
v_i\left(S_i^*, S_i^* + \sum_{j=1}^{m-1} S_j^*\right) \geq v_i\left(S_i, S_i + \sum_{j=1}^{m-1} S_j^*\right), \\
\forall\ C_i \in CS_k; \quad S_i, S_i^* \in [0, R_i]; \quad S_j^* \in [0, R_j]; \quad i, j = 1, 2, \ldots, m; \quad i \neq j,
\end{gathered}
$$

i.e., for every coalition $C_i$ the optimal escapement strategy $S_i^*$ must maximise the coalitional payoff given the optimal escapement strategies of the other coalitions $S_j^*$. In other words, the equilibrium escapement strategy profile of a coalition structure requires that no coalition can get better-off by deviating from its escapement strategy, i.e., optimal escapement strategies are best responses. If the grand coalition is formed, the above decision rule reduces to a single inequality:

$$
v(S^*) \geq v(S), \quad S, S^* \in [0, R],
$$

i.e., the optimal escapement level must maximise the grand coalition's payoff.

In order to determine the equilibrium escapement strategy profile of a coalition structure $CS_k$ the maximisation problem (1.2) as specified in subsection 1.2.2 must be repeatedly solved for every coalition $C_i$ within a coalition structure $CS_k$ until no coalition can further increase its net present value by adjusting its escapement strategy given the escapement strategies of the other coalitions. However, as described in the same subsection, such maximisation problem boils down to a single expression, the "golden-rule", specified in (1.3). Therefore, in order to determine the equilibrium escapement strategy profile of a coalition structure, we solve the following system of $m$ equations:

$$
\begin{aligned}
&\pi_i(S_i) = \gamma\theta_i F'(S)\pi_i[\theta_i(F(S)], \quad \forall C_i \in CS_k; \quad i = 1, 2, \ldots, m, \\
&\text{where } S = \sum_{i=1}^{m} S_i, \quad i = 1, 2, \ldots, m.
\end{aligned} \tag{1.4}
$$

[9] The number of coalition structures $\kappa$ depends on the number of players and is referred to as the Bell number within combinatorial mathematics.

[10] Games where a player's or a coalition's payoff depend only upon its own strategy ($S_i$ in our setting), and a linear aggregate of the full strategy profile ($S$ in our setting) are also called aggregate games, see Martimort and Stole (2012) for additional details and applications.

These equations refer to the "golden-rules" that coalitions within a coalition structure apply in order to determine their escapement strategies. The overall escapement, $S$, is a linear aggregate of the full strategy profile and captures how coalitions interact with each other through their escapement strategies. Note that in the case of the grand coalition the above system of equations consists of only one equation, which is equivalent to the cooperative "golden-rule" (1.1).

It should be obvious by now that the equilibrium escapement strategies depend on the coalition structure that is formed and on the parameters of the model. The coalitions formed are assumed to be asymmetric. They are differentiated by parameter $\theta_i$, the share of mackerel stock that occurs in the EEZ(s) of a coalition, and their marginal cost of harvest, $c_i(x)$. Some coalitions may have equivalent shares, if their members are of the same type, see section 1.4 for additional details. These asymmetries ensure that escapement strategies across coalitions are different and depend upon the form of the coalition structure. Thus, a unique payoff, which depends on the coalition structure, can be computed for every coalition in a coalition structure.

The coalitional payoff, which is equivalent to the net present value of the fishery over an infinite time horizon and depends on the escapement strategy profile of the coalition structure formed, can be written as follows:

$$v_i(S_i^*, S^*) = \sum_{t=0}^{\infty} \gamma^t \Pi_i(R_{it}, S_{it}) = \Pi_i(\theta_i R_0, S_i^*) + \frac{\gamma}{1-\gamma} \Pi_i[\theta_i F(S^*), S_i^*], \tag{1.5}$$

where $R_0$ is the initial recruitment and $S^* = S_i^* + \sum_{j=1}^{m-1} S_j^*$ is the optimal escapement strategy profile of a coalition structure. While specifying the coalitional payoff, it is important to remember that two things are assumed. First, the initial recruitment is high enough to allow for the prescribed harvest strategy in the first period, i.e., $S_i^* \leq \theta_i R_0 \ \forall C_i \in CS_k$. If this is not the case, the stock should not be harvested but allowed to grow until recruitment exceeds escapement. For our mackerel case, the initial recruitment is high enough to sustain all escapement strategies as feasible. Second, the fishing fleet capacity required to implement such harvest strategies (initial depletion and steady state harvest) exists. If the necessary capacity does not exist, the following situations arise: (i) there exist sufficient capacity to harvest the steady state quantity but not to deplete the stock to the steady state in one period, and (ii) no sufficient capacity exists to harvest the steady state quantity.[11,12] If case (i) occurs, then the initial depletion of the stock to the steady state escapement level would take a couple of periods depending on the capacity of the current fishing fleet. On the other hand, if case (ii) occurs, we will never reach the "true" steady state prescribed by the optimal escapement strategy. In the long run, however, a nation would increase its fishing fleet capacity to meet the optimal escapement strategy, either by investing in more fishing vessels or by shifting vessels that operate in less profitable stocks. Since mackerel is one of the most valuable stocks in the Northeast Atlantic region and in order not to complicate things by endogenously determining the fishing fleet capacity, we assume that the necessary capacity for implementing the prescribed strategies exists for all nations.

[11] For a formal analysis of these two cases see Clark (1972).

[12] If a capacity constraint is to be included, then instead of harvesting $\max(R-S, 0)$ our sequence of harvest strategies should satisfy the following: $\max\left[\min(R-S, Cap), 0\right]$, i.e., if $S < R$ then harvest their difference if it is below the fishing fleet capacity $Cap$ or harvest the capacity, otherwise do not harvest and let the stock grow.

### 1.3.2 First stage of coalition formation

Our analysis is in line with the internal and external stability concepts of d'Aspremont et al. (1983) and what is defined as potential internal stability by Eyckmans and Finus (2004). These concepts have been used to test a coalition's stability in both characteristic and partition function games.[13]

We start by introducing the notion of an embedded coalition, which is extensively used throughout this subsection. An embedded coalition is a pair $(C_i, CS_k)$ consisting of a coalition and a coalition structure which contains that coalition, $C_i \in CS_k$. Let $V(C_i, CS_k)$ denote the payoff of an embedded coalition[14] and $V_x(C_i, CS_k)$ denote the payoff received by subcoalition $x$ of the embedded coalition $(C_i, CS_k)$, $x \subset C_i$. The subscript $x$ may refer to an individual player (see internal stability condition below) or a coalition of players (see external stability condition below). The following relationship holds: $\sum_{x \in C_i} V_x(C_i, CS_k) = V(C_i, CS_k)$.

An embedded coalition $(C_i, CS_k)$ is internal stable if none of its members $l$, $l \in C_i$, has incentives to leave and form a singleton coalition $C^l$, where $C^l = \{l\}$. Such condition can be written as follows:

$$V_l(C_i, CS_k) \geq V(C^l, CS_k^l), \quad \forall l \in C_i, \tag{1.6}$$

where $CS_k^l = \{(CS_k \backslash C_i), (C_i \backslash l), (C^l)\}$ stands for a coalition structure formed from the original coalition structure $CS_k$ in which coalition $C_i$ is split into two coalitions: $(C_i \backslash l)$ and $(C^l)$. In other words, given an embedded coalition $(C_i, CS_k)$, the payoff a member $l$ receives as a member of coalition $C_i$ must be higher or equal to the payoff that $l$ can receive if it leaves the coalition in order to form a singleton coalition. If this is true for all the members, then the embedded coalition $(C_i, CS_k)$ is internal stable. Notice that the remaining form of the coalition structure is assumed to be unaffected by $l$'s deviation, i.e., the remaining members of the said coalition do not leave after $l$ leaves and the remaining coalitions in the coalition structure, if any, do not merge or split. This assumption is equivalent to the ceteris paribus assumption. By definition all embedded coalitions which are singletons are always internal stable.

In an open membership game, where membership into a coalition is free for all players, a second condition ensuring that outsiders do not have incentives to join a coalition is needed. Such condition is referred to as external stability. An embedded coalition $(C_i, CS_k)$ is external stable if no other embedded coalition $(C_j, CS_k)$, singleton or not, in the coalition structure $CS_k$ has incentives to join coalition $(C_i, CS_k)$. Such condition can be written as follows:

$$V(C_j, CS_k) \geq V_j(C_j^i, CS_k^j), \quad \forall C_j \in CS_k; \quad C_j \neq C_i, \tag{1.7}$$

where $C_j^i = C_j \cup C_i$ stands for a coalition formed if coalitions $C_i$ and $C_j$ merge, and $CS_k^j = \{(CS_k \backslash (C_j, C_i)), (C_j^i)\}$ stands for a coalition structure formed from the original coalition structure $CS_k$ in which coalitions $C_i$ and $C_j$ are merged into one coalition: $(C_i^j)$. That is to say, given a coalition structure $CS_k$, the payoff an embedded coalition $(C_j, CS_k)$ receives must be higher or equal to the payoff $C_j$ can receive if it joins coalition

[13] See, among others, Pintassilgo et al. (2010) and Liu et al. (2016) for applications of these concepts on fishery games in partition function form.

[14] Note that the payoff of an embedded coalition is equivalent to the coalitional payoff specified in subsection 1.3.2 given that the coalition structure in which the coalitional payoff refers to is the same, i.e., $V(C_i, CS_k) \equiv v_i(S_i^*, S^*)$ if the coalition structure that $v_i$ refers to is equivalent to $CS_k$.

$C_i$ and form a larger coalition. If this is true for all coalitions other than $C_i$ within coalition structure $CS_k$, then the embedded coalition $(C_i, CS_k)$ is external stable. Again, the remaining form of the coalition structure is assumed to be unaffected by the mergence. By definition the grand coalition is always external stable.

So far our analysis has been within the context of d'Aspremont et al. (1983) applied for embedded coalitions. Testing stability within this context requires the division of the coalitional payoff among coalition members. For instance, it is impossible to test for internal stability without knowledge of the individual payoff a coalition member receives (LHS of (1.6)). Likewise, external stability requires information regarding the payoff the merging coalition will receive after the merger takes place (RHS of (1.7)). Hence, a sharing rule is needed in order to split the coalitional payoff. Consequently, the stability of a coalition is going to depend upon such sharing rule.

The existing literature on sharing rules that can be applied to partition function games is not so extensive compared to the one for characteristic function games.[15] Specifying a sharing rule for games in partition form is not an easy undertaking because of the complexity of the partition function. A common issue is that for a given coalition the coalitional payoff is not unique since the same coalition can belong to more than one coalition structures.[16] Some authors have proposed different weighted rules in order to determine a unique coalitional payoff.[17] However, these approaches do not provide a unique solution unless the weight parameters are fully specified.

In order to avoid these issues and since the main objective of this article is to determine the set of stable coalition structures and not to distribute the gains of cooperation among cooperating nations, we adopt Eyckmans and Finus (2004) concept of potential internal stability. An embedded coalition $(C_i, CS_k)$ is potentially internal stable if the sum of the free-riding payoffs of its members $l$, $l \in C_i$, does not exceed its coalitional payoff, i.e.,

$$V(C_i, CS_k) \geq \sum_{l \in C_i} V(C^l, CS_k^l), \tag{1.8}$$

where $C^l = \{l\}$ is a singleton coalition and $CS_k^l = \{(CS_k \backslash C_i), (C_i \backslash l), (C^l)\}$ stands for a coalition structure formed from the original coalition structure $CS_k$ in which coalition $C_i$ is split into two coalitions: $(C_i \backslash l)$ and $(C^l)$. $V(C^l, CS_k^l)$ is the free-riding payoff that a coalition member $l$ can receive if it leaves coalition $C_i$ and form the singleton coalition $C^l$, ceteris paribus. By definition a singleton embedded coalition is always potential internal stable.

A clear advantage of condition (1.8) over (1.6) is that it can test for internal stability in the absence of a sharing rule. If an embedded coalition is potentially internal stable, then there exist some allocation schemes which can ensure internal stability. On the other hand, if potential internal stability does not hold, then no sharing rule can make an embedded coalition internal stable (Pintassilgo et al., 2010).

Clearly, potential internal stability is a necessary condition for internal stability. By the same token, a necessary condition for external stability is needed in order to be able

[15]The coalitional payoff of a game in characteristic form is independent of the coalition structure.

[16]To see this point consider a four player game and the following two coalition structures: $CS_1 = \{12, 3, 4\}$ and $CS_2 = \{12, 34\}$. In both coalition structures players 1 and 2 form a coalition. Players 3 and 4 act as singletons in $CS_1$ and also form a coalition in $CS_2$. The payoff of coalition (12) depends on the coalition structure that it belongs, and the coalition structure that contains coalition (12) is not unique.

[17]See Macho-Stadler et al. (2007), Pham Do and Norde (2007) and De Clippel and Serrano (2008) for examples.

to determine stability in the absence of a sharing rule. An embedded coalition $(C_i, CS_k)$ is potentially external stable if for all other embedded coalitions $(C_j, CS_k)$ the following inequality holds:

$$V(C_j, CS_k) \geq V(C_j^i, CS_k^j) - \sum_{l \in C_i} V(C^l, CS_k^{jl}), \quad \forall C_j \in CS_k; \quad C_j \neq C_i, \tag{1.9}$$

where $C_j^i = C_j \cup C_i$ stands for a coalition formed if coalitions $C_i$ and $C_j$ merge, and $CS_k^j = \{(CS_k \backslash (C_j, C_i)), (C_j^i)\}$ stands for a coalition structure formed from the original coalition structure $CS_k$ in which coalitions $C_i$ and $C_j$ are merged into one coalition: $(C_i^j)$. In addition, $C^l = \{l\}$ is a singleton coalition and $CS_k^{jl} = \{(CS_k^j \backslash C_j^i), (C_j^i \backslash l), (C^l)\}$ stands for a coalition structure formed from coalition structure $CS_k^j$ in which coalition $C_j^i$ is split into two coalitions: $(C_j^i \backslash l)$ and $(C^l)$. $V(C_j^i, CS_k^j)$ is the payoff coalition $C_j^i$ receives after the merger occurs, ceteris paribus (hereinafter the joint payoff). And, $V(C^l, CS_k^{jl})$ is the free-riding payoff that a member $l$ of coalition $C_i$ receives if it leaves coalition $C_j^i$, ceteris paribus. Thus, given a coalition structure $CS_k$, an embedded coalition $(C_i, CS_k)$ is potentially external stable if and only if the payoff of all other embedded coalitions $C_j$ in $CS_k$ is greater than the joint payoff minus the sum of the free-riding payoffs of coalition's $C_i$ members. In other words, in order for coalition $C_j$ not to be willing to merge with coalition $C_i$, its potential share of the joint payoff must be lower than its current payoff. The potential share of the joint payoff that coalition $C_j$ is entitled to is the remainder of the joint payoff after all members of coalition $C_i$ have received their free-riding payoffs. By definition the grand coalition is always potentially external stable.

Having defined the necessary conditions for an embedded coalition to be internal and external stable in the absence of a sharing rule we can now proceed in defining the necessary conditions for a coalition structure to be stable. As in the case of a coalition, stability of a coalition structure in an open membership game requires that the coalition structure is both internal and external stable.

Before we start analysing the two conditions, let us take a step back and visualise what internal and external stability of a coalition structure is. Figure 1.1 depicts the coalition structures for a four-player game. The nodes represent coalition structures. The arcs represent mergers of two coalitions when followed upward and split of a coalition into two subcoalitions when followed downward. In a four-player game there exist four levels in total. A coalition structure level is a subset of the coalition structure set that consists of coalition structures with equal number of coalitions. In our example, the third level subset is composed of coalition structures that have only two coalitions. A stable coalition structure should not move upwards or downwards in the graph but remain in its position. This occurs if all embedded coalitions in a coalition structure do not have incentives to merge or split.

The split part is the easiest to test as it merely requires all embedded coalitions of a coalition structure to be internal stable. If this is true, then the coalition structure cannot be downgraded, i.e., move downwards in the graph. Using the notion of potential internal stability such condition can be written as follows:

$$V(C_i, CS_k) \geq \sum_{l \in C_i} V(C^l, CS_k^l), \quad \forall C_i \in CS_k. \tag{1.10}$$

Therefore, if all embedded coalitions of a coalition structure are potentially internal stable, then the coalition structure is potentially internal stable, which is a necessary condition for internal stability to hold.

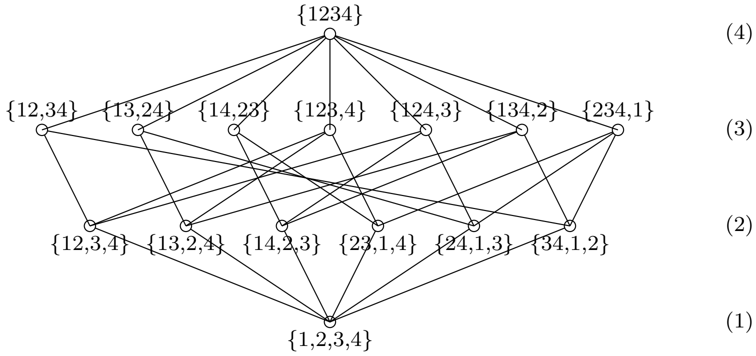



Figure 1.1. Coalition structure graph for a four player game.

On the other hand, the merge part of our argument is not so straightforward to test. This is because it is not equivalent as saying that all embedded coalitions of a coalition structure should be external stable. If we say so, then some externally stable coalition structures will fail to pass the test and considered as externally unstable. To see this point, suppose that external stability of a coalition structure requires all of its embedded coalitions to be external stable. Consider the following coalition structure: $CS_{11} = \{123, 4\}$. According to the aforementioned definition, $CS_{11}$ is external stable if coalitions (123) and (4) are external stable. That is to say that coalition (123) does not want to merge with (4) and coalition (4) does not want to merge with (123). This sounds like a valid definition for a coalition structure to be external stable, and, as a matter of fact, it is. If all embedded coalitions of a coalition structure are external stable, then the coalition structure cannot be upgraded, i.e., move upwards in the graph.

Suppose now that one of the two embedded coalitions of $CS_{11}$ is not external stable. Is this assumption going to upgrade $CS_{11}$ permanently and therefore making it "truly" external unstable? Let coalition (123) be the only external stable coalition. In other words, (4) does not want to merge with (123) but (123) wants to merge with (4). Since not all embedded coalitions are external stable, by definition coalition structure $CS_{11}$ is not external stable. Therefore, upgrade into coalition structure $CS_{15} = \{1234\}$ occurs. But we know that only coalition (123) is better off under the new coalition structure since by assumption it is the only coalition that wants to merge. Thus, coalition (4) deviates and coalition structure $CS_{11} = \{123, 4\}$ forms again.

The question now becomes: is it possible, given a pair of embedded coalitions, that only one has incentives to join the other? The short answer is yes. Typically, games with positive externalities are superadditive, i.e., $V(C_i \cup C_j, CS_k) \geq V(C_i, CS_k^i) + V(C_j, CS_k^j)$, where $CS_k^i = CS_k^j = \{(CS_k \backslash (C_i \cup C_j)), ((C_i \cup C_j) \backslash C_i)\}$. Superadditivity means that a merger between two embedded coalitions generates a payoff at least equal to the sum of the individual payoffs. The superadditivity property may or may not hold across the entire game but it holds for at least some embedded coalitions, at least it does in the game analysed in this article.

Back to our question. Suppose that the superadditive property holds between the embedded coalitions of $CS_{11}$ and $CS_{15}$, i.e., $V(1234, \{1234\}) \geq V(123, \{123, 4\}) + V(4, \{123, 4\})$. If this is true, then coalition (123) is better off under the mergence (strict inequality) or indifferent (equality). This is because the individual payoff of coalition (4) under $CS_{11}$ is also its free-riding payoff. That is, after the mergence occurs, if coalition (4) deviates, it cannot receive a payoff greater than the payoff it already receives. Therefore, after mergence, coalition (123) receives at least its individual payoff. However, after mergence, coalition (4) may not necessarily receive its individual payoff. This is because, coalition (123) must receive a payoff which is at least as high as the sum of the free-riding payoffs of its members, i.e., $V_{123}(1234, \{1234\}) \geq \sum_{l \in (123)} V(l, \{(1234 \backslash l), (l)\})$. Therefore the potential payoff that coalition (4) can receive cannot exceed the difference between the joint payoff and the sum of the free-riding payoffs of coalition (123), i.e., $V_4(1234, \{1234\}) \leq V(1234, \{1234\}) - \sum_{l \in (123)} V(l, \{(1234 \backslash l), (l)\})$. If $V_4(1234, \{1234\})$ is greater than $V(4, \{123, 4\})$ then coalition (4) has incentives to merge otherwise it does not. It should be clear by now, that given a pair of coalitions, $(C_1, C_2)$, the fact that $C_1$ wants to merge with $C_2$ does not imply that $C_2$ also wants to merge with $C_1$. In order for $C_2$ to be willing to merge, its payoff under the mergence should be greater than its individual payoff and this depends on the magnitude of the free-riding payoffs of $C_1$ members.

Even if the entire game is superadditive, i.e., at least some coalitions want to merge, the free-riding effects of these coalitions may be so strong they make it impossible for mergence to occur. And, it is because of strong free-riding effects that superadditive games with externalities cannot necessarily sustain the grand coalition as a stable outcome.

So far we have argued that requiring all embedded coalitions of a coalition structure to be external stable does not necessarily provide us with the set of all external stable coalition structures. So, is there a rule that when applied can give us the set of all external stable coalition structures? The answer is yes. Such condition requires that, given a coalition structure $CS_k$, all possible embedded coalitions pairs $\big[(C_i, CS_k), (C_j, CS_k)\big]$, $\forall C_i, C_j \in CS_k$ and $C_i \neq C_j$, are not willing to merge. An embedded coalition pair is not willing to merge if at least one of its embedded coalitions do not want to merge. Such conditions can be written as follows:

$$\text{A:} \quad V(C_i, CS_k) \geq V(C_i^j, CS_k^i) - \sum_{l \in C_j} V(C^l, CS_k^{il}), \quad C_i \neq C_j; \quad C_i, C_j \in CS_k, \qquad (1.11)$$

$$\text{B:} \quad V(C_j, CS_k) \geq V(C_j^i, CS_k^j) - \sum_{l \in C_i} V(C^l, CS_k^{jl}), \quad C_j \neq C_i; \quad C_j, C_i \in CS_k. \qquad (1.12)$$

Condition A (B) is equivalent to the potential external stability condition (1.9) but only with respect to coalition $C_i$ ($C_j$). That is, if A is true, then $C_i$ does not want to merge with $C_j$, i.e., $C_j$ is potentially external stable with respect to $C_i$. Similarly if B is true, then $C_j$ does not want to merge with $C_i$, i.e., $C_i$ is potentially external stable with respect to $C_j$. If one of the two conditions holds, i.e., A $\vee$ B, then the pair $\big[(C_i, CS_k), (C_j, CS_k)\big]$ will not merge and therefore is considered external stable. If this is true for all possible pairs within a coalition structure, i.e.,

$$\text{A} \ \vee \ \text{B}, \quad \forall C_i, C_j \in CS_k; \quad C_i \neq C_j, \qquad (1.13)$$

then the coalition structure is potentially external stable, which is a necessary condition for external stability to hold. A coalition structure is stable if it is both internal and external stable, i.e., stability of a coalition structure requires conditions (1.10) and (1.13) to hold simultaneously. An illustration of the stability concepts applied in this article is provided through a small numerical example in appendix A.2.

## 1.4 Empirical Model

Before proceeding with the specification of functional forms and parameters we first identify the different coalition structures in the four- and five-player games. The four-

Table 1.1. List of all coalitions for the four player game.

| No. | Coalition | No. | Coalition | No. | Coalition |
|---|---|---|---|---|---|
| 1 | (EU) | 6 | (EU,FO) | 11 | (EU,NO,FO) |
| 2 | (NO) | 7 | (EU,IS) | 12 | (EU,NO,IS) |
| 3 | (FO) | 8 | (NO,FO) | 13 | (EU,FO,IS) |
| 4 | (IS) | 9 | (NO,IS) | 14 | (NO,FO,IS) |
| 5 | (EU,NO) | 10 | (FO,IS) | 15 | (EU,NO,FO,IS) |

Table 1.2. List of all possible coalition structures for the four player game.

| No. | Coalition structure | No. | Coalition structure | No. | Coalition structure |
|---|---|---|---|---|---|
| 1 | (EU),(NO),(FO),(IS) | 6 | (NO,IS),(EU),(FO) | 11 | (EU,NO,FO),(IS) |
| 2 | (EU,NO),(FO),(IS) | 7 | (FO,IS),(EU),(NO) | 12 | (EU,NO,IS),(FO) |
| 3 | (EU,FO),(NO),(IS) | 8 | (EU,NO),(FO,IS) | 13 | (EU,FO,IS),(NO) |
| 4 | (EU,IS),(NO),(FO) | 9 | (EU,FO),(NO,IS) | 14 | (NO,FO,IS),(EU) |
| 5 | (NO,FO),(EU),(IS) | 10 | (EU,IS),(NO,FO) | 15 | (EU,NO,FO,IS) |

player game consists of the following nations: the EU, Norway, the Faroe Islands and Iceland. The total number of coalitions and coalition structures that are likely to occur in a four-player game are 15 and are depicted in tables 1.1 and 1.2. The five-player game consists of the following nations: the EU, the UK, Norway, the Faroe Islands and Iceland. The total number of coalitions and coalition structures that are likely to occur in this game are 31 and 52 and are shown in tables 1.3 and 1.4.

The singleton coalition of EU in the four-player game is treated to be equivalent to the coalition of EU and UK in the five-player game. As a consequence, all of the coalition structures that are likely to occur in the four-player game are also likely to reoccur in the five-player game. For example, $CS_1$ in the four-player game is equivalent to $CS_2$ in the five-player game and etc. However, the set of stable coalition structures is not necessarily equivalent between the two games. This is due to the fact that in the five-player game we allow for the UK to make its own decisions and these decisions may not necessarily be aligned to the ones EU and UK as cooperators may implement. For the remaining of the article and unless explicitly stated all figures related to EU refer to the five-player game and do not take into consideration UK. Table 1.5 provides a concrete list of all the symbols we use in this article.

### 1.4.1 Stock-recruitment relationship

In order to capture the relationship between a period's escapement $S_t$ and next period's recruitment $R_{t+1}$ a function $F(S)$ is needed where $R_{t+1} = F(S_t)$. One functional form, introduced by Ricker (1954) is: $F(S) = aSe^{-bS}$. This function has the property of overcompensation, i.e., it reaches a peak and then descends asymptotically towards $R = 0$, $\lim_{S\to\infty} F(S) = 0$. Another functional form, proposed by Beverton and Holt (1957) is: $F(S) = \frac{aS}{b+S}$. This one does not decrease but instead increases asymptotically towards $R = a$, $\lim_{S\to\infty} F(S) = a$. Both functions are well known among the models that have been developed to fit stock-recruitment curves to data sets.[18] We estimate and make use

[18]See Iles (1994) for a review.

Table 1.3. List of all coalitions for the five player game.

| No. | Coalition | No. | Coalition | No. | Coalition | No. | Coalition |
|---|---|---|---|---|---|---|---|
| 1 | (EU) | 9 | (EU,IS) | 17 | (EU,UK,FO) | 25 | (NO,FO,IS) |
| 2 | (UK) | 10 | (UK,NO) | 18 | (EU,UK,IS) | 26 | (EU,UK,NO,FO) |
| 3 | (NO) | 11 | (UK,FO) | 19 | (EU,NO,FO) | 27 | (EU,UK,NO,IS) |
| 4 | (FO) | 12 | (UK,IS) | 20 | (EU,NO,IS) | 28 | (EU,UK,FO,IS) |
| 5 | (IS) | 13 | (NO,IS) | 21 | (EU,FO,IS) | 29 | (EU,NO,FO,IS) |
| 6 | (EU,UK) | 14 | (NO,FO) | 22 | (UK,NO,FO) | 30 | (UK,NO,FO,IS) |
| 7 | (EU,NO) | 15 | (FO,IS) | 23 | (UK,NO,IS) | 31 | (EU,UK,NO,FO,IS) |
| 8 | (EU,FO) | 16 | (EU,UK,NO) | 24 | (UK,FO,IS) | | |

Table 1.4. List of all possible coalition structures for the five player game.

| No. | Coalition structure | No. | Coalition structure | No. | Coalition structure | No. | Coalition structure |
|---|---|---|---|---|---|---|---|
| 1 | (EU),(UK),(NO),(FO),(IS) | 14 | (EU,FO),(UK,NO),(IS) | 27 | (EU,UK,NO),(FO),(IS) | 40 | (EU,NO,FO),(UK,IS) |
| 2 | (EU,UK),(NO),(FO),(IS) | 15 | (EU,UK),(NO,IS),(FO) | 28 | (EU,UK,FO),(NO),(IS) | 41 | (EU,NO,IS),(UK,FO) |
| 3 | (EU,NO),(UK),(FO),(IS) | 16 | (EU,NO),(UK,IS),(FO) | 29 | (EU,UK,IS),(NO),(FO) | 42 | (EU,FO,IS),(UK,NO) |
| 4 | (EU,FO),(UK),(NO),(IS) | 17 | (EU,IS),(UK,NO),(FO) | 30 | (EU,NO,FO),(UK),(IS) | 43 | (UK,NO,FO),(EU,IS) |
| 5 | (EU,IS),(UK),(NO),(FO) | 18 | (EU,UK),(FO,IS),(NO) | 31 | (EU,NO,IS),(UK),(FO) | 44 | (UK,NO,IS),(EU,FO) |
| 6 | (UK,NO),(EU),(FO),(IS) | 19 | (EU,FO),(UK,IS),(NO) | 32 | (EU,FO,IS),(UK),(NO) | 45 | (UK,FO,IS),(EU,NO) |
| 7 | (UK,FO),(EU),(NO),(IS) | 20 | (EU,IS),(UK,FO),(NO) | 33 | (UK,NO,FO),(EU),(IS) | 46 | (NO,FO,IS),(EU,UK) |
| 8 | (UK,IS),(EU),(NO),(FO) | 21 | (EU,NO),(FO,IS),(UK) | 34 | (UK,NO,IS),(EU),(FO) | 47 | (EU,UK,NO,FO),(IS) |
| 9 | (NO,IS),(EU),(UK),(FO) | 22 | (EU,FO),(NO,IS),(UK) | 35 | (UK,FO,IS),(EU),(NO) | 48 | (EU,UK,NO,IS),(FO) |
| 10 | (NO,FO),(EU),(UK),(IS) | 23 | (EU,IS),(NO,FO),(UK) | 36 | (NO,FO,IS),(EU),(UK) | 49 | (EU,UK,FO,IS),(NO) |
| 11 | (FO,IS),(EU),(UK),(NO) | 24 | (UK,NO),(FO,IS),(EU) | 37 | (EU,UK,NO),(FO,IS) | 50 | (EU,NO,FO,IS),(UK) |
| 12 | (EU,UK),(NO,FO),(IS) | 25 | (UK,FO),(NO,IS),(EU) | 38 | (EU,UK,FO),(NO,IS) | 51 | (UK,NO,FO,IS),(EU) |
| 13 | (EU,NO),(UK,FO),(IS) | 26 | (UK,IS),(NO,FO),(EU) | 39 | (EU,UK,IS),(NO,FO) | 52 | (EU,UK,NO,FO,IS) |

Table 1.5. List of symbols and abbreviations.

| Symbol | Description | Value | Unit |
|---|---|---|---|
| **Sets** | | | |
| $N$ | Players | | |
| $K$ | Coalition structures | | |
| **Subscripts** | | | |
| $n$ | Number of players | $4, 5$ | |
| $m$ | Number of coalitions in a CS | $1, 2, \ldots, n$ | |
| $\kappa$ | Number of CSs | $15, 52$ | |
| $t$ | Time index | $0, 1, 2, \ldots, \infty$ | |
| $l$ | Player index | $1, 2, \ldots, n$ | |
| $i, j$ | Coalition index | $1, 2, \ldots, m$ | |
| $k$ | CS index | $1, 2, \ldots, \kappa$ | |
| **Variables** | | | |
| $S_i$ | Escapement of coalition $i$ in a CS | | $10^3$ tonnes |
| $S$ | Total escapement | | $10^3$ tonnes |
| $R$ | Total recruitment | | $10^3$ tonnes |
| $H$ | Total harvest | | $10^3$ tonnes |
| $V_i$ | NPV of coalition $i$ in a CS (embedded coalition)[a] | | $10^6$ NOK |
| $V_{CS}$ | Total NPV of a CS[b] | | $10^6$ NOK |
| **Parameters** | | | |
| $p$ | Price | 10 | NOK/kg |
| $r$ | Discount rate | 5% | |
| $\theta_l$ | Share of mackerel stock in player's $l$ EEZ | cf. table 1.7 | |
| $a$ | Stock – Recruitment parameter | cf. table 1.6 | |
| $b$ | Stock – Recruitment parameter | cf. table 1.6 | |
| $c_i$ | Cost parameter of coalition $i$ | cf. tables 1.9-1.10 | |
| $\beta$ | Stock elasticity parameter | 1.0, 0.6, 0.3 | |
| $\bar{R}$ | Base year recruitment | 4887 | $10^3$ tonnes |
| $\bar{H}_l$ | Base year harvest of player $l$ | cf. table 1.8 | |
| $\psi$ | Cost – Revenue ratio | 0.78 | |
| **Abbreviations** | | | |
| CS | Coalition structure | | |
| EU | European Union | | |
| UK | United Kingdom | | |
| NO | Norway | | |
| FO | Faroe Islands | | |
| IS | Iceland | | |
| NPV | Net present value | | |

[a] $V_i$ is equivalent to $V(C_i, CS_k)$ and should not be confused with $V_x(C_i, CS_k)$. We make use of compact notation in order to convenience ourselves in the presentation of the results.
[b] $V_{CS} = \sum_{i \in CS_k} V(C_i, CS_k)$.

of both when running our model. By doing so, we are able to test how sensitive the set of stable coalition structures is to the biological constraint of our model.

Both functions are non-linear, thus before proceeding with the regressions we linearise them. The Ricker stock-recruitment relationship becomes:

$$R_t = aS_{t-1}e^{-bS_{t-1}} \Leftrightarrow ln(R_t) = ln(a) + ln(S_{t-1}) - bS_{t-1} \Leftrightarrow ln\left(\frac{R_t}{S_{t-1}}\right) = ln(a) - bS_{t-1}. \quad (1.14)$$

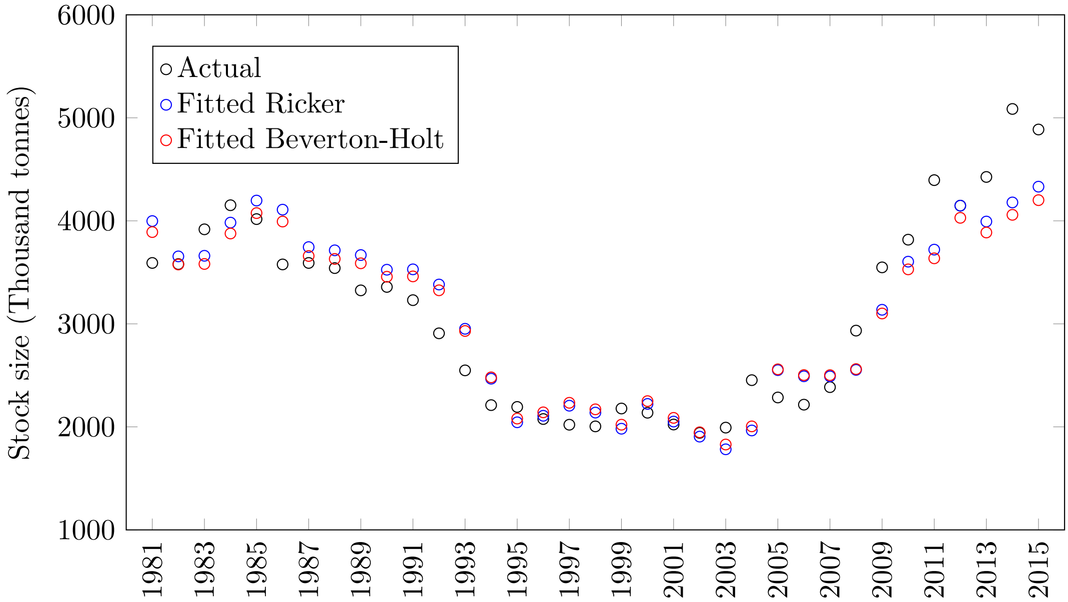



Figure 1.2. Actual and fitted development of the mackerel stock 1981–2015.

Table 1.6. Results from fitting recruitment and escapement data on the Ricker and Beverton-Holt functions.

| Functional form | Parameters $a$ | $b$ | Adjusted $R^2$ |
|---|---|---|---|
| Ricker | 1.6784<br>(0.000) | $9.73 \times 10^{-5}$<br>(0.000) | 0.35 |
| Beverton-Holt | 10,977<br>(0.000) | 5,965<br>(0.000) | 0.88 |

Note: p-values of the transformed regression in parentheses.

Similarly, the Beverton-Holt function becomes:

$$R_t = \frac{aS_{t-1}}{b + S_{t-1}} \Leftrightarrow \frac{1}{R_t} = \frac{1}{a} + \frac{b}{a}\frac{1}{S_{t-1}}. \tag{1.15}$$

We fit Eq. (1.14) and (1.15) using Ordinary Least Squares on recruitment and escapement data. The data used are obtained from ICES (2016a) advice report 9.3.39 table 9.3.39.14. In particular, the following columns covering the period between 1980 and 2015 are used: (i) SSB (Spawning time), and (ii) Landings. According to ICES (2014), SSB means the estimate of the spawning stock biomass at spawning time in the year in which the TAC applies, taking into account of the expected catch (Annex 9.3.17.1 Management plan harvest control rule). In the beginning of section 1.2 of this article, we define the recruitment of a fishery as the unexploited spawning stock biomass at the beginning of a period. If we identify that the beginning of a period occurs when spawning takes place, then the terms recruitment and SSB are equivalent. Moreover, landings refer to the mackerel biomass landed in all ports in the Northeast Atlantic area in a respective year, which is equivalent to the total harvested biomass. Therefore, the difference between SSB and landings represents the escapement of the stock in a particular period/year.

The parameters $a$ and $b$ in Eq. (1.14) and (1.15) are estimated after the time lag

Table 1.7. Shares of mackerel stock in player's $l$ EEZ.

| | EU[a] | UK | NO | FO | IS |
|---|---|---|---|---|---|
| Mackerel share in %, $\theta_l$ | 25.0 | 25.0 | 25.0 | 12.5 | 12.5 |

Note: See table 1.5 for abbreviations.
[a] Mackerel share for EU refers to the five player game, which does not include UK. Mackerel share for EU in the four player game is equivalent to the sum of EU and UK mackerel shares, i.e., 50%.

as well as transformation for variables $R$ and $S$ have been taken into account. The results of the regression are shown in table 1.6. Figure 1.2 shows the actual development of the mackerel stock and the fitted curves for both stock-recruitment functions on the escapement data. Both functions can trace the actual mackerel stock reasonably well.

### 1.4.2 Share of mackerel stock

As we have already mentioned in subsection 1.2.2, $\theta_l$ denotes the share of the mackerel stock that only appears in the EEZ of nation $l$ during the whole year. We believe that the share parameters consists of two dimensions, namely, time and space. Time refers to the percentage of months in a year that mackerel appears in the EEZ of a nation. And, space refers to the percentage of the mackerel stock that appears in the EEZ of a nation. Multiplication of the two percentages for nation $l$ yields parameter $\theta_l$.

For the dimension of time, we base our analysis on the annual migration pattern of the mackerel stock and the time it spends on the respective EEZs of the nations concerned in this article. The migration pattern of mackerel is divided into two elements, namely, a pre-spawning migration and a post-spawning one (ICES, 2016b). From late summer to autumn, the pre-spawning migration starts from the feeding grounds in the North and Nordic seas. This migration phase includes shorter or longer halts in deep waters along the edge of the continental shelf where mackerel shoals overwinter until they reach the spawning grounds south down the west coast of Scotland and Ireland, and along the shelf break waters between Spain and Portugal. The stock is targeted by Norwegian, British and European vessels when it overwinters (fourth quarter) and by European and British vessels afterwards (first quarter). After spawning occurs, the post-spawning migration towards the feeding grounds begins. No significant catches occur during this migration, which takes place in spring (second quarter). During summer the stock is more spread as it feeds in Northern waters. At this time Norwegian, Icelandic and Faroese vessels are active (third quarter).

According to the mackerel migration pattern, we conclude that the stock occurs 50% of the time in the Norwegian EEZ (third and fourth quarters), 50% of the time in the European and British EEZs (fourth and first quarters), and 25% of the time in the Icelandic and Faroese EEZs (third quarter).

For the spatial distribution, unfortunately, no data exist that measures the amount of mackerel that appears in a specific geographical area within the Northeast Atlantic. Therefore, we make the simplifying assumption that approximately half of the stock appears in the EEZ of a nation during mackerel's annual migration pattern. That is, the space percentage that appears in the EEZ of a nation is constant and equal to 50% for all nations. Table 1.7 shows the share of the mackerel stock that appears in the EEZ of the nations we consider in this article, calculated as the product of the two dimensions analysed here. As already mentioned in subsection 1.2.2, the share of the mackerel stock of coalition $i$ is computed as the sum of the individual shares of its members.

### 1.4.3 Unit cost of harvest

As we discuss in section 1.2, the coalitional unit cost of harvest $c_i(x)$ is a continuous and decreasing function with respect to stock size and the total cost within one period is specified as $TC_i(R_{it}, S_{it}) = \int_{S_{it}}^{R_{it}} c_i(x)dx$. Total costs can be also expressed to be proportionate with fishing effort $E_i$, that is $TC_i(E_i) = c_i E_i$, where $c_i$ is a cost parameter. Furthermore, we define the harvest production function of a coalition to be $H_i = E_i x^{\beta}$, where $\beta$ is the stock elasticity and is assumed to be the same for all coalitions. Solving the harvest production function with respect to fishing effort and substituting in the total cost function yields: $TC_i(H_i, x) = c_i H_i x^{-\beta}$. Dividing with harvest, the unit cost of harvest can be expressed as $c_i(x) = c_i x^{-\beta}$. Substituting for the unit cost of harvest in the initial total cost expression and solving the integral provides us with an analytic expression for the total cost of harvest of coalition $i$. Notice that for values of $\beta = 1$ and $\beta \in (0, 1)$ the integral yields different solutions.[19] Thus,

$$TC_i(R_{it}, S_{it}) = \begin{cases} c_i ln\left(\dfrac{R_{it}}{S_{it}}\right), & \beta = 1, \\ c_i \dfrac{1}{1-\beta}(R_{it}^{1-\beta} - S_{it}^{1-\beta}), & 0 < \beta < 1. \end{cases} \tag{1.16}$$

Due to lack of uniformly reported cost data across the nations considered in this article as well as the short-length of some of these series, the cost parameters cannot be estimated through statistical procedures. Instead, the cost coefficients $c_i$ for all coalitions are calibrated at the level which ensures that for base year harvest, $\bar{H}_i = \sum_{l \in C_i} \bar{H}_l$, and base year recruitment, $\bar{R}_i = \theta_i \bar{R}$, total cost is the estimated base year proportion of total revenue $\psi$, i.e.,

$$c_i = \begin{cases} \psi p \bar{H}_i ln\left(\dfrac{\bar{R}_i}{\bar{R}_i - \bar{H}_i}\right)^{-1}, & \beta = 1, \\ \psi p \bar{H}_i (1-\beta)\left[\bar{R}_i^{1-\beta} - (\bar{R}_i - \bar{H}_i)^{1-\beta}\right]^{-1}, & 0 < \beta < 1\ . \end{cases} \tag{1.17}$$

The cost-revenue ratio $\psi$ is equal to 0.78 and is assumed to be equal for all nations. Its computation is based on operating expenses and operating revenues of licensed Norwegian purse seiners for the year 2015 obtained from the report: Profitability survey on the Norwegian fishing fleet, table G 20 (Norwegian Directorate of Fisheries, 2015).

Base year harvest for all nations, $\bar{H}_l$, and base year recruitment for the entire mackerel fishery, $\bar{R}$, are obtained from ICES (2016a) advice report 9.3.39. Recruitment for year 2015 is provided from table 9.3.39.14 of the report and is equivalent to 4,887 thousand tonnes for the entire mackerel fishery. Individual harvest levels for year 2015 are provided from table 9.3.39.12 of the ICES report and are depicted in table 1.8. Base year harvest

[19]For $\beta = 0$ total cost becomes proportional to harvest and the unit cost of harvest is no longer stock dependent. Constant stock density ($\beta = 0$) implies that the equilibrium escapement strategy profile of a coalition structure as specified in subsection 1.3.2 (system of equations (1.4)) cannot be obtained. This is because marginal profit at the beginning and the end of a harvesting period is no longer different and the non-cooperative golden rule becomes $1 = \gamma \theta_i F'(S)$.

Table 1.8. Base year (2015) harvest for European Union, United Kingdom, Norway, Faroe Islands and Iceland. Units: Thousand tonnes.

| | EU[a] | UK | NO | FO | IS |
|---|---|---|---|---|---|
| Base year harvest, $\bar{H}_l$ | 269.929 | 247.986 | 242.231 | 108.412 | 169.333 |

Note: See table 1.5 for abbreviations.
[a] Base year harvest for EU refers to the five player game, which does not include UK. Base year harvest for EU in the four player game is equivalent to the sum of EU and UK base year harvests, i.e., 517.915 thousand tonnes.

Table 1.9. Cost parameters for coalitions $i$ in the four player game for different stock elasticity levels.

| | Cost parameter, $c_i$ | | | |
|---|---|---|---|---|
| Coalition, $C_i$ | $\beta = 1$ | $\beta = 0.6$ | $\beta = 0.3$ | $\beta = 0.1$ |
| (EU) | 17,032.48 | 788.13 | 78.59 | 16.90 |
| (NO) | 8,587.07 | 522.52 | 63.99 | 15.78 |
| (FO) | 4,346.93 | 347.26 | 52.16 | 14.74 |
| (IS) | 4,086.31 | 334.84 | 51.24 | 14.65 |
| (EU,NO) | 25,619.69 | 1,006.85 | 88.83 | 17.60 |
| (EU,FO) | 21,379.94 | 903.27 | 84.14 | 17.28 |
| (EU,IS) | 21,120.84 | 896.80 | 83.84 | 17.26 |
| (NO,FO) | 12,934.16 | 668.07 | 72.35 | 16.44 |
| (NO,IS) | 12,675.85 | 660.17 | 71.93 | 16.40 |
| (FO,IS) | 8,436.17 | 517.08 | 63.66 | 15.75 |
| (EU,NO,FO) | 29,967.05 | 1,106.10 | 93.10 | 17.88 |
| (EU,NO,IS) | 29,708.49 | 1,100.46 | 92.87 | 17.86 |
| (EU,FO,IS) | 25,468.83 | 1,003.35 | 88.68 | 17.59 |
| (NO,FO,IS) | 17,023.72 | 787.89 | 78.58 | 16.90 |
| (EU,NO,FO,IS) | 34,056.20 | 1,194.40 | 96.75 | 18.11 |

Note: See table 1.5 for abbreviations.

for coalition $i$, $\bar{H}_i$, is defined as the sum of the base year quantities of its members $l$, i.e., $\bar{H}_i = \sum_{l \in C_i} \bar{H}_l$. Base year recruitment for coalition $i$, $\bar{R}_i$, is defined as the product of the coalition's share of mackerel stock $\theta_i$ and overall base year recruitment, i.e., $\bar{R}_i = \theta_i \bar{R}$.

The price $p$ is equivalent to 10 NOK/kg. The stock elasticity $\beta$ is not estimated empirically and is therefore varied when running our model in order to capture a range of possibilities. We set $\beta$ equal to 1, 0.6, 0.3 and 0.1.[20] Tables 1.9 and 1.10 in the appendix show the cost parameters for all coalitions in both the four and five player games for all realisations of the stock elasticity.

[20] For models which empirically estimate the stock elasticity see Nøstbakken (2006) and Ekerhovd and Steinshamn (2016).

Table 1.10. Cost parameters for coalitions $i$ in the five player game for different stock elasticity levels.

| | Cost parameter, $c_i$ | | | |
|---|---|---|---|---|
| Coalition, $C_i$ | $\beta = 1$ | $\beta = 0.6$ | $\beta = 0.3$ | $\beta = 0.1$ |
| (EU) | 8,469.55 | 518.29 | 63.73 | 15.76 |
| (UK) | 8,562.75 | 521.65 | 63.94 | 15.77 |
| (NO) | 8,587.07 | 522.52 | 63.99 | 15.78 |
| (FO) | 4,346.93 | 347.26 | 52.16 | 14.74 |
| (IS) | 4,086.31 | 334.84 | 51.24 | 14.65 |
| (EU,UK) | 17,032.48 | 788.13 | 78.59 | 16.90 |
| (EU,NO) | 17,056.91 | 788.80 | 78.62 | 16.90 |
| (EU,FO) | 12,817.18 | 664.50 | 72.16 | 16.42 |
| (EU,IS) | 12,557.15 | 656.52 | 71.73 | 16.39 |
| (UK,NO) | 17,149.83 | 791.33 | 78.75 | 16.91 |
| (UK,FO) | 12,909.92 | 667.33 | 72.32 | 16.43 |
| (UK,IS) | 12,651.26 | 659.41 | 71.89 | 16.40 |
| (NO,IS) | 12,934.16 | 668.07 | 72.35 | 16.44 |
| (NO,FO) | 12,675.85 | 660.17 | 71.93 | 16.40 |
| (FO,IS) | 8,436.17 | 517.08 | 63.66 | 15.75 |
| (EU,UK,NO) | 25,619.69 | 1,006.85 | 88.83 | 17.60 |
| (EU,UK,FO) | 21,379.94 | 903.27 | 84.14 | 17.28 |
| (EU,UK,IS) | 21,120.84 | 896.80 | 83.84 | 17.26 |
| (EU,NO,FO) | 21,404.30 | 903.88 | 84.16 | 17.29 |
| (EU,NO,IS) | 21,145.41 | 897.41 | 83.87 | 17.27 |
| (EU,FO,IS) | 16,905.74 | 784.67 | 78.42 | 16.88 |
| (UK,NO,FO) | 21,497.00 | 906.19 | 84.27 | 17.29 |
| (UK,NO,IS) | 21,238.92 | 899.75 | 83.97 | 17.27 |
| (UK,FO,IS) | 16,999.26 | 787.22 | 78.55 | 16.89 |
| (NO,FO,IS) | 17,023.72 | 787.89 | 78.58 | 16.90 |
| (EU,UK,NO,FO) | 29,967.05 | 1,106.10 | 93.10 | 17.88 |
| (EU,UK,NO,IS) | 29,708.49 | 1,100.46 | 92.87 | 17.86 |
| (EU,UK,FO,IS) | 25,468.83 | 1,003.35 | 88.68 | 17.59 |
| (EU,NO,FO,IS) | 25,493.32 | 1,003.92 | 88.70 | 17.59 |
| (UK,NO,FO,IS) | 25,586.54 | 1,006.08 | 88.79 | 17.60 |
| (EU,UK,NO,FO,IS) | 34,056.20 | 1,194.40 | 96.75 | 18.11 |

Note: See table 1.5 for abbreviations.

## 1.5 Numerical Results and Discussion

Having defined all parameters and functional forms, the solution process of the game is as follows. First, optimal escapement strategies for all coalitions in a coalition structure are computed by solving the system of equations presented in (1.4). The sum of the optimal escapement strategies determines the optimal recruitment through the Ricker (Eq. (1.14)) or the Beverton-Holt (Eq. (1.15)) stock-recruitment function. Then, recruitment and harvest levels for all coalitions in a coalition structure are calculated following the framework described in the beginning of subsection 1.2.2. The coalitional payoff of all coalitions in a coalition structure is determined through Eq. (1.5). This process is repeated for all coalition structures in both games. Finally, internal and external stability of a coalition structure is tested using conditions (1.10) and (1.13).

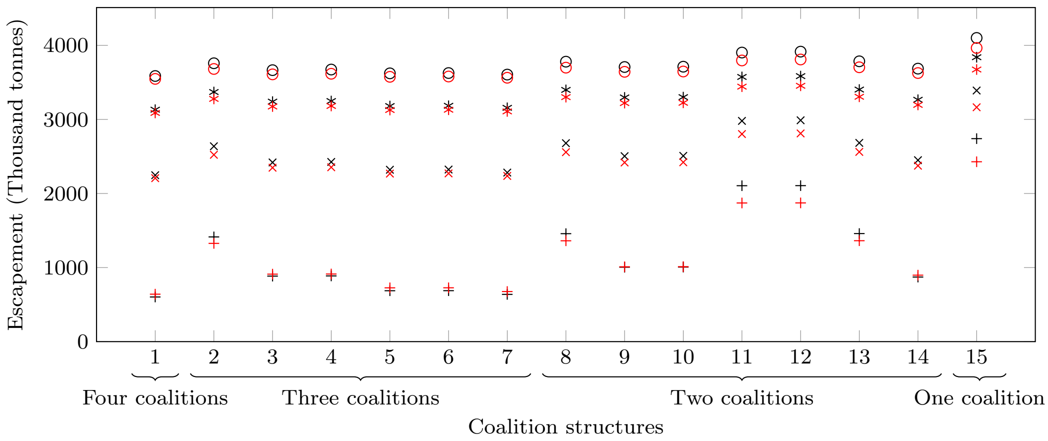
Both games are solved eight times in total, two times for each stock-recruitment function (Ricker and Beverton-Holt) and four times for all the different variations of the stock elasticity parameter. All result tables are placed in the appendix. For the four-player game, tables A.2-A.5 show the results for all stock elasticity levels when the Ricker functional form is applied, and tables A.6-A.9 show the results for the Beverton-Holt functional form. Similarly, tables A.10-A.13 depict the respective results for the five-player game when the Ricker function is used, and tables A.14-A.17 the results for the Beverton-Holt function. The tables are structured as follows: (i) the two first columns represent the coalition structure and its index, (ii) the next four (five) columns show the escapement strategies of the coalitions in a coalition structure, (iii) the next three columns display the aggregate escapement, recruitment and harvest of a coalition structure, (iv) the next four (five) columns show the coalitional payoffs, and (v) the last column is the aggregate value a coalition structure generates.

Before proceeding with the discussion of stable coalition structures, we point out three facts regarding the overall results of these games. First and foremost, our results indicate that positive externalities occur in the mackerel fishery since when coalitions merge to form a larger coalition, outside coalitions not affected by the merger are better off. According to Yi (1997) this result is the defining feature of coalition games with positive externalities. The members of merging coalitions increase the stock level and hence reduce their cost of fishing in order to internalise the positive externality which affects them. Non-cooperating coalitions benefit from the merger by free-riding on the merging coalitions' stock increase.

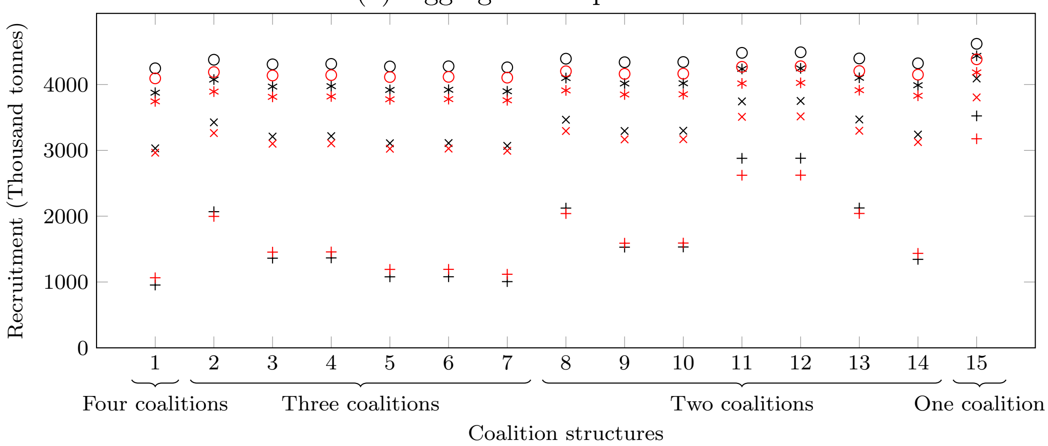
Second, because of this internalisation, aggregate escapement and recruitment increase as the degree of cooperation between coalition structures increases. Figures 1.3a, 1.3b, 1.5 and 1.6 show the escapement and recruitment development across coalition structures in the four- and five-player games for both stock-recruitment functions and all realisations of the stock elasticity. Escapement and recruitment levels are almost the same for both stock-recruitment functions. For stock elasticities equal to 0.3, 0.6 and 1.0 the Ricker function gives slightly higher levels of escapement and recruitment. The opposite is true when stock elasticity is equal to 0.1 for most coalition structures. Furthermore, the lower the stock elasticity the higher the depletion of the stock and thus its growth. This effect is mitigated as the number of coalitions within a coalition structure decreases.

Harvest, which is defined as the difference between recruitment and escapement, is depicted in Figures 1.3c and 1.7. It is not clear whether it increases or not as we move to more cooperative behaviours. For stock elasticities equal to 0.6 and 1.0 it decreases and for stock elasticities equal to 0.1 and 0.3 it increases. This is due to the fact that in stock-recruitment models, as escapement increases, harvest increases from zero to a maximum, i.e., the maximum sustainable yield (MSY) point, and afterwards decreases back to zero, i.e., the carrying capacity point. The MSY points in our model occur at approximately 2,482 and 2,162 thousand tonnes for the Ricker and the Beverton-Holt functions respectively. Thus, all escapement levels before (after) these points lead to an increased (decreased) growth rate and therefore harvest, which explains the change in harvest.

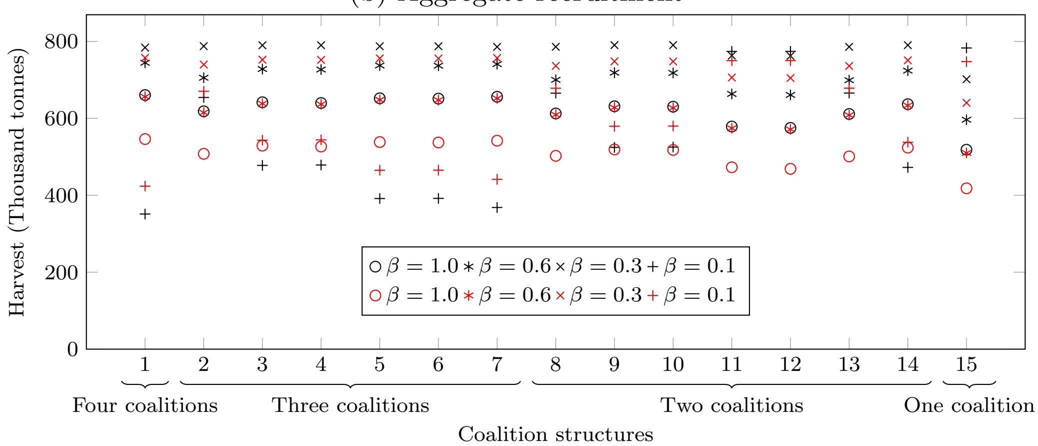
Third, the aggregated value of a coalition structure increases as the number of coalitions within decreases. Figures 1.4a, 1.4b, 1.8 and 1.9 show this increase for both stock-recruitment functions and all realisations of the stock elasticity for both the four- and five-player games. The fact that cooperative behaviours generate more value indicates




(a) Aggregate escapement




(b) Aggregate recruitment

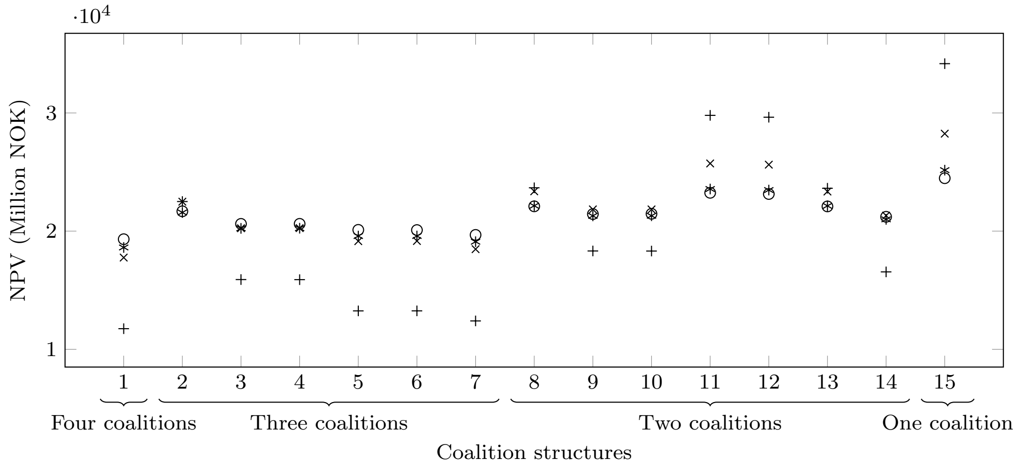



(c) Aggregate harvest

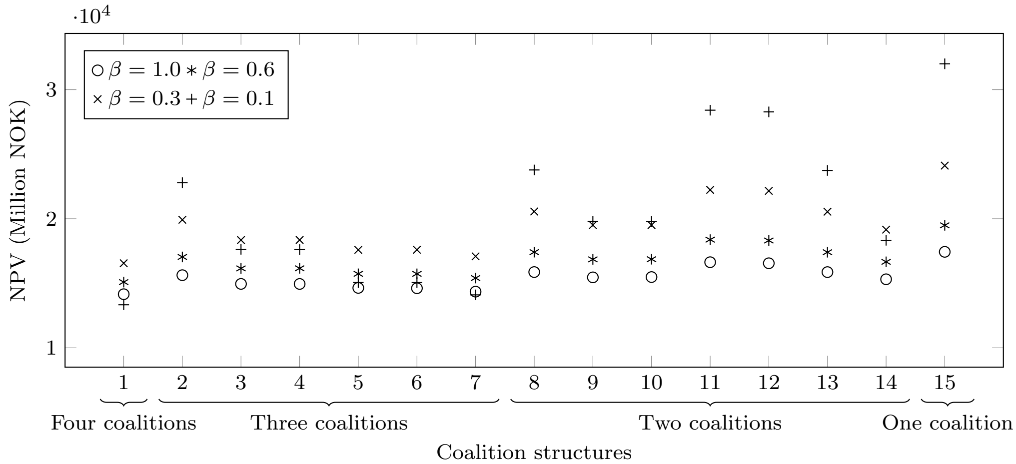
Figure 1.3. Aggregate escapement, recruitment and harvest of a coalition structure for the four player game; Ricker (black) and Beverton-Holt (red) function; and different realisations of the stock elasticity parameter, $\beta$.

that incentives for cooperation among nations exist. However, these incentives must exceed the free-riding benefits in order for cooperation to succeed.

In the four-player game, the grand coalition structure is not a stable outcome in all eight cases. That is, the sum of the free-riding payoffs of the players exceeds the payoff of the grand coalition, thus, making it impossible for any sharing rule to stabilise it. Table




(a) Ricker function




(b) Beverton-Holt function

Figure 1.4. Aggregate NPV of a coalition structure for the four player game; Ricker (a) and Beverton-Holt (b); and different realisations of stock elasticity, $\beta$.

1.11 shows the set of stable coalition structures in the four-player game for all eight cases. The set of stable coalition structures, which is the same for all cases but the Beverton-Holt with $\beta = 1$, consists of all coalition structures that consist of two coalitions, where one of them is a singleton. In addition, the coalition structure representing the current management regime, i.e., $CS_{11}$={(EU,NO,FO),(IS)}, is among the stable ones. Recall, that by stability we mean that in the presence of some but not all sharing rules the coalitions within a coalition structure do not have incentives to merge or split.

In the five-player game, again, the grand coalition structure cannot be sustained as an optimal outcome. The set of stable coalition structures is depicted in table 1.12 for all eight cases. For both stock-recruitment functions and for stock elasticity levels equal to 0.6 and 0.3, all coalition structures consisting of two coalitions, where none of them is a singleton, are stable, namely, $CS_{37}$ to $CS_{46}$.

For the two extreme stock elasticities, the set of stable coalition structures differs across the stock-recruitment functions as well as between the middle elasticities. For $\beta = 1$, $CS_{37}$={(EU,UK,NO),(FO,IS)} is no longer stable for both stock-recruitment functions, but $CS_{27}$={(EU,UK,NO),(FO),(IS)} becomes stable. Thus, according to our results, if the mackerel fishery is uniformly distributed, then Iceland and the Faroe Islands do not have incentives to cooperate with each other any more, given that the remaining

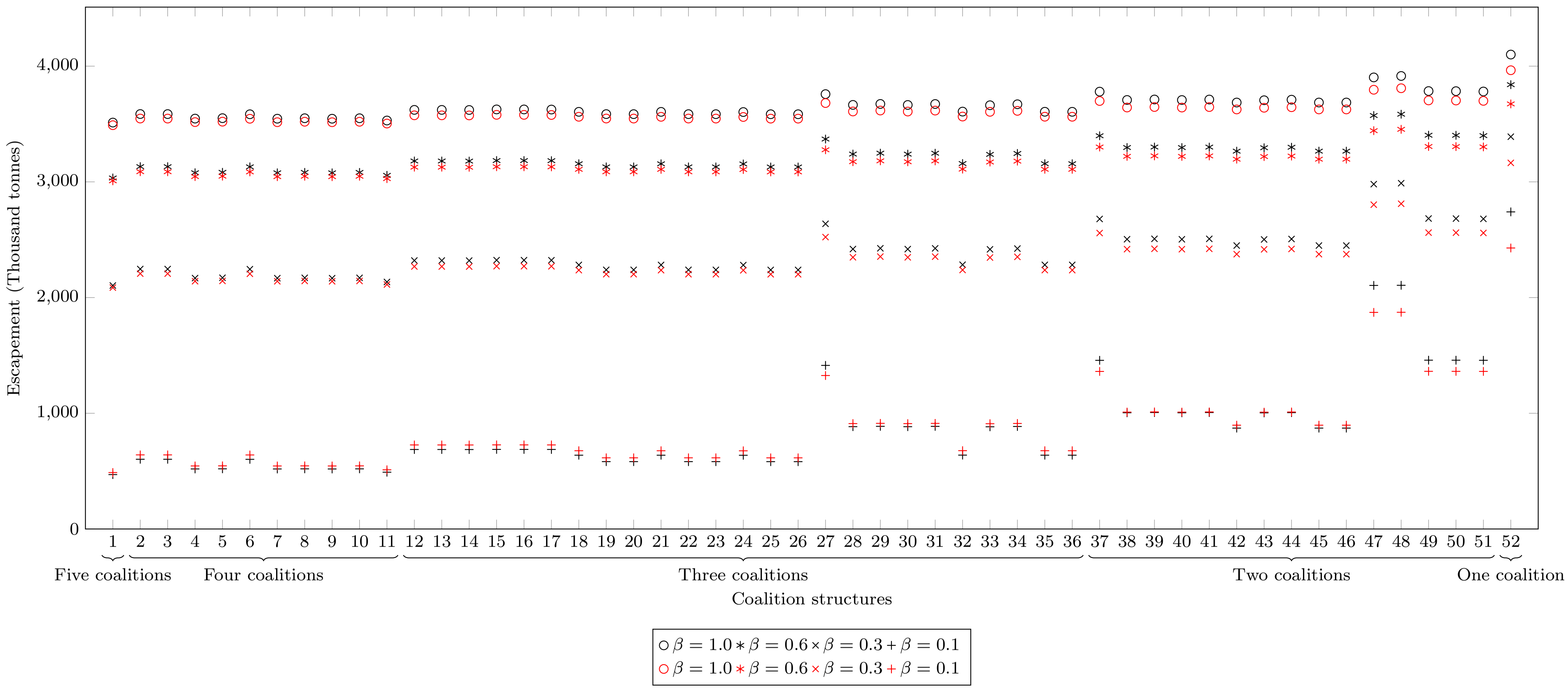



Figure 1.5. Aggregate escapement of a coalition structure for the five player game; Ricker (black) and Beverton-Holt (red) functions, and different realisations of stock elasticity, $\beta$.

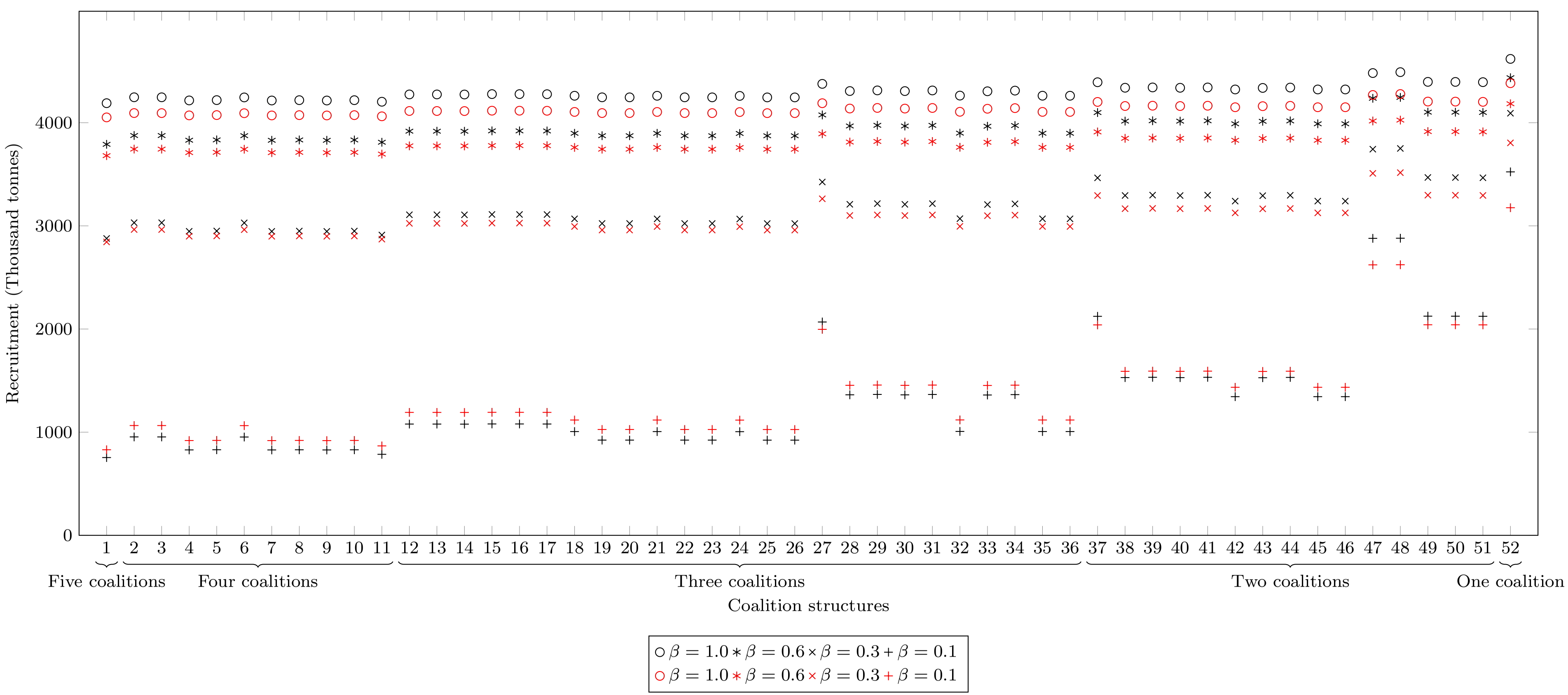



Figure 1.6. Aggregate recruitment of a coalition structure for the five player game; Ricker (black) and Beverton-Holt (red) functions, and different realisations of stock elasticity, $\beta$.

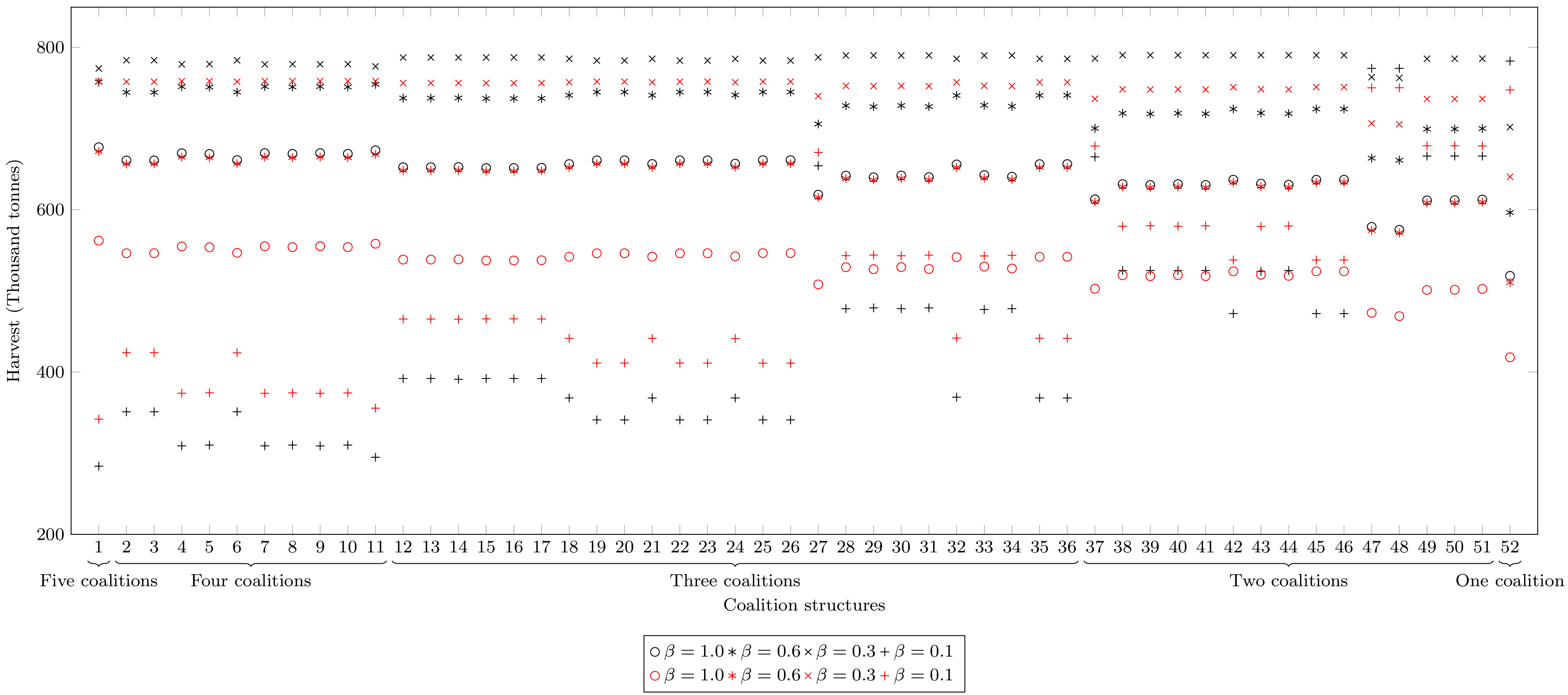



Figure 1.7. Aggregate harvest of a coalition structure for the five player game; Ricker (black) and Beverton-Holt (red) functions, and different realisations of stock elasticity, $\beta$.

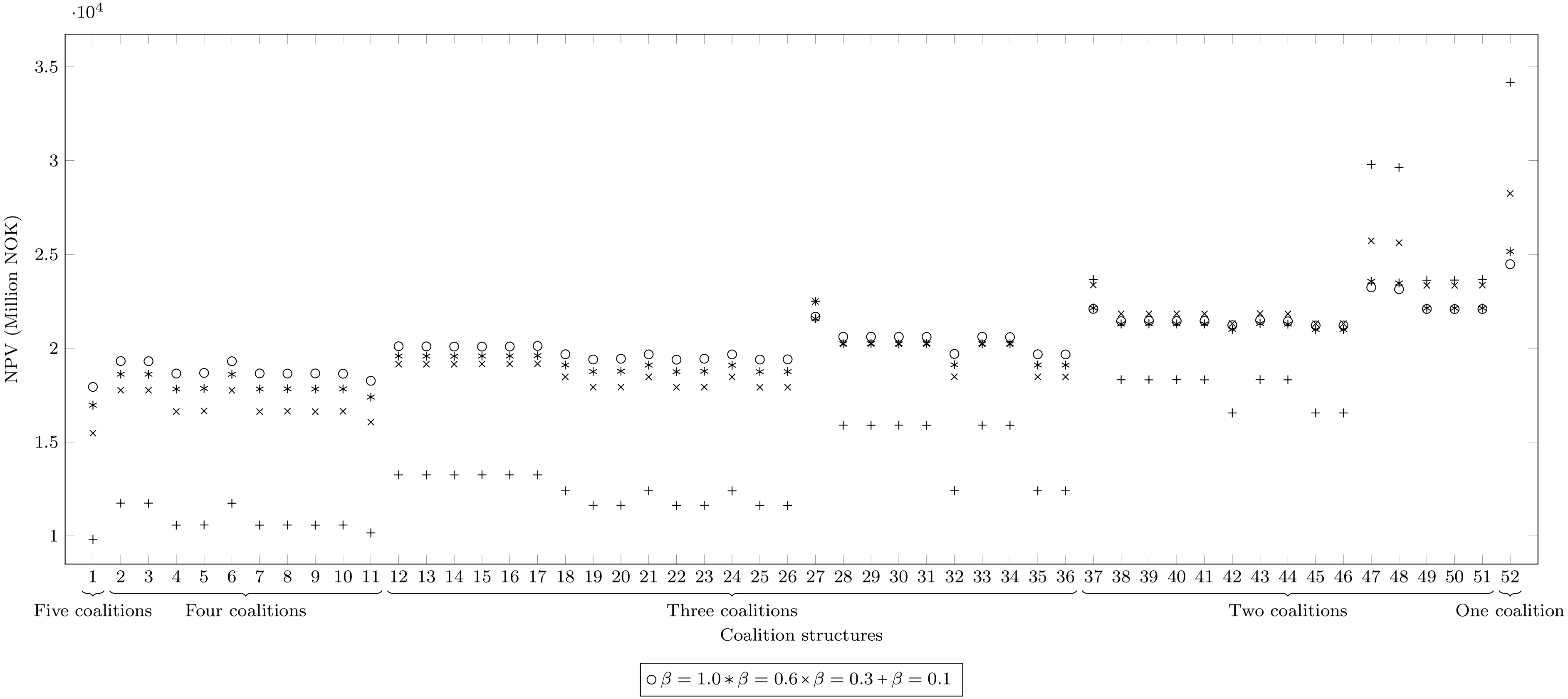



Figure 1.8. Aggregate NPV of a coalition structure for the five player game; Ricker function; and different realisations of stock elasticity, $\beta$.

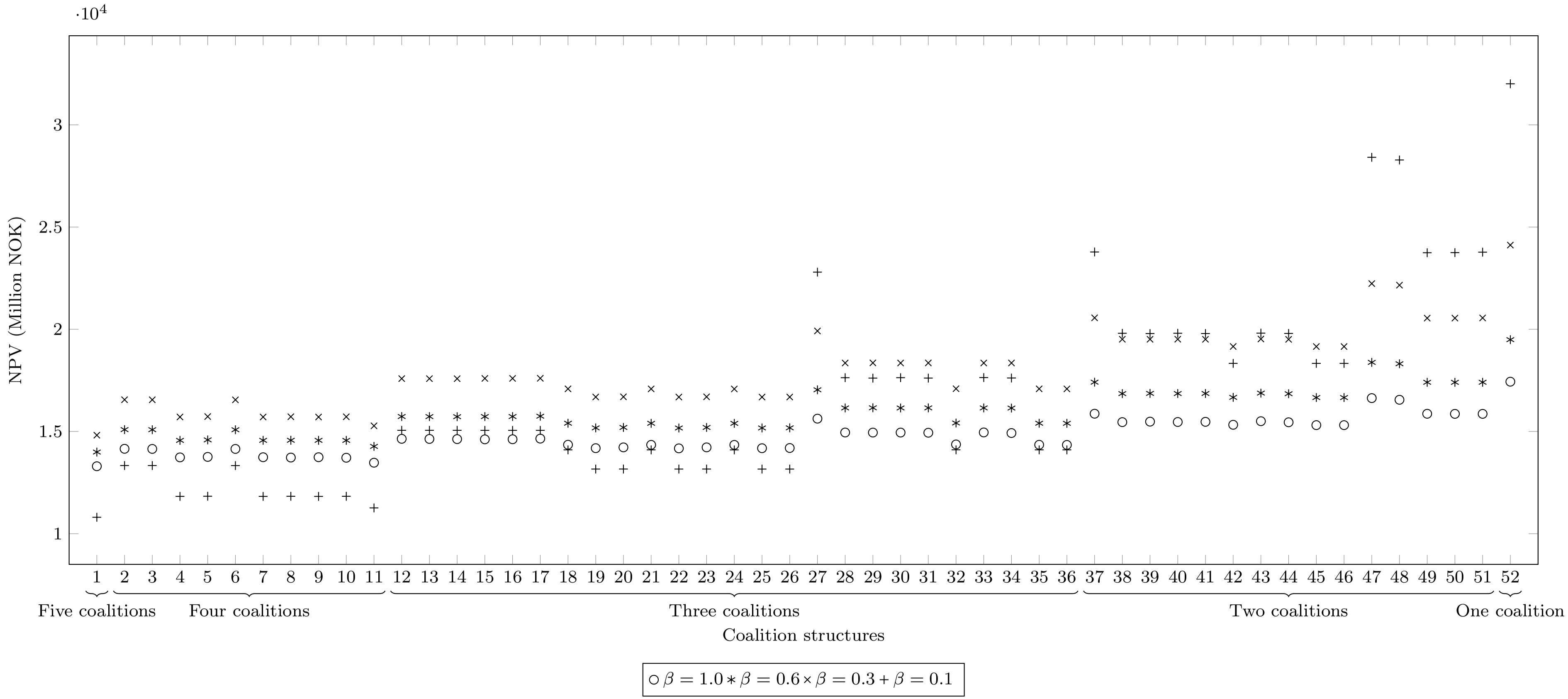



Figure 1.9. Aggregate NPV of a coalition structure for the five player game; Beverton-Holt function; and different realisations of stock elasticity, $\beta$.

Table 1.11. Nash equilibria coalition structures for the four player game for the Ricker and Beverton-Holt stock-recruitment relationships, and different realisations of stock elasticity.

| Ricker | | | | Beverton-Holt | | | |
|---|---|---|---|---|---|---|---|
| $\beta = 1$ | $\beta = 0.6$ | $\beta = 0.3$ | $\beta = 0.1$ | $\beta = 1$ | $\beta = 0.6$ | $\beta = 0.3$ | $\beta = 0.1$ |
| 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 |
| 14 | 14 | 14 | 14 | | 14 | 14 | 14 |

Note: See table 1.2 for which coalition structures the indices refer to.

Table 1.12. Nash equilibria coalition structures for the five player game for the Ricker and Beverton-Holt stock-recruitment relationships, and different realisations of stock elasticity.

| Ricker | | | | Beverton Holt | | | |
|---|---|---|---|---|---|---|---|
| $\beta = 1$ | $\beta = 0.6$ | $\beta = 0.3$ | $\beta = 0.1$ | $\beta = 1$ | $\beta = 0.6$ | $\beta = 0.3$ | $\beta = 0.1$ |
| 27 | 37 | 37 | 37 | 27 | 37 | 37 | 37 |
| 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 |
| 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 |
| 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 |
| 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 |
| 42 | 42 | 42 | 42 | 43 | 42 | 42 | 42 |
| 43 | 43 | 43 | 43 | 44 | 43 | 43 | 43 |
| 44 | 44 | 44 | 44 | | 44 | 44 | 44 |
| 45 | 45 | 45 | 45 | | 45 | 45 | 45 |
| 46 | 46 | 46 | 46 | | 46 | 46 | 46 |
| | | | 47 | | | | 49 |
| | | | 48 | | | | 50 |
| | | | 49 | | | | 51 |
| | | | 50 | | | | |
| | | | 51 | | | | |

Note: See table 1.4 for which coalition structures the indices refer to.

nations cooperate. In addition to $CS_{37}$, coalition structures 42, 45 and 46 are no longer stable when $\beta = 1$ for the Beverton-Holt case. These coalition structures consist of two coalitions where in one coalition a major player (EU, UK or NO) cooperate with the two minors (FO and IS) and in the other the remaining major players cooperate together.

For $\beta = 0.1$, coalitions structures 47 to 51 also become stable for the Ricker case, but only coalition structures 49, 50 and 51 for the Beverton-Holt case. These coalition structures consist of two coalitions, where one of them is a singleton.

Compared to the four-player game, where the set of stable coalition structures remains the same in almost all the cases, in the five-player game stability of some coalition structures is sensitive to the stock-recruitment function and the stock elasticity parameter. Interesting enough, the stable coalition structures in the four-player game are no longer stable for most of the cases in the five-player game. Recall that the singleton coalition of (EU) in the four-player game is equivalent to the coalition of (EU,UK) in the five-player game. The five-player game coalition structures, which are equivalent to the stables ones in the four-player game are: $CS_{11} \equiv CS_{47}$, $CS_{12} \equiv CS_{48}$, $CS_{13} \equiv CS_{49}$ and $CS_{14} \equiv CS_{46}$.

The current management regime, i.e. $CS_{11}$ and $CS_{47}$ in the four- and five-player games respectively, is stable only in one case in the five-player game (Ricker; $\beta = 0.1$), in contrast to the four-player game, where it is stable in all eight cases. This is also true for $CS_{12}$ or $CS_{48}$={(EU,UK,NO,IS),(FO)}. Coalition structure 13, i.e., $CS_{49}$={(EU,UK,FO,IS),(NO)} in the five-player game, occurs only when $\beta = 0.1$ irrespective of the stock-recruitment function. The only four-player game coalition structure that remains stable in all but one (Beverton-Holt; $\beta = 1$) of the five-player game cases is $CS_{14}$, i.e., $CS_{46}$={(NO,FO,IS),(EU,UK)}.

On the other hand, some stable coalition structures in the five-player game are not stable in the four-player game, namely, $CS_{27}$, $CS_{37}$, $CS_{38}$ and $CS_{39}$. The common property of these coalition structures is that the EU and the UK belong to the same coalition. This change in stability between the two games is due to the relative magnitude of the free-riding payoff of the EU in the four-player game and the sum of the free-riding payoffs of the EU and the UK together in the five-player game. In general, the smaller the free-riding payoff, the higher the chance that the external stability condition will not be satisfied, i.e., a coalition will have incentives to merge with another coalition. In the four-player game, the free-riding payoff of the EU is low enough to make it profitable for other coalitions to want to merge with the coalition that it belongs, thus the external stability condition does not hold and therefore the respective coalition structures in the four-player game, i.e., $CS_2$, $CS_8$, $CS_9$ and $CS_{10}$, are not stable. In the five-player game, however, the free-riding payoffs of the EU and the UK together are high enough that is no longer profitable for other coalitions to merge with the coalition that they belong, and therefore making these coalition structures stable.

Having determined the set of stable coalition structures, we now ask ourselves how likely are to form in reality. From the four stable coalition structures of the four-player game, we know that only $CS_{12}$ has been formed in the mackerel fishery. So, from the stable coalition structures of the five-player game, which ones are likely to occur in reality? Or, to put it another way, which ones are unlikely to occur? In what follows, we discuss which coalitions we believe are likely or not to occur post-Brexit based on our intuition of the relations between all five parties.

First, is the cooperation between the EU, the UK, Norway and the Faroe Islands as defined by the current 5-year management plan, likely to continue after the conclusion of the Brexit's negotiations? The agreement itself will cease to exist since the UK will no longer be represented by the EU and therefore must sign its own agreements. In general, after Brexit, the UK will have to negotiate fisheries agreements with other coastal states as well as with the EU. Regarding, straddling and highly migratory fish stocks, such as mackerel, international law requires that all interest parties must cooperate, directly or through RFMOs, that is, the NEAFC in case of Atlantic mackerel. Thus, one possibility is that post-Brexit relationships in the mackerel fishery will be similar or close to existing ones. Of course, the relative TAC shares of the EU, the UK and the other parties, may change depending on the outcome of the negotiations.

After the conclusion of Brexit, the UK will have sovereign control over the resources in its EEZ, and therefore, the principle of equal access[21] will cease to apply in British

[21]Fishing vessels registered in the EU fishing fleet register have equal access to all European waters and resources that are managed under the CFP.

waters and access will now be determined by the criteria set out in UNCLOS. In other words, access in British fisheries will no longer be regulated by European law but by international law. At the moment, EU vessels harvesting in UK's EEZ catch more fish inside British waters than UK vessels catch in the Union's EEZ. Particularly, in 2015, EU vessels caught 683,000 tonnes, i.e., 484 million GBP in revenue, in UK waters, whereas UK vessels caught 111,000 tonnes, i.e., 114 million GBP in revenue, in European waters (Brexit White Paper, 2017).[22] Regarding mackerel, the vast majority of catches taken by the EU occurs within the UK EEZ (Doering et al., 2017). According to a recent study by Le Gallic et al. (2017), if the UK prohibits the EU fleet from accessing fishing stocks within its EEZ, it will cause great loss of revenues for these vessels. Even if the EU redistributes quotas inside its EEZ, it is unlikely that it will compensate for the loss of such important fishing grounds.

From the above, it seems like the UK has all the bargaining power when it comes to negotiating a post-Brexit agreement with the EU. However, this is not true. The UK depends primarily on the EU market for its fishery exports. For the period 2001-2016, 68% on average of the total value generated by fishery exports came from the EU, i.e., 1,204 million EUR. As far as mackerel is concerned, since 2010, on average, more than 60% of UK's annual mackerel exports go to the EU market, generating on average 70 million EUR.[23] Thus, the EU, which is an important trading partner of the UK when it comes to fishery products, might introduce trade barriers, if its access to British waters is limited or denied.

Furthermore, is it possible that cooperation between the EU and Norway will fall apart post-Brexit? Europe and Norway have a long tradition of positive relations, not only in fisheries but across many sectors, and it is doubtful that Norway will act unilaterally, especially if EU and UK agree to cooperate after UK's withdrawal. The bilateral agreement between the EU and Norway covering the North Sea and the Atlantic is the Union's most important international fisheries agreement in terms of both the exchange of fishing opportunities and joint fisheries management measures (Doering et al., 2017).[24] Although this agreement is not related to the management of the mackerel stock,[25] a possible conflict between the EU and Norway regarding the management of mackerel could undermine it. In addition, access of Norwegian fishery products to EU's internal market may also be undermined. As far as Norway is concerned, Brexit is going to make fishery resources in EU waters less attractive, given that when it comes to quota exchanges, stocks in UK waters are more important for Norway than those in EU waters (Sobrino Heredia, 2017). Still, the fact that the EU is a very important trading partner for Norway gives both players more or less equal bargaining power when it comes to negotiating their post-Brexit relationship. The value of Norwegian mackerel exports to the EU excluding the UK were on average 475 million NOK for the period 2007-2016, whereas to the UK

[22]Provisional Statistics – UK Fleet Landings from other EU Member States waters: 2015, Marine Management Organisation, February 2017. These figures do not include fish caught by third country vessels, for example Norway, in UK waters, or fish caught by UK fisherman in third country waters.

[23]The data is obtained from the European Market Observatory for Fisheries and Aquaculture Products (EUMOFA).

[24]The agreement was first enforced on 16 June 1981 for a 10-year period, after that has been tacitly renewed for successive 6-year periods. The last renewal tool place in 2015.

[25]The stocks that this agreement refers to are: cod, plaice and haddock.

for the same period were valued at 62 million NOK on average.[26,27] Thus, making the EU a more significant trading partner for Norway regarding mackerel.

Finally, how are Iceland and the Faroe Islands likely to behave post-Brexit? There has been no indication so far that Iceland is willing to cooperate with the remaining states to jointly determine the mackerel quota. Given its history of disputes, it is highly unlikely that it will cooperate unless it is allowed to maintain its quota or offered something else in exchange for reducing it. However, Iceland may be interested to strengthen its relations with an independent UK and perhaps willing to compromise in the prospect of a future agreement with the UK. As far as the Faroe Islands are concerned, they will most probably keep cooperating with the EU and Norway given that their post-Brexit quota is close to the current one. Like Iceland, they may also be interested to strengthen their relations with the UK. In general, the UK will have to work closely with Norway, Iceland and the Faroe Islands in order to ensure access in one another's waters.

## 1.6 Conclusion

In this article, we analyse how cooperation is likely to occur in the Northeast Atlantic mackerel fishery after the Brexit negotiations are concluded. To do so, we have considered two games: a four-player game, which treats the EU and the UK as one coalition acting together, and a five-player game where the UK is a distinct player acting on its behalf. For our bioeconomic model of the mackerel fishery, we assume a density-dependent stock-recruitment relationship. Both games are solved multiple times for different stock-recruitment functions and levels of the stock elasticity.

We find that positive stock externalities are indeed present in both games since outsiders are better off when a merger between coalitions occurs. The members of a coalition are able to reduce their fishing cost by internalising the positive externality, thus increasing the stock level. This allows outsiders to free-ride on them by benefiting from the increase in the stock. As expected, escapement and recruitment as well as the aggregated value a coalition structure generates increase as the number of coalitions within a coalition structure decreases. That is, cooperation leads to higher profits as well as higher stock preservation. However, in order for cooperation to be achieved the free-riding payoffs of the cooperating nations must not exceed their aggregate coalitional payoff.

In both games, the grand coalition cannot be sustained as an optimal outcome for all scenarios evaluated. The current management regime, however, is found to be a stable outcome in all eight cases of the four-player game, but only in one case of the five-player game. This is also true for all the remaining, but one, stable coalition structures of the four-player game. In addition, some non-stable coalition structures in the four-player game become stable in the five-player game. This occurs because the free-riding payoff of the EU in the four-player game is less than the sum of the free-riding payoffs of the EU and the UK in the five-player game, and therefore making the external stability condition for those coalition structures to only hold in the five-player game. Moreover, in the four-player game, the set of stable coalition structures remains the same in almost all cases,

[26] The data is obtained from Statistics Norway, table: 09283: Exports of fish, by country/trade region/continent.

[27] Value of Norwegian fish, crustaceous animals and mollusc exported to the EU excluding the UK were on average 34,637 million NOK. That is, 90% higher compared to the respective exports in the UK, which were amounted to 3,073 million NOK on average.

whereas in the five-player game stability depends on the stock-recruitment function as well as the magnitude of the stock elasticity.

As far as the future of the mackerel fishery is concerned, we believe that the EU and Norway will keep cooperating post-Brexit. In the event that the UK restricts access to the EU's fleet within its waters, then perhaps Norway will have to give a percentage of its quota to the EU in order to maintain access to the European market. In case of a "hard" Brexit, i.e., no compromises between the EU and the UK during the negotiations, the UK will most likely set its mackerel quota unilaterally. It goes without saying that if this happens, then the pressure on the mackerel stock will increase even more, especially if Iceland continues not to cooperate. However, both the EU and Norway could respond harshly by introducing trade sanctions, as they have already done to Icelandic and Faroese catches in 2013. If a "soft" Brexit occurs, then relationships in the mackerel fishery may be similar or close to existing ones, but the relative shares of the TAC may change depending on the outcome of the negotiations.

A natural extension of the current research is to consider issue-linkage, i.e., link the current cooperative arrangement (exploitation of the mackerel fishery) with a second one, for example, cooperation over the mackerel trade. Then, perhaps, the set of stable coalition structures could indicate that cooperation in the mackerel fishery in presence of externalities is strengthened due to the threat of sanctions in the mackerel trade.

# Acknowledgements

The authors are grateful to Leif K. Sandal, Sturla F. Kvamsdal and two anonymous referees for valuable comments that led to the improvement of the current manuscript. Financial support from the Norwegian Research Council through the MESSAGE project (grant no. 255530/E40) is gratefully acknowledged.

# Appendix

## A.1 Proof of non-cooperative "golden-rule"

The logic of the proof is similar to the one presented by Clark [2010, p. 91]. The profit of coalition $i$ in period $t$ is:

$$\Pi_i(R_{it}, S_{it}) = p(R_{it} - S_{it}) - \int_{S_{it}}^{R_{it}} c_i(x)dx = \int_{S_{it}}^{R_{it}} [p - c_i(x)]dx,$$

where $\pi_i(x) = p - c_i(x)$ is the marginal profit of coalition $i$. Let $\phi_i(x)$ be the antiderivative of $\pi_i(x)$, then we can express the profit of coalition $i$ as:

$$\Pi_i(R_{it}, S_{it}) = \phi_i(R_{it}) - \phi_i(S_{it}).$$

Therefore, the net present value of coalition $i$ becomes:

$$V_i = \sum_{t=0}^{\infty} \gamma^t [\phi_i(R_{it}) - \phi_i(S_{it})].$$

Substituting for the recruitment share of coalition $i$, $R_{it} = \theta_i R_t$, and for the stock-recruitment relationship, $R_t = F(S_{t-1})$ for $t \geq 1$, the first term of the net present value expression yields:

$$\sum_{t=0}^{\infty} \gamma^t \phi_i(R_{it}) = \phi_i(R_{i0}) + \sum_{t=1}^{\infty} \gamma^t \phi_i[\theta_i F(S_{t-1})]$$
$$= \phi_i(R_{i0}) + \sum_{t=0}^{\infty} \gamma^{t+1} \phi_i[\theta_i F(S_t)].$$

Finally, substituting the above term in the net present value of coalition $i$, we obtain:

$$V_i = \phi_i(R_{i0}) + \sum_{t=0}^{\infty} \gamma^{t+1} \phi_i[\theta_i F(S_t)] - \sum_{t=0}^{\infty} \gamma^t \phi_i(S_{it})$$
$$= \phi_i(R_{i0}) + \sum_{t=0}^{\infty} \gamma^t \big[\gamma \phi_i[\theta_i F(S_t)] - \phi_i(S_{it})\big].$$

Now coalition $i$ is enabled to set out the optimal escapement strategy given the escapement strategies of the other coalitions, namely, coalition $i$ to choose the escapement level $S_{it}$ for each time period $t = 0, 1, 2, \ldots, \infty$ by solving the following maximisation problem:

$$\underset{S_{it}}{\text{maximise}} \quad \gamma \phi_i[\theta_i F(S_t)] - \phi_i(S_{it})$$
$$\text{subject to} \quad S_t = S_{it} + \sum_{j=1}^{m-1} S_{jt}, \quad i \neq j.$$

Substituting for $S_t$ in the objective function and taking the first order condition we get:

$$\left[\gamma \phi_i \left[\theta_i F\left(S_{it} + \sum_{j=1}^{m-1} S_{jt}\right)\right] - \phi_i(S_{it})\right]'$$
$$= \gamma \phi_i' \left[\theta_i F\left(S_{it} + \sum_{j=1}^{m-1} S_{jt}\right)\right] \theta_i \frac{dF(S_{it} + \sum_{j=1}^{m-1} S_{jt})}{dS_{it}} - \phi_i'(S_{it})$$
$$= \gamma \pi_i \left[\theta_i F\left(S_{it} + \sum_{j=1}^{m-1} S_{jt}\right)\right] \theta_i \frac{dF(S_{it} + \sum_{j=1}^{m-1} S_{jt})}{dS_{it}} - \pi_i(S_{it}) = 0. \quad \text{(A.1)}$$

It can be shown that the derivative of the stock-recruitment function $F(S)$ with respect to coalition's $i$ escapement $S_i$ is equivalent to the derivative of $F(S)$ with respect to the aggregate escapement $S$. The proof makes use of the chain rule and the fact that the derivative of the aggregate escapement with respect to coalition's $i$ escapement is one, i.e.,

$$\frac{dS}{dS_i} = \frac{d(S_i + \sum_{j=1}^{m-1} S_j)}{dS_i} = 1.$$

Thus,

$$\frac{dF(S_i + \sum_{j=1}^{m-1} S_j)}{dS_i} = \frac{dF(S_i + \sum_{j=1}^{m-1} S_j)}{d(S_i + \sum_{j=1}^{m-1} S_j)} \frac{d(S_i + \sum_{j=1}^{m-1} S_j)}{dS_i} = \frac{dF(S)}{dS} = F'(S).$$

Let $S_{it} = S_i^*$ solve (A.1), then we can re-write it as follows:

$$\pi_i(S_i^*) = \gamma \theta_i F'(S) \pi_i[\theta_i F(S)],$$

where $S = S_i^* + \sum_{j=1}^{m-1} S_j$ is the aggregate escapement and it depends on the optimal escapement strategy of coalition $i$ and the escapement strategies of other coalitions $j$.

## A.2 Illustration of coalition structure stability concepts

Consider a three-player coalition formation game of the class studied in this article. Let $N = \{a, b, c\}$ be the set of players. Table A.1 depicts the payoffs of all embedded coalitions in this game. The property of superadditivity holds for the entire game, i.e., the joint payoff of two embedded coalitions belonging in the same coalition structure is at least as high as their individual payoffs.

Table A.1. Embedded coalition payoffs

| $CS_k$ | $V(C_1, CS_k)$ | $V(C_2, CS_k)$ | $V(C_3, CS_k)$ |
|---|---|---|---|
| $\{a, b, c\}$ | 2 | 4 | 1 |
| $\{ab, c\}$ | 7 | 2 | |
| $\{ac, b\}$ | 4 | 5 | |
| $\{bc, a\}$ | 6 | 4 | |
| $\{abc\}$ | 10 | | |

Suppose we want to test if coalition structure $\{ab, c\}$ is stable. According to subsection 1.3.2 a coalition structure is stable if all of its embedded coalitions are potentially internal and external stable. The tested coalition structure consist of two coalitions: $(ab)$ and $(c)$.

Let us test for potential internal stability first. Coalition $(c)$ is a singleton and therefore is always internal stable. In order for coalition $(ab)$ to be potentially internal stable the payoff of $(ab)$ given coalition structure $\{ab, c\}$ must be greater or equal to the free-riding payoffs of its members, ceteris paribus. The free-riding payoffs are determined as follows. Consider player $a$ first, if player $a$ leaves coalition $(ab)$ then the new coalition structure, ceteris paribus, is $\{a, b, c\}$. Similarly, if player $b$ leaves coalition $(ab)$, then the new coalition structure, ceteris paribus, is $\{a, b, c\}$. Notice that the new coalition structures are the same in both deviations; this is not always the case as we will see in the next case. Having determined the new coalition structures, we can now compare the payoffs and test if coalition $(ab)$ is potentially internal stable.

$$V(ab, \{ab, c\}) \geq V(a, \{a, b, c\}) + V(b, \{a, b, c\}) \Rightarrow 7 \geq 2 + 4 = 6.$$

Since the above inequality holds we can conclude that coalition $(ab)$ is potentially internal stable. Seeing that both coalitions $(ab)$ and $(c)$ are potentially internal stable we can conclude that coalition structure $\{ab, c\}$ is potentially internal stable. We move on to test for potential internal stability.

Coalition structure $\{ab, c\}$ consist of only one pair of embedded coalitions, i.e, $\big[(ab, \{ab, c\}), (c, \{ab, c\})\big]$. In order for $\{ab, c\}$ to be external stable at least one of the two embedded coalitions should not have incentives to merge. Let us start with $(ab)$, if $(ab)$ merges with $(c)$ then the new coalition structure, ceteris paribus, will be $\{abc\}$ but player $c$ must receive at least her free-riding payoff which occurs if she deviates from the new coalition $(abc)$. If player $c$ leaves $(abc)$ the new coalition structure, ceteris paribus, will be $\{ab, c\}$. Thus, the potential external stability condition for coalition $(ab)$ with respect to coalition $(c)$ requires the following:

$$V(ab, \{ab, c\}) \geq V(abc, \{abc\}) - V(c, \{ab, c\}) \Rightarrow 7 \geq 10 - 2 = 8.$$

Since the above inequality does not hold we can conclude that coalition $(ab)$ does have incentives to merge with coalition $(c)$ and therefore $(c)$ is not potentially external stable with respect to $(ab)$. However, coalition structure $\{ab, c\}$ may still be external stable as long as coalition $(c)$ is better off without the mergence. If $(c)$ merges with $(ab)$ then the new coalition structure, ceteris paribus, will be $\{abc\}$ but players $a$ and $b$ must receive at least their free-riding payoffs. If player $a$ leaves $(abc)$ the new coalition structure, ceteris paribus, will be $\{bc, a\}$. Similarly, if player $b$ leaves $(abc)$ the new coalition structure, ceteris paribus, will be $\{ac, b\}$. Thus, the potential external stability condition for coalition $(c)$ with respect to coalition $(ab)$ requires the following:

$$V(c, \{ab, c\}) \geq V(abc, \{abc\}) - V(a, \{bc, a\}) - V(b, \{ac, b\}) \Rightarrow 2 \geq 10 - 4 - 5 = 1.$$

Since the above inequality holds, coalition $(c)$ does not have incentives to merge with coalition $(ab)$ and therefore $(ab)$ is potentially external stable with respect to coalition $(c)$. Since $\big[(ab, \{ab, c\}), (c, \{ab, c\})\big]$ is the only embedded coalition pair of coalition structure $\{ab, c\}$ and $(c, \{ab, c\})$ is not willing to merge, we can conclude that coalition structure $\{ab, c\}$ is potentially external stable. Because coalition structure $\{ab, c\}$ is both potentially internal and external stable we can conclude that $\{ab, c\}$ is a stable coalition structure.

Following the same procedure, it can be showed that coalition structures $\{ac, b\}$ and $\{bc, a\}$ are also stable. The singleton coalition structure $\{a, b, c\}$ is not potentially external stable since all the players have incentives to form a coalition with at least one more player. The grand coalition structure $\{abc\}$ is not potentially internal stable since the sum of the free-riding payoffs of its members exceeds the payoff of the grand coalition, i.e.,

$$V(abc, \{abc\}) \geq V(a, \{bc, a\}) + V(b, \{ac, b\}) + V(c, \{ab, c\}) \Rightarrow 10 \geq 4 + 5 + 2 = 11.$$

This also verifies the fact that superadditive games with externalities cannot necessarily sustain the grand coalition as a stable outcome.

## A.3 Result tables for the four- and five-player games

Table A.2. Optimal solution for the four player game; Ricker function; stock elasticity: $\beta = 1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 1,837 | 890 | 442 | 417 | 3,586 | 4,247 | 661 | 9,133 | 4,640 | 2,127 | 3,418 | 19,317 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 2,898 | 443 | 418 | | 3,758 | 4,377 | 618 | 14,725 | 2,753 | 4,201 | | 21,679 |
| 3 | (EU,FO),(NO),(IS) | 2,356 | 891 | 418 | | 3,665 | 4,307 | 642 | 11,616 | 5,224 | 3,772 | | 20,612 |
| 4 | (EU,IS),(NO),(FO) | 2,340 | 891 | 442 | | 3,673 | 4,313 | 640 | 12,891 | 5,286 | 2,437 | | 20,615 |
| 5 | (NO,FO),(EU),(IS) | 1,365 | 1,839 | 418 | | 3,622 | 4,274 | 652 | 6,880 | 9,648 | 3,578 | | 20,105 |
| 6 | (NO,IS),(EU),(FO) | 1,344 | 1,839 | 442 | | 3,626 | 4,277 | 651 | 8,115 | 9,706 | 2,267 | | 20,089 |
| 7 | (FO,IS),(EU),(NO) | 877 | 1,838 | 890 | | 3,605 | 4,261 | 656 | 5,495 | 9,405 | 4,779 | | 19,679 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 2,900 | 879 | | | 3,780 | 4,392 | 613 | 15,181 | 6,918 | | | 22,099 |
| 9 | (EU,FO),(NO,IS) | 2,360 | 1,347 | | | 3,708 | 4,339 | 631 | 12,366 | 9,078 | | | 21,444 |
| 10 | (EU,IS),(NO,FO) | 2,343 | 1,369 | | | 3,712 | 4,342 | 630 | 13,608 | 7,858 | | | 21,466 |
| 11 | (EU,NO,FO),(IS) | 3,484 | 418 | | | 3,903 | 4,481 | 579 | 18,357 | 4,885 | | | 23,243 |
| 12 | (EU,NO,IS),(FO) | 3,472 | 443 | | | 3,915 | 4,490 | 575 | 19,780 | 3,363 | | | 23,143 |
| 13 | (EU,FO,IS),(NO) | 2,892 | 893 | | | 3,785 | 4,396 | 611 | 15,942 | 6,149 | | | 22,091 |
| 14 | (NO,FO,IS),(EU) | 1,843 | 1,843 | | | 3,686 | 4,323 | 637 | 10,627 | 10,586 | | | 21,212 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 4,100 | | | | 4,100 | 4,619 | 518 | 24,479 | | | | 24,479 |

Note: See table 1.5 for abbreviations.

Table A.3. Optimal solution for the four player game; Ricker function; stock elasticity: $\beta = 0.6$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 1,622 | 771 | 379 | 359 | 3,131 | 3,876 | 744 | 8,733 | 4,608 | 2,185 | 3,092 | 18,618 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 2,630 | 380 | 360 | | 3,370 | 4,075 | 706 | 14,536 | 2,993 | 4,049 | | 21,578 |
| 3 | (EU,FO),(NO),(IS) | 2,108 | 773 | 359 | | 3,240 | 3,968 | 728 | 11,372 | 5,342 | 3,520 | | 20,234 |
| 4 | (EU,IS),(NO),(FO) | 2,095 | 773 | 380 | | 3,248 | 3,975 | 727 | 12,279 | 5,397 | 2,570 | | 20,246 |
| 5 | (NO,FO),(EU),(IS) | 1,195 | 1,627 | 359 | | 3,180 | 3,918 | 737 | 6,943 | 9,358 | 3,283 | | 19,583 |
| 6 | (NO,IS),(EU),(FO) | 1,178 | 1,627 | 380 | | 3,184 | 3,921 | 737 | 7,826 | 9,406 | 2,356 | | 19,588 |
| 7 | (FO,IS),(EU),(NO) | 760 | 1,625 | 771 | | 3,156 | 3,897 | 741 | 5,281 | 9,050 | 4,773 | | 19,105 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 2,634 | 764 | | | 3,398 | 4,098 | 700 | 15,082 | 7,065 | | | 22,146 |
| 9 | (EU,FO),(NO,IS) | 2,115 | 1,182 | | | 3,297 | 4,015 | 719 | 12,269 | 9,023 | | | 21,292 |
| 10 | (EU,IS),(NO,FO) | 2,101 | 1,200 | | | 3,301 | 4,019 | 718 | 13,154 | 8,148 | | | 21,301 |
| 11 | (EU,NO,FO),(IS) | 3,213 | 360 | | | 3,573 | 4,237 | 663 | 18,653 | 4,910 | | | 23,563 |
| 12 | (EU,NO,IS),(FO) | 3,204 | 381 | | | 3,585 | 4,246 | 661 | 19,698 | 3,778 | | | 23,476 |
| 13 | (EU,FO,IS),(NO) | 2,628 | 776 | | | 3,403 | 4,102 | 699 | 15,631 | 6,504 | | | 22,136 |
| 14 | (NO,FO,IS),(EU) | 1,632 | 1,633 | | | 3,265 | 3,989 | 724 | 10,513 | 10,484 | | | 20,997 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 3,840 | | | | 3,840 | 4,436 | 596 | 25,160 | | | | 25,160 |

Note: See table 1.5 for abbreviations.

Table A.4. Optimal solution for the four player game; Ricker function; stock elasticity: $\beta = 0.3$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 1,201 | 541 | 259 | 246 | 2,247 | 3,031 | 784 | 8,148 | 4,579 | 2,258 | 2,777 | 17,762 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 2,129 | 261 | 247 | | 2,638 | 3,425 | 788 | 14,606 | 3,608 | 4,279 | | 22,492 |
| 3 | (EU,FO),(NO),(IS) | 1,627 | 545 | 247 | | 2,419 | 3,209 | 790 | 11,100 | 5,726 | 3,418 | | 20,244 |
| 4 | (EU,IS),(NO),(FO) | 1,620 | 545 | 260 | | 2,426 | 3,216 | 790 | 11,631 | 5,771 | 2,852 | | 20,255 |
| 5 | (NO,FO),(EU),(IS) | 864 | 1,211 | 246 | | 2,320 | 3,108 | 787 | 7,085 | 9,020 | 3,046 | | 19,151 |
| 6 | (NO,IS),(EU),(FO) | 853 | 1,211 | 260 | | 2,323 | 3,111 | 787 | 7,604 | 9,056 | 2,506 | | 19,166 |
| 7 | (FO,IS),(EU),(NO) | 535 | 1,206 | 542 | | 2,282 | 3,068 | 786 | 5,102 | 8,564 | 4,807 | | 18,473 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 2,137 | 543 | | | 2,679 | 3,465 | 786 | 15,382 | 7,987 | | | 23,369 |
| 9 | (EU,FO),(NO,IS) | 1,642 | 862 | | | 2,504 | 3,294 | 790 | 12,375 | 9,464 | | | 21,839 |
| 10 | (EU,IS),(NO,FO) | 1,634 | 874 | | | 2,508 | 3,298 | 790 | 12,898 | 8,940 | | | 21,838 |
| 11 | (EU,NO,FO),(IS) | 2,732 | 248 | | | 2,980 | 3,743 | 763 | 20,025 | 5,701 | | | 25,725 |
| 12 | (EU,NO,IS),(FO) | 2,726 | 262 | | | 2,988 | 3,751 | 762 | 20,682 | 4,942 | | | 25,624 |
| 13 | (EU,FO,IS),(NO) | 2,133 | 550 | | | 2,683 | 3,469 | 786 | 15,718 | 7,633 | | | 23,351 |
| 14 | (NO,FO,IS),(EU) | 1,225 | 1,225 | | | 2,450 | 3,240 | 790 | 10,666 | 10,649 | | | 21,314 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 3,390 | | | | 3,390 | 4,092 | 702 | 28,247 | | | | 28,247 |

Note: See table 1.5 for abbreviations.

Table A.5. Optimal solution for the four player game; Ricker function; stock elasticity: $\beta = 0.1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 362 | 131 | 56 | 54 | 603 | 954 | 351 | 5,312 | 3,136 | 1,594 | 1,702 | 11,745 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 1,295 | 61 | 58 | | 1,414 | 2,068 | 654 | 12,133 | 5,059 | 5,312 | | 22,504 |
| 3 | (EU,FO),(NO),(IS) | 685 | 144 | 56 | | 884 | 1,362 | 478 | 7,930 | 5,171 | 2,801 | | 15,901 |
| 4 | (EU,IS),(NO),(FO) | 684 | 144 | 59 | | 887 | 1,366 | 479 | 8,051 | 5,193 | 2,649 | | 15,894 |
| 5 | (NO,FO),(EU),(IS) | 244 | 389 | 55 | | 687 | 1,079 | 392 | 5,141 | 6,109 | 2,002 | | 13,252 |
| 6 | (NO,IS),(EU),(FO) | 242 | 389 | 57 | | 688 | 1,080 | 392 | 5,258 | 6,116 | 1,880 | | 13,254 |
| 7 | (FO,IS),(EU),(NO) | 131 | 373 | 133 | | 638 | 1,006 | 368 | 3,420 | 5,627 | 3,356 | | 12,403 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 1,305 | 153 | | | 1,458 | 2,123 | 665 | 13,119 | 10,547 | | | 23,666 |
| 9 | (EU,FO),(NO,IS) | 731 | 274 | | | 1,005 | 1,529 | 525 | 9,537 | 8,784 | | | 18,321 |
| 10 | (EU,IS),(NO,FO) | 730 | 277 | | | 1,007 | 1,532 | 525 | 9,675 | 8,641 | | | 18,315 |
| 11 | (EU,NO,FO),(IS) | 2,047 | 58 | | | 2,105 | 2,879 | 774 | 20,960 | 8,836 | | | 29,796 |
| 12 | (EU,NO,IS),(FO) | 2,045 | 61 | | | 2,106 | 2,880 | 774 | 21,151 | 8,487 | | | 29,638 |
| 13 | (EU,FO,IS),(NO) | 1,304 | 155 | | | 1,459 | 2,125 | 666 | 13,215 | 10,412 | | | 23,627 |
| 14 | (NO,FO,IS),(EU) | 436 | 436 | | | 872 | 1,344 | 472 | 8,278 | 8,274 | | | 16,552 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 2,741 | | | | 2,741 | 3,523 | 783 | 34,170 | | | | 34,170 |

Note: See table 1.5 for abbreviations.

Table A.6. Optimal solution for the four player game; Beverton-Holt function; stock elasticity: $\beta = 1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 1,808 | 883 | 440 | 416 | 3,547 | 4,093 | 546 | 6,725 | 3,335 | 1,496 | 2,597 | 14,153 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 2,824 | 441 | 416 | | 3,681 | 4,189 | 508 | 10,651 | 1,878 | 3,098 | | 15,627 |
| 3 | (EU,FO),(NO),(IS) | 2,308 | 884 | 416 | | 3,608 | 4,137 | 529 | 8,437 | 3,691 | 2,822 | | 14,951 |
| 4 | (EU,IS),(NO),(FO) | 2,291 | 884 | 441 | | 3,617 | 4,143 | 527 | 9,517 | 3,741 | 1,689 | | 14,947 |
| 5 | (NO,FO),(EU),(IS) | 1,349 | 1,810 | 416 | | 3,575 | 4,114 | 538 | 4,899 | 7,043 | 2,699 | | 14,641 |
| 6 | (NO,IS),(EU),(FO) | 1,329 | 1,810 | 441 | | 3,579 | 4,117 | 537 | 5,942 | 7,090 | 1,584 | | 14,616 |
| 7 | (FO,IS),(EU),(NO) | 870 | 1,809 | 883 | | 3,563 | 4,105 | 542 | 4,029 | 6,899 | 3,423 | | 14,351 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 2,827 | 872 | | | 3,699 | 4,202 | 503 | 10,945 | 4,924 | | | 15,869 |
| 9 | (EU,FO),(NO,IS) | 2,312 | 1,331 | | | 3,643 | 4,162 | 519 | 8,915 | 6,542 | | | 15,458 |
| 10 | (EU,IS),(NO,FO) | 2,295 | 1,352 | | | 3,647 | 4,165 | 518 | 9,967 | 5,514 | | | 15,482 |
| 11 | (EU,NO,FO),(IS) | 3,379 | 417 | | | 3,796 | 4,269 | 473 | 13,082 | 3,550 | | | 16,633 |
| 12 | (EU,NO,IS),(FO) | 3,368 | 441 | | | 3,809 | 4,278 | 469 | 14,278 | 2,273 | | | 16,551 |
| 13 | (EU,FO,IS),(NO) | 2,818 | 886 | | | 3,704 | 4,205 | 501 | 11,587 | 4,279 | | | 15,866 |
| 14 | (NO,FO,IS),(EU) | 1,813 | 1,813 | | | 3,626 | 4,150 | 524 | 7,672 | 7,638 | | | 15,310 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 3,965 | | | | 3,965 | 4,383 | 418 | 17,435 | | | | 17,435 |

Note: See table 1.5 for abbreviations.

Table A.7. Optimal solution for the four player game; Beverton-Holt function; stock elasticity: $\beta = 0.6$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 1,590 | 764 | 377 | 357 | 3,088 | 3,744 | 656 | 7,148 | 3,684 | 1,726 | 2,533 | 15,091 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 2,541 | 378 | 358 | | 3,277 | 3,892 | 615 | 11,618 | 2,250 | 3,171 | | 17,039 |
| 3 | (EU,FO),(NO),(IS) | 2,052 | 765 | 357 | | 3,174 | 3,813 | 638 | 9,171 | 4,163 | 2,818 | | 16,152 |
| 4 | (EU,IS),(NO),(FO) | 2,039 | 765 | 378 | | 3,182 | 3,818 | 637 | 9,974 | 4,206 | 1,979 | | 16,159 |
| 5 | (NO,FO),(EU),(IS) | 1,177 | 1,593 | 357 | | 3,127 | 3,775 | 648 | 5,507 | 7,563 | 2,661 | | 15,732 |
| 6 | (NO,IS),(EU),(FO) | 1,160 | 1,593 | 378 | | 3,131 | 3,778 | 648 | 6,287 | 7,602 | 1,840 | | 15,729 |
| 7 | (FO,IS),(EU),(NO) | 753 | 1,592 | 764 | | 3,108 | 3,761 | 652 | 4,245 | 7,363 | 3,795 | | 15,404 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 2,545 | 756 | | | 3,301 | 3,911 | 610 | 11,989 | 5,423 | | | 17,412 |
| 9 | (EU,FO),(NO,IS) | 2,057 | 1,163 | | | 3,221 | 3,849 | 628 | 9,776 | 7,077 | | | 16,853 |
| 10 | (EU,IS),(NO,FO) | 2,043 | 1,181 | | | 3,224 | 3,852 | 627 | 10,558 | 6,306 | | | 16,864 |
| 11 | (EU,NO,FO),(IS) | 3,084 | 358 | | | 3,442 | 4,017 | 574 | 14,620 | 3,760 | | | 18,380 |
| 12 | (EU,NO,IS),(FO) | 3,075 | 379 | | | 3,454 | 4,025 | 571 | 15,531 | 2,782 | | | 18,313 |
| 13 | (EU,FO,IS),(NO) | 2,538 | 767 | | | 3,306 | 3,914 | 608 | 12,472 | 4,934 | | | 17,406 |
| 14 | (NO,FO,IS),(EU) | 1,598 | 1,598 | | | 3,196 | 3,830 | 634 | 8,343 | 8,317 | | | 16,660 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 3,674 | | | | 3,674 | 4,184 | 510 | 19,491 | | | | 19,491 |

Note: See table 1.5 for abbreviations.

Table A.8. Optimal solution for the four player game; Beverton-Holt function; stock elasticity: $\beta = 0.3$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 1,171 | 534 | 258 | 244 | 2,207 | 2,965 | 758 | 7,709 | 4,217 | 2,067 | 2,561 | 16,553 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 2,018 | 259 | 245 | | 2,523 | 3,263 | 740 | 13,273 | 3,017 | 3,627 | | 19,918 |
| 3 | (EU,FO),(NO),(IS) | 1,566 | 537 | 245 | | 2,348 | 3,101 | 752 | 10,280 | 5,045 | 3,025 | | 18,350 |
| 4 | (EU,IS),(NO),(FO) | 1,558 | 537 | 258 | | 2,354 | 3,106 | 752 | 10,780 | 5,081 | 2,495 | | 18,356 |
| 5 | (NO,FO),(EU),(IS) | 846 | 1,178 | 244 | | 2,269 | 3,024 | 756 | 6,458 | 8,369 | 2,760 | | 17,587 |
| 6 | (NO,IS),(EU),(FO) | 835 | 1,178 | 258 | | 2,271 | 3,027 | 756 | 6,948 | 8,398 | 2,250 | | 17,595 |
| 7 | (FO,IS),(EU),(NO) | 528 | 1,174 | 535 | | 2,237 | 2,994 | 757 | 4,669 | 8,028 | 4,389 | | 17,086 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 2,024 | 533 | | | 2,558 | 3,294 | 737 | 13,849 | 6,711 | | | 20,559 |
| 9 | (EU,FO),(NO,IS) | 1,576 | 842 | | | 2,418 | 3,167 | 748 | 11,226 | 8,286 | | | 19,512 |
| 10 | (EU,IS),(NO,FO) | 1,568 | 854 | | | 2,422 | 3,170 | 748 | 11,718 | 7,793 | | | 19,511 |
| 11 | (EU,NO,FO),(IS) | 2,557 | 246 | | | 2,803 | 3,509 | 706 | 17,592 | 4,642 | | | 22,235 |
| 12 | (EU,NO,IS),(FO) | 2,551 | 260 | | | 2,811 | 3,516 | 705 | 18,192 | 3,966 | | | 22,158 |
| 13 | (EU,FO,IS),(NO) | 2,020 | 541 | | | 2,561 | 3,298 | 736 | 14,159 | 6,386 | | | 20,545 |
| 14 | (NO,FO,IS),(EU) | 1,187 | 1,188 | | | 2,375 | 3,126 | 751 | 9,585 | 9,569 | | | 19,154 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 3,164 | | | | 3,164 | 3,805 | 640 | 24,118 | | | | 24,118 |

Note: See table 1.5 for abbreviations.

Table A.9. Optimal solution for the four player game; Beverton-Holt function; stock elasticity: $\beta = 0.1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Four coalitions:** | | | | | | | | | | | | | |
| 1 | (EU),(NO),(FO),(IS) | 392 | 136 | 58 | 55 | 640 | 1,064 | 424 | 5,917 | 3,610 | 1,842 | 1,965 | 13,335 |
| **Three coalitions:** | | | | | | | | | | | | | |
| 2 | (EU,NO),(FO),(IS) | 1,208 | 61 | 57 | | 1,326 | 1,997 | 670 | 12,973 | 4,789 | 5,033 | | 22,795 |
| 3 | (EU,FO),(NO),(IS) | 709 | 146 | 57 | | 911 | 1,454 | 543 | 8,823 | 5,718 | 3,090 | | 17,631 |
| 4 | (EU,IS),(NO),(FO) | 708 | 146 | 59 | | 913 | 1,457 | 544 | 8,952 | 5,734 | 2,924 | | 17,610 |
| 5 | (NO,FO),(EU),(IS) | 256 | 415 | 55 | | 726 | 1,192 | 465 | 5,875 | 6,881 | 2,302 | | 15,058 |
| 6 | (NO,IS),(EU),(FO) | 254 | 415 | 58 | | 727 | 1,192 | 465 | 6,004 | 6,887 | 2,164 | | 15,055 |
| 7 | (FO,IS),(EU),(NO) | 136 | 402 | 138 | | 676 | 1,118 | 442 | 3,936 | 6,301 | 3,864 | | 14,101 |
| **Two coalitions:** | | | | | | | | | | | | | |
| 8 | (EU,NO),(FO,IS) | 1,212 | 149 | | | 1,362 | 2,040 | 678 | 13,855 | 9,926 | | | 23,781 |
| 9 | (EU,FO),(NO,IS) | 735 | 275 | | | 1,010 | 1,590 | 580 | 10,430 | 9,374 | | | 19,804 |
| 10 | (EU,IS),(NO,FO) | 734 | 278 | | | 1,012 | 1,592 | 580 | 10,574 | 9,216 | | | 19,790 |
| 11 | (EU,NO,FO),(IS) | 1,815 | 57 | | | 1,872 | 2,622 | 750 | 20,747 | 7,664 | | | 28,412 |
| 12 | (EU,NO,IS),(FO) | 1,812 | 60 | | | 1,873 | 2,623 | 750 | 20,933 | 7,347 | | | 28,280 |
| 13 | (EU,FO,IS),(NO) | 1,211 | 151 | | | 1,363 | 2,041 | 679 | 13,951 | 9,793 | | | 23,744 |
| 14 | (NO,FO,IS),(EU) | 449 | 449 | | | 897 | 1,435 | 538 | 9,168 | 9,163 | | | 18,331 |
| **One coalition:** | | | | | | | | | | | | | |
| 15 | (EU,NO,FO,IS) | 2,428 | | | | 2,428 | 3,176 | 747 | 32,006 | | | | 32,006 |

Note: See table 1.5 for abbreviations.

Table A.10. Optimal solution for the five player game; Ricker function; stock elasticity: $\beta = 1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 878 | 887 | 889 | 442 | 417 | 3,512 | 4,189 | 677 | 4,630 | 4,220 | 4,117 | 1,876 | 3,096 | 17,939 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 1,837 | 890 | 442 | 417 | | 3,586 | 4,247 | 661 | 9,133 | 4,640 | 2,127 | 3,418 | | 19,317 |
| 3 | (EU,NO),(UK),(FO),(IS) | 1,838 | 888 | 442 | 417 | | 3,586 | 4,246 | 661 | 9,022 | 4,747 | 2,126 | 3,416 | | 19,311 |
| 4 | (EU,FO),(UK),(NO),(IS) | 1,353 | 887 | 889 | 417 | | 3,547 | 4,216 | 669 | 6,587 | 4,463 | 4,357 | 3,244 | | 18,652 |
| 5 | (EU,IS),(UK),(NO),(FO) | 1,332 | 887 | 889 | 442 | | 3,550 | 4,219 | 669 | 7,806 | 4,491 | 4,384 | 2,004 | | 18,685 |
| 6 | (UK,NO),(EU),(FO),(IS) | 1,845 | 880 | 442 | 417 | | 3,584 | 4,245 | 661 | 8,609 | 5,170 | 2,120 | 3,409 | | 19,308 |
| 7 | (UK,FO),(EU),(NO),(IS) | 1,360 | 879 | 889 | 417 | | 3,546 | 4,215 | 670 | 6,187 | 4,879 | 4,351 | 3,241 | | 18,658 |
| 8 | (UK,IS),(EU),(NO),(FO) | 1,340 | 879 | 889 | 442 | | 3,550 | 4,219 | 669 | 7,361 | 4,909 | 4,380 | 2,002 | | 18,652 |
| 9 | (NO,IS),(EU),(UK),(FO) | 1,362 | 879 | 887 | 417 | | 3,546 | 4,215 | 670 | 6,084 | 4,877 | 4,456 | 3,240 | | 18,658 |
| 10 | (NO,FO),(EU),(UK),(IS) | 1,342 | 879 | 887 | 442 | | 3,550 | 4,218 | 669 | 7,247 | 4,908 | 4,486 | 2,002 | | 18,642 |
| 11 | (FO,IS),(EU),(UK),(NO) | 876 | 879 | 887 | 889 | | 3,530 | 4,203 | 673 | 4,916 | 4,762 | 4,346 | 4,241 | | 18,265 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 1,839 | 1,365 | 418 | | | 3,622 | 4,274 | 652 | 9,648 | 6,880 | 3,578 | | | 20,105 |
| 13 | (EU,NO),(UK,FO),(IS) | 1,841 | 1,363 | 418 | | | 3,622 | 4,274 | 652 | 9,537 | 6,985 | 3,577 | | | 20,099 |
| 14 | (EU,FO),(UK,NO),(IS) | 1,356 | 1,848 | 418 | | | 3,621 | 4,274 | 652 | 7,395 | 9,122 | 3,574 | | | 20,091 |
| 15 | (EU,UK),(NO,IS),(FO) | 1,839 | 1,344 | 442 | | | 3,626 | 4,277 | 651 | 9,706 | 8,115 | 2,267 | | | 20,089 |
| 16 | (EU,NO),(UK,IS),(FO) | 1,841 | 1,342 | 442 | | | 3,626 | 4,277 | 651 | 9,595 | 8,232 | 2,267 | | | 20,093 |
| 17 | (EU,IS),(UK,NO),(FO) | 1,335 | 1,848 | 442 | | | 3,625 | 4,276 | 651 | 8,685 | 9,175 | 2,263 | | | 20,124 |
| 18 | (EU,UK),(FO,IS),(NO) | 1,838 | 877 | 890 | | | 3,605 | 4,261 | 656 | 9,405 | 5,495 | 4,779 | | | 19,679 |
| 19 | (EU,FO),(UK,IS),(NO) | 1,354 | 1,341 | 890 | | | 3,585 | 4,246 | 661 | 7,004 | 7,764 | 4,635 | | | 19,402 |
| 20 | (EU,IS),(UK,FO),(NO) | 1,333 | 1,362 | 890 | | | 3,585 | 4,246 | 661 | 8,212 | 6,596 | 4,634 | | | 19,441 |
| 21 | (EU,NO),(FO,IS),(UK) | 1,840 | 877 | 888 | | | 3,605 | 4,261 | 656 | 9,293 | 5,492 | 4,888 | | | 19,674 |
| 22 | (EU,FO),(NO,IS),(UK) | 1,354 | 1,343 | 888 | | | 3,585 | 4,246 | 661 | 7,002 | 7,647 | 4,744 | | | 19,393 |
| 23 | (EU,IS),(NO,FO),(UK) | 1,333 | 1,364 | 888 | | | 3,585 | 4,246 | 661 | 8,210 | 6,490 | 4,742 | | | 19,442 |
| 24 | (UK,NO),(FO,IS),(EU) | 1,847 | 877 | 880 | | | 3,603 | 4,260 | 657 | 8,873 | 5,480 | 5,317 | | | 19,671 |
| 25 | (UK,FO),(NO,IS),(EU) | 1,362 | 1,343 | 880 | | | 3,585 | 4,245 | 661 | 6,588 | 7,638 | 5,172 | | | 19,398 |
| 26 | (UK,IS),(NO,FO),(EU) | 1,341 | 1,364 | 880 | | | 3,584 | 4,245 | 661 | 7,753 | 6,484 | 5,172 | | | 19,408 |
| 27 | (EU,UK,NO),(FO),(IS) | 2,898 | 443 | 418 | | | 3,758 | 4,377 | 618 | 14,725 | 2,753 | 4,201 | | | 21,679 |
| 28 | (EU,UK,FO),(NO),(IS) | 2,356 | 891 | 418 | | | 3,665 | 4,307 | 642 | 11,616 | 5,224 | 3,772 | | | 20,612 |
| 29 | (EU,UK,IS),(NO),(FO) | 2,340 | 891 | 442 | | | 3,673 | 4,313 | 640 | 12,891 | 5,286 | 2,437 | | | 20,615 |
| 30 | (EU,NO,FO),(UK),(IS) | 2,358 | 889 | 418 | | | 3,665 | 4,307 | 642 | 11,503 | 5,336 | 3,769 | | | 20,609 |
| 31 | (EU,NO,IS),(UK),(FO) | 2,341 | 889 | 442 | | | 3,673 | 4,313 | 640 | 12,771 | 5,400 | 2,435 | | | 20,606 |
| 32 | (EU,FO,IS),(UK),(NO) | 1,829 | 888 | 890 | | | 3,607 | 4,263 | 656 | 9,998 | 4,905 | 4,794 | | | 19,697 |
| 33 | (UK,NO,FO),(EU),(IS) | 2,364 | 881 | 418 | | | 3,662 | 4,305 | 643 | 11,077 | 5,778 | 3,758 | | | 20,614 |
| 34 | (UK,NO,IS),(EU),(FO) | 2,347 | 881 | 442 | | | 3,671 | 4,311 | 641 | 12,318 | 5,846 | 2,427 | | | 20,591 |
| 35 | (UK,FO,IS),(EU),(NO) | 1,836 | 880 | 890 | | | 3,606 | 4,262 | 656 | 9,559 | 5,336 | 4,783 | | | 19,678 |
| 36 | (NO,FO,IS),(EU),(UK) | 1,837 | 880 | 888 | | | 3,605 | 4,261 | 656 | 9,446 | 5,333 | 4,892 | | | 19,672 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 2,900 | 879 | | | | 3,780 | 4,392 | 613 | 15,181 | 6,918 | | | | 22,099 |
| 38 | (EU,UK,FO),(NO,IS) | 2,360 | 1,347 | | | | 3,708 | 4,339 | 631 | 12,366 | 9,078 | | | | 21,444 |
| 39 | (EU,UK,IS),(NO,FO) | 2,343 | 1,369 | | | | 3,712 | 4,342 | 630 | 13,608 | 7,858 | | | | 21,466 |
| 40 | (EU,NO,FO),(UK,IS) | 2,362 | 1,345 | | | | 3,707 | 4,339 | 631 | 12,252 | 9,199 | | | | 21,451 |
| 41 | (EU,NO,IS),(UK,FO) | 2,345 | 1,367 | | | | 3,711 | 4,342 | 630 | 13,488 | 7,969 | | | | 21,457 |
| 42 | (EU,FO,IS),(UK,NO) | 1,834 | 1,852 | | | | 3,686 | 4,322 | 637 | 11,185 | 10,045 | | | | 21,230 |
| 43 | (UK,NO,FO),(EU,IS) | 2,368 | 1,337 | | | | 3,705 | 4,337 | 632 | 11,822 | 9,668 | | | | 21,490 |
| 44 | (UK,NO,IS),(EU,FO) | 2,351 | 1,359 | | | | 3,710 | 4,341 | 631 | 13,036 | 8,399 | | | | 21,435 |
| 45 | (UK,FO,IS),(EU,NO) | 1,841 | 1,845 | | | | 3,686 | 4,323 | 637 | 10,741 | 10,472 | | | | 21,213 |
| 46 | (NO,FO,IS),(EU,UK) | 1,843 | 1,843 | | | | 3,686 | 4,323 | 637 | 10,627 | 10,586 | | | | 21,212 |
| 47 | (EU,UK,NO,FO),(IS) | 3,484 | 418 | | | | 3,903 | 4,481 | 579 | 18,357 | 4,885 | | | | 23,243 |
| 48 | (EU,UK,NO,IS),(FO) | 3,472 | 443 | | | | 3,915 | 4,490 | 575 | 19,780 | 3,363 | | | | 23,143 |
| 49 | (EU,UK,FO,IS),(NO) | 2,892 | 893 | | | | 3,785 | 4,396 | 611 | 15,942 | 6,149 | | | | 22,091 |
| 50 | (EU,NO,FO,IS),(UK) | 2,893 | 891 | | | | 3,784 | 4,395 | 612 | 15,818 | 6,270 | | | | 22,088 |
| 51 | (UK,NO,FO,IS),(EU) | 2,899 | 882 | | | | 3,781 | 4,393 | 612 | 15,347 | 6,744 | | | | 22,091 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 4,100 | | | | | 4,100 | 4,619 | 518 | 24,479 | | | | | 24,479 |

Note: See table 1.5 for abbreviations.

Table A.11. Optimal solution for the five player game; Ricker function; stock elasticity: $\beta = 0.6$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 760 | 767 | 769 | 379 | 358 | 3,033 | 3,790 | 757 | 4,336 | 4,052 | 3,980 | 1,879 | 2,720 | 16,967 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 1,622 | 771 | 379 | 359 | | 3,131 | 3,876 | 744 | 8,733 | 4,608 | 2,185 | 3,092 | | 18,618 |
| 3 | (EU,NO),(UK),(FO),(IS) | 1,624 | 769 | 379 | 359 | | 3,131 | 3,876 | 744 | 8,656 | 4,684 | 2,184 | 3,090 | | 18,614 |
| 4 | (EU,FO),(UK),(NO),(IS) | 1,182 | 768 | 770 | 359 | | 3,079 | 3,830 | 751 | 6,329 | 4,341 | 4,266 | 2,890 | | 17,826 |
| 5 | (EU,IS),(UK),(NO),(FO) | 1,165 | 768 | 770 | 379 | | 3,082 | 3,833 | 751 | 7,175 | 4,365 | 4,289 | 2,030 | | 17,859 |
| 6 | (UK,NO),(EU),(FO),(IS) | 1,629 | 762 | 379 | 359 | | 3,130 | 3,874 | 745 | 8,365 | 4,981 | 2,179 | 3,085 | | 18,610 |
| 7 | (UK,FO),(EU),(NO),(IS) | 1,188 | 761 | 770 | 359 | | 3,078 | 3,830 | 752 | 6,047 | 4,631 | 4,261 | 2,887 | | 17,827 |
| 8 | (UK,IS),(EU),(NO),(FO) | 1,171 | 761 | 770 | 379 | | 3,082 | 3,833 | 751 | 6,869 | 4,656 | 4,285 | 2,028 | | 17,839 |
| 9 | (NO,IS),(EU),(UK),(FO) | 1,190 | 761 | 768 | 359 | | 3,078 | 3,829 | 752 | 5,975 | 4,630 | 4,335 | 2,887 | | 17,826 |
| 10 | (NO,FO),(EU),(UK),(IS) | 1,173 | 761 | 768 | 379 | | 3,082 | 3,833 | 751 | 6,790 | 4,655 | 4,360 | 2,028 | | 17,833 |
| 11 | (FO,IS),(EU),(UK),(NO) | 758 | 761 | 768 | 769 | | 3,056 | 3,811 | 754 | 4,593 | 4,486 | 4,197 | 4,123 | | 17,398 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 1,627 | 1,195 | 359 | | | 3,180 | 3,918 | 737 | 9,358 | 6,943 | 3,283 | | | 19,583 |
| 13 | (EU,NO),(UK,FO),(IS) | 1,628 | 1,193 | 359 | | | 3,180 | 3,918 | 737 | 9,280 | 7,018 | 3,282 | | | 19,580 |
| 14 | (EU,FO),(UK,NO),(IS) | 1,187 | 1,633 | 359 | | | 3,179 | 3,917 | 738 | 7,309 | 8,986 | 3,279 | | | 19,575 |
| 15 | (EU,UK),(NO,IS),(FO) | 1,627 | 1,178 | 380 | | | 3,184 | 3,921 | 737 | 9,406 | 7,826 | 2,356 | | | 19,588 |
| 16 | (EU,NO),(UK,IS),(FO) | 1,628 | 1,176 | 380 | | | 3,184 | 3,921 | 737 | 9,327 | 7,907 | 2,356 | | | 19,590 |
| 17 | (EU,IS),(UK,NO),(FO) | 1,170 | 1,634 | 380 | | | 3,183 | 3,920 | 737 | 8,222 | 9,031 | 2,353 | | | 19,606 |
| 18 | (EU,UK),(FO,IS),(NO) | 1,625 | 760 | 771 | | | 3,156 | 3,897 | 741 | 9,050 | 5,281 | 4,773 | | | 19,105 |
| 19 | (EU,FO),(UK,IS),(NO) | 1,185 | 1,174 | 771 | | | 3,129 | 3,874 | 745 | 6,815 | 7,346 | 4,594 | | | 18,754 |
| 20 | (EU,IS),(UK,FO),(NO) | 1,167 | 1,191 | 771 | | | 3,129 | 3,874 | 745 | 7,656 | 6,526 | 4,593 | | | 18,775 |
| 21 | (EU,NO),(FO,IS),(UK) | 1,626 | 760 | 770 | | | 3,156 | 3,897 | 741 | 8,971 | 5,279 | 4,851 | | | 19,101 |
| 22 | (EU,FO),(NO,IS),(UK) | 1,185 | 1,175 | 769 | | | 3,129 | 3,874 | 745 | 6,813 | 7,264 | 4,671 | | | 18,748 |
| 23 | (EU,IS),(NO,FO),(UK) | 1,167 | 1,192 | 769 | | | 3,129 | 3,874 | 745 | 7,654 | 6,451 | 4,670 | | | 18,775 |
| 24 | (UK,NO),(FO,IS),(EU) | 1,631 | 760 | 763 | | | 3,155 | 3,896 | 741 | 8,674 | 5,269 | 5,154 | | | 19,097 |
| 25 | (UK,FO),(NO,IS),(EU) | 1,191 | 1,175 | 762 | | | 3,128 | 3,873 | 745 | 6,520 | 7,257 | 4,973 | | | 18,749 |
| 26 | (UK,IS),(NO,FO),(EU) | 1,174 | 1,192 | 762 | | | 3,128 | 3,873 | 745 | 7,336 | 6,445 | 4,973 | | | 18,755 |
| 27 | (EU,UK,NO),(FO),(IS) | 2,630 | 380 | 360 | | | 3,370 | 4,075 | 706 | 14,536 | 2,993 | 4,049 | | | 21,578 |
| 28 | (EU,UK,FO),(NO),(IS) | 2,108 | 773 | 359 | | | 3,240 | 3,968 | 728 | 11,372 | 5,342 | 3,520 | | | 20,234 |
| 29 | (EU,UK,IS),(NO),(FO) | 2,095 | 773 | 380 | | | 3,248 | 3,975 | 727 | 12,279 | 5,397 | 2,570 | | | 20,246 |
| 30 | (EU,NO,FO),(UK),(IS) | 2,109 | 771 | 359 | | | 3,240 | 3,968 | 728 | 11,291 | 5,423 | 3,518 | | | 20,232 |
| 31 | (EU,NO,IS),(UK),(FO) | 2,096 | 771 | 380 | | | 3,248 | 3,974 | 727 | 12,195 | 5,478 | 2,568 | | | 20,241 |
| 32 | (EU,FO,IS),(UK),(NO) | 1,617 | 770 | 772 | | | 3,158 | 3,899 | 741 | 9,466 | 4,866 | 4,786 | | | 19,118 |
| 33 | (UK,NO,FO),(EU),(IS) | 2,114 | 764 | 359 | | | 3,237 | 3,966 | 729 | 10,986 | 5,739 | 3,509 | | | 20,234 |
| 34 | (UK,NO,IS),(EU),(FO) | 2,101 | 764 | 380 | | | 3,245 | 3,973 | 727 | 11,875 | 5,797 | 2,561 | | | 20,232 |
| 35 | (UK,FO,IS),(EU),(NO) | 1,623 | 763 | 771 | | | 3,157 | 3,898 | 741 | 9,158 | 5,170 | 4,777 | | | 19,105 |
| 36 | (NO,FO,IS),(EU),(UK) | 1,624 | 763 | 770 | | | 3,157 | 3,897 | 741 | 9,079 | 5,168 | 4,854 | | | 19,101 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 2,634 | 764 | | | | 3,398 | 4,098 | 700 | 15,082 | 7,065 | | | | 22,146 |
| 38 | (EU,UK,FO),(NO,IS) | 2,115 | 1,182 | | | | 3,297 | 4,015 | 719 | 12,269 | 9,023 | | | | 21,292 |
| 39 | (EU,UK,IS),(NO,FO) | 2,101 | 1,200 | | | | 3,301 | 4,019 | 718 | 13,154 | 8,148 | | | | 21,301 |
| 40 | (EU,NO,FO),(UK,IS) | 2,116 | 1,180 | | | | 3,296 | 4,015 | 719 | 12,188 | 9,108 | | | | 21,296 |
| 41 | (EU,NO,IS),(UK,FO) | 2,102 | 1,198 | | | | 3,301 | 4,019 | 718 | 13,069 | 8,228 | | | | 21,297 |
| 42 | (EU,FO,IS),(UK,NO) | 1,625 | 1,640 | | | | 3,265 | 3,989 | 724 | 10,908 | 10,098 | | | | 21,006 |
| 43 | (UK,NO,FO),(EU,IS) | 2,121 | 1,174 | | | | 3,295 | 4,014 | 719 | 11,878 | 9,440 | | | | 21,318 |
| 44 | (UK,NO,IS),(EU,FO) | 2,107 | 1,192 | | | | 3,299 | 4,017 | 718 | 12,748 | 8,538 | | | | 21,286 |
| 45 | (UK,FO,IS),(EU,NO) | 1,631 | 1,635 | | | | 3,265 | 3,989 | 724 | 10,594 | 10,403 | | | | 20,997 |
| 46 | (NO,FO,IS),(EU,UK) | 1,632 | 1,633 | | | | 3,265 | 3,989 | 724 | 10,513 | 10,484 | | | | 20,997 |
| 47 | (EU,UK,NO,FO),(IS) | 3,213 | 360 | | | | 3,573 | 4,237 | 663 | 18,653 | 4,910 | | | | 23,563 |
| 48 | (EU,UK,NO,IS),(FO) | 3,204 | 381 | | | | 3,585 | 4,246 | 661 | 19,698 | 3,778 | | | | 23,476 |
| 49 | (EU,UK,FO,IS),(NO) | 2,628 | 776 | | | | 3,403 | 4,102 | 699 | 15,631 | 6,504 | | | | 22,136 |
| 50 | (EU,NO,FO,IS),(UK) | 2,629 | 774 | | | | 3,402 | 4,102 | 699 | 15,542 | 6,593 | | | | 22,135 |
| 51 | (UK,NO,FO,IS),(EU) | 2,633 | 767 | | | | 3,399 | 4,099 | 700 | 15,202 | 6,939 | | | | 22,141 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 3,840 | | | | | 3,840 | 4,436 | 596 | 25,160 | | | | | 25,160 |

Note: See table 1.5 for abbreviations.

Table A.12. Optimal solution for the five player game; Ricker function; stock elasticity: $\beta = 0.3$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 531 | 535 | 536 | 258 | 245 | 2,104 | 2,878 | 774 | 3,905 | 3,750 | 3,710 | 1,824 | 2,283 | 15,472 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 1,201 | 541 | 259 | 246 | | 2,247 | 3,031 | 784 | 8,148 | 4,579 | 2,258 | 2,777 | | 17,762 |
| 3 | (EU,NO),(UK),(FO),(IS) | 1,202 | 540 | 259 | 246 | | 2,246 | 3,030 | 784 | 8,105 | 4,622 | 2,257 | 2,776 | | 17,760 |
| 4 | (EU,FO),(UK),(NO),(IS) | 848 | 537 | 538 | 245 | | 2,168 | 2,947 | 779 | 5,903 | 4,133 | 4,091 | 2,501 | | 16,627 |
| 5 | (EU,IS),(UK),(NO),(FO) | 837 | 537 | 538 | 259 | | 2,171 | 2,950 | 779 | 6,371 | 4,151 | 4,109 | 2,023 | | 16,654 |
| 6 | (UK,NO),(EU),(FO),(IS) | 1,205 | 535 | 259 | 246 | | 2,245 | 3,029 | 784 | 7,940 | 4,791 | 2,253 | 2,772 | | 17,755 |
| 7 | (UK,FO),(EU),(NO),(IS) | 851 | 533 | 538 | 245 | | 2,168 | 2,947 | 779 | 5,745 | 4,294 | 4,087 | 2,499 | | 16,625 |
| 8 | (UK,IS),(EU),(NO),(FO) | 841 | 533 | 538 | 259 | | 2,171 | 2,950 | 779 | 6,204 | 4,313 | 4,105 | 2,022 | | 16,645 |
| 9 | (NO,IS),(EU),(UK),(FO) | 852 | 533 | 537 | 245 | | 2,167 | 2,946 | 779 | 5,704 | 4,293 | 4,128 | 2,498 | | 16,624 |
| 10 | (NO,FO),(EU),(UK),(IS) | 842 | 533 | 537 | 259 | | 2,171 | 2,950 | 779 | 6,161 | 4,313 | 4,147 | 2,021 | | 16,642 |
| 11 | (FO,IS),(EU),(UK),(NO) | 530 | 532 | 536 | 537 | | 2,135 | 2,911 | 776 | 4,150 | 4,091 | 3,931 | 3,890 | | 16,063 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 1,211 | 864 | 246 | | | 2,320 | 3,108 | 787 | 9,020 | 7,085 | 3,046 | | | 19,151 |
| 13 | (EU,NO),(UK,FO),(IS) | 1,212 | 863 | 246 | | | 2,320 | 3,108 | 787 | 8,976 | 7,129 | 3,045 | | | 19,149 |
| 14 | (EU,FO),(UK,NO),(IS) | 859 | 1,215 | 246 | | | 2,320 | 3,107 | 787 | 7,297 | 8,807 | 3,043 | | | 19,146 |
| 15 | (EU,UK),(NO,IS),(FO) | 1,211 | 853 | 260 | | | 2,323 | 3,111 | 787 | 9,056 | 7,604 | 2,506 | | | 19,166 |
| 16 | (EU,NO),(UK,IS),(FO) | 1,212 | 852 | 260 | | | 2,323 | 3,111 | 787 | 9,011 | 7,650 | 2,505 | | | 19,166 |
| 17 | (EU,IS),(UK,NO),(FO) | 847 | 1,215 | 260 | | | 2,322 | 3,110 | 787 | 7,828 | 8,841 | 2,503 | | | 19,171 |
| 18 | (EU,UK),(FO,IS),(NO) | 1,206 | 535 | 542 | | | 2,282 | 3,068 | 786 | 8,564 | 5,102 | 4,807 | | | 18,473 |
| 19 | (EU,FO),(UK,IS),(NO) | 853 | 846 | 541 | | | 2,240 | 3,023 | 784 | 6,545 | 6,843 | 4,534 | | | 17,921 |
| 20 | (EU,IS),(UK,FO),(NO) | 842 | 857 | 541 | | | 2,240 | 3,023 | 784 | 7,015 | 6,381 | 4,534 | | | 17,929 |
| 21 | (EU,NO),(FO,IS),(UK) | 1,207 | 535 | 541 | | | 2,282 | 3,068 | 786 | 8,519 | 5,100 | 4,852 | | | 18,471 |
| 22 | (EU,FO),(NO,IS),(UK) | 853 | 847 | 539 | | | 2,240 | 3,023 | 784 | 6,543 | 6,797 | 4,579 | | | 17,919 |
| 23 | (EU,IS),(NO,FO),(UK) | 842 | 858 | 539 | | | 2,239 | 3,023 | 784 | 7,013 | 6,338 | 4,578 | | | 17,928 |
| 24 | (UK,NO),(FO,IS),(EU) | 1,210 | 535 | 536 | | | 2,281 | 3,066 | 786 | 8,349 | 5,092 | 5,026 | | | 18,466 |
| 25 | (UK,FO),(NO,IS),(EU) | 857 | 847 | 535 | | | 2,239 | 3,023 | 784 | 6,375 | 6,791 | 4,750 | | | 17,917 |
| 26 | (UK,IS),(NO,FO),(EU) | 846 | 858 | 535 | | | 2,239 | 3,023 | 784 | 6,835 | 6,333 | 4,750 | | | 17,919 |
| 27 | (EU,UK,NO),(FO),(IS) | 2,129 | 261 | 247 | | | 2,638 | 3,425 | 788 | 14,606 | 3,608 | 4,279 | | | 22,492 |
| 28 | (EU,UK,FO),(NO),(IS) | 1,627 | 545 | 247 | | | 2,419 | 3,209 | 790 | 11,100 | 5,726 | 3,418 | | | 20,244 |
| 29 | (EU,UK,IS),(NO),(FO) | 1,620 | 545 | 260 | | | 2,426 | 3,216 | 790 | 11,631 | 5,771 | 2,852 | | | 20,255 |
| 30 | (EU,NO,FO),(UK),(IS) | 1,628 | 544 | 247 | | | 2,419 | 3,209 | 790 | 11,053 | 5,774 | 3,416 | | | 20,243 |
| 31 | (EU,NO,IS),(UK),(FO) | 1,621 | 544 | 260 | | | 2,425 | 3,215 | 790 | 11,582 | 5,820 | 2,850 | | | 20,253 |
| 32 | (EU,FO,IS),(UK),(NO) | 1,201 | 541 | 542 | | | 2,284 | 3,070 | 786 | 8,799 | 4,865 | 4,818 | | | 18,482 |
| 33 | (UK,NO,FO),(EU),(IS) | 1,631 | 540 | 247 | | | 2,417 | 3,207 | 790 | 10,873 | 5,962 | 3,408 | | | 20,243 |
| 34 | (UK,NO,IS),(EU),(FO) | 1,624 | 540 | 260 | | | 2,423 | 3,213 | 790 | 11,396 | 6,009 | 2,844 | | | 20,249 |
| 35 | (UK,FO,IS),(EU),(NO) | 1,205 | 536 | 542 | | | 2,283 | 3,069 | 786 | 8,625 | 5,039 | 4,810 | | | 18,475 |
| 36 | (NO,FO,IS),(EU),(UK) | 1,206 | 536 | 541 | | | 2,282 | 3,068 | 786 | 8,580 | 5,037 | 4,855 | | | 18,472 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 2,137 | 543 | | | | 2,679 | 3,465 | 786 | 15,382 | 7,987 | | | | 23,369 |
| 38 | (EU,UK,FO),(NO,IS) | 1,642 | 862 | | | | 2,504 | 3,294 | 790 | 12,375 | 9,464 | | | | 21,839 |
| 39 | (EU,UK,IS),(NO,FO) | 1,634 | 874 | | | | 2,508 | 3,298 | 790 | 12,898 | 8,940 | | | | 21,838 |
| 40 | (EU,NO,FO),(UK,IS) | 1,642 | 861 | | | | 2,504 | 3,294 | 790 | 12,326 | 9,515 | | | | 21,841 |
| 41 | (EU,NO,IS),(UK,FO) | 1,634 | 873 | | | | 2,507 | 3,298 | 790 | 12,848 | 8,988 | | | | 21,837 |
| 42 | (EU,FO,IS),(UK,NO) | 1,220 | 1,230 | | | | 2,450 | 3,240 | 790 | 10,896 | 10,422 | | | | 21,318 |
| 43 | (UK,NO,FO),(EU,IS) | 1,645 | 857 | | | | 2,502 | 3,293 | 790 | 12,141 | 9,710 | | | | 21,852 |
| 44 | (UK,NO,IS),(EU,FO) | 1,637 | 869 | | | | 2,506 | 3,296 | 790 | 12,659 | 9,175 | | | | 21,834 |
| 45 | (UK,FO,IS),(EU,NO) | 1,224 | 1,226 | | | | 2,450 | 3,240 | 790 | 10,713 | 10,601 | | | | 21,314 |
| 46 | (NO,FO,IS),(EU,UK) | 1,225 | 1,225 | | | | 2,450 | 3,240 | 790 | 10,666 | 10,649 | | | | 21,314 |
| 47 | (EU,UK,NO,FO),(IS) | 2,732 | 248 | | | | 2,980 | 3,743 | 763 | 20,025 | 5,701 | | | | 25,725 |
| 48 | (EU,UK,NO,IS),(FO) | 2,726 | 262 | | | | 2,988 | 3,751 | 762 | 20,682 | 4,942 | | | | 25,624 |
| 49 | (EU,UK,FO,IS),(NO) | 2,133 | 550 | | | | 2,683 | 3,469 | 786 | 15,718 | 7,633 | | | | 23,351 |
| 50 | (EU,NO,FO,IS),(UK) | 2,134 | 549 | | | | 2,683 | 3,469 | 786 | 15,663 | 7,689 | | | | 23,353 |
| 51 | (UK,NO,FO,IS),(EU) | 2,136 | 544 | | | | 2,680 | 3,466 | 786 | 15,456 | 7,908 | | | | 23,364 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 3,390 | | | | | 3,390 | 4,092 | 702 | 28,247 | | | | | 28,247 |

Note: See table 1.5 for abbreviations.

Table A.13. Optimal solution for the five player game; Ricker function; stock elasticity: $\beta = 0.1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 121 | 122 | 122 | 54 | 52 | 470 | 754 | 284 | 2,454 | 2,426 | 2,419 | 1,220 | 1,301 | 9,820 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 362 | 131 | 56 | 54 | | 603 | 954 | 351 | 5,312 | 3,136 | 1,594 | 1,702 | | 11,745 |
| 3 | (EU,NO),(UK),(FO),(IS) | 362 | 131 | 56 | 54 | | 603 | 954 | 351 | 5,304 | 3,145 | 1,593 | 1,702 | | 11,744 |
| 4 | (EU,FO),(UK),(NO),(IS) | 216 | 125 | 125 | 53 | | 519 | 828 | 309 | 3,810 | 2,668 | 2,660 | 1,437 | | 10,574 |
| 5 | (EU,IS),(UK),(NO),(FO) | 214 | 125 | 126 | 55 | | 520 | 830 | 310 | 3,896 | 2,673 | 2,665 | 1,349 | | 10,582 |
| 6 | (UK,NO),(EU),(FO),(IS) | 362 | 130 | 56 | 54 | | 602 | 953 | 351 | 5,271 | 3,178 | 1,592 | 1,700 | | 11,741 |
| 7 | (UK,FO),(EU),(NO),(IS) | 216 | 124 | 125 | 53 | | 519 | 828 | 309 | 3,781 | 2,697 | 2,658 | 1,437 | | 10,573 |
| 8 | (UK,IS),(EU),(NO),(FO) | 215 | 124 | 126 | 55 | | 520 | 829 | 310 | 3,866 | 2,703 | 2,664 | 1,348 | | 10,580 |
| 9 | (NO,IS),(EU),(UK),(FO) | 217 | 124 | 125 | 53 | | 519 | 828 | 309 | 3,773 | 2,697 | 2,666 | 1,436 | | 10,573 |
| 10 | (NO,FO),(EU),(UK),(IS) | 215 | 124 | 125 | 55 | | 520 | 829 | 310 | 3,858 | 2,702 | 2,671 | 1,348 | | 10,580 |
| 11 | (FO,IS),(EU),(UK),(NO) | 122 | 122 | 123 | 123 | | 491 | 785 | 295 | 2,564 | 2,553 | 2,524 | 2,517 | | 10,158 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 389 | 244 | 55 | | | 687 | 1,079 | 392 | 6,109 | 5,141 | 2,002 | | | 13,252 |
| 13 | (EU,NO),(UK,FO),(IS) | 389 | 244 | 55 | | | 687 | 1,079 | 392 | 6,099 | 5,150 | 2,002 | | | 13,252 |
| 14 | (EU,FO),(UK,NO),(IS) | 243 | 389 | 55 | | | 687 | 1,078 | 391 | 5,186 | 6,063 | 2,001 | | | 13,250 |
| 15 | (EU,UK),(NO,IS),(FO) | 389 | 242 | 57 | | | 688 | 1,080 | 392 | 6,116 | 5,258 | 1,880 | | | 13,254 |
| 16 | (EU,NO),(UK,IS),(FO) | 389 | 241 | 57 | | | 688 | 1,080 | 392 | 6,107 | 5,267 | 1,880 | | | 13,254 |
| 17 | (EU,IS),(UK,NO),(FO) | 240 | 390 | 57 | | | 687 | 1,079 | 392 | 5,305 | 6,070 | 1,878 | | | 13,253 |
| 18 | (EU,UK),(FO,IS),(NO) | 373 | 131 | 133 | | | 638 | 1,006 | 368 | 5,627 | 3,420 | 3,356 | | | 12,403 |
| 19 | (EU,FO),(UK,IS),(NO) | 227 | 226 | 130 | | | 582 | 923 | 341 | 4,279 | 4,337 | 3,012 | | | 11,628 |
| 20 | (EU,IS),(UK,FO),(NO) | 225 | 228 | 130 | | | 582 | 923 | 341 | 4,370 | 4,247 | 3,012 | | | 11,628 |
| 21 | (EU,NO),(FO,IS),(UK) | 373 | 131 | 133 | | | 638 | 1,006 | 368 | 5,617 | 3,419 | 3,365 | | | 12,402 |
| 22 | (EU,FO),(NO,IS),(UK) | 227 | 226 | 129 | | | 582 | 923 | 341 | 4,278 | 4,328 | 3,021 | | | 11,627 |
| 23 | (EU,IS),(NO,FO),(UK) | 225 | 228 | 129 | | | 582 | 923 | 341 | 4,370 | 4,238 | 3,021 | | | 11,628 |
| 24 | (UK,NO),(FO,IS),(EU) | 374 | 131 | 132 | | | 637 | 1,005 | 368 | 5,582 | 3,415 | 3,401 | | | 12,399 |
| 25 | (UK,FO),(NO,IS),(EU) | 228 | 226 | 129 | | | 582 | 923 | 341 | 4,245 | 4,326 | 3,055 | | | 11,626 |
| 26 | (UK,IS),(NO,FO),(EU) | 225 | 228 | 129 | | | 582 | 923 | 341 | 4,335 | 4,236 | 3,055 | | | 11,626 |
| 27 | (EU,UK,NO),(FO),(IS) | 1,295 | 61 | 58 | | | 1,414 | 2,068 | 654 | 12,133 | 5,059 | 5,312 | | | 22,504 |
| 28 | (EU,UK,FO),(NO),(IS) | 685 | 144 | 56 | | | 884 | 1,362 | 478 | 7,930 | 5,171 | 2,801 | | | 15,901 |
| 29 | (EU,UK,IS),(NO),(FO) | 684 | 144 | 59 | | | 887 | 1,366 | 479 | 8,051 | 5,193 | 2,649 | | | 15,894 |
| 30 | (EU,NO,FO),(UK),(IS) | 685 | 143 | 56 | | | 884 | 1,362 | 478 | 7,918 | 5,183 | 2,800 | | | 15,902 |
| 31 | (EU,NO,IS),(UK),(FO) | 684 | 144 | 59 | | | 887 | 1,366 | 479 | 8,040 | 5,206 | 2,648 | | | 15,894 |
| 32 | (EU,FO,IS),(UK),(NO) | 373 | 133 | 133 | | | 638 | 1,007 | 369 | 5,675 | 3,371 | 3,361 | | | 12,407 |
| 33 | (UK,NO,FO),(EU),(IS) | 685 | 142 | 56 | | | 883 | 1,360 | 477 | 7,875 | 5,232 | 2,796 | | | 15,902 |
| 34 | (UK,NO,IS),(EU),(FO) | 684 | 142 | 59 | | | 886 | 1,364 | 478 | 7,996 | 5,254 | 2,644 | | | 15,894 |
| 35 | (UK,FO,IS),(EU),(NO) | 373 | 132 | 133 | | | 638 | 1,006 | 368 | 5,639 | 3,407 | 3,357 | | | 12,404 |
| 36 | (NO,FO,IS),(EU),(UK) | 373 | 132 | 133 | | | 638 | 1,006 | 368 | 5,630 | 3,406 | 3,367 | | | 12,403 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 1,305 | 153 | | | | 1,458 | 2,123 | 665 | 13,119 | 10,547 | | | | 23,666 |
| 38 | (EU,UK,FO),(NO,IS) | 731 | 274 | | | | 1,005 | 1,529 | 525 | 9,537 | 8,784 | | | | 18,321 |
| 39 | (EU,UK,IS),(NO,FO) | 730 | 277 | | | | 1,007 | 1,532 | 525 | 9,675 | 8,641 | | | | 18,315 |
| 40 | (EU,NO,FO),(UK,IS) | 731 | 274 | | | | 1,004 | 1,529 | 525 | 9,524 | 8,798 | | | | 18,322 |
| 41 | (EU,NO,IS),(UK,FO) | 730 | 277 | | | | 1,006 | 1,532 | 525 | 9,662 | 8,654 | | | | 18,316 |
| 42 | (EU,FO,IS),(UK,NO) | 435 | 437 | | | | 872 | 1,344 | 472 | 8,335 | 8,217 | | | | 16,552 |
| 43 | (UK,NO,FO),(EU,IS) | 731 | 273 | | | | 1,004 | 1,528 | 524 | 9,475 | 8,851 | | | | 18,326 |
| 44 | (UK,NO,IS),(EU,FO) | 730 | 276 | | | | 1,006 | 1,531 | 525 | 9,612 | 8,705 | | | | 18,318 |
| 45 | (UK,FO,IS),(EU,NO) | 436 | 436 | | | | 872 | 1,344 | 472 | 8,290 | 8,262 | | | | 16,552 |
| 46 | (NO,FO,IS),(EU,UK) | 436 | 436 | | | | 872 | 1,344 | 472 | 8,278 | 8,274 | | | | 16,552 |
| 47 | (EU,UK,NO,FO),(IS) | 2,047 | 58 | | | | 2,105 | 2,879 | 774 | 20,960 | 8,836 | | | | 29,796 |
| 48 | (EU,UK,NO,IS),(FO) | 2,045 | 61 | | | | 2,106 | 2,880 | 774 | 21,151 | 8,487 | | | | 29,638 |
| 49 | (EU,UK,FO,IS),(NO) | 1,304 | 155 | | | | 1,459 | 2,125 | 666 | 13,215 | 10,412 | | | | 23,627 |
| 50 | (EU,NO,FO,IS),(UK) | 1,304 | 154 | | | | 1,459 | 2,124 | 666 | 13,200 | 10,433 | | | | 23,633 |
| 51 | (UK,NO,FO,IS),(EU) | 1,305 | 153 | | | | 1,458 | 2,123 | 666 | 13,140 | 10,517 | | | | 23,657 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 2,741 | | | | | 2,741 | 3,523 | 783 | 34,170 | | | | | 34,170 |

Note: See table 1.5 for abbreviations.

Table A.14. Optimal solution for the five player game; Beverton-Holt function; stock elasticity: $\beta = 1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 872 | 880 | 882 | 440 | 416 | 3,490 | 4,052 | 562 | 3,450 | 3,101 | 3,013 | 1,344 | 2,391 | 13,300 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 1,808 | 883 | 440 | 416 | | 3,547 | 4,093 | 546 | 6,725 | 3,335 | 1,496 | 2,597 | | 14,153 |
| 3 | (EU,NO),(UK),(FO),(IS) | 1,810 | 881 | 440 | 416 | | 3,547 | 4,093 | 546 | 6,631 | 3,425 | 1,495 | 2,596 | | 14,148 |
| 4 | (EU,FO),(UK),(NO),(IS) | 1,338 | 881 | 883 | 416 | | 3,516 | 4,071 | 555 | 4,837 | 3,250 | 3,160 | 2,486 | | 13,734 |
| 5 | (EU,IS),(UK),(NO),(FO) | 1,317 | 881 | 883 | 440 | | 3,520 | 4,074 | 554 | 5,879 | 3,272 | 3,182 | 1,424 | | 13,757 |
| 6 | (UK,NO),(EU),(FO),(IS) | 1,816 | 873 | 440 | 416 | | 3,545 | 4,092 | 547 | 6,281 | 3,784 | 1,491 | 2,590 | | 14,147 |
| 7 | (UK,FO),(EU),(NO),(IS) | 1,345 | 872 | 883 | 416 | | 3,516 | 4,071 | 555 | 4,498 | 3,604 | 3,156 | 2,483 | | 13,741 |
| 8 | (UK,IS),(EU),(NO),(FO) | 1,324 | 872 | 883 | 440 | | 3,520 | 4,073 | 554 | 5,496 | 3,628 | 3,178 | 1,422 | | 13,725 |
| 9 | (NO,IS),(EU),(UK),(FO) | 1,347 | 872 | 880 | 416 | | 3,515 | 4,070 | 555 | 4,411 | 3,603 | 3,245 | 2,482 | | 13,742 |
| 10 | (NO,FO),(EU),(UK),(IS) | 1,326 | 872 | 881 | 440 | | 3,519 | 4,073 | 554 | 5,398 | 3,627 | 3,268 | 1,422 | | 13,715 |
| 11 | (FO,IS),(EU),(UK),(NO) | 869 | 872 | 880 | 882 | | 3,504 | 4,062 | 558 | 3,666 | 3,534 | 3,180 | 3,091 | | 13,473 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 1,810 | 1,349 | 416 | | | 3,575 | 4,114 | 538 | 7,043 | 4,899 | 2,699 | | | 14,641 |
| 13 | (EU,NO),(UK,FO),(IS) | 1,812 | 1,347 | 416 | | | 3,575 | 4,114 | 538 | 6,949 | 4,988 | 2,699 | | | 14,635 |
| 14 | (EU,FO),(UK,NO),(IS) | 1,340 | 1,818 | 416 | | | 3,574 | 4,113 | 539 | 5,334 | 6,598 | 2,696 | | | 14,627 |
| 15 | (EU,UK),(NO,IS),(FO) | 1,810 | 1,329 | 441 | | | 3,579 | 4,117 | 537 | 7,090 | 5,942 | 1,584 | | | 14,616 |
| 16 | (EU,NO),(UK,IS),(FO) | 1,812 | 1,327 | 441 | | | 3,579 | 4,116 | 537 | 6,995 | 6,041 | 1,584 | | | 14,620 |
| 17 | (EU,IS),(UK,NO),(FO) | 1,319 | 1,819 | 441 | | | 3,578 | 4,116 | 538 | 6,430 | 6,640 | 1,581 | | | 14,651 |
| 18 | (EU,UK),(FO,IS),(NO) | 1,809 | 870 | 883 | | | 3,563 | 4,105 | 542 | 6,899 | 4,029 | 3,423 | | | 14,351 |
| 19 | (EU,FO),(UK,IS),(NO) | 1,339 | 1,325 | 883 | | | 3,547 | 4,094 | 546 | 5,100 | 5,748 | 3,336 | | | 14,184 |
| 20 | (EU,IS),(UK,FO),(NO) | 1,318 | 1,346 | 883 | | | 3,547 | 4,093 | 546 | 6,133 | 4,755 | 3,334 | | | 14,223 |
| 21 | (EU,NO),(FO,IS),(UK) | 1,811 | 870 | 881 | | | 3,562 | 4,104 | 542 | 6,804 | 4,027 | 3,515 | | | 14,345 |
| 22 | (EU,FO),(NO,IS),(UK) | 1,339 | 1,327 | 881 | | | 3,547 | 4,093 | 546 | 5,099 | 5,648 | 3,427 | | | 14,174 |
| 23 | (EU,IS),(NO,FO),(UK) | 1,318 | 1,348 | 881 | | | 3,547 | 4,093 | 546 | 6,132 | 4,666 | 3,426 | | | 14,224 |
| 24 | (UK,NO),(FO,IS),(EU) | 1,817 | 870 | 873 | | | 3,561 | 4,103 | 542 | 6,448 | 4,017 | 3,879 | | | 14,345 |
| 25 | (UK,FO),(NO,IS),(EU) | 1,346 | 1,327 | 873 | | | 3,546 | 4,093 | 546 | 4,749 | 5,641 | 3,791 | | | 14,181 |
| 26 | (UK,IS),(NO,FO),(EU) | 1,325 | 1,348 | 873 | | | 3,546 | 4,093 | 546 | 5,739 | 4,661 | 3,791 | | | 14,191 |
| 27 | (EU,UK,NO),(FO),(IS) | 2,824 | 441 | 416 | | | 3,681 | 4,189 | 508 | 10,651 | 1,878 | 3,098 | | | 15,627 |
| 28 | (EU,UK,FO),(NO),(IS) | 2,308 | 884 | 416 | | | 3,608 | 4,137 | 529 | 8,437 | 3,691 | 2,822 | | | 14,951 |
| 29 | (EU,UK,IS),(NO),(FO) | 2,291 | 884 | 441 | | | 3,617 | 4,143 | 527 | 9,517 | 3,741 | 1,689 | | | 14,947 |
| 30 | (EU,NO,FO),(UK),(IS) | 2,310 | 882 | 416 | | | 3,608 | 4,137 | 529 | 8,342 | 3,786 | 2,820 | | | 14,947 |
| 31 | (EU,NO,IS),(UK),(FO) | 2,293 | 882 | 441 | | | 3,616 | 4,143 | 527 | 9,415 | 3,836 | 1,688 | | | 14,939 |
| 32 | (EU,FO,IS),(UK),(NO) | 1,800 | 881 | 883 | | | 3,565 | 4,106 | 541 | 7,404 | 3,528 | 3,434 | | | 14,366 |
| 33 | (UK,NO,FO),(EU),(IS) | 2,315 | 874 | 416 | | | 3,605 | 4,135 | 530 | 7,984 | 4,159 | 2,811 | | | 14,954 |
| 34 | (UK,NO,IS),(EU),(FO) | 2,299 | 874 | 441 | | | 3,614 | 4,141 | 528 | 9,030 | 4,212 | 1,681 | | | 14,924 |
| 35 | (UK,FO,IS),(EU),(NO) | 1,806 | 873 | 883 | | | 3,563 | 4,105 | 542 | 7,030 | 3,894 | 3,426 | | | 14,349 |
| 36 | (NO,FO,IS),(EU),(UK) | 1,808 | 873 | 881 | | | 3,563 | 4,105 | 542 | 6,933 | 3,891 | 3,518 | | | 14,343 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 2,827 | 872 | | | | 3,699 | 4,202 | 503 | 10,945 | 4,924 | | | | 15,869 |
| 38 | (EU,UK,FO),(NO,IS) | 2,312 | 1,331 | | | | 3,643 | 4,162 | 519 | 8,915 | 6,542 | | | | 15,458 |
| 39 | (EU,UK,IS),(NO,FO) | 2,295 | 1,352 | | | | 3,647 | 4,165 | 518 | 9,967 | 5,514 | | | | 15,482 |
| 40 | (EU,NO,FO),(UK,IS) | 2,313 | 1,329 | | | | 3,642 | 4,162 | 519 | 8,819 | 6,645 | | | | 15,464 |
| 41 | (EU,NO,IS),(UK,FO) | 2,296 | 1,350 | | | | 3,647 | 4,165 | 518 | 9,865 | 5,607 | | | | 15,472 |
| 42 | (EU,FO,IS),(UK,NO) | 1,804 | 1,822 | | | | 3,626 | 4,150 | 524 | 8,146 | 7,181 | | | | 15,327 |
| 43 | (UK,NO,FO),(EU,IS) | 2,319 | 1,321 | | | | 3,640 | 4,160 | 520 | 8,458 | 7,045 | | | | 15,503 |
| 44 | (UK,NO,IS),(EU,FO) | 2,303 | 1,343 | | | | 3,645 | 4,164 | 518 | 9,481 | 5,969 | | | | 15,450 |
| 45 | (UK,FO,IS),(EU,NO) | 1,811 | 1,815 | | | | 3,626 | 4,150 | 524 | 7,770 | 7,542 | | | | 15,311 |
| 46 | (NO,FO,IS),(EU,UK) | 1,813 | 1,813 | | | | 3,626 | 4,150 | 524 | 7,672 | 7,638 | | | | 15,310 |
| 47 | (EU,UK,NO,FO),(IS) | 3,379 | 417 | | | | 3,796 | 4,269 | 473 | 13,082 | 3,550 | | | | 16,633 |
| 48 | (EU,UK,NO,IS),(FO) | 3,368 | 441 | | | | 3,809 | 4,278 | 469 | 14,278 | 2,273 | | | | 16,551 |
| 49 | (EU,UK,FO,IS),(NO) | 2,818 | 886 | | | | 3,704 | 4,205 | 501 | 11,587 | 4,279 | | | | 15,866 |
| 50 | (EU,NO,FO,IS),(UK) | 2,820 | 884 | | | | 3,703 | 4,204 | 501 | 11,482 | 4,380 | | | | 15,862 |
| 51 | (UK,NO,FO,IS),(EU) | 2,825 | 875 | | | | 3,700 | 4,202 | 502 | 11,085 | 4,777 | | | | 15,862 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 3,965 | | | | | 3,965 | 4,383 | 418 | 17,435 | | | | | 17,435 |

Note: See table 1.5 for abbreviations.

Table A.15. Optimal solution for the five player game; Beverton-Holt function; stock elasticity: $\beta = 0.6$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 753 | 760 | 762 | 377 | 357 | 3,009 | 3,681 | 672 | 3,588 | 3,332 | 3,267 | 1,525 | 2,282 | 13,994 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 1,590 | 764 | 377 | 357 | | 3,088 | 3,744 | 656 | 7,148 | 3,684 | 1,726 | 2,533 | | 15,091 |
| 3 | (EU,NO),(UK),(FO),(IS) | 1,591 | 762 | 377 | 357 | | 3,088 | 3,744 | 656 | 7,079 | 3,751 | 1,725 | 2,532 | | 15,088 |
| 4 | (EU,FO),(UK),(NO),(IS) | 1,165 | 761 | 763 | 357 | | 3,046 | 3,710 | 665 | 5,184 | 3,525 | 3,458 | 2,397 | | 14,565 |
| 5 | (EU,IS),(UK),(NO),(FO) | 1,148 | 761 | 763 | 377 | | 3,049 | 3,713 | 664 | 5,943 | 3,544 | 3,476 | 1,626 | | 14,589 |
| 6 | (UK,NO),(EU),(FO),(IS) | 1,597 | 755 | 377 | 357 | | 3,086 | 3,743 | 657 | 6,820 | 4,017 | 1,722 | 2,527 | | 15,085 |
| 7 | (UK,FO),(EU),(NO),(IS) | 1,171 | 754 | 763 | 357 | | 3,045 | 3,710 | 665 | 4,932 | 3,785 | 3,455 | 2,395 | | 14,567 |
| 8 | (UK,IS),(EU),(NO),(FO) | 1,154 | 754 | 763 | 377 | | 3,049 | 3,713 | 664 | 5,667 | 3,805 | 3,474 | 1,624 | | 14,570 |
| 9 | (NO,IS),(EU),(UK),(FO) | 1,173 | 754 | 761 | 357 | | 3,045 | 3,710 | 665 | 4,868 | 3,784 | 3,521 | 2,394 | | 14,567 |
| 10 | (NO,FO),(EU),(UK),(IS) | 1,156 | 754 | 761 | 377 | | 3,048 | 3,713 | 664 | 5,596 | 3,804 | 3,540 | 1,624 | | 14,565 |
| 11 | (FO,IS),(EU),(UK),(NO) | 751 | 754 | 761 | 762 | | 3,028 | 3,696 | 668 | 3,786 | 3,690 | 3,430 | 3,365 | | 14,271 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 1,593 | 1,177 | 357 | | | 3,127 | 3,775 | 648 | 7,563 | 5,507 | 2,661 | | | 15,732 |
| 13 | (EU,NO),(UK,FO),(IS) | 1,595 | 1,175 | 357 | | | 3,127 | 3,775 | 648 | 7,494 | 5,574 | 2,661 | | | 15,729 |
| 14 | (EU,FO),(UK,NO),(IS) | 1,169 | 1,600 | 357 | | | 3,126 | 3,775 | 648 | 5,833 | 7,233 | 2,658 | | | 15,724 |
| 15 | (EU,UK),(NO,IS),(FO) | 1,593 | 1,160 | 378 | | | 3,131 | 3,778 | 648 | 7,602 | 6,287 | 1,840 | | | 15,729 |
| 16 | (EU,NO),(UK,IS),(FO) | 1,595 | 1,158 | 378 | | | 3,131 | 3,778 | 648 | 7,532 | 6,360 | 1,840 | | | 15,731 |
| 17 | (EU,IS),(UK,NO),(FO) | 1,152 | 1,600 | 378 | | | 3,130 | 3,777 | 648 | 6,642 | 7,268 | 1,837 | | | 15,748 |
| 18 | (EU,UK),(FO,IS),(NO) | 1,592 | 753 | 764 | | | 3,108 | 3,761 | 652 | 7,363 | 4,245 | 3,795 | | | 15,404 |
| 19 | (EU,FO),(UK,IS),(NO) | 1,167 | 1,156 | 763 | | | 3,087 | 3,743 | 656 | 5,512 | 5,987 | 3,678 | | | 15,177 |
| 20 | (EU,IS),(UK,FO),(NO) | 1,150 | 1,173 | 763 | | | 3,087 | 3,743 | 657 | 6,265 | 5,255 | 3,678 | | | 15,198 |
| 21 | (EU,NO),(FO,IS),(UK) | 1,593 | 753 | 762 | | | 3,108 | 3,760 | 652 | 7,293 | 4,243 | 3,864 | | | 15,400 |
| 22 | (EU,FO),(NO,IS),(UK) | 1,167 | 1,158 | 762 | | | 3,087 | 3,743 | 657 | 5,510 | 5,914 | 3,747 | | | 15,171 |
| 23 | (EU,IS),(NO,FO),(UK) | 1,150 | 1,175 | 762 | | | 3,087 | 3,743 | 657 | 6,264 | 5,188 | 3,746 | | | 15,198 |
| 24 | (UK,NO),(FO,IS),(EU) | 1,598 | 753 | 755 | | | 3,107 | 3,759 | 653 | 7,029 | 4,235 | 4,133 | | | 15,398 |
| 25 | (UK,FO),(NO,IS),(EU) | 1,173 | 1,158 | 755 | | | 3,086 | 3,743 | 657 | 5,250 | 5,908 | 4,016 | | | 15,173 |
| 26 | (UK,IS),(NO,FO),(EU) | 1,156 | 1,175 | 755 | | | 3,086 | 3,743 | 657 | 5,980 | 5,184 | 4,016 | | | 15,179 |
| 27 | (EU,UK,NO),(FO),(IS) | 2,541 | 378 | 358 | | | 3,277 | 3,892 | 615 | 11,618 | 2,250 | 3,171 | | | 17,039 |
| 28 | (EU,UK,FO),(NO),(IS) | 2,052 | 765 | 357 | | | 3,174 | 3,813 | 638 | 9,171 | 4,163 | 2,818 | | | 16,152 |
| 29 | (EU,UK,IS),(NO),(FO) | 2,039 | 765 | 378 | | | 3,182 | 3,818 | 637 | 9,974 | 4,206 | 1,979 | | | 16,159 |
| 30 | (EU,NO,FO),(UK),(IS) | 2,053 | 763 | 357 | | | 3,174 | 3,812 | 638 | 9,100 | 4,234 | 2,816 | | | 16,150 |
| 31 | (EU,NO,IS),(UK),(FO) | 2,040 | 763 | 378 | | | 3,181 | 3,818 | 637 | 9,899 | 4,278 | 1,978 | | | 16,154 |
| 32 | (EU,FO,IS),(UK),(NO) | 1,584 | 762 | 764 | | | 3,110 | 3,762 | 652 | 7,734 | 3,876 | 3,805 | | | 15,415 |
| 33 | (UK,NO,FO),(EU),(IS) | 2,058 | 756 | 357 | | | 3,172 | 3,810 | 639 | 8,831 | 4,513 | 2,809 | | | 16,153 |
| 34 | (UK,NO,IS),(EU),(FO) | 2,045 | 757 | 378 | | | 3,179 | 3,816 | 637 | 9,615 | 4,559 | 1,972 | | | 16,146 |
| 35 | (UK,FO,IS),(EU),(NO) | 1,590 | 755 | 764 | | | 3,109 | 3,761 | 652 | 7,460 | 4,146 | 3,798 | | | 15,404 |
| 36 | (NO,FO,IS),(EU),(UK) | 1,591 | 755 | 762 | | | 3,109 | 3,761 | 652 | 7,389 | 4,144 | 3,867 | | | 15,400 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 2,545 | 756 | | | | 3,301 | 3,911 | 610 | 11,989 | 5,423 | | | | 17,412 |
| 38 | (EU,UK,FO),(NO,IS) | 2,057 | 1,163 | | | | 3,221 | 3,849 | 628 | 9,776 | 7,077 | | | | 16,853 |
| 39 | (EU,UK,IS),(NO,FO) | 2,043 | 1,181 | | | | 3,224 | 3,852 | 627 | 10,558 | 6,306 | | | | 16,864 |
| 40 | (EU,NO,FO),(UK,IS) | 2,059 | 1,162 | | | | 3,220 | 3,848 | 628 | 9,704 | 7,153 | | | | 16,856 |
| 41 | (EU,NO,IS),(UK,FO) | 2,045 | 1,179 | | | | 3,224 | 3,851 | 627 | 10,483 | 6,376 | | | | 16,859 |
| 42 | (EU,FO,IS),(UK,NO) | 1,591 | 1,606 | | | | 3,196 | 3,830 | 634 | 8,693 | 7,976 | | | | 16,669 |
| 43 | (UK,NO,FO),(EU,IS) | 2,063 | 1,155 | | | | 3,219 | 3,847 | 629 | 9,431 | 7,447 | | | | 16,878 |
| 44 | (UK,NO,IS),(EU,FO) | 2,050 | 1,173 | | | | 3,223 | 3,850 | 628 | 10,198 | 6,649 | | | | 16,848 |
| 45 | (UK,FO,IS),(EU,NO) | 1,596 | 1,600 | | | | 3,196 | 3,830 | 634 | 8,415 | 8,245 | | | | 16,660 |
| 46 | (NO,FO,IS),(EU,UK) | 1,598 | 1,598 | | | | 3,196 | 3,830 | 634 | 8,343 | 8,317 | | | | 16,660 |
| 47 | (EU,UK,NO,FO),(IS) | 3,084 | 358 | | | | 3,442 | 4,017 | 574 | 14,620 | 3,760 | | | | 18,380 |
| 48 | (EU,UK,NO,IS),(FO) | 3,075 | 379 | | | | 3,454 | 4,025 | 571 | 15,531 | 2,782 | | | | 18,313 |
| 49 | (EU,UK,FO,IS),(NO) | 2,538 | 767 | | | | 3,306 | 3,914 | 608 | 12,472 | 4,934 | | | | 17,406 |
| 50 | (EU,NO,FO,IS),(UK) | 2,539 | 765 | | | | 3,305 | 3,914 | 609 | 12,393 | 5,011 | | | | 17,404 |
| 51 | (UK,NO,FO,IS),(EU) | 2,544 | 759 | | | | 3,302 | 3,911 | 609 | 12,095 | 5,313 | | | | 17,407 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 3,674 | | | | | 3,674 | 4,184 | 510 | 19,491 | | | | | 19,491 |

Note: See table 1.5 for abbreviations.

Table A.16. Optimal solution for the five player game; Beverton-Holt function; stock elasticity: $\beta = 0.3$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 525 | 530 | 531 | 257 | 243 | 2,086 | 2,844 | 758 | 3,746 | 3,594 | 3,555 | 1,738 | 2,185 | 14,819 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 1,171 | 534 | 258 | 244 | | 2,207 | 2,965 | 758 | 7,709 | 4,217 | 2,067 | 2,561 | | 16,553 |
| 3 | (EU,NO),(UK),(FO),(IS) | 1,172 | 533 | 258 | 244 | | 2,207 | 2,964 | 758 | 7,667 | 4,259 | 2,066 | 2,560 | | 16,551 |
| 4 | (EU,FO),(UK),(NO),(IS) | 834 | 531 | 532 | 244 | | 2,141 | 2,900 | 758 | 5,613 | 3,892 | 3,851 | 2,354 | | 15,710 |
| 5 | (EU,IS),(UK),(NO),(FO) | 823 | 531 | 533 | 257 | | 2,144 | 2,902 | 758 | 6,066 | 3,907 | 3,866 | 1,892 | | 15,731 |
| 6 | (UK,NO),(EU),(FO),(IS) | 1,175 | 529 | 258 | 244 | | 2,206 | 2,963 | 758 | 7,508 | 4,421 | 2,063 | 2,556 | | 16,548 |
| 7 | (UK,FO),(EU),(NO),(IS) | 838 | 527 | 532 | 244 | | 2,141 | 2,899 | 758 | 5,460 | 4,049 | 3,848 | 2,352 | | 15,709 |
| 8 | (UK,IS),(EU),(NO),(FO) | 827 | 527 | 533 | 257 | | 2,144 | 2,902 | 758 | 5,904 | 4,064 | 3,863 | 1,891 | | 15,722 |
| 9 | (NO,IS),(EU),(UK),(FO) | 839 | 527 | 531 | 244 | | 2,141 | 2,899 | 758 | 5,421 | 4,048 | 3,888 | 2,351 | | 15,708 |
| 10 | (NO,FO),(EU),(UK),(IS) | 828 | 527 | 531 | 257 | | 2,143 | 2,902 | 758 | 5,862 | 4,063 | 3,904 | 1,891 | | 15,719 |
| 11 | (FO,IS),(EU),(UK),(NO) | 525 | 526 | 531 | 532 | | 2,113 | 2,871 | 758 | 3,950 | 3,893 | 3,737 | 3,697 | | 15,277 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 1,178 | 846 | 244 | | | 2,269 | 3,024 | 756 | 8,369 | 6,458 | 2,760 | | | 17,587 |
| 13 | (EU,NO),(UK,FO),(IS) | 1,179 | 845 | 244 | | | 2,268 | 3,024 | 756 | 8,326 | 6,500 | 2,760 | | | 17,585 |
| 14 | (EU,FO),(UK,NO),(IS) | 841 | 1,182 | 244 | | | 2,268 | 3,024 | 756 | 6,661 | 8,165 | 2,758 | | | 17,583 |
| 15 | (EU,UK),(NO,IS),(FO) | 1,178 | 835 | 258 | | | 2,271 | 3,027 | 756 | 8,398 | 6,948 | 2,250 | | | 17,595 |
| 16 | (EU,NO),(UK,IS),(FO) | 1,179 | 834 | 258 | | | 2,271 | 3,027 | 756 | 8,355 | 6,992 | 2,250 | | | 17,596 |
| 17 | (EU,IS),(UK,NO),(FO) | 830 | 1,182 | 258 | | | 2,270 | 3,026 | 756 | 7,163 | 8,192 | 2,248 | | | 17,602 |
| 18 | (EU,UK),(FO,IS),(NO) | 1,174 | 528 | 535 | | | 2,237 | 2,994 | 757 | 8,028 | 4,669 | 4,389 | | | 17,086 |
| 19 | (EU,FO),(UK,IS),(NO) | 838 | 830 | 534 | | | 2,202 | 2,960 | 758 | 6,106 | 6,393 | 4,189 | | | 16,688 |
| 20 | (EU,IS),(UK,FO),(NO) | 826 | 842 | 534 | | | 2,202 | 2,960 | 758 | 6,559 | 5,948 | 4,188 | | | 16,696 |
| 21 | (EU,NO),(FO,IS),(UK) | 1,175 | 528 | 534 | | | 2,237 | 2,994 | 757 | 7,985 | 4,667 | 4,432 | | | 17,084 |
| 22 | (EU,FO),(NO,IS),(UK) | 838 | 831 | 533 | | | 2,202 | 2,960 | 758 | 6,105 | 6,349 | 4,231 | | | 16,685 |
| 23 | (EU,IS),(NO,FO),(UK) | 826 | 843 | 533 | | | 2,202 | 2,959 | 758 | 6,558 | 5,907 | 4,231 | | | 16,695 |
| 24 | (UK,NO),(FO,IS),(EU) | 1,179 | 528 | 529 | | | 2,236 | 2,993 | 757 | 7,822 | 4,661 | 4,598 | | | 17,081 |
| 25 | (UK,FO),(NO,IS),(EU) | 841 | 831 | 528 | | | 2,201 | 2,959 | 758 | 5,944 | 6,345 | 4,396 | | | 16,684 |
| 26 | (UK,IS),(NO,FO),(EU) | 830 | 843 | 528 | | | 2,201 | 2,959 | 758 | 6,388 | 5,903 | 4,396 | | | 16,687 |
| 27 | (EU,UK,NO),(FO),(IS) | 2,018 | 259 | 245 | | | 2,523 | 3,263 | 740 | 13,273 | 3,017 | 3,627 | | | 19,918 |
| 28 | (EU,UK,FO),(NO),(IS) | 1,566 | 537 | 245 | | | 2,348 | 3,101 | 752 | 10,280 | 5,045 | 3,025 | | | 18,350 |
| 29 | (EU,UK,IS),(NO),(FO) | 1,558 | 537 | 258 | | | 2,354 | 3,106 | 752 | 10,780 | 5,081 | 2,495 | | | 18,356 |
| 30 | (EU,NO,FO),(UK),(IS) | 1,567 | 536 | 245 | | | 2,348 | 3,100 | 752 | 10,235 | 5,091 | 3,023 | | | 18,349 |
| 31 | (EU,NO,IS),(UK),(FO) | 1,559 | 536 | 258 | | | 2,354 | 3,106 | 752 | 10,733 | 5,127 | 2,494 | | | 18,354 |
| 32 | (EU,FO,IS),(UK),(NO) | 1,170 | 534 | 535 | | | 2,239 | 2,995 | 757 | 8,254 | 4,442 | 4,397 | | | 17,092 |
| 33 | (UK,NO,FO),(EU),(IS) | 1,570 | 532 | 245 | | | 2,346 | 3,099 | 753 | 10,065 | 5,268 | 3,017 | | | 18,350 |
| 34 | (UK,NO,IS),(EU),(FO) | 1,562 | 532 | 258 | | | 2,352 | 3,104 | 752 | 10,558 | 5,305 | 2,489 | | | 18,351 |
| 35 | (UK,FO,IS),(EU),(NO) | 1,173 | 529 | 535 | | | 2,237 | 2,994 | 757 | 8,087 | 4,608 | 4,391 | | | 17,086 |
| 36 | (NO,FO,IS),(EU),(UK) | 1,174 | 529 | 534 | | | 2,237 | 2,994 | 757 | 8,044 | 4,607 | 4,434 | | | 17,084 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 2,024 | 533 | | | | 2,558 | 3,294 | 737 | 13,849 | 6,711 | | | | 20,559 |
| 38 | (EU,UK,FO),(NO,IS) | 1,576 | 842 | | | | 2,418 | 3,167 | 748 | 11,226 | 8,286 | | | | 19,512 |
| 39 | (EU,UK,IS),(NO,FO) | 1,568 | 854 | | | | 2,422 | 3,170 | 748 | 11,718 | 7,793 | | | | 19,511 |
| 40 | (EU,NO,FO),(UK,IS) | 1,577 | 841 | | | | 2,418 | 3,166 | 748 | 11,180 | 8,334 | | | | 19,514 |
| 41 | (EU,NO,IS),(UK,FO) | 1,568 | 853 | | | | 2,421 | 3,169 | 748 | 11,671 | 7,839 | | | | 19,510 |
| 42 | (EU,FO,IS),(UK,NO) | 1,183 | 1,192 | | | | 2,375 | 3,126 | 751 | 9,803 | 9,354 | | | | 19,158 |
| 43 | (UK,NO,FO),(EU,IS) | 1,580 | 837 | | | | 2,417 | 3,165 | 748 | 11,007 | 8,518 | | | | 19,524 |
| 44 | (UK,NO,IS),(EU,FO) | 1,572 | 849 | | | | 2,420 | 3,168 | 748 | 11,493 | 8,014 | | | | 19,507 |
| 45 | (UK,FO,IS),(EU,NO) | 1,186 | 1,189 | | | | 2,375 | 3,126 | 751 | 9,630 | 9,524 | | | | 19,154 |
| 46 | (NO,FO,IS),(EU,UK) | 1,187 | 1,188 | | | | 2,375 | 3,126 | 751 | 9,585 | 9,569 | | | | 19,154 |
| 47 | (EU,UK,NO,FO),(IS) | 2,557 | 246 | | | | 2,803 | 3,509 | 706 | 17,592 | 4,642 | | | | 22,235 |
| 48 | (EU,UK,NO,IS),(FO) | 2,551 | 260 | | | | 2,811 | 3,516 | 705 | 18,192 | 3,966 | | | | 22,158 |
| 49 | (EU,UK,FO,IS),(NO) | 2,020 | 541 | | | | 2,561 | 3,298 | 736 | 14,159 | 6,386 | | | | 20,545 |
| 50 | (EU,NO,FO,IS),(UK) | 2,021 | 540 | | | | 2,561 | 3,297 | 736 | 14,108 | 6,438 | | | | 20,547 |
| 51 | (UK,NO,FO,IS),(EU) | 2,023 | 535 | | | | 2,559 | 3,295 | 736 | 13,917 | 6,638 | | | | 20,555 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 3,164 | | | | | 3,164 | 3,805 | 640 | 24,118 | | | | | 24,118 |

Note: See table 1.5 for abbreviations.

Table A.17. Optimal solution for the five player game; Beverton-Holt function; stock elasticity: $\beta = 0.1$. The unit for all escapement, recruitment and harvest is thousand tonnes; and for NPV is million NOK.

| No. | Coalition Structure | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S$ | $R$ | $H$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_{CS}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Five coalitions:** | | | | | | | | | | | | | | | |
| 1 | (EU),(UK),(NO),(FO),(IS) | 126 | 127 | 127 | 56 | 53 | 488 | 830 | 342 | 2,696 | 2,665 | 2,657 | 1,348 | 1,439 | 10,804 |
| **Four coalitions:** | | | | | | | | | | | | | | | |
| 2 | (EU,UK),(NO),(FO),(IS) | 392 | 136 | 58 | 55 | | 640 | 1,064 | 424 | 5,917 | 3,610 | 1,842 | 1,965 | | 13,335 |
| 3 | (EU,NO),(UK),(FO),(IS) | 392 | 136 | 58 | 55 | | 640 | 1,064 | 424 | 5,907 | 3,620 | 1,842 | 1,965 | | 13,334 |
| 4 | (EU,FO),(UK),(NO),(IS) | 230 | 131 | 131 | 54 | | 545 | 919 | 374 | 4,222 | 2,995 | 2,986 | 1,623 | | 11,827 |
| 5 | (EU,IS),(UK),(NO),(FO) | 228 | 131 | 131 | 56 | | 546 | 920 | 374 | 4,319 | 3,001 | 2,991 | 1,522 | | 11,833 |
| 6 | (UK,NO),(EU),(FO),(IS) | 393 | 135 | 58 | 55 | | 640 | 1,063 | 424 | 5,870 | 3,659 | 1,840 | 1,963 | | 13,332 |
| 7 | (UK,FO),(EU),(NO),(IS) | 231 | 130 | 131 | 54 | | 545 | 918 | 374 | 4,189 | 3,029 | 2,985 | 1,622 | | 11,826 |
| 8 | (UK,IS),(EU),(NO),(FO) | 229 | 130 | 131 | 56 | | 546 | 920 | 374 | 4,285 | 3,035 | 2,990 | 1,522 | | 11,831 |
| 9 | (NO,IS),(EU),(UK),(FO) | 231 | 130 | 131 | 54 | | 545 | 918 | 374 | 4,180 | 3,029 | 2,994 | 1,622 | | 11,825 |
| 10 | (NO,FO),(EU),(UK),(IS) | 229 | 130 | 131 | 56 | | 545 | 920 | 374 | 4,276 | 3,034 | 2,999 | 1,521 | | 11,831 |
| 11 | (FO,IS),(EU),(UK),(NO) | 127 | 127 | 128 | 129 | | 512 | 867 | 355 | 2,843 | 2,831 | 2,798 | 2,790 | | 11,261 |
| **Three coalitions:** | | | | | | | | | | | | | | | |
| 12 | (EU,UK),(NO,FO),(IS) | 415 | 256 | 55 | | | 726 | 1,192 | 465 | 6,881 | 5,875 | 2,302 | | | 15,058 |
| 13 | (EU,NO),(UK,FO),(IS) | 415 | 256 | 55 | | | 726 | 1,192 | 465 | 6,870 | 5,886 | 2,302 | | | 15,058 |
| 14 | (EU,FO),(UK,NO),(IS) | 255 | 416 | 55 | | | 726 | 1,191 | 465 | 5,927 | 6,829 | 2,301 | | | 15,057 |
| 15 | (EU,UK),(NO,IS),(FO) | 415 | 254 | 58 | | | 727 | 1,192 | 465 | 6,887 | 6,004 | 2,164 | | | 15,055 |
| 16 | (EU,NO),(UK,IS),(FO) | 415 | 253 | 58 | | | 727 | 1,192 | 465 | 6,876 | 6,015 | 2,164 | | | 15,055 |
| 17 | (EU,IS),(UK,NO),(FO) | 252 | 416 | 58 | | | 727 | 1,192 | 465 | 6,058 | 6,835 | 2,162 | | | 15,055 |
| 18 | (EU,UK),(FO,IS),(NO) | 402 | 136 | 138 | | | 676 | 1,118 | 442 | 6,301 | 3,936 | 3,864 | | | 14,101 |
| 19 | (EU,FO),(UK,IS),(NO) | 241 | 239 | 135 | | | 615 | 1,026 | 411 | 4,830 | 4,896 | 3,437 | | | 13,163 |
| 20 | (EU,IS),(UK,FO),(NO) | 239 | 242 | 135 | | | 615 | 1,026 | 411 | 4,934 | 4,794 | 3,437 | | | 13,164 |
| 21 | (EU,NO),(FO,IS),(UK) | 402 | 136 | 138 | | | 676 | 1,118 | 441 | 6,291 | 3,935 | 3,875 | | | 14,101 |
| 22 | (EU,FO),(NO,IS),(UK) | 241 | 240 | 135 | | | 615 | 1,026 | 411 | 4,830 | 4,886 | 3,447 | | | 13,162 |
| 23 | (EU,IS),(NO,FO),(UK) | 239 | 242 | 135 | | | 615 | 1,026 | 411 | 4,933 | 4,784 | 3,447 | | | 13,163 |
| 24 | (UK,NO),(FO,IS),(EU) | 403 | 136 | 137 | | | 676 | 1,117 | 441 | 6,252 | 3,932 | 3,916 | | | 14,100 |
| 25 | (UK,FO),(NO,IS),(EU) | 242 | 240 | 134 | | | 615 | 1,026 | 411 | 4,792 | 4,884 | 3,486 | | | 13,162 |
| 26 | (UK,IS),(NO,FO),(EU) | 239 | 242 | 134 | | | 615 | 1,026 | 411 | 4,894 | 4,782 | 3,486 | | | 13,162 |
| 27 | (EU,UK,NO),(FO),(IS) | 1,208 | 61 | 57 | | | 1,326 | 1,997 | 670 | 12,973 | 4,789 | 5,033 | | | 22,795 |
| 28 | (EU,UK,FO),(NO),(IS) | 709 | 146 | 57 | | | 911 | 1,454 | 543 | 8,823 | 5,718 | 3,090 | | | 17,631 |
| 29 | (EU,UK,IS),(NO),(FO) | 708 | 146 | 59 | | | 913 | 1,457 | 544 | 8,952 | 5,734 | 2,924 | | | 17,610 |
| 30 | (EU,NO,FO),(UK),(IS) | 709 | 145 | 57 | | | 911 | 1,454 | 543 | 8,811 | 5,732 | 3,089 | | | 17,632 |
| 31 | (EU,NO,IS),(UK),(FO) | 708 | 145 | 59 | | | 913 | 1,457 | 544 | 8,940 | 5,748 | 2,923 | | | 17,611 |
| 32 | (EU,FO,IS),(UK),(NO) | 401 | 138 | 138 | | | 677 | 1,119 | 442 | 6,355 | 3,880 | 3,868 | | | 14,103 |
| 33 | (UK,NO,FO),(EU),(IS) | 709 | 144 | 57 | | | 910 | 1,453 | 543 | 8,764 | 5,786 | 3,086 | | | 17,637 |
| 34 | (UK,NO,IS),(EU),(FO) | 708 | 144 | 59 | | | 912 | 1,456 | 544 | 8,893 | 5,803 | 2,920 | | | 17,616 |
| 35 | (UK,FO,IS),(EU),(NO) | 402 | 137 | 138 | | | 676 | 1,118 | 442 | 6,315 | 3,921 | 3,865 | | | 14,102 |
| 36 | (NO,FO,IS),(EU),(UK) | 402 | 137 | 138 | | | 676 | 1,118 | 442 | 6,305 | 3,920 | 3,876 | | | 14,101 |
| **Two coalitions:** | | | | | | | | | | | | | | | |
| 37 | (EU,UK,NO),(FO,IS) | 1,212 | 149 | | | | 1,362 | 2,040 | 678 | 13,855 | 9,926 | | | | 23,781 |
| 38 | (EU,UK,FO),(NO,IS) | 735 | 275 | | | | 1,010 | 1,590 | 580 | 10,430 | 9,374 | | | | 19,804 |
| 39 | (EU,UK,IS),(NO,FO) | 734 | 278 | | | | 1,012 | 1,592 | 580 | 10,574 | 9,216 | | | | 19,790 |
| 40 | (EU,NO,FO),(UK,IS) | 735 | 275 | | | | 1,010 | 1,590 | 580 | 10,416 | 9,389 | | | | 19,805 |
| 41 | (EU,NO,IS),(UK,FO) | 734 | 278 | | | | 1,012 | 1,592 | 580 | 10,561 | 9,231 | | | | 19,791 |
| 42 | (EU,FO,IS),(UK,NO) | 447 | 450 | | | | 897 | 1,435 | 538 | 9,230 | 9,102 | | | | 18,332 |
| 43 | (UK,NO,FO),(EU,IS) | 736 | 274 | | | | 1,010 | 1,589 | 579 | 10,364 | 9,447 | | | | 19,811 |
| 44 | (UK,NO,IS),(EU,FO) | 734 | 277 | | | | 1,011 | 1,591 | 580 | 10,509 | 9,287 | | | | 19,796 |
| 45 | (UK,FO,IS),(EU,NO) | 448 | 449 | | | | 897 | 1,435 | 538 | 9,181 | 9,151 | | | | 18,331 |
| 46 | (NO,FO,IS),(EU,UK) | 449 | 449 | | | | 897 | 1,435 | 538 | 9,168 | 9,163 | | | | 18,331 |
| 47 | (EU,UK,NO,FO),(IS) | 1,815 | 57 | | | | 1,872 | 2,622 | 750 | 20,747 | 7,664 | | | | 28,412 |
| 48 | (EU,UK,NO,IS),(FO) | 1,812 | 60 | | | | 1,873 | 2,623 | 750 | 20,933 | 7,347 | | | | 28,280 |
| 49 | (EU,UK,FO,IS),(NO) | 1,211 | 151 | | | | 1,363 | 2,041 | 679 | 13,951 | 9,793 | | | | 23,744 |
| 50 | (EU,NO,FO,IS),(UK) | 1,212 | 151 | | | | 1,362 | 2,041 | 679 | 13,935 | 9,814 | | | | 23,750 |
| 51 | (UK,NO,FO,IS),(EU) | 1,212 | 150 | | | | 1,362 | 2,040 | 678 | 13,876 | 9,896 | | | | 23,772 |
| **One coalition:** | | | | | | | | | | | | | | | |
| 52 | (EU,UK,NO,FO,IS) | 2,428 | | | | | 2,428 | 3,176 | 747 | 32,006 | | | | | 32,006 |

Note: See table 1.5 for abbreviations.

# References

Agnello, R. J. and Donnelley, L. P. (1976). Externalities and property rights in the fisheries, *Land Economics*, Vol. 52, 518-529.

Bailey, M., Sumaila, U. R. and Lindroos, M. (2010). Application of game theory to fisheries over three decades, *Fisheries Research*, Vol. 102, 1-8.

Balton, D. A. and Koehler, H. R. (2006). Reviewing the United Nations Fish Stocks Treaty, *Sustainable Development Law & Policy*, Vol. 7, 5-9.

Beverton, R.J.H. and Holt, S.J. (1957). *On the Dynamics of Exploited Fish Populations*, Ministry of Agriculture, Fisheries and Food (London) Fisheries Investigations Series, Vol. 2, 19.

Bjørndal, T. (1987). Production Economics and Optimal Stock Size in a North Atlantic Fishery, *The Scandinavian Journal of Economics*, Vol. 89, 145-164.

Bjørndal, T. and Munro, G. R. (2003) The Management of high seas fisheries, In Folmer, H. and Tietenberg, T. (Eds.) *The International Yearbook of Environmental and Resource Economics 2003/2004*, Elgar, Cheltenham, UK: 1-35.

Bloch, F. (1996). Sequential formation of coalitions in games with externalities and fixed payoff division, *Games and Economic Behavior*, Vol. 14, 90-123.

Brexit White Paper (2017). *The United Kingdom's exit from and new partnership with the European Union*, Department for Exiting the European Union and The Rt Hon David Davis MP, February 2017, Available at: `https://www.gov.uk/government/publications/the-united-kingdoms-exit-from-and-new-partnership-with-the-european-union-white-paper`

Churchill, R. R. and Owen, D. (2010). *The EU Common Fisheries Policy: Law and Practice*, Oxford University Press, 640pp.

Clark, C.W. (1972). The Dynamics of Commercially Exploited Natural Animal Populations, *Mathematical Biosciences*, Vol. 13, 149-164.

Clark, C.W. (1973). Profit Maximization and the Extinction of Animal Species, *The Journal of Political Economy*, Vol. 81, 950-961.

Clark, C. W. and Munro, G. R. (1975). The economics of fishing and modern capital theory: a simplified approach, *Journal of environmental economics and management*, Vol. 2, 92-106.

Clark, C.W. (2010). *Mathematical Bioeconomics: The Mathematics of Conservation*, Wiley, New Jersey.

d'Aspremont, C., Jacquemin, A., Gabszewicz, J. J. and Weymark, J. A. (1983). On the stability of collusive price leadership, *Canadian Journal of economics*, Vol. 16, 17-25.

De Clippel, G. and Serrano, R. (2008). Marginal Contributions and Externalities in the Value, *Econometrica*, Vol. 76, 1413-1436.

Doering, R., Kempf, A., Belschner, T., Berkenhagen, J., Bernreuther, M., Hentsch, S., Kraus, G., Raetz, H.-J., Rohlf, N., Simons, S., Stransky, C., Ulleweit, J. (2017). *Research for PECH Committee - Brexit Consequences for the Common Fisheries Policy-Resources and Fisheries: a Case Study*, European Parliament, Policy Department for Structural and Cohesion Policies, Brussels.

Ekerhovd, N. A. and Steinshamn, S. I. (2016). Economic benefits of multi-species management: The pelagic fisheries in the Northeast Atlantic, *Marine Resource Economics*, Vol. 31, 193-210.

Ellefsen, H. (2013). The Stability of Fishing Agreements with Entry: The Northeast Atlantic Mackerel, *Strategic Behavior and the Environment*, Vol. 3, 67-95.

Eyckmans, J. and Finus, M. (2004). An Almost Ideal Sharing Scheme for Coalition Games with Externalities, FEEM Working Paper No. 155.04, Available at SSRN: `https://ssrn.com/abstract=643641`

FAO (2003). *Code of Conduct for Responsible Fisheries*, Rome.

Gulland, J.A. (1980). *Some Problems of the Management of Shared Stocks*, FAO Fisheries Technical Paper No. 206, Rome.

Hannesson, R. (2011). Game theory and fisheries, *Annu. Rev. Resour. Econ.*, Vol. 3, 181-202.

Hannesson, R. (2012). Sharing the Northeast Atlantic mackerel, *ICES Journal of Marine Science*, Vol. 70, 256-269.

Hannesson, R. (2013). Sharing a Migrating Fish Stock, *Marine Resource Economics*, Vol. 28, 1-17.

ICES CM (1996). Report of the Working Group on the assessment of mackerel, horse mackerel, sardine and anchovy, Copenhagen, 10-19 October 1995. ICES Doc. C.M. 1996/Assess:7.

ICES (2014). Mackerel in the Northeast Atlantic (combined Southern, Western, and North Sea spawning components), ICES Advice 2014, Book 9, Section 3.17b.

ICES (2016a). Mackerel (Scomber scombrus) in subareas 1-7 and 14, and in divisions 8.a-e and 9.a (Northeast Atlantic), ICES Advice 2016, Book 9, Section 3.39.

ICES (2016b). Stock Annex: Mackerel (Scomber scombrus) in subareas 1-7 and 14 and divisions 8.a-e, 9.a (the Northeast Atlantic and adjacent waters), ICES, Available at: `http://www.ices.dk/sites/pub/Publication%20Reports/Stock%20Annexes/2016/mac-nea_SA.pdf`

Iles, T.C. (1994). A review of stock-recruitment relationships with reference to flatfish populations, *Netherlands Journal of Sea Research*, Vol. 32, 399-420.

Jaquette, D.L. (1974). A discrete time population control model with setup cost, *Operations Research*, Vol. 22, 298-303.

Jensen, F., Frost, H., Thøgersen, T., Andersen, P. and Andersen, J. L. (2015). Game theory and fish wars: the case of the Northeast Atlantic mackerel fishery, *Fisheries Research*, Vol. 172, 7-16.

Kaitala, V. and Munro, G.R. (1993). The management of high seas fisheries, *Marine Resource Economics*, Vol. 8, 313-29.

Kaitala, V. and Lindroos, M. (2007). Game theoretic applications to fisheries, In Weintraub, A., Romero, C., Bjørndal, T. and Epstein, R. (Eds.) *Handbook of operations research in natural resources*, Springer.

Kennedy, J. (2003). Scope for efficient multinational exploitation of Nort-East Atlantic Mackerel, *Marine Resource Economics*, Vol. 18, 55-60.

Kronbak, L.G. and Lindroos, M. (2007). Sharing rules and stability in coalition games with externalities, *Marine Resource Economics*, Vol. 22, 137-154.

Le Gallic, B., Mardle, S. and Metz, S. (2017). *Research for PECH Committee - Common fisheries Policy and Brexit - Trade and economic related issues*, European Parliament, Policy Department for Structural and Cohesion Policies, Brussels.

Lindroos, M., Kaitala, V. and Kronbak, L.G. (2007). Coalition Games in Fisheries Economics, In Bjørndal, T., Gordon, D., Arnason, R. and Sumaila, U. R. (Eds.) *Advances in Fisheries Economics: Festschrift in Honour of Professor Gordon Munro*, Blackwell.

Liu, X., Lindroos, M. and Sandal, L. (2016). Sharing a fish stock when distribution and harvest costs are density dependent, *Environmental and Resource Economics*, Vol. 63, 665-686.

Macho-Stadler, I., Perez-Castrillo, D. and Wettstein, D. (2007). Sharing the surplus: an extension of the Shapley value for environments with externalities, *Journal of Economic Theory*, Vol. 135, 339-356.

Martimort, D. and Stole, L. (2012). Representing equilibrium aggregates in aggregate games with applications to common agency. *Games and Economic Behavior*, Vol. 76, 753-772.

McKelvey, R. W., Sandal, L. K. and Steinshamn, S. I. (2002). Fish Wars on the High Seas: A Straddling Stock Competitive Model, *International Game Theory Review*, Vol. 4, 53-69.

Munro, G. R. (2003). On the management of shared fish stocks, Papers presented at the Norway-FAO expert consultation on the management of shared fish stocks, *FAO Fisheries Report*, No. 695, 2-29.

Munro, G. R. (2008). Game theory and the development of resource management policy: the case of international fisheries, *Environment and Development Economics*, Vol. 14, 7-27.

Norwegian Directorate of Fisheries (2015). *Profitability survey on the Norwegian fishing fleet*, Norwegian Directorate of Fisheries, Bergen, Norway, Available at: `http://www.fiskeridir.no/Yrkesfiske/Statistikk-yrkesfiske/Statistiske-publikasjoner/Loennsomhetsundersoekelse-for-fiskefartoey`.

Nøstbakken, L. (2006). Cost Structure and Capacity in Norwegian Pelagic Fisheries, *Applied Economics*, Vol. 38, 1877-87.

Pavlov, A.K., Tverberg, V., Ivanov, B.V., Nilsen, F., Falk-Petersen, S. and Granskog, M.A. (2013). Warming of Atlantic Water in two west Spitsbergen fjords over the last century (1912-2009), *Polar Research*, Vol. 32.

Pham Do, K. and Folmer, H. (2003). International fisheries agreements: The feasibility and impacts of partial cooperation, (CentER Discussion Paper; Vol. 2003-52). Tilburg: Microeconomics.

Pham Do, K. and Norde H. (2007). The Shapley value for partition function form games, *International Game Theory Review*, Vol. 9, 353-360.

Pintassilgo, P. (2003). A coalition approach to the management of high seas fisheries in the presence of externalities, *Natural Resource Modeling*, Vol. 16, 175-197.

Pintassilgo, P., Finus, M. and Lindroos, M. (2010). Stability and success of regional fisheries management organisations, *Environmental Resource Economics*, Vol. 46, 377-402.

Ray, D. and Vohra, R. (1999). A theory of endogenous coalition structures, *Games and Economic Behavior*, Vol. 26, 286-336.

Reed, W.J. (1974). A stochastic model for the economic management of a renewable resource animal, *Mathematical Biosciences*, Vol. 22, 313-337.

Ricker, W.E. (1954). Stock and Recruitment, *Journal of the Fisheries Research Board of Canada*, Vol. 11, 559-623.

Sobrino Heredia, J. M. (2017). *Research for PECH Committee - Common Fisheries Policy and BREXIT - Legal framework for governance*, European Parliament, Policy Department for Structural and Cohesion Policies, Brussels.

Smith, V. L. (1969). On models of commercial fishing, *Journal of political economy*, Vol. 77, 181-198.

Thrall, R. M. and Lucas, W. F. (1963). N-person games in partition function form, *Naval Research Logistics (NRL)*, Vol. 10, 281-298.

United Nations (1982). *United Nations Convention on the Law of the Sea*, UN Doc. A/Conf.62/122.

United Nations (1995). *United Nations Conference on Straddling fish Stocks and Highly Migratory Fish Stocks. Agreement for the Implementation of the Provisions of the United Nations Convention on the Law of the Sea of 10 December 1982 Relating to the Conservation and MAnagement of Straddling Fish Stocks and Highly Migratory Fish Stocks*, UN Doc. A/Conf./164/37.

Yi, S. S. (1997). Stable coalition structures with externalities, *Games and economic behavior*, Vol. 20, 201-237.

Yi, S. S. and Shin, H. (1995). Endogenous Formation of Coalitions in Oligopoly, Dartmouth College Department of Economics WP No. 95-2.

# Chapter 2

# Optimal infinite-horizon feedback policies for single-leader multi-follower seasonal fishery games

Evangelos Toumasatos[a,b]

[a]SNF - Centre for Applied Research, Norwegian School of Economics
[b]Department of Business and Management Science, Norwegian School of Economics

## Abstract

The focus of this paper is on the study of optimal fishing strategies for infinitely repeated seasonal fisheries games within a Stackelberg framework. Seasonality is an important feature of many commercial fisheries since both biological processes and human activities occur on a seasonal instead of an annual basis, as is often assumed. This work expands on the seasonal model of Ni and Sandal (2019), who consider a single agent exploiting a fishery with seasonal dynamics, by introducing strategic interaction between an incumbent leader and multiple potential entrants (followers). The game consists of two subgames: a simultaneous game played by $n$ followers, and a sequential game played by all players. The leader maximises the net present value of the fishery given the behaviour of the followers and the seasonal stock dynamics. The followers, on the other hand, behave myopically and maximise current profits. The feedback Nash equilibrium for the $n$-follower game is derived analytically and used as input into the optimisation process of the leader. A numerical scheme based on recursion is used to derive the dynamic feedback policies of the leader. The results are compared to the benchmark case without strategic interaction. In presence of multiple followers, the leader adopts a more aggressive fishing strategy in all seasons. As a consequence, entry for some followers is delayed or not even realised. However, this increases the pressure on the stock and therefore the long-term biomass equilibrium is reduced. In addition, there is an almost 50% value reduction for the leader along the state space, implying rent dissipation.

*Keywords:* Dynamic games; feedback Nash equilibrium; feedback Stackelberg equilibrium; bioeconomics; seasonal fisheries management.

Subject Classification: C72, C73, Q22, Q57.

## 2.1 Introduction

Most fishing stocks worldwide are managed on an annual basis despite the fact they exhibit periodic or seasonal variations with respect to their biological, environmental and economic characteristics. These include, but are not limited to, reproducing, feeding, migrating, and harvesting (Clark, 2010; Bjørndal and Munro, 2012). For example, many commercial species undergo extensive annual migrations between their spawning and feeding grounds, which make them more vulnerable to being captured on a seasonal basis. Some of those species include Arctic Cod (Hannesson et al., 2010; Hermansen and Dreyer, 2010), Atlantic Mackerel (Hannesson, 2013), and Norwegian spring-spawning Herring (Liu et al., 2016). On top of that, most governments regulate fishing activity by setting annual total allowable catches (TACs), which they allot to different vessel groups without limiting the seasons of the year they can fish. This allocation, which is usually based on political rather than bioeconomic criteria (Armstrong and Sumaila, 2001), can become very problematic for fisheries with strong seasonal variability, especially when no clear comprehension of the within-season biomass dynamics exists (Ben-Hasan et al., 2019). Therefore, seasonality or periodicity has to be incorporated to a larger and deeper extent within our studies in order to acquire a better understanding of ecological and fisheries systems.

However, most researchers and policy makers in fisheries management do not consider it explicitly; not because they are not aware of it, its importance and implications, but because of the complexities and difficulties associated with its nature. Perhaps the most practical issue in incorporating seasonality in the management of real fisheries is the lack of season-specific data. Although economic information, like harvested quantities and market prices, for some commercial species exist on a sub-annual basis, biological data are typically collected and analysed on an annual basis. Agencies like the International Council for the Exploration of the Sea (ICES) provide annual advices regarding the status of fishing stocks and recommend reference points, like fishing mortalities, that are based on annual projections. Thus, addressing seasonality in a systematic way, when managing real fisheries, would imply increased managerial costs due to more frequent occurrence of the various routines, like monitoring of the stocks.

Even if all the required data were available, modelling periodicity explicitly is not an easy undertaking, since it entails modifying existing bioeconomic models in order to account for it in the parameters, variables and functional forms. Furthermore, it renders the problem as non-autonomous, which is more difficult to solve. Not to mention the curse of dimensionality associated with the increase of control and state variables. Therefore, in many applications periodicity is frequently treated in some specially appointed way or skipped at all (Kvamsdal et al., 2016).

In their general form, non-autonomous problems in fisheries management date back to 1975 and the influential paper of Clark and Munro. Since then, many authors have approached the topic, however, most of them have not dealt with periodicity in a systematic and explicit way. Using time-varying parameters in the stock-recruitment relationship, Parma (1990) concluded that optimal escapement strategies should be adjusted according to the anticipated environmental conditions these parameters reflect. In the same spirit, Carson, Granger, Jackson and Schlenker (2009) replaced the constant growth rate in the Gordon-Schaefer fishing model with a cyclical growth rate. They showed that the optimal harvest rate is periodic and it follows the fluctuations of the biological parameters. Similar studies include Arnason (1991), Hannesson and Steinshamn (1991), Walters and

Parma (1996), and Castilho and Srinivasu (2007).

Another branch of the literature, which goes back to Smith (1969), has approached seasonality by studying within-season behaviour of fishermen while disregarding intra-seasonal biological growth. Clark (1980) demonstrated that annual fishing quotas are suboptimal in the presence of seasonal variability in the stock and argued that the problem could be solved if the quota allocations follow the variations. Boyce (1992) showed that an individual transferable quota (ITQ) scheme is incapable of solving production externalities, like within-season stock effects. Extending this framework to a game theoretic setting, Costello and Deacon (2007) argued that, in the presence of seasonality, harvesters in an ITQ managed fishery will no longer be indifferent to when or where they exercise their quotas. Similar game theoretic studies include Fell (2009), who modelled within-season fishing as a differential game with open-loop information structure, and Valcu and Wenginger (2003), who used Markov-perfect strategies in a dynamic intra-seasonal game. For a more comprehensive review of intra-seasonal effects on fisheries management see Smith (2012).

The implications of seasonality have also been investigated in several applied and empirical studies. Önal et al. (1991) modelled the annual life cycle of the Texas shrimp fishery as a one-year equilibrium model. They determined an optimal seasonal harvesting pattern, compared it with actual fishing effort and concluded that there was excess fishing pressure in spring and early summer. Larkin and Sylvia (2004) investigated the effect of within-season fluctuations on resource rent generation in the Pacific whiting fishery. Pelletier et al. (2009) developed a generic multi-species, multi-fleet bioeconomic model, which, among other things, allowed for spatial and seasonal fluctuations. Their purpose was to assess and compare alternative fisheries management scenarios through the use of extensive what-if analyses. With the integration of game theory in the management of migratory and straddling fish stocks, harvest patterns of highly mobile species that exhibit strong seasonal migrations are also been studied (Bailey et al. 2010; Hannesson 2011).

Although many researchers have tried to integrate periodicity to the standard fishery management framework (Clark, 2010), most of the studies that approached the topic have done so under very special circumstances and settings. However, in a relatively recent series of papers, Kvamsdal, Maroto, Morán and Sandal (2016, 2017, 2020) have proposed a new theoretical framework that internalises periodicity in discrete-time infinite-horizon optimal control problems. Their idea, which is simple although not trivial, is to model the fishery as repeated cycles of multiple intervals with differing characteristics. They extended the classical Bellman problem (Bellman, 1957; Bertsekas, 2001) to periodic problems, which they modelled as a system of coupled Bellman equations and showed that such extension is both feasible and practical. Their approach, although developed with fisheries in mind, can be applied to many infinite-horizon optimization problems characterised by cyclical variations (see Kvamsdal et al., 2020, and references therein).

The modelling of the seasonal fishery in this work most closely resembles that of Ni and Sandal (2019). Their method provides a useful framework for the regulation of seasonal fisheries; some of their proposed measures include periodic moratoriums as well as seasonal and fleet-specific TACs. A key finding of theirs is the existence of an equilibrium cycle during which a naturally occurring seasonal closure of the fishery takes place. What this means is that without imposing any restrictions on the harvest activity, letting the stock to recover in one season and harvest it in another emerges as an optimal equilibrium strategy for the managing authority.

Compared to this paper, both models assume a two-state two-season bioeconomic model in an attempt to explicitly address periodicity and derive optimal feedback policies for each season. However, the model assumptions and respective goals of the research are quite different. Ni and Sandal assume that harvesting activities are coordinated in the sense that the fishery is exploited by a single agent. In contrast, this work applies a game-theoretic setting with one leader and multiple potential followers, where the focus is on the strategic interaction among them and its effect on the long-term state of the resource.

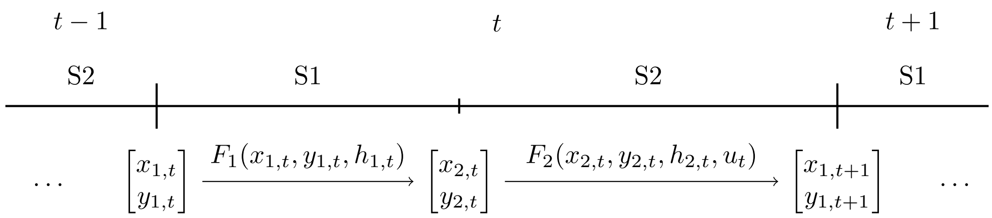
The organisation of the paper is as follows. The next section introduces the conceptual seasonal and game-theoretic model. In section 3, the Nash solution for the followers' subgame is analytically derived. The numerical solution for the leader's subgame and discussion of the results are presented in section 4. Finally, section 5 summarises the main findings and concludes the paper.

## 2.2 Bioeconomic model

### 2.2.1 Seasonal dynamics

The seasonal bioeconomic model employed here is an extension of the annual stock-recruitment model introduced by Clark (1973). In contrast to annual models, where biological and human processes, like growth and fishing, occur only once a year, this seasonal model considers multiple seasons with different characteristics, where the stock undergoes changes in all of them. In order to put this into context, it seems relevant to start by discussing the biological traits of the fishery considered here.

Suppose that there exist a species of fish that migrates between its spawning and feeding grounds on an annual basis, like Arctic cod or Atlantic mackerel. To depart from the standard framework, where the stock biomass is aggregated and described by a single variable, in this study the stock biomass consists of two cohorts or groups, a mature and an immature one, denoted by $x$ and $y$ respectively. This makes the model more informative, while at the same time remains tractable and convenient to work with.[1] Moreover, there exist time periods, e.g. a year, where one period consists of two seasons of different length: a spawning season (S1) during which the stock reproduces and a feeding season (S2) where the stock feeds and further grows. In line with many real fisheries, where spawning seasons are typically of shorter duration, S1 is considered to be smaller than S2. The two seasons follow each other and this pattern repeats itself indefinitely. The schema below illustrates the stock dynamics between seasons.




[1]The traditional single-state biomass approach, although convenient to analyse, is considered by some as an oversimplification of the species population structure (see Tahvonen, 2010, and references therein). Its critics favour age-structured models that group the species population into age-classes and track their development. These models can be more informative, however, the accuracy of their results depends on the availability and quality of necessary information.

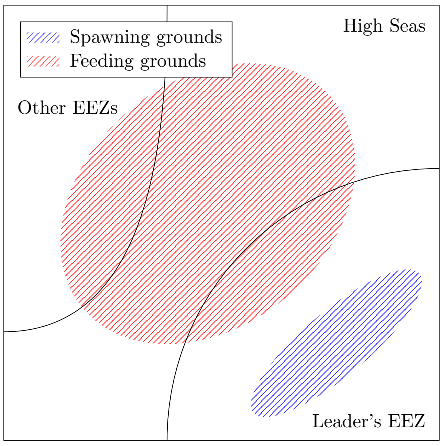



Figure 2.1. Spatial distribution of the seasonal fishery studied in the paper.

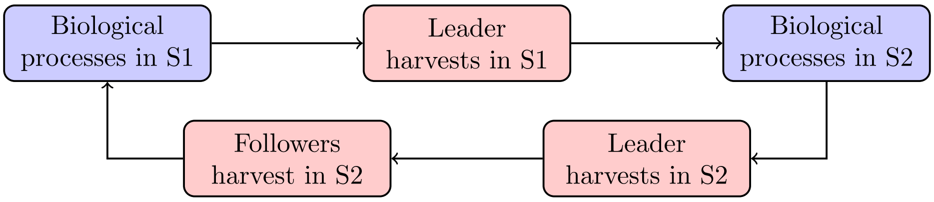



Figure 2.2. Sequence of biological processes and human activities within a periodic cycle.

For a given period $t = 0, \ldots, \infty$, the stock biomass is represented by $(x_{1,t}, y_{1,t})$ at the beginning of S1, and by $(x_{2,t}, y_{2,t})$ at the beginning of S2. Vector functions $F_1(\cdot)$ and $F_2(\cdot)$ describe the transition dynamics of the states between seasons. Because this seasonal pattern repeats itself indefinitely, we ease on the notation and skip the time argument when this is obvious. Thus, $x_i \equiv x_{i,t}$ and $y_i \equiv y_{i,t}$, $i = 1, 2$, will represent the current period's states, and $X_1 \equiv x_{1,t+1}$ and $Y_1 \equiv y_{1,t+1}$ the next period's states before spawning. Variables $h_{1,t}$, $h_{2,t}$ and $u_t$ represent fishing activities, and are introduced below.

The spatial distribution of the fishery is as follows: the spawning grounds are confined within the exclusive economic zone (EEZ) of a coastal state, hereinafter the leader of the game. The feeding grounds, on the other hand, are distributed over a vast area that overlaps the leader's EEZ, the high seas and the EEZs of other nations, thus making it possible for distant water fleets (DWFS) and other coastal states, hereinafter the followers of the game, to harvest the resource when it moves to the feeding grounds. Real fisheries that exhibit similar distributional characteristics are Arctic cod and Norwegian spring-spawning Herring, both of which spawn alongside the Norwegian coast before their annual feeding migration. Figure 2.1 depicts such spatial setting.

Based on this, the leader can target the resource in both seasons, whereas the followers can do so only during the feeding season. In addition, players can only target the mature biomass. This reflects fisheries managers attempts to regulate fishing effort in order to conserve juvenile and immature populations. The economic objectives, timing of decisions and information structure of the players are discussed in the next subsection. For the moment, let $h_1$ and $h_2$ denote the leader's harvest strategies in S1 and S2 respectively, and $u$ the aggregate harvest of the followers during S2. Finally, the biological processes precede human activities, i.e., in each season the stock biomass first changes due to growth and spawning, and then gets harvested because of fishing. This sequential process of events is depicted in figure 2.2.

Having provided some context about the type of the fishery investigated here, it is time to move on and introduce the state transition functions $F_1(\cdot)$ and $F_2(\cdot)$. The specification follows that of Ni and Sandal (2019). The biomass change in S1 is described by the following system:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = F_1(x_1, y_1, h_1) = \begin{bmatrix} x_1 - h_1 \\ \min(a_1 x_1 + (a_2 + a_3 x_1) y_1, y_{\max}) \end{bmatrix}. \tag{2.1}$$

During S1, which is the shorter season, the net growth of the mature cohort is assumed to be very small and thus ignored. The loss in mature biomass is only attributed to the leader's harvest pattern, $h_1$. The spawning dynamics of the immature group is linear in $y_1$ and bounded from above at $y_{\max}$. Both the slope and the intercept depend on the mature population. The intercept, $a_1 x_1$, reflects the biomass gain through spawning, whereas the slope, $a_2 + a_3 x_1$, the biomass growth of the existent immature population. The biomass growth includes weight gained through growth and weight loss due to natural mortality. This structure implies that a larger mature population gives birth to more offspring, which creates a larger immature density and allows for faster growth rate of the immature population according to the Allee effect (Allee and Warder, 1931).

The biomass change in S2 is described by the following system:

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = F_2(x_2, y_2, h_2, u) = \begin{bmatrix} \alpha x_2 + \beta y_2 - h_2 - u \\ \dfrac{\phi_1 y_2}{1 + \phi_2 y_2} - \beta y_2 - \gamma x_2 y_2 \end{bmatrix}. \tag{2.2}$$

During S2, the mature biomass changes because of net growth, maturity and harvest. The mature group exhibits a linear net growth, $\alpha x_2$, is strengthened by a fixed percentage of the immature group, $\beta y_2$, and is depleted by the fishing activities of all players, $h_2 + u$. On the other hand, the immature biomass changes because of net growth, maturity and cannibalism. The net growth of the immature group follows a non-linear relationship that resembles the Beverton-Holt recruitment function (Beverton and Holt, 1957). In addition, its biomass is depleted by the equivalent percentage that is transited to the mature group. Finally, when both groups coexist in the feeding grounds, there is some biomass loss due to cannibalistic behaviour, $\gamma x_2 y_2$.

Figure 2.3 shows the state transition of the immature group in S1 (left) and S2 (right). In S1, the biomass change of the immature is higher for most of the state-space and is bounded by the ecosystem's carrying capacity for high states. Moreover, as can be seen from the isocurves, the mature group plays a more crucial role in the biomass change of the immature during spawning. That is, higher mature states in the beginning of S1 lead to higher immature biomass in the beginning of S2 for all immature states. Figure 2.4 depicts the seasonal biomass development in the absence of fishing for the next 150 years or 300 seasons. As in Ni and Sandal (2019), the stock biomass moves towards a cyclical equilibrium state, namely $(x_1^*, y_1^*) = (5596, 4980)$ and $(x_2^*, y_2^*) = (5596, 6995)$, which occurs within the state-space. The numerical specification of the transition functions is given in Table 2.1. The parameters of the model do not describe a particular fishery, instead they are stylised representations of a hypothetical, but still possible, fishery with meaningful characteristics. Having discussed the biological traits and specified the seasonal dynamics, it is time to move on with the economic specification and description of the players' behaviour.

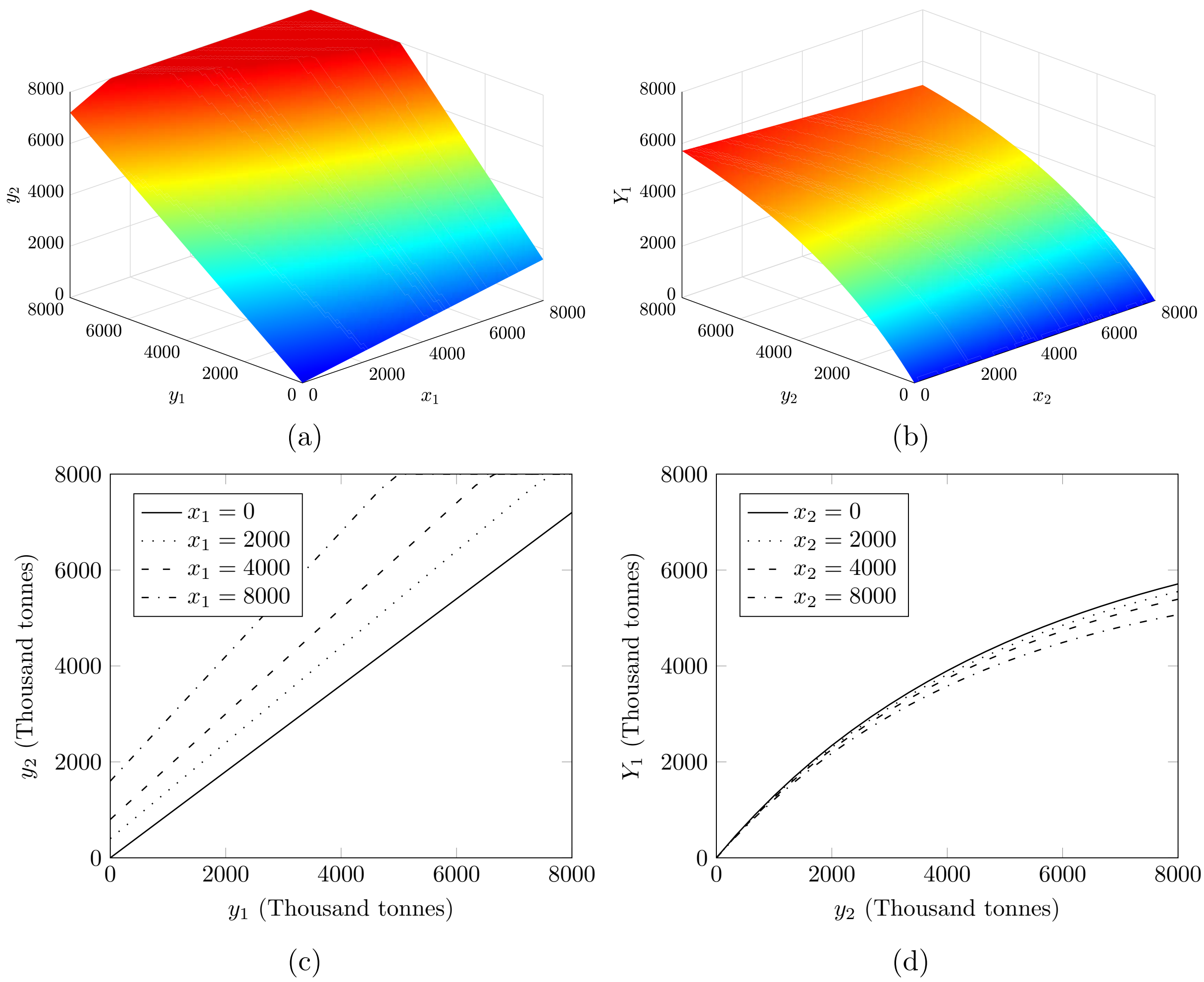



Figure 2.3. Immature biomass at the end of S1 (left) and S2 (right): (a) and (b) state-space surfaces; (c) and (d) isocurves for $x_1 = x_2 = 0, 2000, 4000, 8000$.

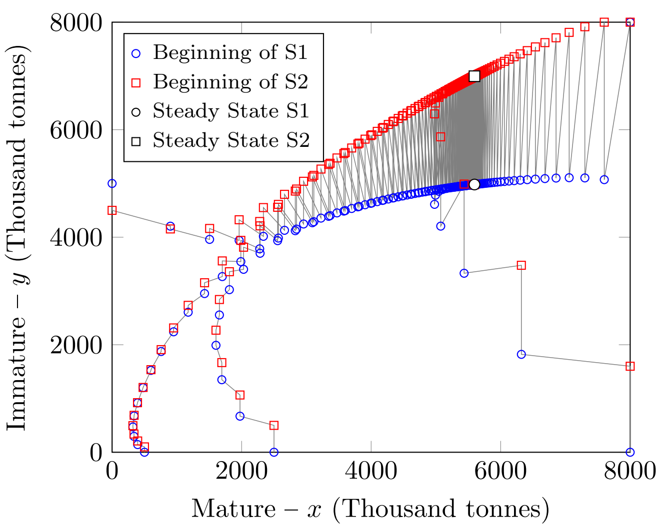



Figure 2.4. Biomass development in the absence of fishing. Time horizon 150 years. Initial $(x, y)$ states: (500,0), (2500,0), (8000,0), (0,5000) and (8000,8000).

Table 2.1. List of symbols and parameter values used in the study.

| Symbol | Description | Value | Unit |
|---|---|---|---|
| **Variables** | | | |
| $x, y$ | Mature and Immature stock biomass | | $10^3$ tonnes |
| $x_1, y_1$ | Stock biomass at the start of S1 | | $10^3$ tonnes |
| $x_2, y_2$ | Stock biomass at the start of S2 | | $10^3$ tonnes |
| $X_1, Y_1$ | Stock biomass at the end (start) of S2 (next S1) | | $10^3$ tonnes |
| $h_1, h_2$ | Leader's harvest strategy in each season | | $10^3$ tonnes |
| $u_i$ | Follower $i$'s harvest strategy in S2, $i = 1, \ldots, n$ | | $10^3$ tonnes |
| $\theta_i$ | Follower $i$'s harvest share | | |
| $\Pi_1^L, \Pi_2^L$ | Leader's profit in each season | | $10^6$ NOK |
| $\Pi_i^F$ | Follower $i$'s profit in S1 | | $10^6$ NOK |
| $V_1, V_2$ | Leader's value function in each season | | $10^6$ NOK |
| **Parameters** | | | |
| $x_{\max}, y_{\max}$ | Maximum stock biomass | $8000, 8000$ | $10^3$ tonnes |
| $a_1, a_2, a_3$ | Growth parameters for immature group in S1 | $0.9, 0.2, 0.5 \times 10^{-5}$ | |
| $\phi_1, \phi_2$ | Growth parameters for immature group in S2 | $1.645, 1 \times 10^{-4}$ | |
| $\alpha$ | Growth parameter for mature group in S2 | 0.75 | |
| $\beta$ | Biomass transition between groups in S2 | 0.2 | |
| $\gamma$ | Cannibalistic behaviour between groups in S2 | $1 \times 10^{-5}$ | |
| $p_1, p_2$ | Maximum prices in each season | $15, 19$ | NOK/kg |
| $\eta$ | Price sensitivity parameter | 0.0016 | |
| $c_1^L, c_2^L$ | Leader's cost parameters in each season | $6, 21000$ | |
| $n$ | Number of potential followers | 3 | |
| $c_i^F$ | Follower $i$'s cost parameters in S2, $i = 1, 2, 3$ | $[17, 19, 21] \times 10^3$ | |
| $\delta_1, \delta_2$ | Discount factors in each season | $0.95^{1/3}, 0.95^{2/3}$ | |

### 2.2.2 Economic model

As already mentioned, the resource can be targeted in S1 by the leader, and in S2 by both the leader and the followers with the leader having a first-mover advantage. The leader is interested in the long-term preservation of the fishery, and thus its objective is to maximise the present value of all future benefits, while considering the effect of both its actions and those of the followers on the state of the stock. The followers, who are assumed to target the resource simultaneously, behave myopically, i.e., they maximise their individual current profit. The game analysed here has two subgames: a simultaneous game played by $n$ followers, and a sequential game played by all players. The relevant solution concepts are the Nash and Stackelberg equilibrium. Before moving to the optimisation problems, the market for the resource and cost structure of the players are discussed.

In each season, demand for fish is described by the following downward and non-linear inverse demand relations:

$$P_1(h_1) = p_1 e^{-\eta h_1},$$
$$P_2(h_2 + u) = p_2 e^{-\eta(h_2+u)},$$

where parameters $p_1$ and $p_2$ are the respective seasonal maximum prices, and $\eta > 0$ is a sensitivity parameter that describes the rate at which market price drops as quantity increases, i.e., $P_i'/P_i = -\eta$, $i = 1, 2$.[2] Here it is assumed that $\eta$ is equivalent for both

[2]The inverse demand elasticities of $P_1(h_1)$ and $P_2(h_2 + u)$ are given by $-\eta h_1$ and $-\eta(h_2 + u)$.

seasons, whereas the maximum prices differ. In particular, it is assumed that prices are higher in feeding season, i.e., $p_2 > p_1$, which can be attributed either to higher demand, e.g., taste preferences and sustainability consciousness,[3] limited supply, e.g., less intensive fishing during the longer season, or both.

With regard to fishing costs, it has been a common practice in the literature to differentiate between density dependent and independent costs of harvest. This distinction is due to the underlying assumptions about the production activity of fishing vessels, which is typically specified as a Cobb-Douglas production function with two inputs: fishing effort and available stock biomass.[4] Effort elasticity of fishing is often considered equal to one, implying constant returns of scale with respect to it. The stock elasticity, on the other hand, ranges between zero and one, and serves as a proxy for fish density within a geographical area. A fishery is said to be uniformly distributed, if the stock elasticity is equal to one. As the stock elasticity decreases, a fishery becomes more concentrated, with zero implying pure schooling behaviour. Because the more dense the fish are within an area, the easier it is to target them, it is only natural to assume that fishing costs depend on both stock size and stock elasticity. Therefore, in many studies, fishing costs are assumed to be density dependent for positive stock elasticities, and density independent for zero. Density dependent costs represent trawling fishing technology and imply that the resource is economically protected since at low stock levels fishing costs become very high (Maroto et al., 2012). On the other hand, density independent costs represent purse seine fishing technology and signify constant unit costs, which is a common assumption for fisheries that exhibit some sort of schooling behaviour (Tahvonen et al., 2013). For a comprehensive analysis on stock and effort elasticities in bioeconomic models see Steinshamn (2011). Some empirical studies that deal with estimation of output elasticities include Bjørndal and Conrad (1987) and Nøstbakken (2006).

In this study, stock elasticities are assumed to vary between seasons, with the fishery being more concentrated when in the spawning grounds. For simplicity, it is assumed that the stock elasticity is zero in S1 and one in S2. It should be mentioned, however, that using any in between value does not pose any challenge with respect to both the leader's and the followers' optimisation problems, and thus can easily be incorporated into the model. Having said that, the unit cost of fishing for the leader is constant and equal to $c_1^L$ in S1, and a function of the stock biomass in S2, i.e.,

$$C(x_2, y_2) = \frac{c_2^L}{G(x_2, y_2)},$$

where $G(x_2, y_2) = \alpha x_2 + \beta y_2$ describes the state of mature biomass in S2 before any harvesting takes place. Since the followers are active only in S2, their respective fishing costs are also stock dependent; unit cost for follower $i$ is specified as:

$$\psi_i(z) = \frac{c_i^F}{z},$$

[3]Some consumers may develop negative preferences towards fish consumption during spawning seasons. This may result from either fish tasting different due to chemical changes in their bodies, or an effort to reduce the pressure on the stock at this critical time. In an empirical study, Asche, Chen and Smith (2015) found evidence that Norwegian cod prices are higher between May and December, i.e., the post-spawning months.

[4]Schaefer (1957) was the first to introduce this production function in the fisheries literature, and thus it is often referred to as the Schaefer harvest or production function. Its specification is given by $h = e^{b_1} x^{b_2}$, where $e$ is fishing effort, $x$ stock biomass, and $b_1$ and $b_2$ non-negative output elasticities.

where $z$ is a temporary variable that represents the state of the mature stock right before the followers harvest. Notice, that in both cases, the computation of unit costs is based on the state of the stock prior to harvest. This implies that fishing effort remains constant during fishing. And although this underestimates costs, especially when harvest is significant, it provides some convenience in deriving analytical solutions for the followers' subgame.[5]

Finally, for all players, net profit from fishing activity is defined as the difference between gross revenue and total fishing costs. The respective profit functions for the leader in S1 and S2, and follower $i$ are given by

$$\begin{aligned}\Pi_1^L(h_1) &= [P_1(h_1) - c_1^L]h_1,\\ \Pi_2^L(x, y, h_2, u) &= [P_2(h_2 + u) - C(x, y)]h_2,\\ \Pi_i^F(z, h_2, u_i, \mathbf{u}_{-i}) &= [P_2(h_2 + u) - \psi_i(z)]u_i,\end{aligned}$$

where $u_i$ is follower $i$'s harvest and $\mathbf{u}_{-i}$ the harvest vector of all other followers except $i$. Hereinafter all variables or sets with the $-i$ subscript refer to all other folowers except $i$. Notice that follower $i$'s net profit does not depend on the state of the immature group. This is because followers are myopic and only need to observe the state of the mature group right before they harvest. In contrast, the leader has to keep track of both states, since a higher immature state at the beginning of S2 will, ceteris paribus, lead to a higher mature state right before the leader harvests.

#### 2.2.2.1 Follower $i$'s optimisation problem

Before deciding how much to harvest, follower $i$ has at its disposal the following information: the leader's harvest strategy, $h_2$, the remaining mature biomass, $z$, and the fact that all other followers determine their strategies simultaneously and have access to the same information. Therefore, follower $i$ chooses a myopic harvest strategy, $u_i$, that maximises its net profit function, $\Pi_i^F$, given the harvest strategies of the other followers and the resource availability constraint. Such maximisation problem is time-independent and can be expressed as follows:

$$\begin{aligned}\max_{u_i} \quad & J_i^F = \Pi_i^F(z, h_2, u_i, \mathbf{u}_{-i})\\ \text{s.t.} \quad & \sum_{i=1}^{n} u_i \leq z, \;\; u_i \geq 0.\end{aligned} \tag{2.3}$$

For any stock level and harvest choice of the leader in S2, let $\tilde{u}_i(z, h_2)$ be a myopic feedback strategy, $\tilde{\mathcal{U}}_i$ the set of all feedback strategies for follower $i$, and $\tilde{\mathcal{U}} = \tilde{\mathcal{U}}_1 \times \cdots \times \tilde{\mathcal{U}}_n$ the Cartesian product of all such sets. Hereinafter all variables with the tilde icon refer to their corresponding feedback rules. A feedback strategy vector $\tilde{\mathbf{u}}^* = (\tilde{u}_1^*, \ldots, \tilde{u}_n^*) \in \tilde{\mathcal{U}}$ is a feedback Nash equilibrium if and only if

$$\begin{aligned}J_i^F(z, h_2, \tilde{u}_i^*(z, h_2), \tilde{u}_{-i}^*(z, h_2)) \geq J_i^F(z, h_2, \tilde{u}_i(z, h_2), \tilde{u}_{-i}^*(z, h_2)),\\ \forall \tilde{u}_i \in \tilde{\mathcal{U}}_i, \;\forall (z, h_2) \in \mathbb{R}_+^2, \;\forall i = 1, \ldots, n.\end{aligned}$$

[5]The alternative would have been to allow for changes in effort during fishing and specify total costs as a function of the stock before and after harvest. This would mean to keep tract of unit costs as the stock gets depleted, i.e., $\int_S^R c(x)dx$, where $c(x)$ is the density-dependent unit cost of fishing, and $R$ and $S$ the respective states before and after fishing. However, doing so adds unnecessary complexity to the analysis. For a study with such cost specification see Ekerhovd and Steinshamn (2016).

Finally, let the aggregate harvest of all followers at Nash equilibrium be given by $\tilde{u}^*(z, h_2) = \sum_{i=1}^n \tilde{u}_i^*(z, h_2)$. Additional details regarding the followers' subgame including the derivation of the feedback Nash equilibrium are given in section 2.3.

#### 2.2.2.2 Leader's optimisation problem

The leader is perfectly aware of the biomass states of the fishery in both S1 and S2, and possesses perfect information regarding the followers' information set and behaviour. That is, the leader anticipates that in S2 the followers will act both simultaneously and myopically. Based on this information structure, the leader chooses seasonal harvest strategies, $h_{1,t}$ and $h_{2,t}$ for $t = 0, \ldots, \infty$, that maximise its net present value. Such maximisation problem is time-dependent and can be expressed as follows:

$$\begin{aligned}
\max_{h_{1,t}, h_{2,t}} \quad & J^L = \sum_{t=0}^{\infty} \delta^t [\delta_1 \Pi_{1,t}^L(h_{1,t}) + \delta_1 \delta_2 \Pi_{2,t}^L(x_{2,t}, y_{2,t}, h_{2,t}, \tilde{u}^*(z_{2,t}, h_{2,t}))] \\
\text{s.t.} \quad & [x_{2,t}\ y_{2,t}]^{\mathrm{T}} = F_1(x_{1,t}, y_{1,t}, h_{1,t}) \\
& [x_{1,t+1}\ y_{1,t+1}]^{\mathrm{T}} = F_2(x_{2,t}, y_{2,t}, h_{2,t}, \tilde{u}^*(z_{2,t}, h_{2,t})) \\
& z_{2,t} = G(x_{2,t}, y_{2,t}) - h_{2,t} \\
& x_{i,t} \geq 0,\ y_{i,t} \geq 0,\ h_{i,t} \geq 0, \quad \forall i = 1, 2,
\end{aligned} \tag{2.4}$$

where T stands for transpose, $\delta$, $\delta_1$ and $\delta_2$ are the respective annual and seasonal discount factors with $\delta = \delta_1 \delta_2$, and $z_{2,t}$ is a temporary variable that describes the state of the mature biomass in S2 following the leader's but preceding the followers' harvest activities. Keep in mind that although variables $z$ and $z_{2,t}$ look like they are referring to the same state, they are not. Variable $z$ is used in the followers' optimisation and refers to all states that can possible occur. In contrast, variable $z_{2,t}$ is used in the leader's optimisation and refers to specific states that follow from the states in the beginning of S2.

Kvamsdal et al. (2016, 2020) showed that the solution of a class of problems like the one specified above can be obtained by solving an equivalent system of coupled Bellman equations using standard dynamic programming techniques. Such system is specified as follows:

$$\begin{aligned}
V_1(x_1, y_1) &= \max_{h_1} \{\delta_1 \Pi_1^L(h_1) + \delta_1 V_2(F_1(x_1, y_1, h_1))\}, \\
V_2(x_2, y_2) &= \max_{h_2} \{\delta_2 \Pi_2^L(x_2, y_2, h_2, \tilde{u}^*(z_2, h_2)) + \delta_2 V_1(F_2(x_2, y_2, h_2, \tilde{u}^*(z, h_2)))\},
\end{aligned} \tag{2.5}$$

with $z_2 = G(x_2, y_2) - h_2$. The value functions $V_1$ and $V_2$ represent the largest possible value the leader can derive from this infinite-horizon optimisation process, when the initial state starts at S1 and S2 respectively. What is important to understand here is that $V_1$ (or $V_2$ for that matter) does not represent the maximum net present value the leader obtains from its S1 (S2) fishing activity, but the aggregate maximum net present value from both seasons starting at S1 (S2).

For any state of the mature and immature biomass, let $\tilde{h}_1(x, y)$ and $\tilde{h}_2(x, y)$ be feedback harvest strategies for the leader in S1 and S2, and let $\tilde{\mathcal{H}}_1$ and $\tilde{\mathcal{H}}_2$ be the respective sets. Moreover, the realised harvest for follower $i$ is given by $\tilde{u}_i^*(\tilde{z}_2(x, y), \tilde{h}_2(x, y))$, where $\tilde{z}_2(x, y) = G(x, y) - \tilde{h}_2(x, y)$. The feedback strategies $(\tilde{h}_1^*, \tilde{h}_2^*, \tilde{\mathbf{u}}^*) \in \tilde{\mathcal{H}}_1 \times \tilde{\mathcal{H}}_2 \times \tilde{\mathcal{U}}$ constitute a feedback Stackelberg equilibrium if and only if

$$J^L(\cdot, \tilde{h}_1^*(\cdot), \tilde{h}_2^*(\cdot), \tilde{u}^*(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot))) \geq J^L(\cdot, \tilde{h}_1(\cdot), \tilde{h}_2(\cdot), \tilde{u}^*(\tilde{z}_2(\cdot), \tilde{h}_2(\cdot))),$$
$$\forall(\tilde{h}_1, \tilde{h}_2) \in \tilde{\mathcal{H}}_1 \times \tilde{\mathcal{H}}_2, \ \forall(x, y) \in \times\mathbb{R}_+^2,$$
$$J_i^F(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot), \tilde{u}_i^*(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot)), \tilde{u}_{-i}^*(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot))) \geq$$
$$J_i^F(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot), \tilde{u}_i(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot)), \tilde{u}_{-i}^*(\tilde{z}_2^*(\cdot), \tilde{h}_2^*(\cdot))), \quad \forall\tilde{u}_i \in \tilde{\mathcal{U}}_i, \ \forall(x, y) \in \mathbb{R}_+^2, \ \forall i = 1, \ldots, m,$$

where $(\cdot)$ refers to the mature and immature states $(x, y)$. For a description of the dynamic programming algorithm used to solve problem (2.5) see appendix A.1.

## 2.3 Myopic followers' game

In this section, the feedback Nash equilibrium for the $n$-follower game is derived. From the $n$ potential followers, let $m \leq n$ be the number of active ones. An active follower is defined as one who has a positive harvest strategy, i.e., $u_i > 0$. Moreover, let $\psi_1(z) \leq \psi_2(z) \leq \cdots \leq \psi_n(z)$ be the ranking of unit fishing costs for all $n$ followers, which remains constant for all stock levels. This implies that the first follower is the most efficient cost-wise whereas the $n$-th one is the least efficient. This setting is similar to Sandal and Steinshamn (2004), where the authors investigated myopic behaviour of multiple Cournot competing fishing agents. To proceed, we split problem (2.3) in two subproblems, where in the first the resource constraint is slack and in the second it is binding.

Let $u = \sum_{i=1}^m u_i < z$, then if follower $i$ is active, its optimal harvest is given by the first order condition of its profit function:

$$\frac{\partial \Pi_i^F}{\partial u_i} = 0 \Leftrightarrow (1 - \eta u_i) = \psi_i(z) k e^{\eta u}, \quad \forall i = 1, \ldots, m, \tag{2.6}$$

with $k = e^{\eta h_2}/p_2$ and $u_i < 1/\eta$.[6] This corresponds to a system of $m$ equations and $m$ unknowns for all active followers. Summing (2.6) over all active followers yields an implicit expression in terms of $u$ and $m$ for all $(z, h_2) \in \mathbb{R}_+^2$:

$$(m - \eta u) = \sum_{i=1}^m \psi_i(z) k e^{\eta u}, \tag{2.7}$$

with $u < m/\eta$. Using (2.7), expression (2.6) can be re-written as follows:

$$(1 - \eta u_i) = \frac{\psi_i}{\sum_{i=1}^m \psi_i}(m - \eta u), \quad \forall i = 1, \ldots, m. \tag{2.8}$$

For convenience, the $z$ argument in the unit cost term is dropped for the moment. Because followers are ordered according to their fishing costs, it follows from (2.8) that $u_1 \geq u_2 \geq \cdots \geq u_m > 0$. It is thus enough to find the aggregate quantity $u$ that satisfies $u_m > 0$ on the threshold. That is follower $m$ is active but follower $m + 1$, if exists, is inactive.

To determine the number of active followers, $m$, and solve (2.7) for their aggregate harvest, $u$, the following procedure is recommended. Let $w = \eta u$ and $w_i = \eta u_i$ be scaled variables, and $\bar{w} = w/m$ and $\bar{\Psi} = \sum_{i=1}^m \psi_i/m$ be the means of the total harvest and the

[6] In order for the solution to represent a global maximum, the profit function has to be strictly concave. This is equivalent to $u_i < 2/\eta$, which is always true when follower $i$ is active, since $u_i < 1/\eta$.

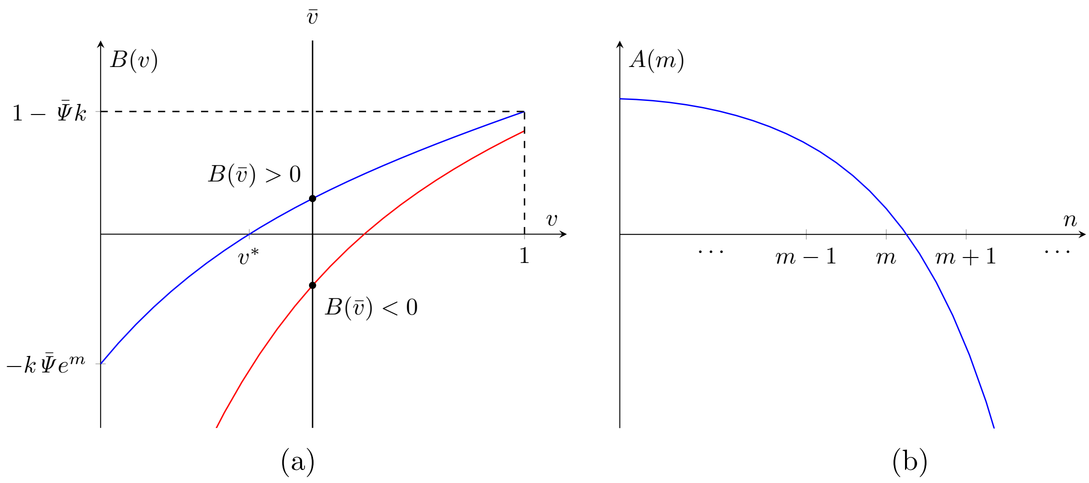



Figure 2.5. Graphical representation of the solution to equations (2.9) and (2.10). (a) Function $B(v)$ in the feasible region. The total harvest for $m$ active followers is given when $B(v)=0$ if and only if $v^*<\bar{v}$; the blue and red lines show the relative position of the $B(v)$ function when follower $m$ is active and inactive, respectively. (b) Activity function $A(m)$, the last active follower, $m$, is the maximum integer value that satisfies $A(m)>0$.

cumulative sum of costs, respectively. Moreover, let $v=1-\bar{w}$ and $v_i=1-w_i$, then (2.7) and (2.8) can be re-written as follows:

$$v = k\,\bar{\Psi}e^{m(1-v)}, \tag{2.9}$$

$$v_i = \frac{\psi_i}{\bar{\Psi}}v, \quad \forall i = 1,\dots,m. \tag{2.10}$$

By definition variables $w$ and $v$ are bounded in the open interval $(0,1)$. Next, let $B(v)\equiv v-k\,\bar{\Psi}e^{m(1-v)}$, which is a strictly increasing function. If at least one follower is active, i.e., $m\geq 1$, then the optimal total harvest, $u=m(1-v)/\eta$, can be determined from the solution of $B(v)=0$ for the correct number of active followers.

To figure out the proper $m$, expression (2.10) is utilised. The $m$-th follower is active if $u_m>0$ or by (2.10) when $v_m<1$. Let $\bar{v}:v_m=1$ denote the threshold that follower $m$ is indifferent between being active or inactive, i.e., $\bar{v}=\bar{\Psi}/\psi_m$.[7] In order for follower $m$ to be active, the solution of $B(v)=0$ must lie on the left of the $\bar{v}$ threshold (figure 2.5 left). Since $B(v)$ is strictly increasing the previous statement holds true whenever $B(\bar{v})>0$ or

$$A(m) = 1 - k\psi_m \exp\left\{ m\left(1-\frac{\bar{\Psi}}{\psi_m}\right)\right\} > 0. \tag{2.11}$$

The activity function $A(m)$ is strictly decreasing with respect to the number of followers (figure 2.5 right). The maximum integer value that satisfies $A(m)>0$, determines the number of active followers. If the activity function evaluated at one is negative, then all followers are inactive. This means that the harvest quantity of the most efficient follower is negative and thus infeasible to implement. The process after determining $m$ is straightforward: solve for the root of $B(v)$ and substitute the solution to (2.8) or (2.10) depending on whether $v$ is rescaled back to $u$ or not.

[7]It follows from the cost ordering assumption that $\bar{v}<1$.

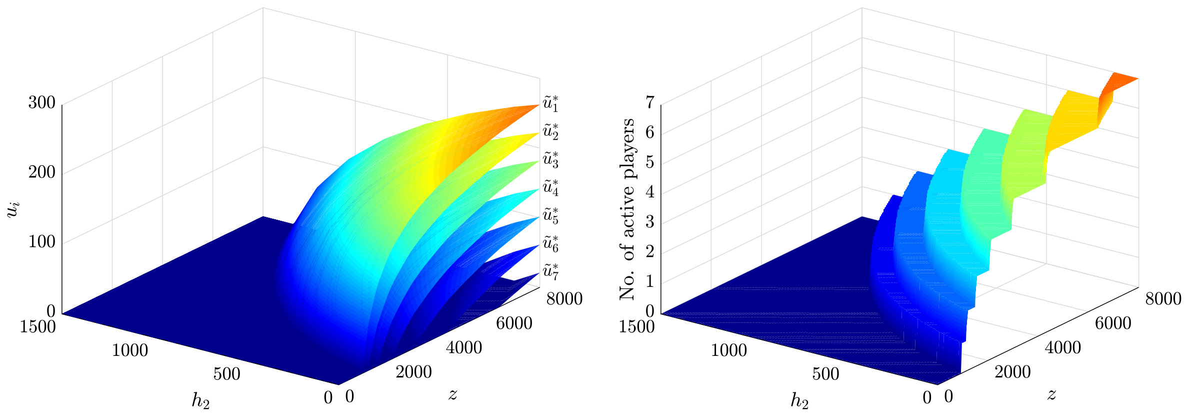



Figure 2.6. (a) Myopic feedback rules for all active followers. (b) The number of active followers as a function of the stock biomass and the leader's harvest in S2. The parameters used in plotting are: $m = 20$, $p_2 = 19$, $\eta = 0.0016$, and $c_i^F = 18000 + 2000(i - 1)$.

Moving on to the case where the resource constraint is binding, i.e., $\sum_{i=1}^{m} u_i = z$, any combination of strategies that satisfy it can be a potential equilibrium. This is because no follower can unilaterally improve its position by changing its strategy. Therefore, unless the rules of interaction among the followers are more clear, an infinite number of equilibria can occur when the resource constraint binds. To deal with this, the following assumption is made: at any point in time the active followers deplete the resource simultaneously and the rate of depletion is proportional to their first-best harvest strategies. This means that at the point in time where the stock is depleted, each follower will have already harvested a percentage of it, which is proportional to what they would have harvested if there had been sufficient biomass available. Thus, follower $i$'s harvest is given by $\theta_i z$, where $\theta_i = u_i/u$ is its first-best share. This concludes the solution of the followers' subgame. Follower $i$'s feedback Nash equilibrium harvest strategy can be specified as follows:

$$\tilde{u}_i^*(z, h_2) = \begin{cases} u_i & \text{if } i \leq m \wedge u < z, \\ z\theta_i & \text{if } i \leq m \wedge u \geq z, \\ 0 & \text{if } i > m, \end{cases} \qquad \forall (z, h_2) \in \mathbb{R}_+^2,\ \forall i = 1, \ldots, n, \tag{2.12}$$

where $m = 0, \ldots, n$ is the maximum integer value that satisfies inequality (2.11), and $u$ and $u_i$ are the scaled solutions of (2.9) and (2.10), respectively.

Figure 2.6 illustrates the number of active followers and their respective feedback harvest strategies for all biomass states and possible harvest strategies of the leader in S2. For a given set of parameters, from the twenty potential followers, at most seven can be active. The more efficient a follower is, the smaller the stock biomass is where it becomes active and the higher the quantity it harvests, ceteris paribus. As the leader's harvest increases, the stock biomass threshold of entry follows a convex curve.

## 2.4 Numerical results and discussion

In this section, the leader's infinite horizon repeated game is solved numerically. A number of results are presented to demonstrate the kind of insights that can be obtained using the model and evaluate the impact of economic parameters on long-term harvest strategies. The analysis is done in two steps. First, optimal feedback harvest policies are computed using the procedure described in appendix A.1. Second, stock development and fishing behaviour are simulated for a time horizon of 150 years, i.e., 300 seasons.

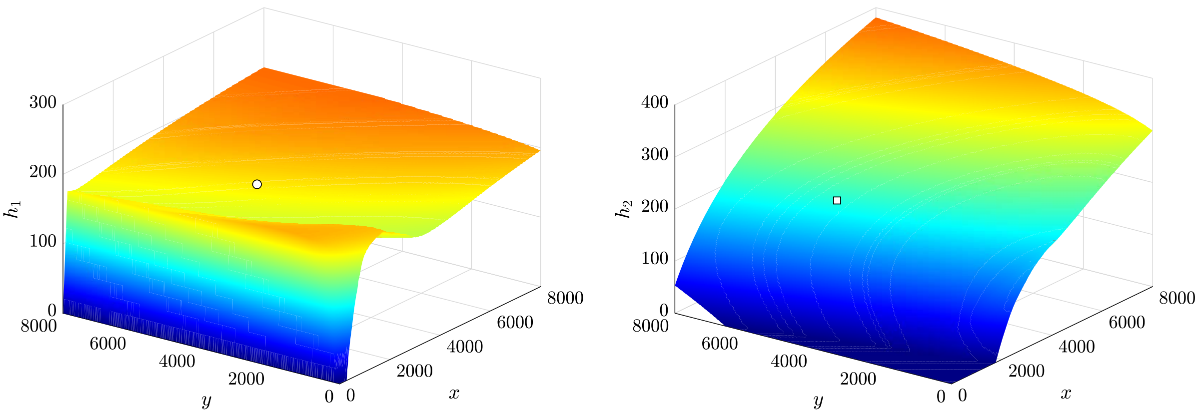
Two main cases are considered. In the first one, there are no potential followers, $n = 0$, which translates to the leader being the sole participant in the fishery. This case will serve as benchmark and be referred to as first-best because it yields the best possible outcome both economically and biologically, i.e., higher net present value for the leader and lower pressure on the long-term stock biomass. In the second case, also referred to as second-best, three potential followers are considered, where some of them have a cost advantage over the leader. The model is solved and simulated multiple times for a variety of parameter values and initial biomass scenarios. The parameter values used for the base runs are presented in Table 2.1.

### 2.4.1 Seasonal feedback strategies

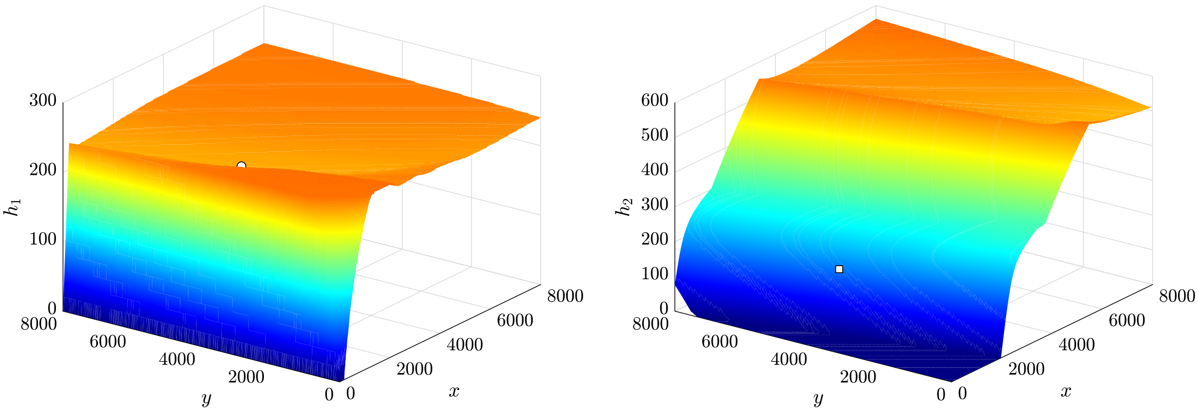
Figure 2.7 shows the seasonal feedback fishing strategies for the leader in the presence and absence of followers. A feedback strategy takes as input the states of the system, in this context the mature and immature stock biomass, and in return prescribes courses of action, i.e., fishing policies. In both cases, optimal harvest in S1 follows more or less the same pattern: a sharp increase for low mature states, followed by a decrease that creates a valley, which eventually starts increasing at a very slow rate. On the other hand, the optimal harvest pattern in S2 differs considerably between cases. In the absence of followers, there is a gradual increase in harvest along the entire state space. In the presence of followers, however, there is a sharp increase followed by a gradual increase and then again a sharp increase until a plateau is reached. Compared to S1, it is optimal not to harvest for low stock levels in S2 in both cases. This follows from the cost structure, which is density dependent in S2 and thus very high when stock abundance is low.

The optimal harvest for the three followers when the leader acts optimally is depicted in figures 2.8.a to 2.8.c. The harvest pattern of the more efficient followers is characterised by no fishing for low states, a gradual increase followed by a decrease, and an increase again for higher states. The biomass states where harvest decreases correspond to the ones where the leader starts fishing. Positive amounts of harvest for the least efficient follower are confined to high biomass states.

To get a better understanding of the leader's feedback rules, the harvest isocurves across the diagonal states together with the development of unit profits for both seasons and cases are displayed in figure 2.9. In both cases, the decline in harvest during S1 coincides with the start of fishing in S2, implying a trade-off between seasonal profits. Seasonal unit profits are higher in the first-best scenario, with the unit profit in S2 to surpass the one in S1 for states above 2000, which is an indication of the more intense fishing in S2. Although such indication is absent in the presence of followers, since, for high states, harvest in S2 exceeds that of S1 but the respective unit profit is significantly lower. This can be explained by the following. First, rent dissipation attributed to their existence, which leads to what is often referred to as "the race to fish". Second, trade-off between current and future seasonal profits. This means that although it seems more profitable to exploit the resource in S1, it will lead to less long-term benefits for the leader. The reason is that increased fishing in S1, will further decrease the stock biomass, which will lead to a weaker seasonal growth, and thus higher costs and less fishing opportunities in S2.




(a) $\tilde{h}_1^*(x, y)$ when $n = 0$ (b) $\tilde{h}_2^*(x, y)$ when $n = 0$



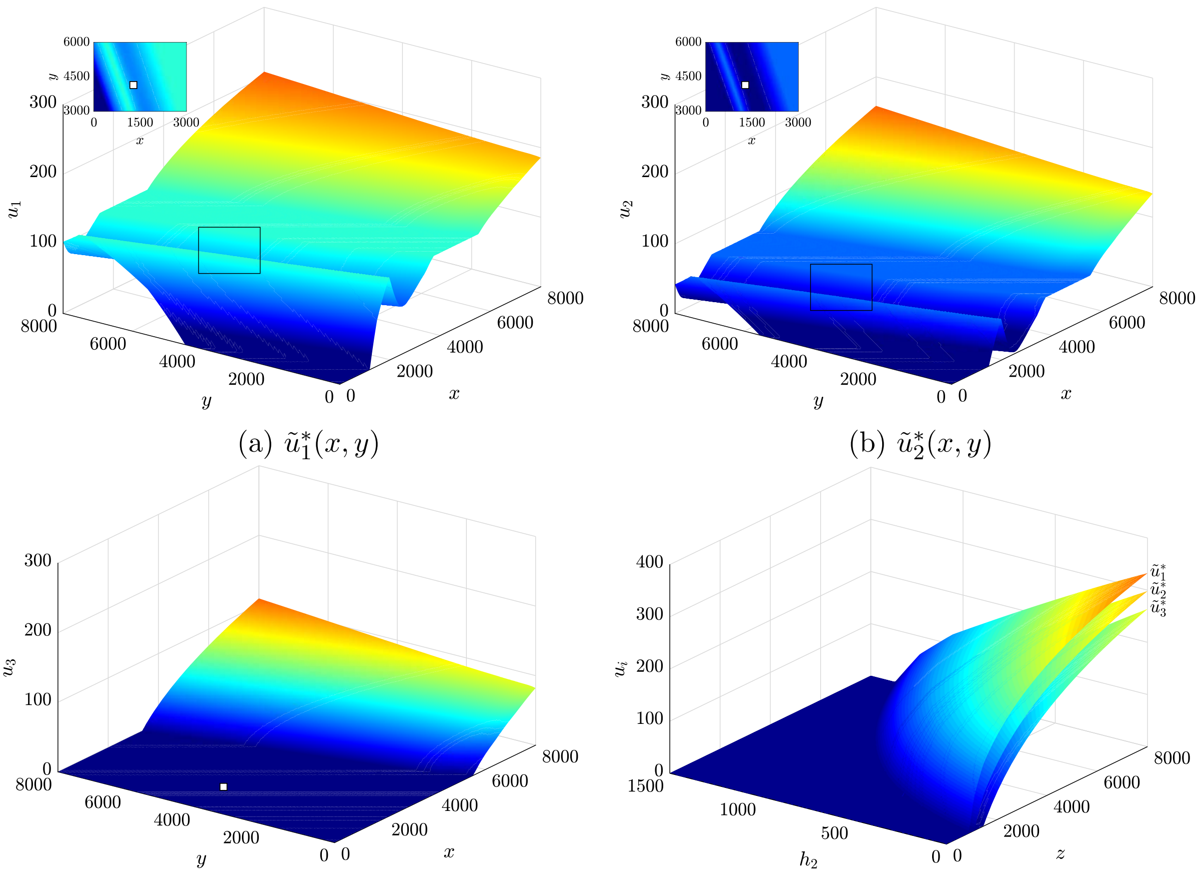

(c) $\tilde{h}_1^*(x, y)$ when $n = 3$ (d) $\tilde{h}_2^*(x, y)$ when $n = 3$

Figure 2.7. Leader's first- and second-best feedback harvest strategies in S1 and S2. The white circles and squares point out the respective S1 and S2 equilibrium states and policy levels. The units for all states and policies are in thousand tonnes.

Without any followers, the leader is the sole participant in the fishery and therefore can strike a balance between exploitation and sustainability. This balance implies that the marginal benefit from harvesting an additional unit is equivalent to the marginal benefit of letting it grow in order to be harvested in the future. However, in the threat of potential followers, who can also target the resource, any additional unit of fish left by the leader is one more unit for the followers to harvest. This creates distortions with regard to the leader's harvest activity, and thus its optimal harvest becomes substantially different between the two cases. By adopting a more aggressive fishing strategy, the leader is able to delay entry or even exclude some followers from the fishery. This is because the cost of fishing in S2 depends on the magnitude of the mature biomass, which the leader reduces first and as a consequence increases the fishing cost of the followers.[8]

Figure 2.10 illustrates this point for the diagonal states in S2. In the case where the leader does not engage into fishing in S2, the three followers enter the fishery at the following biomass states: 925, 1166 and 1528. After entry, their harvest increases at a decreasing rate with the stock biomass (dashed lines in figure 2.10 left). However, when the leader is active, it utilises its first mover advantage to delay the entry of the third follower, who now enters when the stock biomass is at 4261, and to reduce the overall

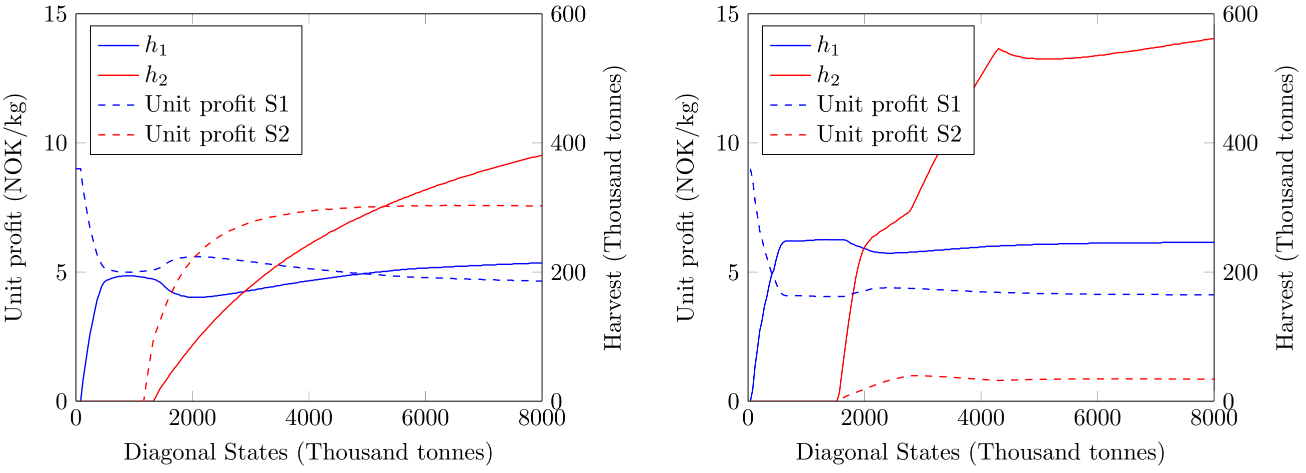
[8]The inverse relationship between stock biomass and fishing costs is a type of negative production externalities, also known as stock externalities (Smith, 1969; Bjørndal, 1987).




(a) $\tilde{u}_1^*(x, y)$ (b) $\tilde{u}_2^*(x, y)$

(c) $\tilde{u}_3^*(x, y)$ (d) $\tilde{u}_i(h_2, z)$

Figure 2.8. (a)-(c) Followers' realised harvest strategies based on leader's optimal harvest. The white squares point out the equilibrium states and policies in S2. (d) Followers' potential harvest strategies. The units for all states and policies are in thousand tonnes.




(a) $n = 0$ (b) $n = 3$

Figure 2.9. Harvest isocurves and realised unit profits for the leader in the absence and presence of followers across the diagonal of the state space.

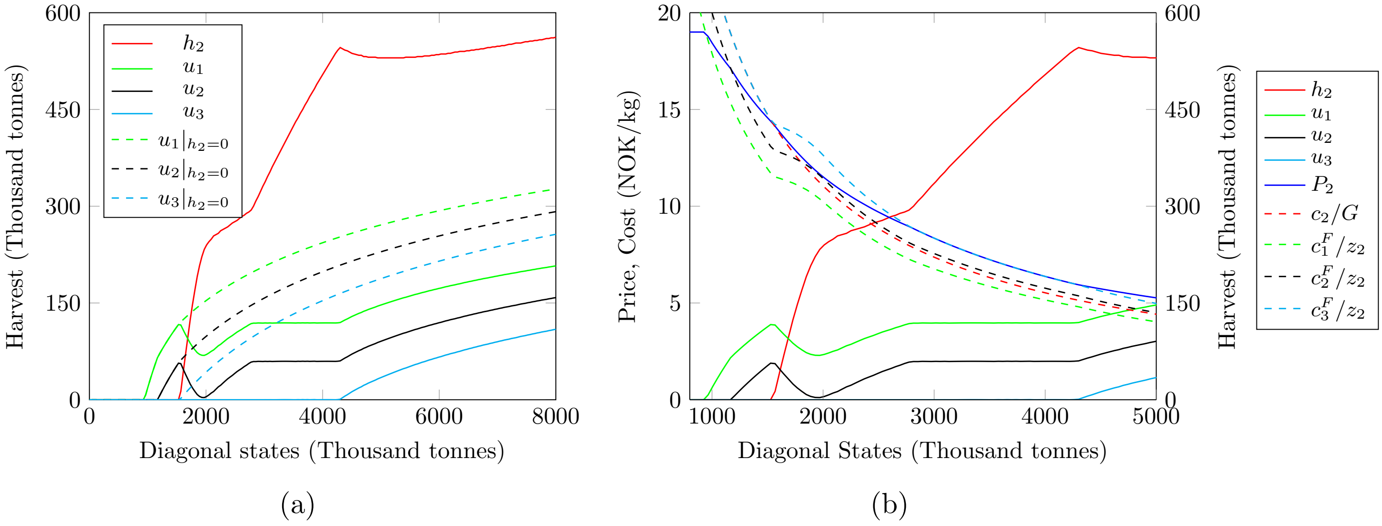



Figure 2.10. (a) Harvest strategies in S2 for the diagonal of the state space in the presence of three followers. The dashed lines represent the respective followers' strategies when the leader's does not enter the fishery in S2. (b) Development of prices and unit costs of fishing for all players in S2 along the diagonal of the state space.

amount of catches all followers take.

To further investigate such behaviour, the right part of figure 2.10 displays the price and cost development for all players along the diagonal. For low stock levels, unit costs of fishing exceed the seasonal maximum price, $p_2 = 19$, and thus no player is active. As the stock biomass increases, the unit costs decrease and players begin to enter the fishery. Follower one enters first followed by follower two. Their harvest pattern is equivalent as if the leader was inactive. When the leader enters, the stock biomass is reduced and the unit cost for all followers increases. This is reflected upon the decrease in harvest for the already active followers and the delay of entry for the last follower. Depending on the relative cost efficiency between the leader and a follower, it is possible that the leader drives a follower out of the fishery despite the fact that it was already active. Notice how the realised fishing strategy for the second follower almost touches zero (black line).

As the stock biomass increases, followers one and two harvest slowly increases, until it flattens out, and then increases again. Meanwhile, the leader's harvest increases slowly and then fast, until it reaches a plateau. Notice that the unit cost of fishing for the least efficient follower (cyan dashed line in figure 2.10 right) becomes equivalent to the realised price (blue line) right before the leader's second rapid increase in harvest, and remains so until the plateau is reached. This indicates that the leader increases its harvest quantity such that it becomes unprofitable for the least efficient follower to enter the fishery. Moreover, the remaining followers are confined to harvest a relatively steady quantity. In other words, as the stock biomass increases, and cost decreases, the leader harvests what the followers would have, nevertheless, harvested. Eventually there is a point where it is no longer beneficial to keep increasing the pressure in the stock. This is characterised by the entry of the last follower, a slight decrease in the leader's harvest, and an increase in the followers' harvest.

A final remark concerns the comparison of the leader's net present value between cases. Figure 2.11 shows the total value from the solution of problem (2.4) for all initial states starting at S1. In the presence of followers, the value has reached almost 50% reduction for most initial states, suggesting rent dissipation due to multiple competing

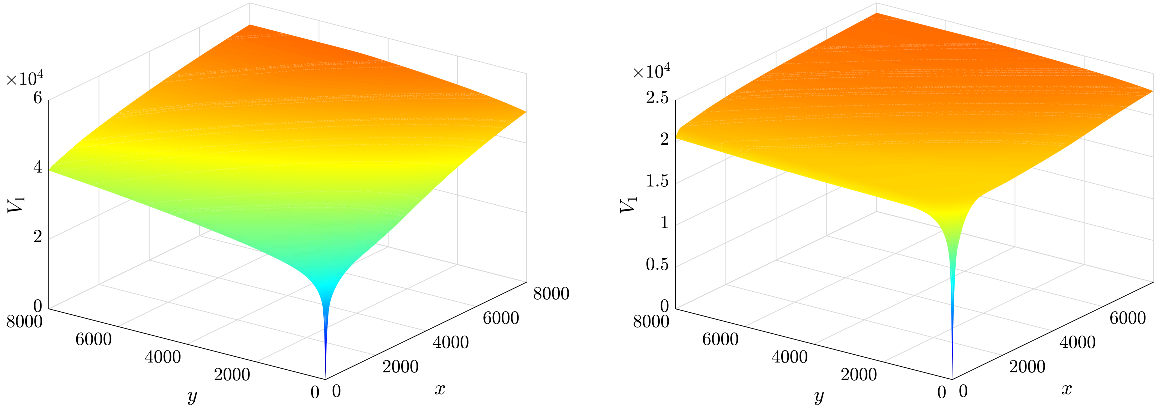



Figure 2.11. Leader's net present value with and without followers for all biomass states starting at S1. The units for all states are in thousand tonnes. The monetary units are in million NOK.

agents and myopic behaviour.

### 2.4.2 Long-term biomass development

To see the development of the stock biomass in time and test its sensitivity to initial conditions, the transition systems (2.1) and (2.2) are simulated for a time horizon of 150 years (300 seasons). Figure 2.12 shows the seasonal biomass development for the two cases analysed here. Only several initial states are presented, however, the results remain robust for all initial states tested. The long-term status of the stock is given by an annual equilibrium cycle where the stock is targeted by the leader in both seasons, and, if present, followers one and two in S2.

In the absence of followers, the cyclical equilibrium occurs at $(x_1^*, y_1^*) = (2841, 4453)$ and $(x_2^*, y_2^*) = (2664, 5209)$, with the leader harvesting 177 and 199 thousand tonnes in S1 and S2, respectively. With three potential followers, on the other hand, the long-term seasonal biomass states are lower and occur at $(x_1^*, y_1^*) = (1511, 3937)$ and $(x_2^*, y_2^*) = (1282, 4143)$. The leader harvest 229 and 176 thousand tonnes in S1 and S2, respectively. Followers one and two harvest 83 and 20 thousand tonnes, respectively, in S2, and follower three is inactive. The white circles and squares in figures 2.7 and 2.8 point out the respective S1 and S2 equilibrium states and policy levels within the feedback surfaces.

In both cases, there seems to be some undershooting towards the equilibrium cycle for low mature and immature states. That is, there is a reduction in the biomass development of the mature group, which becomes more evident the lower its initial biomass is, for example, see initial states (500,0) and (2500,0) in figure 2.12. This undershooting phenomenon becomes less significant for higher immature states or as the immature biomass recovers through time.

To determine how many years are needed for the stock biomass to approach the equilibrium cycle, three circles with a radius that represents a 10%, 15% and 20% deviation from its mean distance are drawn.[9] Then, for all grid squares A1, . . . ,D4 (see figure 2.12), 2500 uniformly spaced nodes are generated and used as initial states in the simulation runs. The minimum, maximum and average number of years before the stock biomass

[9] The radius of a circle with center $(x, y) = \left(\frac{x_1^*+x_2^*}{2}, \frac{y_1^*+y_2^*}{2}\right)$ and 10% deviation is $\sqrt{(0.1x)^2 + (0.1y)^2}$.

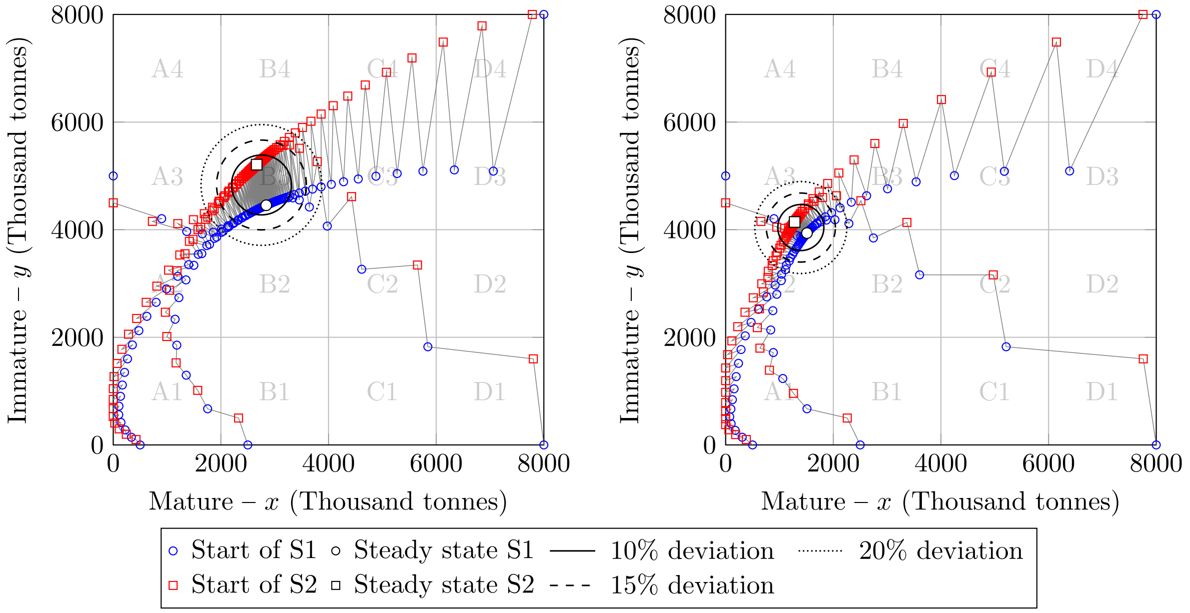



Figure 2.12. Biomass development with fishing: without (left) and with followers (right). Time horizon 150 years. Initial $(x, y)$ states: (500,0), (2500,0), (8000,0), (0,5000) and (8000,8000). The circles around the equilibrium cycle represent a 10%, 15% and 20% deviation from its mean distance. The A1, . . . ,D4 combinations are grid square references.

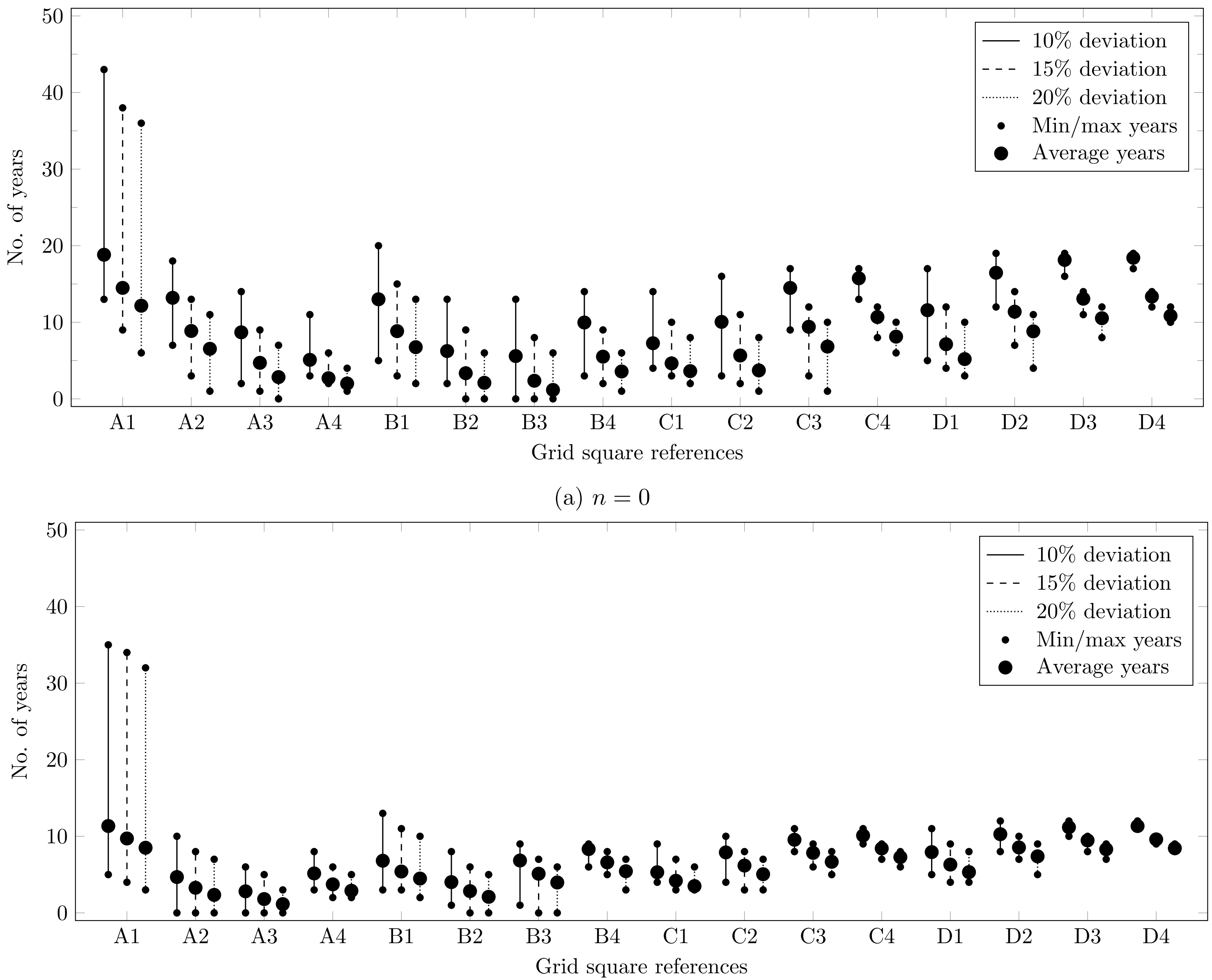
approaches the equilibrium circles are depicted in figure 2.13 for both cases. For very low mature and immature biomass levels (grid square A1), the resource would need, in the presence of followers, up to 35 years before getting in the vicinity of the equilibrium cycle at 10% tolerance. This number is higher and equal to 43 years, when the leader is in by itself, but the equilibrium cycle is also higher in this scenario. As the initial biomass increases the number of years required to get closer to the equilibrium cycle are significantly reduced. For example, the next grid square that takes the longest to approach the equilibrium cycle is B1, which needs 11 and 15 years, with and without followers, respectively. This is a considerable drop from 35 and 43. From all initial states in the state-space, it would require on average around 8 years to approach it when follower are present and 12 years when they are not. All years reported are based on the 10% deviation scenario. Clearly, as the circle radius around the equilibrium cycle is relaxed, the number of years required to reach it are reduced.

## 2.4.3 Sensitivity analysis

Moving on, the impact of the maximum price parameters, $p_1$ and $p_2$, and the sensitivity parameter, $\eta$, on the long-term harvest strategies and biomass states is identified. The analysis here is focused on how varying parameter values affects the equilibrium steady states, while the value of all other parameters remains the same as in the base cases (ceteris paribus). The model is repeatedly solved and simulated with changes in one parameter at a time.

### 2.4.3.1 Impact of maximum price parameters

The maximum price parameters reflect upon the attractiveness of harvesting and selling the resource in a particular season. From the leader's point of view, a higher maximum

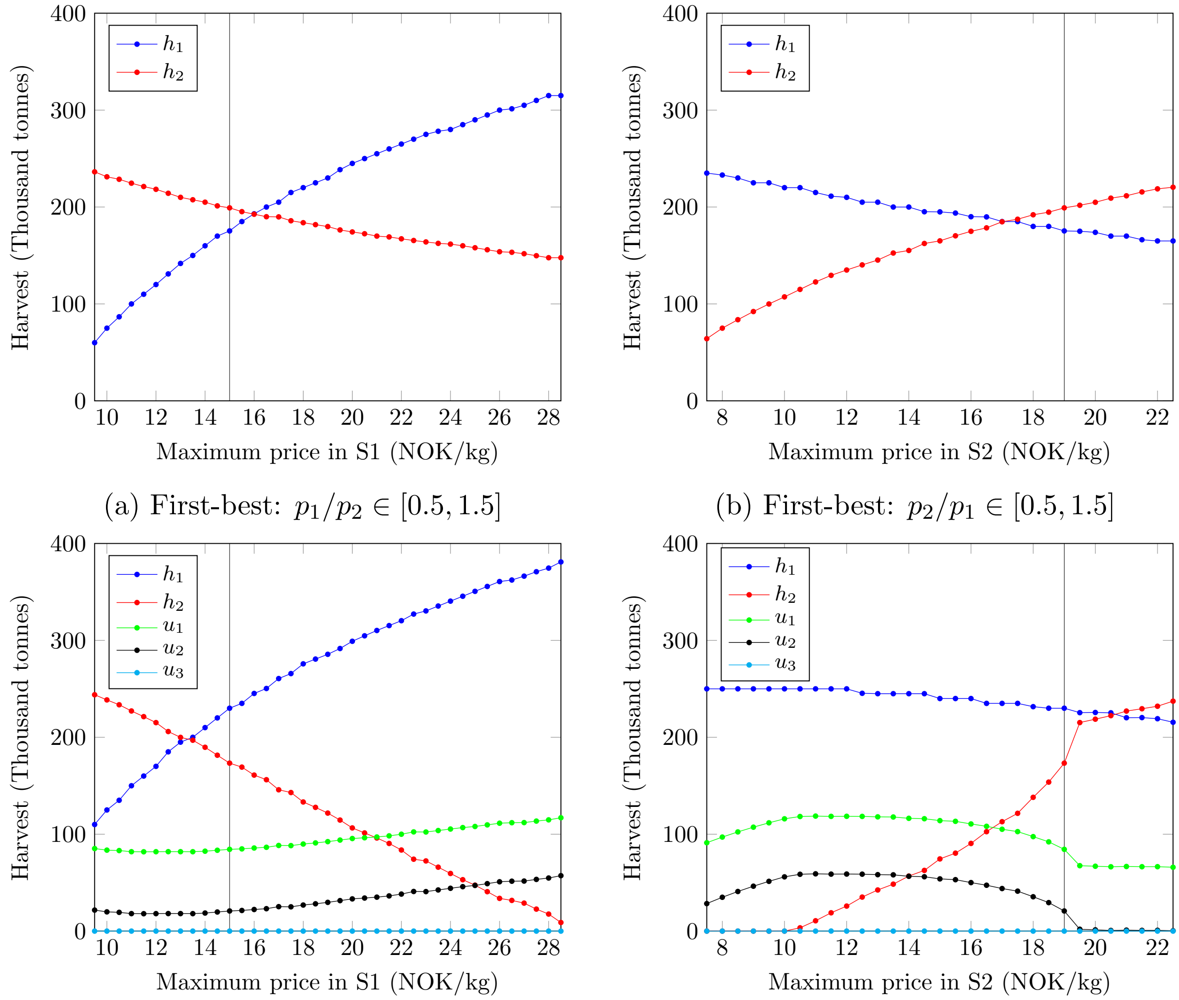



(a) $n = 0$



(b) $n = 3$

Figure 2.13. Minimum, maximum and average number of years required to approach the equilibrium cycle for 10%, 15% and 20% deviation from its mean. For each grid square, 2500 uniformly distributed nodes are simulated.




(a) First-best: $p_1/p_2 \in [0.5, 1.5]$

(b) First-best: $p_2/p_1 \in [0.5, 1.5]$

(c) Second-best: $p_1/p_2 \in [0.5, 1.5]$

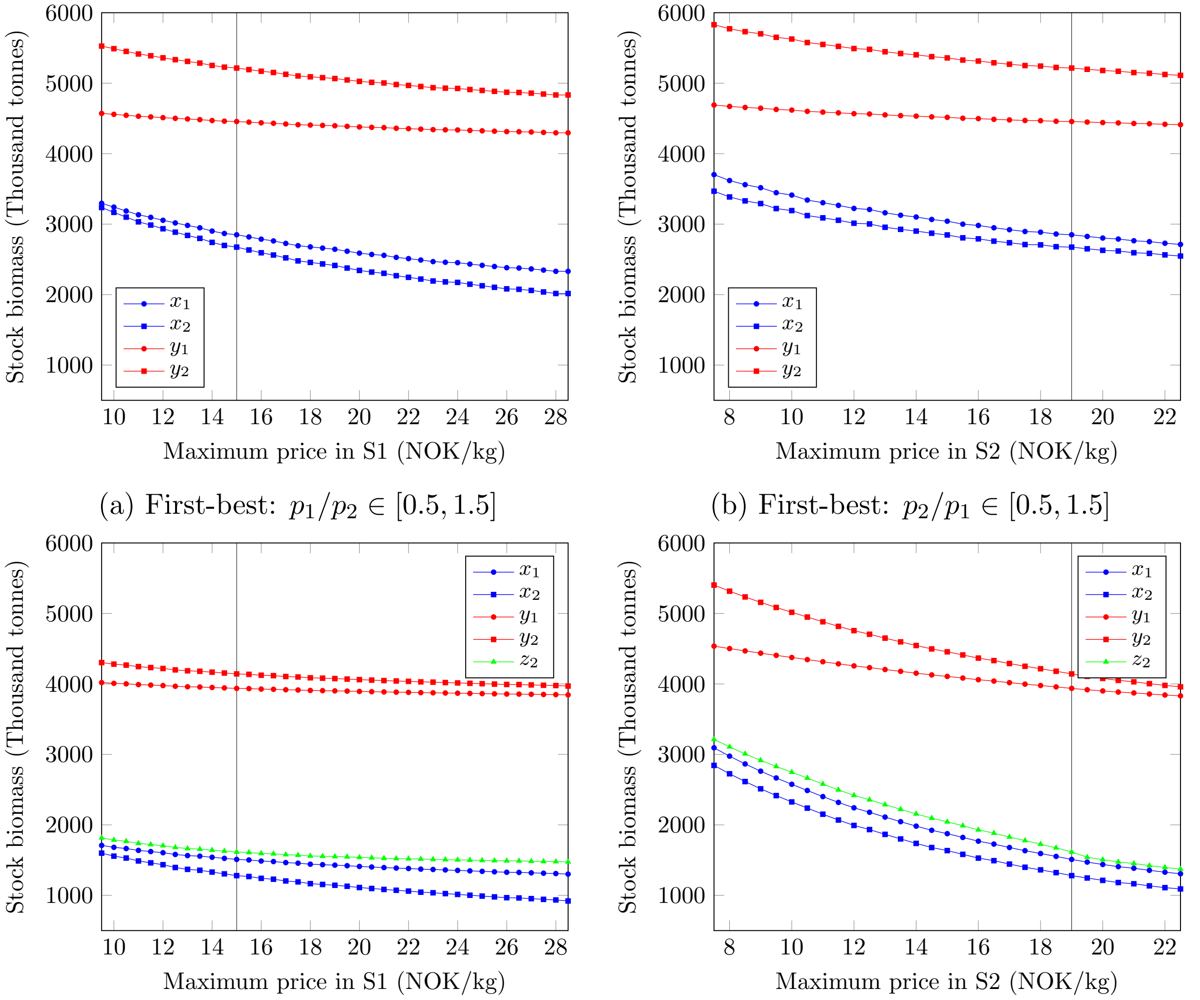
(d) Second-best: $p_2/p_1 \in [0.5, 1.5]$

Figure 2.14. Long-term harvest levels for different values of the maximum price parameters, $p_1$ and $p_2$, for both cases, ceteris paribus. The black vertical lines reflect the base run values $p_1 = 15$ and $p_2 = 19$. For all other parameter values see table 2.1.

price in S1, or S2, implies a higher seasonal marginal profit, and a stronger trade-off between seasonal fishing. For example, a ceteris paribus increase in $p_1$, increases the marginal profit the leader receives from an additional unit of fish sold in S1, and therefore creates a preference towards harvesting more in S1 relatively to S2.

Figures 2.14 and 2.15 show the effect of ceteris paribus changes in $p_1$ and $p_2$ on steady state harvest and stock biomass for the first- and second-best cases. The reference values are $p_1 = 15$ and $p_2 = 19$, and are represented by the black vertical lines. The respective ranges for $p_1$ and $p_2$ are calculated as a varying ratio, between 0.5 and 1.5, relative to the other season's fixed maximum price. For example, the lower and upper limits for $p_1$ are given by $0.5 \times 19 = 9.5$ and $1.5 \times 19 = 28.5$.

As expected, an increase in $p_1$ ($p_2$), increases the respective seasonal harvest of the leader and decreases its harvest in the subsequent season for both cases, i.e., $h_1$ ($h_2$) increases whereas $h_2$ ($h_1$) decreases. It is interesting, however, that in the presence of followers, the rate of decrease in $h_1$ as $p_2$ increases (blue curve in 2.14.d) is significantly lower than the rate of decrease in $h_2$ as $p_1$ increases (red curve in 2.14.c), whereas the respective $h_2$ and $h_1$ (red and blue curves in 2.14.d and 2.14.c) are increasing at almost




(a) First-best: $p_1/p_2 \in [0.5, 1.5]$

(b) First-best: $p_2/p_1 \in [0.5, 1.5]$

(c) Second-best: $p_1/p_2 \in [0.5, 1.5]$

(d) Second-best: $p_2/p_1 \in [0.5, 1.5]$

Figure 2.15. Long-term stock biomass levels for different values of the maximum price parameters, $p_1$ and $p_2$, for both cases, ceteris paribus. The black vertical lines reflect the base run values $p_1 = 15$ and $p_2 = 19$. For all other parameter values see table 2.1.

the same rate. It can be seen from figures 2.14.a and 2.14.b, that without followers, the rates of increase and decrease in seasonal fishing associated with the increase of seasonal maximum prices are more or less equivalent.

The reason for this difference lies in the steady state size of the stock biomass in S2 (curves with square marks in figure 2.15), which is always higher in the first-best case (vertical comparison). Remember that the mature stock biomass right before the leader harvests in S2 is given by $G(x_2, y_2)$, which is a non-decreasing function. Because in the presence of followers the steady state stock biomass in S2 is lower, $G(x_2, y_2)$ is also lower, and therefore the leader's steady state fishing cost is higher since costs in S2 are inversely proportional to the available mature biomass. Moreover, as the seasonal maximum prices increase this effect becomes more significant and consequently increases the leader's preference to harvest in S1 where its fishing cost is independent of stock biomass. In other words, there is an indirect effect on seasonal marginal profits associated with higher seasonal maximum prices, which becomes more evident in the presence of followers, where the leader faces a higher steady state fishing cost in S2. This explains why, compared to the case without followers, the rate of decrease in $h_1$ is lower when $p_2$

increases, and similarly the rate of decrease in $h_2$ is higher when $p_1$ increases.

The impact of maximum prices on the followers' steady state harvest can be inferred from the direct effect, which occurs only when $p_2$ varies, and the indirect effects on leader's harvest and available stock biomass. In general, the higher $h_2$, the lower the true maximum price the followers face, i.e., $p_2^{-\eta h_2}$, and thus the lower their profit margin, ceteris paribus. Similarly, the higher the available stock biomass before they harvest, i.e., $z_2$, the lower their fishing cost, and thus the higher their profit margin, ceteris paribus.

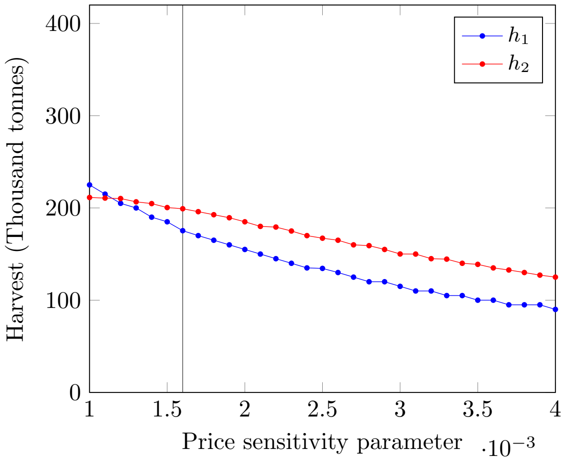
The sensitivity results suggest that as $p_1$ increases, the steady state harvest of the most efficient followers increases, although there is a very small decrease for low $p_1$ values (green and black curves in figure 2.14.c). In addition, the least efficient follower remains inactive for all situations tested. Because $p_2$ remains constant, the change in active followers harvest is caused only by indirect effects. As $p_1$ increases, $h_2$ decreases, and the followers' profit margins increase. At the same time, $z_2$ decreases implying that their margins decrease. For high $p_1$ values the net effect must be positive since the harvest of the active ones increases. That is, the benefits from the reduction in leader's harvest outweigh the losses from the decrease in available biomass. Although this is true for all followers including the least efficient one, the fact that it remains inactive simply implies that its initial marginal profit remains negative despite the positive change.

If $p_1$ is constant, the steady state harvest of the followers increases for low $p_2$ values, but decreases as $p_2$ increases. The decrease coincides with the leader's entry in the fishery. When the leader is inactive, the positive direct effect from higher maximum price in S2 offsets the negative indirect effect from the reduction in available stock biomass, and thus increases the active followers' harvest. Once the leader becomes active, the positive effect from higher prices no longer outweighs the aggregate negative effects from higher $h_2$ and less $z_2$, which eventually makes the second follower inactive.

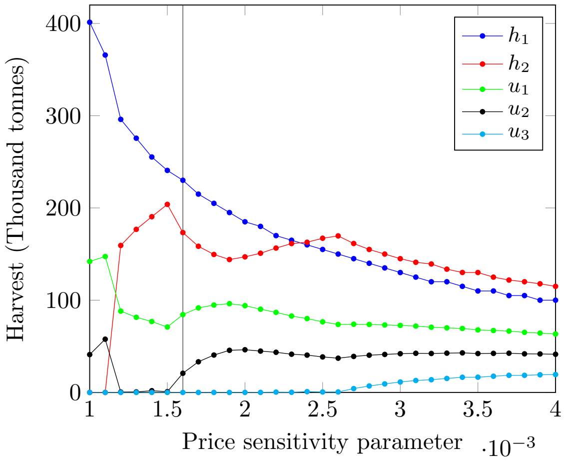
#### 2.4.3.2 Impact of price sensitivity parameter

The price sensitivity parameter, $\eta$, describes the rate at which the market price drops as quantity supplied increases. The higher $\eta$ is, the faster the price will drop, and thus the less profitable will be for all players to supply additional units in the market. In other words, the market is cleared at smaller quantities because marginal profits diminish faster, which leads to less intensive fishing. Although this is observed when the leader is alone, in the presence of followers there seems to be an inverse effect on their fishing strategies, which is counterintuitive.

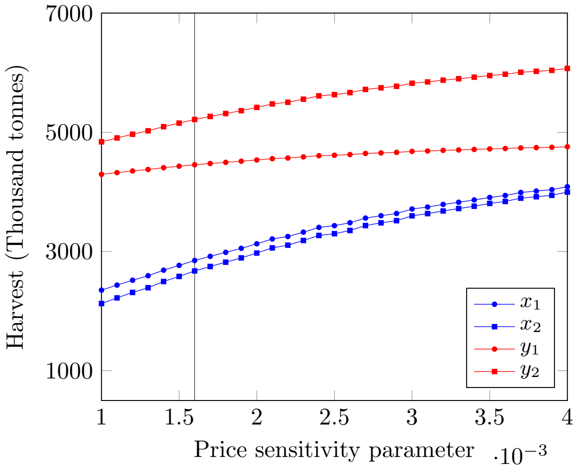
Figure 2.16 shows the effect on steady state harvest and stock biomass for different $\eta$ values, ceteris paribus. The reference value is $\eta = 0.0016$ and the range varies from 0.001 to 0.004. In the absence of followers there is a gradually decline in leader's harvest in both seasons, which is expected since price drops faster as $\eta$ increases making it unprofitable to harvest more in both seasons. A direct consequence of this is the strengthen of stock biomass overall. But what is interesting is the development of all fishing strategies when followers are present. For low $\eta$ values the leader is better off targeting the resource only in S1, which is not true in the first-best scenario. This is because it is no longer the sole participant in the market in S2, and therefore has to consider the trade-off between participating and lowering the price even further, or take everything during S1 where in addition its fishing cost is independent of the biomass size. The latter appears to be true when $\eta$ is small. As $\eta$ increases, this trade-off shifts towards harvesting in both seasons. For values left of the reference point, the second follower becomes inactive in steady state,



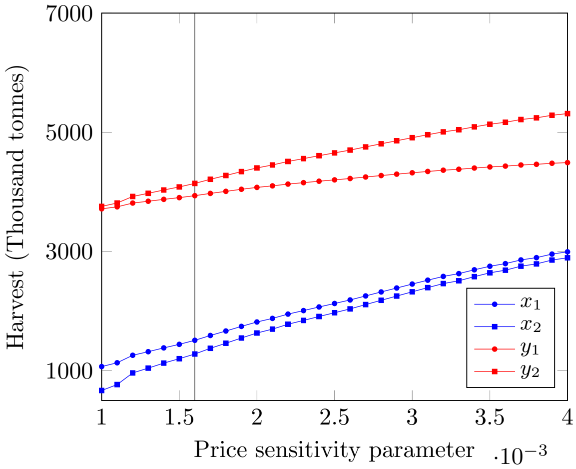

(a) First-best: $\eta \in [1, 4] \times 10^{-3}$




(b) Second-best: $\eta \in [1, 4] \times 10^{-3}$




(c) First-best: $\eta \in [1, 4] \times 10^{-3}$




(d) Second-best: $\eta \in [1, 4] \times 10^{-3}$

Figure 2.16. Long-term harvest levels (a)-(b) and stock biomass (c)-(d) for different values of the price sensitivity parameter, $\eta$, for both cases, ceteris paribus. The black vertical lines reflect the base run value $\eta = 0.0016$. For all other parameter values see table 2.1.

but as $\eta$ keeps increasing it becomes active again. Surprisingly, for high $\eta$ values all three followers are active. This is because the market saturates at a much lower quantity that allows the stock biomass to increase, which reduces fishing costs in S2 and allows the least efficient follower to remain active in steady state.

## 2.5 Conclusion

This paper expands on the recent seasonal model of Ni and Sandal (2019) by introducing strategic interaction between multiple fishing agents. The behaviour of the seasonal fishery modelled here resembles that of straddling species, which undergo seasonal migrations between feeding and spawning grounds, and are targeted in a seasonal basis. Some examples include Atlantic mackerel, Arctic cod, and Norwegian spring-spawning herring. Two seasons are considered with distinct dynamics where biological processes precede human activities. The fishing agents include an incumbent leader and multiple myopic followers. The model is solved and simulated multiple times to illustrate the kind of insights that can be obtained when applied to the management of seasonal fisheries where non-cooperation is the norm. The solution comprises of a set of optimal feedback strategies for the leader, which take as input the state of the stock biomass, and prescribe optimal courses of action, i.e., fishing policies. The induced myopic strategies of all followers can also be inferred, among other things, from the leader's feedback rules.

Compared to the benchmark case where followers are not present, the leader's optimal feedback strategies call for more intense fishing in both seasons. This is attributed to the devaluation of the fishery because of the followers' myopic behaviour, which leads to a race to fish. In the absence of followers, the leader is the sole participant and therefore achieves a balance between exploitation and sustainability. However, when they are present, this balance is distorted because the leader has no longer incentives to preserve a relatively higher stock biomass when knowing that it will be harvested down by the followers. By fishing more aggressively the leader is able to delay entry or even exclude some followers from the fishery. As a consequence, its net present value from the fishery is reduced by almost 50% for most initial biomass states, suggesting that, indeed, rents are dissipated due to multiple competing agents and myopic behaviour. In this context, the presence of followers can be perceived as posing a negative externality for both the leader, who no longer exploits the resource according to its first-best strategy, and the stock biomass, which, compared to the benchmark case, is less in the long-term as shown in the simulations.

The long-term status of the stock is described by an annual equilibrium cycle where the stock is targeted by the leader in both seasons, and, if present, a subset of followers in S2, which is comprised of the most efficient ones. To get an understanding of how long long-term is, the time needed to approach the steady-state equilibrium cycle starting from almost all initial states within the defined state space is computed. For very low biomass levels the recovery of the fish population and its approach towards the equilibrium cycle can take up to 35 and 43 years, with and without followers, respectively. However, for slightly higher biomass levels the time needed drops significantly to 11 and 15 years, respectively. For all initial states tested, it would on average require around 8 years when followers are present and 12 years when not. So, unless the fishery is initially at risk, these are reasonable time horizons for reaching a biomass level, which is both profitable to exploit and at the same time self-sustainable.

From a strategic perspective, the current analysis can be enriched by considering fixed or sunk costs associated with followers entering into the fishery. In the current structure, followers become active from the first NOK they earn, however, fishing trips are often associated with sunk costs, like crew wages, fuel costs, etc. Taking these into account would require followers to enter only if they can at least break even these additional costs. This makes the analysis more realistic, but it does not come free of challenges since ranking followers according to their fishing efficiencies is no longer helpful. For example, an efficient follower with high fixed costs may become inactive whereas an inefficient follower with low fixed costs may become active. This induces multiple feasible equilibria for the followers' Nash game.

# Acknowledgements

The author is grateful to Leif K. Sandal and Stein I. Steinshamn for fruitful discussions that led to this work. Financial support from the Norwegian Research Council through the MESSAGE Project (Grant No. 255530/E40) is gratefully acknowledged.

# Appendix

## A.1 Dynamic programming algorithm

A numerical solution to the dynamic problem (2.5) requires the discretisation of the two-dimensional state space and a recursive approach to solve the system of Bellman equations. The numerical scheme applied here includes a combination of policy and value iterations while taking advantage of interpolation. The algorithm steps are as follows:

1. Define all necessary parameters and functional forms, initialise the grid and create all storing objects. Let $\mathcal{X} = [0, x_{\max}]$ and $\mathcal{Y} = [0, y_{\max}]$, with step $s_1$, be the set admissible values for the state variables $x$ and $y$. Let $V_1^k$, $V_2^k$, $h_1^k$, $h_2^k$, $u_i^k$, and $u^k$ be matrices mapped on the state space $\mathcal{X} \times \mathcal{Y}$, superscript $k$ refers to the $k$-th iteration.

2. At policy iteration $k$, for all $x_i, y_j \in \mathcal{X} \times \mathcal{Y}$, let $\mathcal{H}_1 = [0, x_i]$, with grid step $s_2$,[10] be the set of admissible values for $h_1^k$. Evaluate $\{\delta_1 \Pi_1^L(\mathcal{H}_1) + \delta_1 V_2^k(F_1(x_i, y_j, \mathcal{H}_1))\}$ and store the max in $V_1^k(x_i, y_i)$ and the respective control in $h_1^k(x_i, y_i)$.

3. For all $x_i, y_i \in \mathcal{X} \times \mathcal{Y}$, let $\mathcal{H}_2 = [0, G(x_i, y_i)]$, with grid step $s_2$, be the set of admissible values for $h_2^k$, $\mathcal{Z}_2 = [0, G(x_i, y_i) - \mathcal{H}_2]$ be the set of admissible values for $z_2$, and $\mathcal{U}_i = \tilde{u}_i(\mathcal{Z}_2, \mathcal{H}_2)$ be the set of admissible values for $u_i^k$, with $\mathcal{U} = \sum_i^n \mathcal{U}_i$. Evaluate $\{\delta_2 \Pi_2^L(x_i, y_j, \mathcal{H}_2, \mathcal{U}) + \delta_2 V_2^k(F_2(x_i, y_j, \mathcal{H}_2, \mathcal{U}))\}$ and store the max in $V_2^k(x_i, y_j)$, the leader's policy in $h_2^k(x_i, y_j)$, and the realised follower $i$'s policy in $u_i^k(x_i, y_i)$.

4. Let $W_1^l = V_1^k$ and $W_2^l = V_2^k$, and consider the operators $T_1$ and $T_2$ that evaluate the RHS of the Bellman equations in (2.5) at the $k$-th feedback rules and map them back to themselves, i.e., $T_i W_j^l = W_j^l|_{h_1^k, h_2^k, u^k}$ for $i, j = 1, 2 : i \neq j$. Update the value functions by applying the simultaneous mapping: $W_1^{l+1} = T_1 W_2^l$, $W_2^{l+1} = T_2 W_1^l$. Do so until $||(W_1^{l+1} - W_1^l|| \leq \epsilon_1$ and $||W_2^{l+1} - W_2^l|| \leq \epsilon_1$, where $\epsilon_1$ is a predefined tolerance level.[11] After convergence, store the new value matrices back to $V_1^k$ and $V_2^k$. If $||h_1^k - h_1^{k-1}|| \leq \epsilon_2$ and $||h_2^k - h_2^{k-1}|| \leq \epsilon_2$ stop. Otherwise, set $k = k + 1$ and return to step 2.

Upon convergence, the functions $V_1^k$, $V_2^k$, $h_1^k$, $h_2^k$, and $u_i^k$ are approximations to the respective value function and feedback rules. The quality of the approximation depends on the grid sizes, the tolerance level, and the interpolation scheme applied.

# References

Allee, W. C. and Warder, C. (1931). *Animal aggregations, a study in general sociology*, Chicago: The University of Chicago Press.

Asche, F., Chen, Y. and Smith, M. D. (2015). "Economic incentives to target species and fish size: prices and fine-scale product attributes in Norwegian fisheries", *ICES Journal of Marine Science*, Vol. 72, 733-740.

[10] Notice that the step for the state space grid and the step for the policy feasible grid differ, with the later to be finer ($s_2 < s_1$). A linear interpolation scheme is applied for the evaluation of functionals outside the state space grid points.

[11] To speed things up, especially at the beginning of the iterative process, an exit criterion can be put in place, e.g., exit after L value iterations.

Armstrong, C. W. and Sumaila, U. R. (2001). "Optimal allocation of TAC and the implications of implementing an ITQ management system for the north-east Arctic cod", *Land Economics*, Vol. 77, 350-359.

Arnason, R. (1991). "On the optimal harvesting of migratory species", In Arnason, R. and Bjørndal, T. (Eds.), *Essays on the economics of migratory fish stocks*, Berlin: Springer-Verlag.

Bailey, M., Sumaila, U. R. and Lindroos, M. (2010). "Application of game theory to fisheries over three decases", *Fisheries Research*, Vol. 102, 1-8.

Bellman, R. (1957). *Dynamic Programming*, Princeton University Press, US.

Ben-Hasan, A., Walters, C. and Sumaila, R. (2019). "Effects of Management on the Profitability of Seasonal Fisheries", *Frontiers in Marine Science*, Vol. 6, 1-10.

Bertsekas, D. P. (2001). *Dynamic Programming and Optimal Control*, Athena Scientific, US.

Beverton, R. J. H. and Holt S. J. (1957). *On the Dynamics of Exploited Fish Populations*, Ministry of Agriculture, Fisheries and Food (London) Fisheries Investigations Series, Vol. 2, 19.

Bjørndal, T. and Conrad, J. M. (1987). "The dynamics of an open access fishery", *Canadian Journal of Economics*, Vol. 20, 74-85.

Bjørndal, T. and Munro, G. R. (2012). *The Economics and Management of World Fisheries*, Oxford University Press, Oxford.

Boyce, J. R. (1992). "Individual transferable quotas and production externalities in a fishery", *Natural Resource Modeling*, Vol. 6, 385-408.

Castilho, C. and Srinivasu, P. D. N. (2007). "Bio-economics of a renewable resource in a seasonally varying environment", *Mathematical Biosciences*, Vol. 205, 1-18.

Carson, R. T., Granger, C. W. J., Jackson, J. B. C. and Schlenker, W. (2009). "Fisheries management under cyclical population dynamics", *Environmental and Resource Economics*, Vol. 42, 379-410.

Clark, C. W. (1973). "Profit Maximization and the Extinction of Animal Species", *The Journal of Political Economy*, Vol. 81, 950-961.

Clark, C. W. (1980). "Towards a predictive model for the economic regulation of commercial fisheries", *Canadian Journal of Fisheries and Aquatic Sciences*, Vol. 37, 1111-1129.

Clark, C. W. (2010). *Mathematical Bioeconomics: The Mathematics of Conservation*, John Wiley & Sons, New Jersey.

Clark, C. W. and Munro, G. R. (1975). "The Economics of Fishing and Modern Capital Theory: A Simplified Approach", *Journal of Environmental Economics and Management*, Vol. 2, 92-106.

Costello, C. and Deacon, R. (2007). "The efficiency gains from fully delineating rights in an ITQ fishery", *Marine Resource Economics*, Vol. 22, 347-361.

Ekerhovd, N. A. and Steinshamn, S. I. (2016). "Economic benefits of multi-species management: The pelagic fisheries in the Northeast Atlantic", *Marine Resource Economics*, Vol. 31, 193-210.

Fell, H. (2009). "Ex-vessel pricing and IFQs: A strategic approach", *Marine Resource Economics*, Vol. 24, 311-328.

Hannesson, R. (2011). "Game theory and fisheries", *Annual Review of Resource Economics*, Vol. 3, 181-202.

Hannesson, R. and Steinshamn, S. I. (1991). "How to set catch quotas: constant effort or constant catch?" *Journal of Environmental Economics and Management*, Vol. 70, 71-91.

Hannesson, R., Salvanes, K. G. and Squires, D. (2010). "Technological change and the tragedy of the commons: the Lofoten fishery over 130 years", *Land Economics*, Vol. 86, 746-765.

Hannesson, R. (2013). "Sharing a migrating fish stock", *Marine Resource Economics*, Vol. 28, 1-17.

Hermansen, Ø. and Dreyer, B. (2010). “Challenging spatial and seasonal distribution of fish landings - the experiences from rural community quotas in Norway”, *Marine Policy*, Vol. 34, 567-574.

Kvamsdal, S. F., Maroto, J. M., Morán, M. and Sandal, L. K. (2016). *A Bellman approach to periodic optimization problems*, FOR Working paper. Retrived from `https://openaccess.nhh.no/nhh-xmlui/handle/11250/2423706`

Kvamsdal, S. F., Maroto, J. M., Morán, M. and Sandal, L. K. (2017). “A bridge between continuous and discrete-time bioeconomic models: Seasonality in fisheries”, *Ecological Modelling*, Vol. 364, 124-131.

Kvamsdal, S. F., Maroto, J. M., Morán, M. and Sandal, L. K. (2020). “Bioeconomic modeling of seasonal fisheries”, *European Journal of Operational Research*, Vol. 281, 332-340.

Larkin, S. L. and Sylvia, G. (2004). “Generating enhanced fishery rents by internalizing product quality characteristics”, *Environmental and Resource Economics*, Vol. 28, 101-122.

Liu, X., Lindroos, M. and Sandal, L. (2016). “Sharing a fish stock when distribution and harvest costs are density dependent”, *Inverionmental and Resource Economics*, Vol. 63, 665-686.

Maroto, J. M., Moran, M., Sandal, L. K. and Steinshamn, S. I. (2012). “Potential collapse in fisheries with increasing returns and stock-dependent costs”, *Marine Resource Economics*, Vol. 27, 43-63.

Ni, Y. and Sandal, L. K. (2019). “Seasonality matters: A multi-season, multi-state dynamic optimization in fisheries”, *European Journal of Operational Research*, Vol. 275, 648-658.

Nøstbakken, L. (2006). “Cost Structure and Capacity in Norwegian Pelagic Fisheries”, *Applied Economics*, Vol. 38, 1877-1887.

Önal, H., McCarl, B. A., Griffin, W. L., Matlock, G. and Clark, J. (1991). “A bioeconomic analysis of the Texas shrimp fishery and its optimal management”, *American Journal of Agricultural Economics*, Vol. 73, 1161-1170.

Parma, A. M. (1990). “Optimal harvesting of fish populations with non-stationary stock-recruitment relationships”, *Natural Resource Modeling*, Vol. 4, 39-76.

Pelletier, D., Mahevas, S., Drouineau, H., Vermard, Y., Thebaud, O., Guyader, O. and Poussin, B. (2009). “Evaluation of the bioeconomic sustainability of multi-species multi-fleet fisheries under a wide range of policy options using ISIS-Fish”, *Ecological Modelling*, Vol. 220, 1013-1033.

Sandal, L. K. and Steinshamn, S. I. (2004). “Dynamic Cournot-competitive harvesting of a common pool resource”, *Journal of Economic Dynamics and Control*, Vol. 28, 1781-1799.

Schaefer, M. B. (1957). “Some considerations of population dynamics in economics in relation to the management of marine fisheries”, *Journal of the Fisheries Research Board of Canada*, Vol. 14, 669-681.

Smith, V. L. (1969). “On Models of Commercial Fishing”, *Journal of Political Economy*, Vol. 77, 181-198.

Smith, M. D. (2012). “The new fisheries economics: incentives across many margins”, *Annu. Rev. Resour. Econ.*, Vol. 4, 379-402.

Steinshamn, S. I (2011). “A conceptional analysis of dynamics and production in bioeconomic models”, *American Journal of Agricultural Economics*, Vol. 93, 803-812.

Tahvonen, O. (2010). “Age structured optimization models in fisheries bioeconomics: A survey”, In Boucekkine, R, Hritonenko, N. and Yatsenko, Y. (Eds.), *Optimal control of age-structured populations in economy, demography, and the environment.*, Abingdon: Routledge.

Tahvonen, O., Quaas, M. F., Schmidt, J. O. and Voss, R. (2013). “Optimal harvesting of an age-structured schooling fishery”, *Environmental and Resource Economics*, Vol. 54, 21-39.

Valcu, A. and Weninger, Q. (2013). "Markov-Perfect rent dissipation in rights-based fisheries", *Marine Resource Economics*, Vol. 28, 111-131.

Walters, C. and Parma A. M. (1996). "Fixed exploitation rate strategies for coping with effects of climate change", *Canadian Journal of Fisheries and Aquatic Sciences*, Vol. 53, 148-158.

# Chapter 3

# Keep it in house or sell it abroad? Fishery rent maximisation in a two-market Cournot duopoly

Evangelos Toumasatos,[a,b] Leif Kristoffer Sandal[b] and Stein Ivar Steinshamn[b]

[a]SNF - Centre for Applied Research, Norwegian School of Economics
[b]Department of Business and Management Science, Norwegian School of Economics

## Abstract

The motivation for this research is to understand and quantify on what basis fisheries agreements are drawn up. For example, the European Union (EU) with the so-called sustainable fisheries partnership agreements (SFPAs) gives financial and technical support in exchange for fishing rights in non-EU countries, like Mauritania. For that purpose, a game theoretic model is proposed where a country with some sort of property right over a fishing resource is faced with the following dilemma: freely grant fishing quotas to a domestic firm or sell them to a foreign agent in return for an endogenously determined price. To keep our analysis general, the foreign agent can act either on behalf of a foreign country, union of countries, or a firm. Either way, all purchasing quotas are freely transferred to the foreign firm. Both firms exploit the resource according to their quotas and have the option to sell their harvest in two markets, one at home and one abroad. The game where the total allowable catch (TAC) is exogenous consists of three sequential subgames: a quota pricing subgame, a quota purchasing subgame, and a sales subgame. In a symmetric equilibrium, both firms are active when the foreign agent is welfare-maximiser, but only the home firm is active when the foreign agent is profit-maximiser. Under cost asymmetry, the home country has a pool of mutually exclusive price strategies depending on the market parameters and the firms' fishing costs. For the case of endogenous TAC, an additional subgame is introduced on top of the initial game. This affects the shape of the optimal price areas and increases the home country's flexibility in terms of its pricing options.

*Keywords:* International fisheries agreements; resource rent maximisation; sequential games; Nash equilibrium; Stackelberg equilibrium; two-market Cournot duopoly.

Subject Classification: C72, Q22, Q28, Q38.

## 3.1 Introduction

The 1982 United Nations Convention on the Law of the Sea (UNCLOS) has given coastal states sovereign rights to explore, exploit, conserve and manage natural resources found within 200 nautical-miles of their baselines, i.e., in their exclusive economic zones (EEZ) (United Nations, 1982). This regime change has placed almost 90% of the marine resources worldwide in the control of coastal states and thus has excluded fishing vessels that had traditionally engaged into fishing activities within foreign EEZs (Gorez, 2006). As a consequence, a number of nations have entered into bilateral agreements over access to fishing stocks that occurred beyond their sovereignty.

Today the most known, perhaps, agreements of such type are the so-called sustainable fisheries partnership agreements (SFPAs) between the European Union (EU) and non-EU coastal states, like Mauritania, Maroco, and Guinea-Bissau, among others. SFPAs, which were introduced during the latest common fisheries policy (CFP) reform in 2013, allow EU vessels to fish in the signatory countries' EEZs, and in exchange, the EU provides both financial and sectoral support, including employment opportunities and food security, towards the partner countries. The estimated annual financial contribution in 2014 was around 180 million Euro, of which 30 million Euro went to the development and governance of the partner states fisheries sectors (European Commission, 2017).

The SFPAs were not the EU's first attempt to access fishery resources in other countries. Since the adoption of UNCLOS, the EU has concluded more than 30 bilateral agreements mainly with developing nations in Africa, the Caribbean and the Pacific (ACP countries). Historically, the evolution of international fisheries agreements conducted by the EU has been categorised from "pay, fish and go" agreements to fisheries partnership agreements (FPAs) introduced in 2003, and from FPAs to SFPAs introduced in 2014. For a comprehensive examination of the different types of agreements and relevant legal information see Heredia and Oanta (2015).

Prior to FPAs, the EU fishing agreements with developing countries have extensively been criticized on the grounds that they were unfair towards the partner states (Nagel and Gray, 2012). Kaczynski and Fluharty (2002) argued that these early agreements secured employment for the EU fleet and processing industry and a steady stream of supply to the EU market while overfishing the partner countries' fisheries resources. Criticism has continued even after the introduction of FPAs. Cullberg and Lövin (2009) stated that although these new partnerships sounded good in theory, sustainable exploitation of the partner countries' fisheries and enhancement of the their fisheries sectors were questionable in practice. Despite the EU's efforts to promote sustainable and responsible fisheries partnerships internationally with SFPAs, Okafor-Yarwood and Belhabib (2019) advocate that subsidies towards third countries under the framework of SFPAs contradict the provisions of the CFP, as the exploitation of fully exploited or overexploited species in the partner regions continues.

The purpose of this study is not to criticize the nature of international fisheries agreements or debate the EU code of conduct with regard to them, but rather to introduce a framework from which to better understand and quantify the basis upon which these agreements are being drawn up. In this spirit, we propose a model of strategic interaction that captures, among other things, how much it is worth for a coastal state to give access to its fishery resources to foreign fleets. To the best of our knowledge, this paper is the first to provide a tool for evaluating the outcome of international fisheries agreements.

Our game theoretic model consists of four players. A coastal state with some sort of property right over a fishing resource, hereinafter the home country or resource owner. A foreign agent who wants to gain access to the home country's resource. And two firms: a home one and a foreign one, which fish the resource on behalf of the home country and the foreign agent, respectively. In the context of SFPAs, the foreign agent is the EU, the home country is the signatory partner, and the firms represent the fleets of the respective parties harvesting the resource.

The home country is faced with the following dilemma: freely grant fishing quotas to the domestic firm or sell them to the foreign agent in return for an endogenously determined price. In the context of SFPAs, the quota price is the equivalent of the financial cost the EU has to bear in order for its fleet to access and fish in a coastal state's EEZ. To keep our analysis general and also applicable to other situations, e.g., when firms enter into bilateral agreements with governments, we assume that the foreign agent can act either on behalf of a foreign country or a firm. The difference will lie in the objective function of the foreign agent as we shall see in the next section. In the case of the EU the foreign agent is a country, which is a common assumption in the literature and stems from the fact that all of its members abide by the CFP.

Irrespective of the foreign agent's incentives, all purchasing quotas are freely transferred to the foreign firm. The remaining total allowable catch (TAC) is then freely given to the domestic firm. In our model both firms are assumed to harvest according to their quotas despite the fact that this may imply suboptimal or even negative profits, i.e., they may earn more by not fishing their entire quota. However, for the firms to fish accordingly will always be optimal for the home country and the foreign agent. In case of negative profits, the firms can be compensated internally through redistribution of the benefits.

Although firms are forced to harvest as instructed, they are free to choose in which market to sell. In this setting we consider two markets, one at home and one abroad, where consumers willingness to pay differ between them. In addition, home and foreign inverse demand functions are endogenously determined and depend on aggregate home and foreign supply, which is defined as the sum of fish sold in each market by both firms. If both firms are active, then they compete à la Cournot. If only one firm is active, then it enjoys a monopoly position. Being active in this context depends on whether a firm receives a positive amount of quotas or not. Firms are asymmetric with respect to fishing costs and fixed costs are assumed to be zero. Moreover, they do not incur any transport or transaction cost in supplying either market.

To focus on the strategic interaction between the players, we start by disregarding the problem of optimal fishing, which has been analysed in detail within the literature (Clark, 1973; Clark and Munro, 1975; Hannesson, 1983; Sandal and Steinshamn, 1997), and assume that for any fixed period of time, e.g., a year, the TAC is exogenous. This means that the problems of how much to fish and who should fish can now be dealt and analysed separately. Once all possible strategic outcomes are identified, it is possible to determine the optimal fishing policy by optimising over them. This is illustrated at the end of the paper by allowing the TAC to be endogenous for the optimal sustainable exploitation level of a single-species fishery.

The remaining of the paper is organised as follows. The basic model and the objectives of all players are introduced in the next section. In sections 3, 4 and 5, we derive analytical solutions for each subgame when the TAC is exogenous and provide some numerical insights. The case of endogenous TAC is described in section 6. Finally, section 7 summarises our findings and concludes the paper.

## 3.2 The basic model

### 3.2.1 Preliminaries

Let $U$ denote the TAC and $U_i$ firm $i$'s, $i = 1, 2$, individual quotas such that $U = U_1 + U_2$. Moreover, let $Q_i$ be the quantity firm $i$ sells on its own market and $U_i - Q_i$ the quantity it sells on the other market. Aggregate supply in market $i$ is then given by $S_i = Q_i + U_j - Q_j$ for all $i, j = 1, 2 : i \neq j$. Hereinafter and unless otherwise stated, $i = 1$ will be associated with the home entities and $i = 2$ with the foreign entities. Inverse demand functions, i.e., market prices, are given by $P_i(S_i)$. They depend on total supply in each market and are downward slopping. To keep the analysis tractable and derive analytical solutions we assume the following linear specification:

$$P_i(S_i) = a_i\left(1 - \frac{S_i}{b_i}\right), \quad \forall i = 1, 2,$$

where $a_i > 0$ is the maximum price or willingness to pay parameter in market $i$, and $b_i > 0$ is the maximum quantity market $i$ can absorb before the price becomes negative also referred to as the market saturation quantity. In addition, both markets together are able to at least absorb the entire quota, i.e., $b_1 + b_2 \geq U$.

Firm $i$'s total variable cost is denoted by $C_i(U_i)$, is increasing in production, and defined as:

$$C_i(U_i) = c_i U_i, \quad \forall i = 1, 2,$$

where $c_i > 0$ is the unit cost of fishing for firm $i$. It has been a common practice in the literature to assume that the unit cost of production associated with fishing activity is stock dependent and decreasing, i.e., $c_i(x)$ with $c_i'(x) < 0$ where $x$ denotes the stock or state variable (Clark and Munro, 1975). This implies that the fishery is economically protected because production costs escalate at low biomass levels, which makes it unprofitable to operate (Maroto et al., 2012). In our model parameter $c_i$ is seemingly constant and this emanates from the fact that the TAC is exogenous. If the TAC is to be endogenous, then it will typically be given by a feedback rule of the form $U(x)$ meaning that the optimal policy will depend on the state of the stock biomass. In this paper, what we really mean when we assume that the TAC is exogenous is that the optimal TAC is exogenous and therefore both $U$ and $x$ are fixed. Thus, when the TAC is exogenous, the cost parameter is constant, which is not to be confused with the case of schooling fisheries where fishing costs are constant because they are stock independent.

### 3.2.2 Players' objectives

**Firms** Total profit for firm $i$ is defined as the sum of sales revenue in the two markets minus the cost of production as follows:

$$\Pi_i = P_i(S_i)Q_i + P_j(S_j)(U_i - Q_i) - C_i(U_i), \quad \forall i, j = 1, 2 : i \neq j.$$

However, the cost of production is sunk and does not affect the firms' selling decisions. This is because when firms choose in which markets to sell, individual quotas are already decided. Thus, firms determine optimal sales by maximising the total revenue, $R_i$. Firm's $i$ maximisation problem is defined as follows:

$$\max_{Q_i \in [0, U_i]} \quad R_i = P_i(S_i)Q_i + P_j(S_j)(U_i - Q_i), \quad \forall i, j = 1, 2 : i \neq j. \tag{3.1}$$

A corner solution, i.e., $Q_i = 0$ or $Q_i = U_i$, implies that firm $i$ sells everything in the foreign or domestic market, whereas an inner one means that firm $i$ serves both markets.

**Home country** The home country decides on the price of quotas, $p$, by maximising total welfare defined as the sum of consumer and producer surplus and the revenue from selling fishing quotas. The consumer surplus for country $i$ is defined as the difference between the gross benefit of consumers, i.e., the area under the inverse demand curve, and the value of the total quantity sold in the market as follows:

$$CS_i = \int_0^{S_i} P_i(v)dv - P_i(S_i)S_i, \quad \forall i = 1, 2.$$

Since no fixed costs exist, producer surplus for country $i$ is equivalent to the firm's profit, that is $PS_i = \Pi_i$ for all $i = 1, 2$. In the case where the TAC is exogenous, the home country's maximisation problem can be expressed as follows:

$$\max_{p \in \mathbb{R}} \quad V_1 = CS_1 + PS_1 + pU_2. \tag{3.2}$$

Details regarding the maximisation problem of the home country for the case of endogenous TAC are provided in section 3.6.

**Foreign agent** As mentioned in the introduction, the foreign agent can act either on behalf of a foreign country or a firm. The difference lies in the objective functions and particular in the consumer surplus, which is not considered if the foreign agent represents a firm. Let $\xi$ be a binary variable that takes the value of zero if the foreign agent represents a firm and one if not. The foreign agent decides how many quotas to buy by solving the following optimisation problem:

$$\max_{U_2 \in [0, U]} \quad V_2 = \xi CS_2 + PS_2 - pU_2. \tag{3.3}$$

### 3.2.3 Sequence of events

The game with exogenous TAC consists of three subgames the timing and information structure of each are as follows. First, given the TAC, the home country decides on the price of quotas by solving (3.2) while taking into consideration the responses of all other players in the game, namely, how the foreign agent is going to react to the quota price, and how the firms are going to react to their individual quotas. This is a Stackelberg game in the price of quota against the foreign agent and in the quota shares against the firms. Next, given the quota price and knowing that any unpurchased quotas will be granted to the home firm, the foreign agent chooses to buy quotas by solving (3.3) with the full knowledge of how this will influence the firms' selling choices in the subsequent stage. This is a Stackelberg game in the quota shares against the firms. Finally, given the individual quotas, the firms harvest them and simultaneously choose how much to supply in each market by solving (3.1). This is a Nash game in selling quantities between the firms. In the case of endogenous TAC, the game is supplemented by an additional subgame at the very top where the home country decides on the optimal TAC while knowing how this will affect the quota price, the foreign agent's purchasing quantities, and the firms' selling decisions. Figure 3.1 provides a schematic representation of the different decisions when the TAC is both exogenous and endogenous.

### 3.2.4 Rescaling

To make the analysis more convenient, we rescale the objective functions by dividing them with the TAC and one of the maximum prices. Changing the scale changes the interpretation of the parameters and the decision variables as follows.

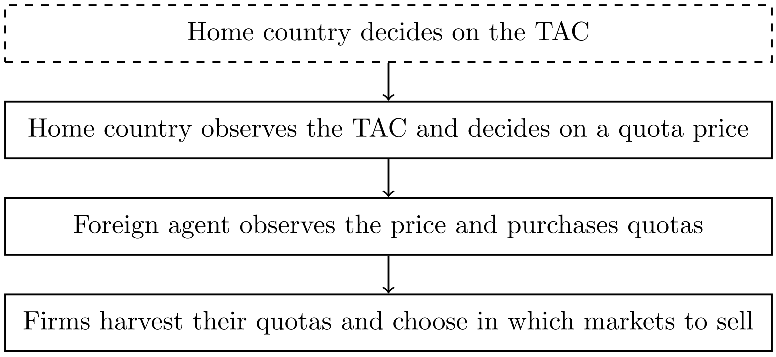



Figure 3.1. The sequential process of decisions. The dashed node occurs only when the TAC is endogenous.

Dividing with $U$ normalises total production to one and expresses individual production strategies as percentages or shares of total production, i.e., $u_i = U_i/U$. This also affects the selling quantities. The scaled sales variables are $q_i = Q_i/U$ and express firm $i$'s domestic sales as a ratio of the total production. To reflect on these changes, the market saturation parameters are adjusted as follows: $\beta_i = b_i/U$ with $\beta_1 + \beta_2 \geq 1$.

Dividing with $a_i$ reduces the number of parameters by one, namely, the maximum price that we divide. Here we choose to divide with $a_2$ because we are going to restrict our attention to the cases where $a_2 > a_1$ and thus it is convenient to bound their ratio in the zero-one interval. The following scaled parameters and variables are introduced. The relative maximum price: $\alpha = a_1/a_2$, with $\alpha \in (0, 1)$ when the maximum price in the foreign market exceeds the one in the home market and $\alpha = 1$ when both maximum prices are equal. The scaled cost parameters $\psi_i = c_i/a_2$, and the scaled quota price variable $\rho = p/a_2$.

All results in the paper can be traced back to the initial parameters and units using the above conversions. A detailed list of the most frequently used symbols is provided in table 3.1.

## 3.3 Firms' sales subgame

Turning first to the firms' sales subgame, we wish to determine the equilibrium behaviour of the firms while knowing the outcome in the previous subgame, i.e., the TAC shares of the firms. Let $\bar{R}_i = R_i/(a_2 U)$ be the scaled objective function,[1] and $u_2 \in [0, 1]$ with $u_1 = 1 - u_2$ the individual quota shares. The firms choose which markets to serve by simultaneously solving the following maximisation problems:[2]

$$\max_{q_1 \in [0, u_1]} \quad \bar{R}_1 = \alpha\left(1 - \frac{q_1 + u_2 - q_2}{\beta_1}\right) q_1 + \left(1 - \frac{q_2 + u_1 - q_1}{\beta_2}\right)(u_1 - q_1), \tag{3.4}$$

$$\max_{q_2 \in [0, u_2]} \quad \bar{R}_2 = \left(1 - \frac{q_2 + u_1 - q_1}{\beta_2}\right) q_2 + \alpha\left(1 - \frac{q_1 + u_2 - q_2}{\beta_1}\right)(u_2 - q_2). \tag{3.5}$$

In order to be more general, we disregard the resource constraint, i.e., $u_1 + u_2 = 1$, and solve the above problems for all possible combinations of $u_1 \times u_2 \in [0, 1]^2$. The resource

[1] Hereinafter all functions with the bar icon refer to their corresponding scaled forms.

[2] The cases where the foreign agent buys everything or nothing are degenerate cases of the problem analysed in this section. In that case, no strategic interaction between firms exists and the sales can be determined by straightforward optimisation of the active firm's objective.

Table 3.1. List of symbols used in the study.

| Symbol | Description | Relation/Value |
|---|---|---|
| **Subscripts** | | |
| $i, j$ | Market and firm indices | $1, 2$ |
| $k, l$ | The markets firm $i$ serves, both or primary (paired only) | b,p |
| **Superscripts** | | |
| $k$ | Firms' distinct equilibrium action index | 1,2,3 |
| $l$ | Foreign agent's distinct equilibrium index | $1, \ldots, 5$ |
| $m$ | Home country's distinct equilibrium index | $1, \ldots, 12$ |
| **Functions** | | |
| $R_i, \bar{R}_i$ | Objective of firm $i$, level and scaled | $\bar{R}_i = R_i/a_2$ |
| $V_2, \bar{V}_2$ | Objective of the foreign agent, level and scaled | $\bar{V}_2 = V_2/a_2$ |
| $V_1, \bar{V}_1$ | Objective of the home country, level and scaled | $\bar{V}_1 = V_1/a_2$ |
| **Variables** | | |
| $U$ | Total allowable catch (TAC) | $U = U_1 + U_2$ |
| $U_i, u_i$ | Individual quota and share of firm $i$ | $u_i = U_i/U$ |
| $Q_i, q_i$ | Quantity firm $i$ sells in its own market, level and scaled | $q_i = Q_i/U$ |
| $p, \rho$ | Price of quotas, level and scaled | $\rho = p/a_2$ |
| **Parameters** | | |
| $a_i, \alpha$ | Maximum price in market $i$ and relative price | $\alpha = a_1/a_2$ |
| $b_i, \beta_i$ | Maximum quantity market $i$ absorb, level and scaled | $\beta_i = b_i/U$ |
| $c_i, \psi_i$ | Unit cost of firm $i$, level and scaled | $\psi_i = c_i/a_2$ |
| $\xi$ | Type of the foreign agent | $0, 1$ |
| $\phi_i$ | Market efficiency parameter of market $i$ | cf. Eq. (3.7) |
| **Equilibria** | | |
| ${}_i\mathbf{q}_{kl}$, $\mathbf{q}_i^k(u_2)$, $\mathbf{q}^*(u_2)$ | Firms' equilibrium sales vector[a] | |
| $u_2^l(\rho)$, $u_2^*(\rho)$ | Foreign agent's optimal quota strategy[b] | |
| $\rho^m$, $\rho^*$ | Home country's optimal pricing policy when $U$ is exogenous[b] | |
| **Thresholds**[c] | | |
| $p_i, \rho_i$ | Price, level and scaled, $i = 1, \ldots, 9$ | |
| $\Lambda_i, \lambda_i$ | Cost difference when $\xi = 1$, level and scaled, $i = 1, \ldots, 12$ | |
| $M_i, \mu_i$ | Cost difference when $\xi = 0$, level and scaled, $i = 1, \ldots, 7$ | |

[a] Subscript $i$ refers to the primary market. The left notation describes a distinct equilibrium pair. The mid notation groups the equilibria according to market parameters. The right notation groups all equilibria together.
[b] The left notation groups the equilibria according to market parameters. The right notation groups all equilibria together.
[c] The thresholds are scaled lump sum parameters associated with the equilibrium solutions of the foreign agent's quota and the home country's price subgames. In particular, they represent the conditions that make players indifferent between a pair of strategies.

constraint is incorporated at the end of our analysis in order to categorise the equilibria. Both objectives are continuous, concave and bounded in the $[0, u_i]$ interval. Their solution is a continuous best response function given by

$$B_i(q_j) = \min(\max(0, z_i(q_j)), u_i) = \begin{cases} u_i, & q_j \geq \bar{q}_j. \\ z_i(q_j), & \underline{q}_j \leq q_j \leq \bar{q}_j, \\ 0, & q_j \leq \underline{q}_j, \end{cases} \quad \forall i, j = 1, 2 : i \neq j, \tag{3.6}$$

where $z_i(q_j)$ is firm $i$'s inner solution as a function of firm $j$'s domestic sales. The bounds $\underline{q}_j$ and $\bar{q}_j$ are the thresholds that bound the inner solution within the feasible region, and are given by $z_i(q_j) = 0$ and $z_i(q_j) = u_i$ respectively. To keep the main text clear and precise, all explicit formulas are presented in the respective appendices, see appendix A for this section's explicit formulas.

A pair of strategies $\mathbf{q}^* \equiv (q_1^*, q_2^*) \in [0, u_1] \times [0, u_2]$ constitutes a Nash equilibrium of

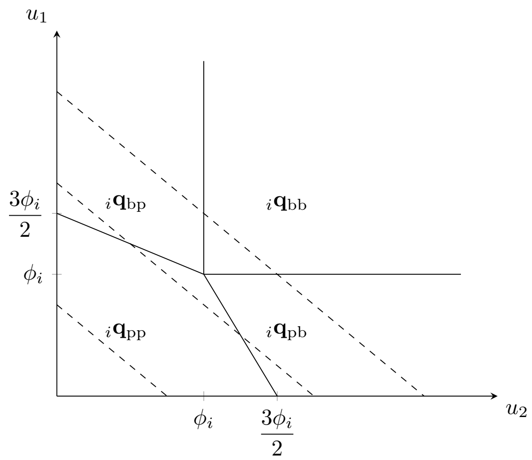



Figure 3.2. Firms' sales subgame distinct equilibria. Parameter $\phi_i$ describes the speed of market clearing. The dashed lines represent possible positions of the quota constraint. At $\phi_i = 0$, both markets are equally preferred and the sole equilibrium is ${}_i\mathbf{q}_{\text{bb}}$.

the firms optimisation problems (3.4) and (3.5) if and only if

$$\bar{R}_i(q_i^*, q_j^*) \geq \bar{R}_i(q_i, q_j^*), \quad \forall q_i \in [0, u_i], \quad \forall i, j = 1, 2 : i \neq j.$$

Let ${}_i\mathbf{q}_{kl} = (q_1^*, q_2^*)$ be the distinct equilibrium strategy vector. The left subscript, $i = 1, 2$, refers to the primary or preferred market, i.e., the one with the greater maximum price. The right subscripts indicate which market(s) firms one and two respectively serve, $k, l = \{\text{b}, \text{p}\}$, where 'b' stands for both and 'p' for primary. For example, ${}_1\mathbf{q}_{\text{bp}}$ reads as follows: at equilibrium the home firm serves both markets whereas the foreign firm serves only market one, which is the primary one.

For any given initial combination of $(u_1, u_2, \alpha, \beta_1, \beta_2)$, there exists a unique Nash equilibrium. All Nash equilibria are specified in Tables 3.2 and 3.3. Figure 3.2 graphically illustrates the regions for the different equilibria on the $u_1 \times u_2$ parameter space. The dashed lines depict possible positions of the quota constraint, $u_1 + u_2 = 1$, which, depending on the market conditions, is positioned distinctively within the regions giving rise to three unique outcomes.[3] The equilibrium regions depend on the market parameters, and the firms individual quota shares. The market parameters are lumped and described through the single parameter $\phi_i$ given by

$$\phi_i = \frac{\beta_i}{3}\left(1 - \frac{a_j}{a_i}\right), \quad \forall i, j = 1, 2 : i \neq j, \tag{3.7}$$

which describes the circumstances under which the two markets are served. Parameter $\phi_i$ is strictly positive when market $i = 1, 2$ is the preferred one, i.e., $a_i = \max(a_1, a_2)$, zero when both markets are equally preferred, and negative otherwise.

The quantity $\phi_i$ we shall refer to as the speed of market clearing or the market efficiency parameter. The idea behind is that for any sales allocation of the other firm, a firm distributes its product to the markets based on the decision rule that its marginal revenue across them is equal. Since both firms act simultaneously, both markets are cleared when all marginal revenues are equal. The magnitude of $\phi_i$ determines whether the primary market can clear the entire production by itself or not. The higher $\phi_i$ is, the larger the quantity sold in the primary market by both firms. The market efficiency

[3] Market parameters $\alpha$, $\beta_1$ and $\beta_2$ affect the size but not the shape of the equilibrium regions.

parameter is increasing in $\beta_i$ and $a_i$, and decreasing in $a_j$. The following proposition provides a complete characterisation of the firms' sales subgame equilibria.

**Proposition 1.** Let $\mathbf{q}^*(u_2)$ denote the equilibrium solution for the firms' sales subgame, which is a function of the foreign firm's share. The solution consists of three mutually exclusive equilibria that depend on the preferred market's efficiency parameter:

$$\mathbf{q}^*(u_2) = (q_1^*, q_2^*) = \begin{cases} \mathbf{q}_{i,3}(u_2), & \phi_i \geq 2/3, \\ \mathbf{q}_{i,2}(u_2), & 1/2 \leq \phi_i \leq 2/3, \\ \mathbf{q}_{i,1}(u_2), & 0 < \phi_i \leq 1/2, \end{cases} \quad \forall i = 1, 2 : a_i = \max(a_1, a_2),\ \forall u_2 \in [0, 1],$$

where $\mathbf{q}_{i,k}$ represents the exclusive equilibrium $k = 1, 2, 3$ and is specified as follows:

$$\mathbf{q}_i^1(u_2) = \begin{cases} {}_i\mathbf{q}_{\text{pb}}, & 1 - \phi_i \leq u_2 \leq 1, \\ {}_i\mathbf{q}_{\text{bb}}, & \phi_i \leq u_2 \leq 1 - \phi_i, \\ {}_i\mathbf{q}_{\text{bp}}, & 0 \leq u_2 \leq \phi_i, \end{cases} \quad \mathbf{q}_i^2(u_2) = \begin{cases} {}_i\mathbf{q}_{\text{pb}}, & 3\phi_i - 1 \leq u_2 \leq 1, \\ {}_i\mathbf{q}_{\text{pp}}, & 2 - 3\phi_i \leq u_2 \leq 3\phi_i - 1, \\ {}_i\mathbf{q}_{\text{bp}}, & 0 \leq u_2 \leq 2 - 3\phi_i, \end{cases}$$

$$\mathbf{q}_i^3(u_2) = {}_i\mathbf{q}_{\text{pp}}, \quad \forall i = 1, 2 : a_i = \max(a_1, a_2), \quad \forall u_2 \in [0, 1].$$

At $\phi_i = 0$, both markets are equally preferred and the unique equilibrium follows from ${}_i\mathbf{q}_{\text{bb}}$ and is given by $(q_1^*, q_2^*) = ((1 - u_2)\beta_1/(\beta_1 + \beta_2), u_2\beta_2/(\beta_1 + \beta_2))$ for all $u_2 \in [0, 1]$. See appendix A.1 for the proof.

As long as $\phi_i$ is greater or equal to two-thirds, the primary market alone clears the aggregate supply and the inferior market is not served. This implies that the market-clearing price in the preferred market is greater or equal to the maximum price in the other market, i.e., $P_i(U) \geq a_j$. The preferred market also clears the entire production when $\phi_i$ is greater or equal to one-half and the foreign firm's share is between $2 - 3\phi_i$ and $3\phi_i - 1$. In all other cases, both markets are required in order for the total output to be cleared. In other words, the primary market is served at all equilibria by both firms when active. The other market is served: a) by the firm with the higher quota share, as long as it is sufficiently high, and b) by both firms when the market efficiency parameter is between zero and one-half, and the foreign firm's share is between $\phi_i$ and $1 - \phi_i$.

This concludes the firms' sales subgame analysis. In what follows, we restrict attention to the cases where the foreign market is preferred, i.e., $a_2 > a_1$, which implies that $\alpha \in (0, 1)$. The reason is twofold. First and foremost, it is the most interesting case to analyse from a strategic point of view, i.e., a resource abundant nation responsible for the management of its resource prefers to sell the product in the foreign market. This diminishes somehow the advantage the home country has from being the leader in the subsequent Stackelberg game. Second, it shortens the length of the paper.

## 3.4 Foreign agent's quota subgame

After observing the quota selling price, the foreign agent buys quotas with the full knowledge of how it influences the outcome in the subsequent subgame. In addition, the foreign agent is aware that any unpurchased quotas will be granted to and harvested by the home firm, i.e, $u_1 = 1 - u_2$. Let $\bar{V}_2 = V_2/(a_2U)$ be the scaled objective function, the foreign

Table 3.2. Firms' sales subgame equilibria: market one is the primary market, i.e., $\alpha \geq 1$, and $u_1 = 1 - u_2$.

| Symbol | Parameter conditions | Firm's 1 strategy ($q_1^*$) | Firm's 2 strategy ($q_2^*$) |
|---|---|---|---|
| ${}_1\mathbf{q}_{\text{bb}}$ | $\phi_1 \in [0, 1/2]$ and $u_2 \in [\phi_1, 1 - \phi_1]$ | $\frac{\beta_1(3u_1 - (1-\alpha)\beta_2)}{3(\alpha\beta_2 + \beta_1)}$ | $\frac{\beta_2(3\alpha u_2 + (1-\alpha)\beta_1)}{3(\alpha\beta_2 + \beta_1)}$ |
| ${}_1\mathbf{q}_{\text{pp}}$ | $\phi_1 \in [1/2, 2/3]$ and $u_2 \in [2 - 3\phi_1, 3\phi_1 - 1]$<br>$\phi_1 \geq 2/3$ and $u_2 \in [0, 1]$ | $u_1$ | 0 |
| ${}_1\mathbf{q}_{\text{pb}}$ | $\phi_1 \in [0, 1/2]$ and $u_2 \in [1 - \phi_1, 1]$<br>$\phi_1 \in [1/2, 2/3]$ and $u_2 \in [3\phi_1 - 1, 1]$ | $u_1$ | $\frac{\alpha\beta_2(u_1 + 2u_2) - (\alpha - 1)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}$ |
| ${}_1\mathbf{q}_{\text{bp}}$ | $\phi_1 \in [0, 1/2]$ and $u_2 \in [0, \phi_1]$<br>$\phi_1 \in [1/2, 2/3]$ and $u_2 \in [0, 2 - 3\phi_1]$ | $\frac{2\beta_1 u_1 - \alpha\beta_2 u_2 + (\alpha - 1)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}$ | 0 |

Table 3.3. Firms' sales subgame equilibria: market two is the primary market, i.e., $\alpha \in (0, 1]$, and $u_1 = 1 - u_2$.

| Symbol | Parameter conditions | Firm's 1 strategy ($q_1^*$) | Firm's 2 strategy ($q_2^*$) |
|---|---|---|---|
| ${}_2\mathbf{q}_{\text{bb}}$ | $\phi_2 \in [0, 1/2]$ and $u_2 \in [\phi_2, 1 - \phi_2]$ | $\frac{\beta_1(3u_1 - (1-\alpha)\beta_2)}{3(\alpha\beta_2 + \beta_1)}$ | $\frac{\beta_2(3\alpha u_2 + (1-\alpha)\beta_1)}{3(\alpha\beta_2 + \beta_1)}$ |
| ${}_2\mathbf{q}_{\text{pp}}$ | $\phi_2 \in [1/2, 2/3]$ and $u_2 \in [2 - 3\phi_2, 3\phi_2 - 1]$<br>$\phi_2 \geq 2/3$ and $u_2 \in [0, 1]$ | 0 | $u_2$ |
| ${}_2\mathbf{q}_{\text{pb}}$ | $\phi_2 \in [0, 1/2]$ and $u_2 \in [1 - \phi_2, 1]$<br>$\phi_2 \in [1/2, 2/3]$ and $u_2 \in [3\phi_2 - 1, 1]$ | 0 | $\frac{2\alpha\beta_2 u_2 - \beta_1 u_1 + (1-\alpha)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}$ |
| ${}_2\mathbf{q}_{\text{bp}}$ | $\phi_2 \in [0, 1/2]$ and $u_2 \in [0, \phi_2]$<br>$\phi_2 \in [1/2, 2/3]$ and $u_2 \in [0, 2 - 3\phi_2]$ | $\frac{\beta_1(2u_1 + u_2) - (1-\alpha)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}$ | $u_2$ |

The subscripts right of **q** indicate which market(s) firms one and two respectively serve, where 'b' stands for both and 'p' for primary.

agent chooses how many quotas to buy by solving the following maximisation problem:

$$\begin{aligned} \max_{u_2\in[0,1]} \quad \bar{V}_2(u_2,\mathbf{q}^*(u_2)) &= \begin{cases} \bar{V}_2(u_2,\mathbf{q}_2^3(u_2)), & \phi_2 \geq 2/3, \\ \bar{V}_2(u_2,\mathbf{q}_2^2(u_2)), & 1/2 \leq \phi_2 \leq 2/3, \\ \bar{V}_2(u_2,\mathbf{q}_2^1(u_2)), & 0 \leq \phi_2 \leq 1/2, \end{cases} \\ \text{where} \quad \bar{V}_2(u_2,\mathbf{q}_2^k(u_2)) &= \xi\frac{(q_2^*+1-u_2-q_1^*)^2}{2\beta_2} + \left(1-\frac{q_2^*+1-u_2-q_1^*}{\beta_2}\right)q_2^* \\ &+ \alpha\left(1-\frac{q_1^*+u_2-q_2^*}{\beta_1}\right)(u_2-q_2^*) - (\psi_2+\rho)u_2, \quad \forall k=1,2,3. \end{aligned} \tag{3.8}$$

The subscript 2 in $\mathbf{q}_2^k$ refers to the equilibrium strategy when the foreign market is primary and will be dropped for the remainder of the paper since it remains fixed, i.e., $\mathbf{q}^k \equiv \mathbf{q}_2^k$. The superscript $k$ refers to the firms' sales subgame distinct equilibrium solution. The $\xi$ parameter reflects the type of the foreign agent, i.e., profit- or welfare-maximiser, depending on whether its value is zero or one, respectively. The difference lies in the consumer surplus, which is strictly positive when the foreign market is the preferred one, and non-negative otherwise.

The strategies $(u_2^*,\mathbf{q}^*(u_2^*)) \in [0,1]\times[0,u_1]\times[0,u_2]$ constitute a Stackelberg equilibrium of the foreign agent's quota subgame if and only if

$$\begin{aligned} &\bar{V}_2\big(u_2^*,\mathbf{q}^*(u_2^*)\big) \geq \bar{V}_2\big(u_2,\mathbf{q}^*(u_2)\big), && \forall u_2\in[0,1], \\ &\bar{R}_i\big(u_2^*,q_i^*(u_2^*),q_j^*(u_2^*)\big) \geq \bar{R}_i\big(u_2^*,q_i(u_2^*),q_j^*(u_2^*)\big), && \forall q_i\in[0,u_i], \quad \forall i,j=1,2: i\neq j. \end{aligned}$$

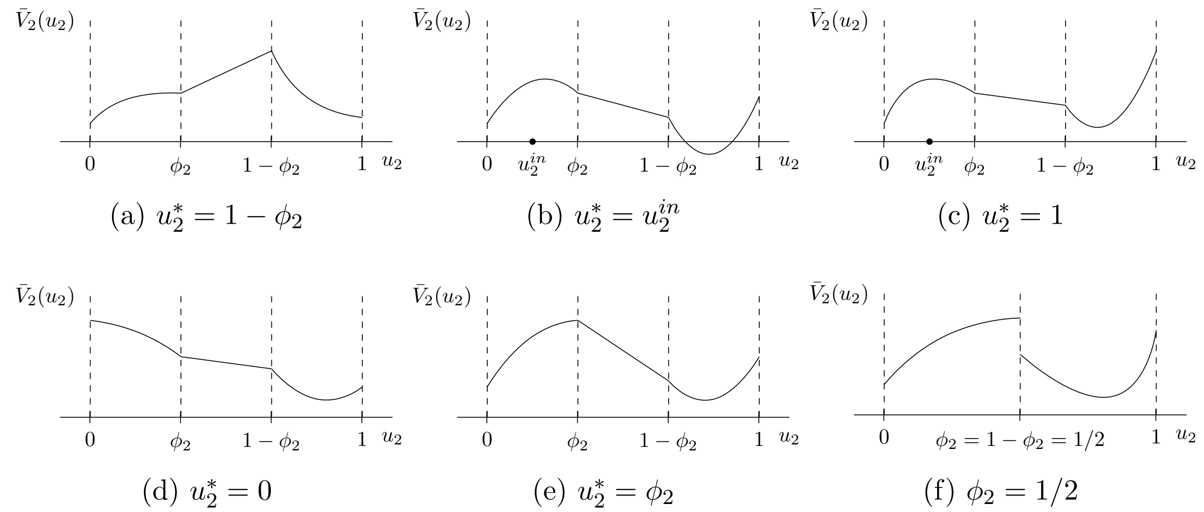
For a given set of parameter values, solving problem (3.8) through enumeration is straightforward. However, our objective here is to provide a complete categorisation of the different equilibria. Therefore, we zoom into each branch of the foreign agent's piecewise objective and derive the respective Stackelberg equilibria together with the necessary conditions for their existence. In addition, we show that all equilibria are mutually exclusive, i.e., unique given that the conditions of existence are satisfied. Based on the magnitude of the market efficiency parameter, we distinguish between three cases: low, medium and high efficiency.

Because the solution procedure between cases is repetitive, we describe it in detail only for the first case, i.e., when market efficiency is low, $\phi_2 \leq 1/2$. For the remaining cases, we summarise the main results and refer the reader to the appendices for more details. The following algorithm provides a general description of the steps involved. For every branch, based on the market efficiency parameter, do the following:

1. Determine the branches, if any, with respect to the foreign agent's quota purchasing variable, $u_2$. Check for the curvature of the individual branches in order to determine the potential equilibrium candidates.

2. Compare the payoff generated by the equilibrium candidates both within and across branches in order to identify all possible outcomes.

At the end of this process all distinct equilibria are categorised based on the foreign agent's type and the foreign market efficiency parameter. A distinct equilibrium strategy prescribes a quota purchasing plan for the foreign agent that depends on the quota price.




(a) $u_2^* = 1-\phi_2$ (b) $u_2^* = u_2^{in}$ (c) $u_2^* = 1$

(d) $u_2^* = 0$ (e) $u_2^* = \phi_2$ (f) $\phi_2 = 1/2$

Figure 3.3. Possible plottings of the foreign agent's objective. Plots (a)-(e) give rise to each one of the five equilibrium candidates when foreign market efficiency is low. The last plot illustrates the boundary case, i.e., $\phi_2 = 1/2$. The plots are merely for illustration and reflect no real parameter values.

### 3.4.1 Low market efficiency

In case the foreign market efficiency parameter is low, i.e., $\phi_2 \in (0, 1/2]$,[4] the equilibrium outcome in the firms' sales subgame is given by $\mathbf{q}^1(u_2)$, see proposition 1. This solution implies that the foreign market cannot clear the entire quantity by itself, and thus both markets are served. In particular, for medium quota shares, i.e., between $\phi_2$ and $1-\phi_2$, both firms serve both markets. Otherwise, only the firm with the majority of quotas serve both markets. Substituting for $\mathbf{q}^1(u_2)$ in $\bar{V}_2$ yields the following piecewise continuous function:

$$\bar{V}_2(u_2, \mathbf{q}^1(u_2)) = \begin{cases} \bar{V}_{2a} = \bar{V}_2(u_2, {}_2\mathbf{q}_{\text{pb}}), & 1-\phi_2 \le u_2 \le 1, \\ \bar{V}_{2b} = \bar{V}_2(u_2, {}_2\mathbf{q}_{\text{bb}}), & \phi_2 \le u_2 \le 1-\phi_2, \\ \bar{V}_{2c} = \bar{V}_2(u_2, {}_2\mathbf{q}_{\text{bp}}), & 0 \le u_2 \le \phi_2. \end{cases} \tag{3.9}$$

For the remainder of the paper, we shall use the compact notation when referring to the different sub-functions, for example $\bar{V}_{2a}$ or $\bar{V}_{2a}(u_2)$. The top branch is strictly convex, the middle is linear, and the lower is strictly concave.[5] Two remarks regarding the linear part. First, if the slope is zero, then $\bar{V}_{2b}$ becomes constant making the foreign agent indifferent between all $u_2 \in [\phi_2, 1-\phi_2]$. However, as we shall see in the next section, the home country is always better off in one of the corners. Second, if $\phi_2$ is one-half, the linear function is absorbed by the other two. This creates a discontinuity when the slope is non-zero (see subfigure 3.3 (f)).

Based on the curvature of the three branches and as long as $\bar{V}_{2b}$ is not constant, there exist five equilibrium candidates, namely, one, $1-\phi_2$, $\phi_2$, $u_2^{in}(\rho)$, and zero. Strategy $u_2^{in}$ is the inner candidate of the concave function $\bar{V}_{2c}$, i.e., $u_2^{in}(\rho) = \text{argmax}\, \bar{V}_{2c}$. Moreover, it is linear on $\rho$ and downward slopping, i.e., $du_2^{in}/d\rho = k_2 < 0$. This implies that the higher the quota price the lower the foreign firm's quota share, which makes perfect sense

[4]The case of $\phi_2 = 0$ is of no interest because there is no market differentiation with respect to the maximum price.

[5]The second order conditions are: $\bar{V}''_{2a}(u_2) = (2\alpha\beta_1\beta_2 + (\xi+2)\beta_1^2)/(4\beta_2(\alpha\beta_2+\beta_1)^2) > 0$, $\bar{V}''_{2b}(u_2) = 0$, $\bar{V}''_{2c}(u_2) = -(4\alpha\beta_1\beta_2 + (4-\xi)\beta_1^2)(4\beta_2(\alpha\beta_2+\beta_1)^2) < 0$.

from an economic point of view. For the explicit definition of $k_2$ and all other explicit formulas used in this subsection see appendix B.1.

Figure 3.3 shows some possible plottings of the objective where each one of the five candidates is drawn as optimal. Notice that in subfigures (a), (d) and (e) strategy $u_2 = u_2^{in}$ is outside the feasible region. Because of the piecewise nature of the objective, we need to compare the value generated by all equilibrium candidates in order to determine the optimal foreign policy. We start by comparing the ones that occur in the same branch, e.g., branch $\bar{V}_{2c}$ with candidates zero, $u_2^{in}$ and $\phi_2$. If necessary, we compare the candidates across branches. Since the quota price $\rho$ enters the foreign agent's problem exogenously, we categorise the different equilibria with respect it. In what follows we derive price intervals that make the equilibria candidates mutually exclusive. We begin with the concave function, $\bar{V}_{2c}$. Lemma 1 provides the optimal policy in the $u_2 \in [0, \phi_2]$ subregion.

**Lemma 1.** Let $\rho_1$ and $\rho_2$ given by $u_2^{in}(\rho) = 0$ and $u_2^{in}(\rho) = \phi_2$, respectively, be quota price thresholds.Then, the optimal policy $u_2^*$ in the $[0, \phi_2]$ region is given by

$$u_2^*(\rho) = \min(\max(0, u_2^{in}), \phi_2) = \begin{cases} 0, & \rho \geq \rho_1, \\ u_2^{in}(\rho), & \rho_2 \leq \rho \leq \rho_1, \\ \phi_2, & \rho \leq \rho_2. \end{cases}$$

In addition, $\rho_1 > \rho_2$.
See appendix B.1 for the proof.

Lemma 1 tells us that for any price below $\rho_1$, which is the price that induces the zero-quota quantity, the foreign agent purchases a positive amount of quotas. If the price exceeds $\rho_2$, its share of quotas is determined by $u_2^{in}(\rho)$. Similarly, if the price falls short of $\rho_2$, the foreign agent buys according to $\phi_2$. Because $0 \leq u_2^{in}(\rho) \leq \phi_2$, it is implied that lower prices increase the number of quotas purchased, which makes sense from an economic point of view. Next, we investigate the linear branch. Lemma 2 provides the optimal policy in the $u_2 \in [\phi_2, 1 - \phi_2]$ subregion.

**Lemma 2.** Let $\rho_3$ given by $\bar{V}'_{2b}(u_2) = 0$ be a quota price threshold. Then, the optimal policy $u_2^*$ in the $[\phi_2, 1 - \phi_2]$ region is given by

$$u_2^*(\rho) = \begin{cases} \phi_2, & \rho > \rho_3, \\ u_2 \in [\phi_2, 1 - \phi_2], & \rho = \rho_3, \\ 1 - \phi_2, & \rho < \rho_3. \end{cases}$$

In addition, $\rho_3 < \rho_2$ for all $\alpha \in (0, 1)$.
See appendix B.1 for the proof.

Lemma 2 tells us that the foreign agent prefers to purchase $1 - \phi_2$ quotas when the slope is positive, $\phi_2$ quotas when the slope is negative, and is indifferent between any $u_2 \in [\phi_2, 1 - \phi_2]$ when the slope is zero. So far we have categorised four out of five equilibrium candidates, and the fact that the thresholds are descending, i.e., $\rho_1 > \rho_2 > \rho_3$, implies that for all $\rho > \rho_3$ the four candidates are mutually exclusive and the optimal policy follows from lemmas 1 and 2. Next, we compare the candidates of the convex function. Lemma 3 provides the optimal policy in the $u_2 \in [1 - \phi_2, 1]$ subregion.

**Lemma 3.** Let $\rho_4$ given by $\bar{V}_{2a}(1-\phi_2) = \bar{V}_{2a}(1)$ be a quota price threshold. Then, the optimal policy $u_2^*$ in the $[1-\phi_2, 1]$ is given by

$$u_2^*(\rho) = \begin{cases} 1-\phi_2, & \rho \geq \rho_4, \\ 1, & \rho \leq \rho_4. \end{cases}$$

The relative position of $\rho_4$ depends on the market parameters and $\xi$ as follows:
a. Let $\xi = 1$ and $\phi_2 \in (0, \max(0, 2/5-\theta)]$, where $\theta = \beta_1(1-\alpha)/(10\alpha)$. Then, $\rho_4 \leq \rho_3$.
b. Let $\xi = 1$ and $\phi_2 \in (\max(0, 2/5-\theta), 1/2]$. Then, $\rho_4 \in (\rho_3, \rho_2)$.
c. Let $\xi = 0$. Then $\rho_4 \in (\rho_2, \rho_1)$ for all $\alpha \in (0,1)$.
See appendix B.1 for the proof.

In contrast to the previous price thresholds, the relative position of $\rho_4$ depends on the market efficiency parameter and the type of the foreign agent. This implies that for every distinct position of $\rho_4$ there exist a unique equilibrium solution, which is a function of $\rho$. To determine these equilibria, we start by defining the notions of strict and weak dominance with regard to suboptimal strategies. That is, given a pair of strategies, the value generated by a strictly dominated strategy is always less than that of its counterpart, whereas the value generated by a weakly dominated strategy is less or equal to that of its counterpart.

Suppose that $\rho_4 \leq \rho_3$. This is true when $\xi = 1$ and $\phi_2 \in (0, \max(0, 2/5-\theta)]$. It then follows from lemmas 1-3 that $\rho_1 > \rho_2 > \rho_3 \geq \rho_4$. This implies that that all strategies occur in distinct $\rho$ regions. Therefore, a complete equilibrium exists. Note that at $\phi_2 = 2/5-\theta$, price thresholds $\rho_4$ and $\rho_3$ are equal and strategy $u_2 = 1-\phi_2$ becomes weakly dominated by strategies $u_2 = 1$ and $u_2 = \phi_2$, this is a degeneration.

Suppose that $\rho_4 > \rho_3$. This is true when $\xi = 0$, or $\xi = 1$ and $\phi_2 \in (\max(0, 2/5-\theta), 1/2]$. Then, the foreign agent prefers to play $u_2 = \phi_2$ instead of $u_2 = 1-\phi_2$ for any $\rho \geq \rho_4 > \rho_3$, see lemma 2. In addition, from lemma 3, $u_2 = 1$ strictly dominates $u_2 = 1-\phi_2$ for all $\rho < \rho_4$. Thus, $u_2 = 1-\phi_2$ is a strictly dominated strategy either by $u_2 = \phi_2$ or $u_2 = 1$ when $\rho_4 > \rho_3$, and therefore we need to compare them in order to determine the complete equilibrium strategy. The following lemma gives the optimal policy between $u_2 = \phi_2$ and $u_2 = 1$ when $\rho_4$ exceeds $\rho_3$.

**Lemma 4.** Suppose $\rho_4 > \rho_3$ and let $\rho_5$ given by $\bar{V}_{2c}(\phi_2) = \bar{V}_{2a}(1)$ be a quota price threshold. Then, the optimal policy $u_2^*$ is given by

$$u_2^*(\rho) = \begin{cases} \phi_2, & \rho \geq \rho_5, \\ 1, & \rho \leq \rho_5. \end{cases}$$

The relative position of $\rho_5$ depends on the market parameters and on $\xi$ as follows:
a. Let $\xi = 1$ and $\phi_2 \in (\max(0, 2/5-\theta), 1/2]$. Then $\rho_5 \in (\rho_3, \rho_4]$.
b. Let $\xi = 0$ and $\phi_2 \in (0, 2/7]$. Then $\rho_5 \in (\rho_3, \rho_2]$.
c. Let $\xi = 0$ and $\phi_2 \in (2/7, 1/2]$. Then $\rho_5 \in (\rho_2, \rho_1)$.
See appendix B.1 for the proof.

Suppose that $\rho_5 \leq \rho_2$. It then follows from lemmas 1-4 that $\rho_1 > \rho_2 > \rho_5$ when $\xi = 1$ and $\phi_2 \in (\max(0, 2/5-\theta), 1/2]$, and $\rho_1 > \rho_2 \geq \rho_5$ when $\xi = 0$ and $\phi_2 \in (0, 2/7]$. This implies that the feasible equilibrium strategies, namely, zero, one, $u_2^{in}(\rho)$, and $\phi_2$, occur in distinct $\rho$ regions. This is the second complete equilibrium and occurs for different market conditions irrespective of the foreign agent's type. Note that when $\xi = 0$ and

$\phi_2 = 2/7$, price thresholds $\rho_5$ and $\rho_2$ are equal and strategy $u_2 = \phi_2$ is weakly dominated by $u_2 = 1$ and $u_2 = u_2^{in}$, this is a degeneration.

Suppose that $\rho_5 > \rho_2$. This is true when the foreign agent is profit-maximiser and the market efficiency parameter is in the $(2/7, 1/2]$ region. Then, the foreign agents prefers to play $u_2 = u_2^{in}$ instead of $u_2 = \phi_2$ for any $\rho \geq \rho_5 > \rho_2$, see lemma 1. In addition, from lemma 4, $u_2 = 1$ is the preferred strategy for all $\rho < \rho_5$. Thus, $u_2 = \phi_2$ is a strictly dominated strategy either by $u_2 = u_2^{in}$ or $u_2 = 1$ when $\rho_5 > \rho_2$, and therefore we need to compare them in order to determine the complete equilibrium strategy. Lemma 5 provides the optimal policy between $u_2 = u_2^{in}$ and $u_2 = 1$ when $\rho_5$ exceeds $\rho_2$.

**Lemma 5.** Suppose $\rho_5 > \rho_2$ and let and $\rho_6$ given by $\bar{V}_{2c}(u_2^{in}) = \bar{V}_{2a}(1)$ be a quota price threshold. Then, the optimal policy $u_2^*$ is given by

$$u_2^*(\rho) = \begin{cases} u_2^{in}, & \rho \geq \rho_6, \\ 1, & \rho \leq \rho_6. \end{cases}$$

In addition, $\rho_6 \in (\rho_2, \rho_5)$ when $\xi = 0$ and $\phi_2 \in (2/7, 1/2]$.
See appendix B.1 for the proof.

The third and final equilibrium for the case of low market efficiency occurs when $\rho_5 > \rho_2$. It follows from lemma 1-5 that $\rho_1 > \rho_6$, and the feasible equilibrium strategies, namely, zero, one, and $u_2^{in}(\rho)$, are mutually exclusive for different $\rho$ levels. The following proposition summarises the unique foreign agent's equilibria when the foreign market efficiency parameter is low.

**Proposition 2.** Let $u_2^*(\rho)$ denote the equilibrium solution for the foreign agent's quota purchasing subgame, which is a function of the quota price. Moreover, let $\phi_2 \in (0, 1/2]$. Then, there exist three mutually exclusive equilibria that depend on the type of the foreign agent and the foreign market efficiency parameter as follows:

$$u_2^*(\rho)|_{\xi=1} = \begin{cases} u_2^1(\rho), & \phi_2 \in (0, \max(0, 2/5 - \theta)], \\ u_2^2(\rho), & \phi_2 \in (\max(0, 2/5 - \theta), 1/2], \end{cases} \quad u_2^*(\rho)|_{\xi=0} = \begin{cases} u_2^2(\rho), & \phi_2 \in (0, 2/7], \\ u_2^3(\rho), & \phi_2 \in (2/7, 1/2], \end{cases}$$

where $\theta = \beta_1(1-\alpha)/(10\alpha)$. For the specification of the equilibria see appendix B.4.

### 3.4.2 Medium and high market efficiency

In case the foreign market efficiency parameter is medium, i.e., $\phi_2 \in [1/2, 2/3]$, the equilibrium outcome in the firms' sales subgame is given by $\mathbf{q}^2(u_2)$, see proposition 1. This solution implies that there are circumstances where the foreign market can clear the entire quantity by itself. In particular, for all quota shares between $2 - 3\phi_2$ and $3\phi_2 - 1$, both firms serve only the foreign market. Otherwise, the firm with the majority of quotas serve both markets. Substituting for $\mathbf{q}^2(u_2)$ in $\bar{V}_2$ yields the following piecewise continuous function:

$$\bar{V}_2(u_2, \mathbf{q}^2(u_2)) = \begin{cases} \bar{V}_{2a} = \bar{V}_2(u_2, {}_2\mathbf{q}_{\text{pb}}), & 3\phi_2 - 1 \leq u_2 \leq 1, \\ \bar{V}_{2d} = \bar{V}_2(u_2, {}_2\mathbf{q}_{\text{pp}}), & 2 - 3\phi_2 \leq u_2 \leq 3\phi_2 - 1, \\ \bar{V}_{2c} = \bar{V}_2(u_2, {}_2\mathbf{q}_{\text{bp}}), & 0 \leq u_2 \leq 2 - 3\phi_2. \end{cases} \tag{3.10}$$

The top and bottom sub-functions are the same as in the previous case, but the middle one differs. The inner bounds are also different. The upper bound of the concave function is $2 - 3\phi_2$ instead of $\phi_2$, and the lower bound of the convex function is $3\phi_2 - 1$ instead of $1 - \phi_2$. The curvature remains the same since $\bar{V}_{2d}$ is also linear. Again, if the slope of $\bar{V}_{2d}$ is zero, any $u_2 \in [2 - 3\phi_2, 3\phi_2 - 1]$ makes the foreign agent indifferent. Also, at $\phi_2 = 1/2$, the linear function is absorbed by the other two, which creates a discontinuity if the slope is non-zero. At $\phi_2 = 2/3$, the concave and the convex functions are absorbed by the linear. Then, the optimal strategy becomes a bang-bang, i.e., either buy everything or buy nothing.

This last point is also true when the foreign market efficiency parameter is high, i.e., $\phi_2 \geq 2/3$. The equilibrium outcome in the firms' sales subgame is then given by $\mathbf{q}^3(u_2)$, see proposition 1, and it implies that only the primary market, which in analysis is the foreign one, is served. Substituting for $\mathbf{q}^3(u_2)$ in $\bar{V}_2$ yields $\bar{V}_2(u_2, {}_2\mathbf{q}_{\text{pp}})$, which is equivalent to $\bar{V}_{2d}$. Thus, the case of high market efficiency degenerates from the medium one.

As in the low efficiency case, there exist five equilibrium candidates when $\bar{V}_{2d}$ is not constant, namely, one, $3\phi_2-1$, $2-3\phi_2$, $u_2^{in}(\rho)$, and zero. Because the process of identifying the unique equilibria is very similar to the previous case, we have placed all details on appendix B.2 and proceed with proposition 3 that summarises the solution.

**Proposition 3.** Let $u_2^*(\rho)$ denote the equilibrium solution for the foreign agent's quota purchasing subgame, which is a function of the quota price. Moreover, let $\phi_2 \geq 1/2$. Then, there exist three mutually exclusive equilibria that depend on the type of the foreign agent and the foreign market efficiency parameter as follows:

$$u_2^*(\rho)|_{\xi=1} = \begin{cases} u_2^4(\rho), & \phi_2 \in [1/2, 2/3), \\ u_2^5(\rho), & \phi_2 \geq 2/3, \end{cases} \qquad u_2^*(\rho)|_{\xi=0} = \begin{cases} u_2^3(\rho), & \phi_2 \in [1/2, 2/3), \\ u_2^5(\rho), & \phi_2 \geq 2/3. \end{cases}$$

For the specification of the equilibria see appendix B.4.

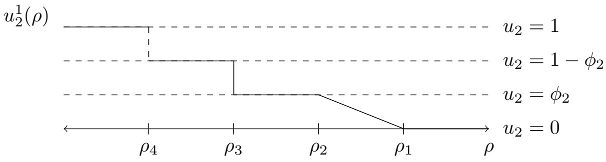
### 3.4.3 Complete solution

The complete solution of the foreign agent's purchasing quota subgame follows from propositions 2 and 3 and is given by the following piecewise continuous function:

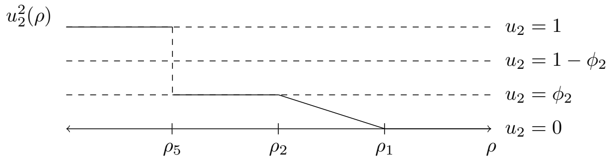
$$u_2^*(\rho)|_{\xi=1} = \begin{cases} u_2^1(\rho), & \phi_2 \in (0, \max(0, 2/5 - \theta)], \\ u_2^2(\rho), & \phi_2 \in (\max(0, 2/5 - \theta), 1/2], \\ u_2^4(\rho), & \phi_2 \in [1/2, 2/3), \\ u_2^5(\rho), & \phi_2 \geq 2/3, \end{cases} \qquad u_2^*(\rho)|_{\xi=0} = \begin{cases} u_2^2(\rho), & \phi_2 \in (0, 2/7], \\ u_2^3(\rho), & \phi_2 \in (2/7, 2/3), \\ u_2^5(\rho), & \phi_2 \geq 2/3, \end{cases}$$

where $\theta = \beta_1(1 - \alpha)/(10\alpha)$. Figure 3.4 depicts all possible equilibria that occur in the foreign agent's quota purchasing subgame. The optimal policy in all equilibria is decreasing in the quota price. Functions $u_2^1(\rho)$, $u_2^2(\rho)$, $u_2^3(\rho)$, and $u_2^4(\rho)$ are discontinuous at $\rho_4$, $\rho_5$, $\rho_6$ and $\rho_9$ respectively. At the point of discontinuity, the foreign agent is indifferent between the nearby options. For example, at $\rho = \rho_4$, the foreign agent is indifferent between buying the entire quota or the majority of it prescribed by $1 - \phi_2$, see subfigure (a). Moreover, in subfigures (a), (d) and (e), price thresholds $\rho_3$ and $\rho_8$ make the foreign agent indifferent between all nearby options. For example, when the market efficiency parameter exceeds two-thirds (subfigure (e)), any quota share makes the foreign agent indifferent at $\rho = \rho_8$.

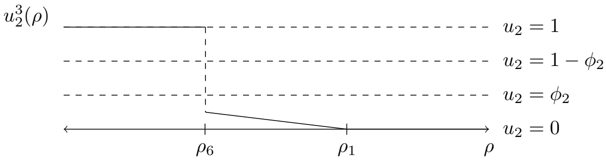
[6]The magnitude of the price thresholds differ between $\xi = 0$ and $\xi = 1$, however the shape of the equilibrium policy remains the same.



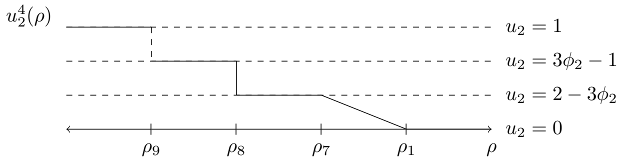

(a) $\xi = 1$ and $\phi_2 \in (0, \max(0, 2/5 - \theta)]$.



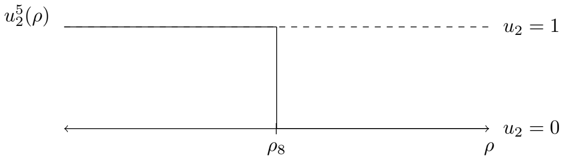

(b) $\xi = 1$ and $\phi_2 \in (\max(0, 2/5 - \theta), 1/2]$, or, $\xi = 0$ and $\phi_2 \in (0, 2/7]$.[6]




(c) $\xi = 0$ and $\phi_2 \in (2/7, 2/3)$.




(d) $\xi = 1$ and $\phi_2 \in [1/2, 2/3)$.




(e) $\phi_2 \geq 2/3$.

Figure 3.4. Complete characterisation of the foreign agent's optimal quota purchasing strategy. The type of the foreign agent and the market parameters determine the feasible equilibrium strategy, which depends on the quota price $\rho$.

As a final remark, we describe how the optimal purchasing quota strategy changes as $\phi_2$ increases for the case of a welfare-maximiser foreign agent, i.e., $\xi = 1$. For values of $\phi_2$ close to zero the optimal policy is given by subfigure (a). As $\phi_2$ increases price thresholds $\rho_4$ and $\rho_3$ get closer, as do strategies $\phi_2$ and $1 - \phi_2$. At $\phi_2 = 2/5 - \theta$, price thresholds $\rho_3$, $\rho_4$ and $\rho_5$ coincide. For values of $\phi_2$ greater than $2/5 - \theta$ the optimal policy is given by subfigure (b), strategy $1 - \phi_2$ becomes strictly dominated and $\rho_5$ becomes the switching price threshold between strategies $\phi_2$ and one. Once $\phi_2$ hits one-half, strategies $2 - 3\phi_2$ and $3\phi_2 - 1$ become feasible. Moreover, $\phi_2 = 1 - \phi_2 = 2 - 3\phi_2 = 3\phi_2 - 1 = 1/2$ and thresholds $\rho_5$ and $\rho_9$ coincide. For values of $\phi_2$ greater than one-half the optimal strategy is given by subfigure (d). As $\phi_2$ keeps increasing all price thresholds $\rho_i$, $i = 1, 7, 8, 9$, converge towards $\rho_8$. In addition, strategies $2 - 3\phi_2$ and $3\phi_2 - 1$ move towards zero and one, respectively. For values of $\phi_2$ greater or equal to two-thirds the optimal policy is given by sbufigure (e). Now strategies zero and one are strictly dominant and price threshold $\rho_8$ becomes the sole switching price.

## 3.5 Home country's price subgame

The home country sets the quota price with the full knowledge of how it influences the outcome in all subsequent subgames, i.e., both the foreign agent's purchasing strategy and the firms' selling decisions. In addition, it is perfectly informed about the foreign agent's incentives, i.e., it knows whether the foreign agent is profit- or welfare-maximiser. Let $\bar{V}_1 = V_1/(a_2 U)$ be the scaled objective function, the home country decides on the quota price by solving the following maximisation problem for each foreign agent type:

$$\max_{\rho\in\mathbb{R}} \quad \bar{V}_1\big(\rho, u_2^*(\rho), \mathbf{q}^*(u_2^*(\rho))\big)\Big|_{\xi=1} = \begin{cases} \bar{V}_1(\rho, u_2^1(\rho), \mathbf{q}^1(u_2^1(\rho))), & \phi_2 \in (0, \max(0, 2/5-\theta)], \\ \bar{V}_1(\rho, u_2^2(\rho), \mathbf{q}^1(u_2^2(\rho))), & \phi_2 \in (\max(0, 2/5-\theta), 1/2], \\ \bar{V}_1(\rho, u_2^4(\rho), \mathbf{q}^2(u_2^4(\rho))), & \phi_2 \in (1/2, 2/3), \\ \bar{V}_1(\rho, u_2^5(\rho), \mathbf{q}^3(u_2^5(\rho))), & \phi_2 \geq 2/3, \end{cases}$$

$$\max_{\rho\in\mathbb{R}} \quad \bar{V}_1\big(\rho, u_2^*(\rho), \mathbf{q}^*(u_2^*(\rho))\big)\Big|_{\xi=0} = \begin{cases} \bar{V}_1(\rho, u_2^2(\rho), \mathbf{q}^1(u_2^2(\rho))), & \phi_2 \in (0, 2/7], \\ \bar{V}_1(\rho, u_2^3(\rho), \mathbf{q}^1(u_2^3(\rho))), & \phi_2 \in (2/7, 2/3), \\ \bar{V}_1(\rho, u_2^5(\rho), \mathbf{q}^3(u_2^5(\rho))), & \phi_2 \geq 2/3, \end{cases}$$

$$\text{where} \quad \bar{V}_1\big(\rho, u_2^l(\rho), \mathbf{q}^k(u_2^l(\rho))\big) = \frac{\alpha(q_1^* + u_2^* - q_2^*)^2}{2\beta_1} + \alpha\left(1 - \frac{q_1^* + u_2^* - q_2^*}{\beta_1}\right)q_1^*$$
$$+ \left(1 - \frac{q_2^* + 1 - u_2^* - q_1^*}{\beta_2}\right)(1 - u_2^* - q_1^*) - \psi_1(1 - u_2^*) + \rho u_2^*,$$
$$\forall k = 1, 2, 3, \quad \forall l = 1, \ldots, 5. \tag{3.11}$$

The superscripts $l$ and $k$ refer to the respective foreign agent's quota and firms' sales distinct equilibrium solutions. Parameter $\xi$ affects the home country's objective indirectly through the foreign agent's equilibria and corresponding price thresholds. The correct relationship between $u_2^l$ and $q^k(u_2^l)$ follows from propositions 1, 2 and 3.

The strategies $(\rho^*, u_2^*(\rho^*), \mathbf{q}^*(u_2^*(\rho^*))) \in \mathbb{R} \times [0,1] \times [0, u_1] \times [0, u_2]$ constitute a Stackelberg equilibrium of the home country's price subgame if and only if

$$\bar{V}_1\big(\rho^*, u_2^*(\rho^*), \mathbf{q}^*(u_2^*(\rho^*))\big) \geq \bar{V}_1\big(\rho, u_2^*(\rho), \mathbf{q}^*(u_2^*(\rho))\big), \qquad \forall \rho \in \mathbb{R},$$
$$\bar{V}_2\big(\rho^*, u_2^*(\rho^*), \mathbf{q}^*(u_2^*(\rho^*))\big) \geq \bar{V}_2\big(\rho^*, u_2(\rho^*), \mathbf{q}^*(u_2(\rho))\big), \qquad \forall u_2(\rho^*) \in [0,1],$$
$$\bar{R}_i\big(\rho^*, u_2^*(\rho^*), q_i^*(u_2^*(\rho^*)), q_j^*(u_2^*(\rho^*))\big) \geq \bar{R}_i\big(\rho^*, u_2^*(\rho^*), q_i(u_2^*(\rho^*)), q_j^*(u_2^*(\rho^*))\big),$$
$$\forall q_i \in [0, u_i], \quad \forall i, j = 1, 2 : i \neq j.$$

Similar to the previous subgame, solving problem (3.11) for a given set of parameter values through enumeration is straightforward. But doing so does not provide a complete categorisation of the different equilibria. Thus, again, we zoom into the branches of the home country's piecewise objective and derive the respective Stackelberg equilibria as well as all necessary conditions for their existence. Moreover, we show that all equilibria are mutually exclusive, i.e., unique. In total there are six cases that we need to evaluate for both foreign agent types. This is because the functional specification for high market efficiency, i.e., $\phi_2 \geq 2/3$, is the same for both types. In this section, the different cases are categorised on the basis of the foreign agent's type and the magnitude of the market efficiency parameter. In particular, when the foreign agent is welfare-maximiser, we distinguish between very low, low, medium and high market efficiency. Similarly, when the foreign agent is profit-maximiser, we discern between low, medium and high efficiency.

As in the previous subgame, the solution procedure between cases is repetitive, and thus we describe it in detail only for the first case, namely, when the foreign agent is welfare-maximiser and the market efficiency is very low, $\phi_2 \in (0, 2/5 - \theta]$. For the remaining cases, we summarise the main results and refer the reader to the appendices for more details. The following algorithm provides a general description of the steps involved. For every branch, based on the market efficiency parameter, do the following:

1. Determine the branches with respect to quota price variable, $\rho$. Check for the curvature of the individual branches in order to determine the potential equilibrium candidates.

2. Compare the payoff generated by the equilibrium candidates both within and across branches in order to identify all possible outcomes.

3. Derive the necessary conditions for an optimal solution to guarantee the home country a non-negative payoff, i.e., $\bar{V}_1(\rho^*) \geq 0$.

At the end of this process all distinct equilibria are categorised in the entire parameter space. A distinct equilibrium strategy prescribes a quota pricing scheme for the home country that depends on the parameter values when the TAC is exogenous, and on the TAC and stock biomass levels when it is endogenous.

## 3.5.1 Welfare-maximiser foreign agent

### 3.5.1.1 Very low market efficiency

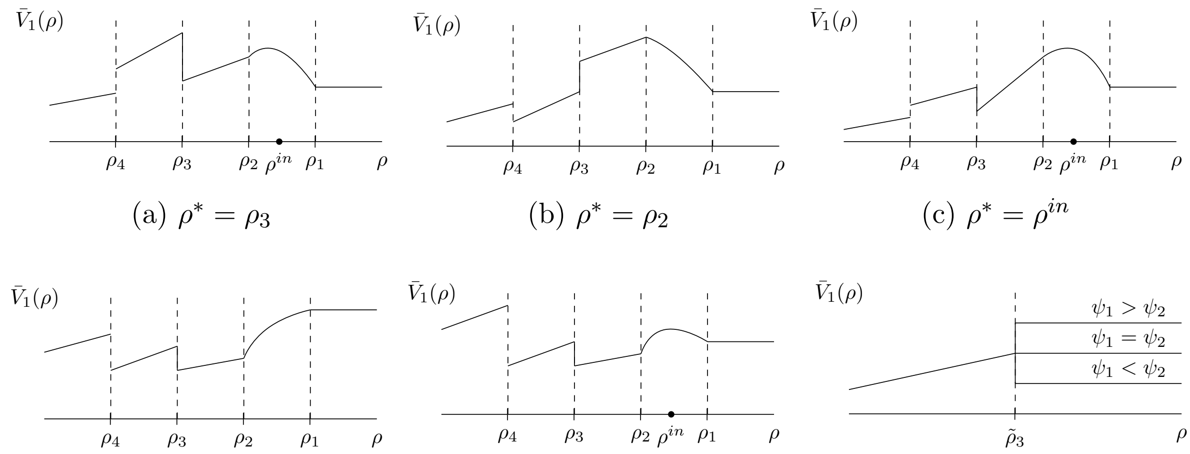
In case the foreign agent is welfare-maximiser and the foreign market parameter is very low, i.e., $\theta < 2/5$ and thus $\phi_2 \in (0, 2/5 - \theta]$, the sequential equilibrium outcome in the foreign agent's quota and the firms' sales subgames is given by $u_2^1(\rho)$ and $\mathbf{q}^1(u_2(\rho))$, respectively, see propositions 1 and 2. Substituting in $\bar{V}_1$ yields the following piecewise multi-valued function:

$$\bar{V}_1(\rho, u_2^1(\rho), \mathbf{q}^1(u_2^1(\rho))) = \begin{cases} \bar{V}_{1a} = \bar{V}_1(\rho, 0, {}_2\mathbf{q}_{\text{bp}}), & \rho \geq \rho_1, \\ \bar{V}_{1b} = \bar{V}_1(\rho, u_2^{in}(\rho), {}_2\mathbf{q}_{\text{bp}}), & \rho_2 \leq \rho \leq \rho_1, \\ \bar{V}_{1c} = \bar{V}_1(\rho, \phi_2, {}_2\mathbf{q}_{\text{bp}}), & \rho_3 \leq \rho \leq \rho_2, \\ \bar{V}_1(\rho_3, u_2, {}_2\mathbf{q}_{\text{bb}}), \forall u_2 \in [\phi_2, 1-\phi_2], & \rho = \rho_3, \\ \bar{V}_{1d} = \bar{V}_1(\rho, 1-\phi_2, {}_2\mathbf{q}_{\text{pb}}), & \rho_4 \leq \rho \leq \rho_3, \\ \bar{V}_{1e} = \bar{V}_1(\rho, 1, {}_2\mathbf{q}_{\text{pb}}), & \rho \leq \rho_4, \end{cases} \tag{3.12}$$

which is discontinuous at $\rho_4$, and at $\rho_3$ its value is associated with multiple elements, namely all $u_2$ in the $[\phi_2, 1-\phi_2]$ region. In addition, starting from the top, the first branch is constant, the second is concave, the forth is a vertical line, and the remaining three are linear and increasing.[7] This implies that there exist five equilibrium candidates, namely, $\rho_1$, $\rho^{in}$, $\rho_2$, $\rho_3$, and $\rho_4$. Candidate $\rho^{in}$ is the inner candidate of the concave function $\bar{V}_{1b}$, i.e., $\rho^{in} = \text{argmax}\ \bar{V}_{1b}$.

Figure 3.5 depicts possible plottings of the objective where each one of the five candidates is plotted as optimal. Notice how prices $\rho_3$ and $\rho_4$, which make the foreign agent indifferent between multiple strategies, make the home country better off at a distinct strategy, see subfigures (a) and (e). For example suppose that $\rho^* = \rho_4$, then the home country is better off when the foreign agent purchases according to $u_2 = 1$ (right corner of the line left of $\rho_4$ in (e)), instead of $u_2 = 1 - \phi_2$ (left corner of the line right of $\rho_4$ in (a)). In this context, we assume that the home country can induce the preferred quantity either by charging slightly below, i.e., $\rho_3^-$ and $\rho_4^-$, or by offering the foreign agent a bundle that specifies both a price and a quantity, for instance $(\rho, u_2) = (\rho_4, 1)$ when $\rho^* = \rho_4$. In other words, in order for the equilibrium to be well defined, we follow the convention that in case of indifference between strategies, the follower selects the one that favours the leader. This is observed in all cases where a discontinuity occurs or the objective is vertical, i.e., multi-valued.

[7]The second order conditions are: $\bar{V}_{1b}''(\rho) = -(4\beta_2(5\alpha\beta_2 + 4\beta_1)(\alpha\beta_2 + \beta_1^2)^2)/(\beta_1(4\alpha\beta_2 + 3\beta_1)^2) < 0$, $\bar{V}_{1a}''(\rho) = \bar{V}_{1c}''(\rho) = \bar{V}_{1d}''(\rho) = \bar{V}_{1e}''(\rho) = 0$. The first order conditions of the linear functions are: $\bar{V}_{1a}'(\rho) = 0$, $\bar{V}_{1c}'(\rho) = \phi_2 > 0$, $\bar{V}_{1d}'(\rho) = 1 - \phi_2 > 0$, $\bar{V}_{1e}'(\rho) = 1$.




(a) $\rho^* = \rho_3$ (b) $\rho^* = \rho_2$ (c) $\rho^* = \rho^{in}$

(d) $\rho^* = \rho_1$ (e) $\rho^* = \rho_4$ (f) $\phi_2 \geq 2/3$

Figure 3.5. Possible plottings of the home country's objective. Plots (a)-(e) depict the five equilibrium candidates as optimal when the foreign agent is welfare-maximiser and the foreign market efficiency is very low. The last plot illustrates the case of high market efficiency. The plots are merely for illustration and reflect no real parameter values.

Because of the piecewise nature of the objective, we need to compare the payoff generated by all equilibrium candidates in order to determined the optimal pricing policy. We start with the concave part of the objective. This categorises three candidates, namely, $\rho_1$, $\rho^{in}$ and $\rho_2$. Because the linear branches are increasing the optimal candidate is their right bound. But this is not enough for specifying the global optimum and thus we need to compare candidates across branches until all solutions are identified. We distinguish between the different optimality regions using the cost parameters, and specifically their difference, i.e., $\Delta\psi = \psi_1 - \psi_2$, which we consider a measurement of efficiency associated with the exploitation of the resource, and sometimes refer to as the firms' efficiency level. The home firm is more efficient or has a cost advantage when $\Delta\psi < 0$, and vice-versa when $\Delta\psi > 0$. Throughout the analysis, we use $\lambda_i$ to denote the cost difference thresholds when $\xi = 1$ and $\mu_i$ when $\xi = 0$. Lemma 6 provides the optimal policy in the $[\rho_1, \rho_2]$ region.

**Lemma 6.** Let $\Delta\psi = \lambda_1$ and $\Delta\psi = \lambda_2$ given by $\rho^{in} = \rho_1$ and $\rho^{in} = \rho_2$, respectively, be cost difference thresholds.[8] Then, the optimal policy $\rho^*$ in the $[\rho_2, \rho_1]$ region is given by

$$\rho^* = \min(\max(\rho_2, \rho^{in}), \rho_1) = \begin{cases} \rho_2, & \Delta\psi \geq \lambda_2, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_2, \\ \rho_1, & \Delta\psi \leq \lambda_1. \end{cases}$$

In addition, $\lambda_1 < 0$ and $\lambda_2 > 0$ for all $\alpha \in (0, 1)$. See appendix C.1.1 for the proof.

Lemma 6 tells us that for any efficiency level below $\lambda_1$, the home country issues a price equal to $\rho_1$. This induces the foreign agent to buy zero quotas, and as a consequence the home firm takes the entire quota. Since $\lambda_1$ is negative, it is implied that the home firm exploits the resource by itself only when it has a substantial cost advantage such that $\psi_1 \leq \psi_2 + \lambda_1$. If the foreign firm is more efficient, or when the home firm's cost advantage does not exceed $|\lambda_1|$, both firms are active. For efficiency levels between $\lambda_1$ and $\lambda_2$, the optimal pricing strategy is $\rho^{in}$ inducing the foreign firm to buy according to

[8] For explicit definitions of the cost difference thresholds associated with this case see appendix C.1.1.

$u_2^{in}(\rho^{in})$, which is the least amount of quotas it can get. Finally, when the foreign firm has a cost advantage that exceeds $\lambda_2$, the optimal price is $\rho^* = \rho_2$ and the optimal quota shares are $u_2^* = \phi_2$ and $u_1^* = 1 - \phi_2$, with $u_1^* > u_2^*$ since $\phi_2 < 1/2$.

Moving on, we compare the welfare generated at $\rho_4$ and $\rho_3$, which are local optima of $\bar{V}_{1e}$ and $\bar{V}_{1d}$, respectively. If $\bar{V}_{1e}(\rho_4)$ exceeds $\bar{V}_{1d}(\rho_3)$, then the optimal policy occurs at $\rho_4$ and the home country prefers the foreign agent to buy everything (the foreign agent is indifferent between $u_2 = 1$ and $u_2 = 1 - \phi_2$ at $\rho_4$). Otherwise, the optimal policy occurs at $\rho_3$ and the home country prefers the foreign agent to buy $u_2 = 1 - \phi_2$ (the foreign agent is indifferent between all $u_2 \in [\phi_2, 1 - \phi_2]$ at $\rho_3$). Lemma 7 gives the optimal policy.

**Lemma 7.** Let $\Delta\psi = \lambda_3$ given by $\bar{V}_{1e}(\rho_4) = \bar{V}_{1d}(\rho_3)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_4, & \Delta\psi \geq \lambda_3, \\ \rho_3, & \Delta\psi \leq \lambda_3. \end{cases}$$

In addition, $\lambda_3 > \lambda_2$ when $\phi_2 \in (0, 2/5 - \theta]$. See appendix C.1.1 for the proof.

Next, we compare the welfare generated at $\rho_3$ and $\rho_2$, which are local optima of $\bar{V}_{1d}$ and $\bar{V}_{1c}$. If $\bar{V}_{1d}(\rho_3)$ exceeds $\bar{V}_{1c}(\rho_2)$, then the optimal policy occurs at $\rho_3$ and the home country prefers the foreign agent to buy $u_2 = 1 - \phi_2$, who is indifferent between all $u_2 \in [\phi_2, 1 - \phi_2]$. Otherwise, the optimal policy occurs at $\rho_2$ where the foreign agent always buys $u_2 = \phi_2$. The following lemma gives the optimal policy.

**Lemma 8.** Let $\Delta\psi = \lambda_4$ given by $\bar{V}_{1d}(\rho_3) = \bar{V}_{1c}(\rho_2)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_3, & \Delta\psi \geq \lambda_4, \\ \rho_2, & \Delta\psi \leq \lambda_4. \end{cases}$$

Let $\beta_1/(\alpha\beta_2) > -(34\phi_2^3 - 39\phi_2^2 + 22\phi_2 - 4)/(28\phi_2^3 - 14\phi_2^2 + 3\phi_2)$ be condition (C.1) and $\beta_1(8\phi_2^2 - \phi_2)/(\alpha\beta_2) + 6\phi_2^2 \geq 0$ condition (C.2). The relative position of $\lambda_4$ depends on (C.1) and (C.2) as follows:
a. Let condition (C.1) to hold. Then, $\lambda_4 > \lambda_3$.
b. Let condition (C.1) to fail (C.2) to hold. Then, $\lambda_4 \in [\lambda_2, \lambda_3]$.
c. Let conditions (C.1) and (C.2) to fail. Then, $\lambda_4 \in (\lambda_1, \lambda_2)$.
See appendix C.1.1 for the proof.

From the last lemma we conclude that there exist cost thresholds, e.g., $\lambda_4$, with multiple relative positions. This implies that for every distinct position of $\lambda_4$ the home country has a mutually exclusive equilibrium strategy that depends on the market parameters and the type of the foreign agent.

Suppose that (C.1) fails but (C.2) holds, then $\lambda_4$ is positioned in the $[\lambda_2, \lambda_3]$ region. And it follows from lemmas 6-8 that $\lambda_1 < \lambda_2 \leq \lambda_4 \leq \lambda_3$. This implies that all strategies occur in distinct $\Delta\psi$ regions and a complete equilibrium exists. If condition (C.2) binds, then $\lambda_4 = \lambda_2$ and strategy $\rho = \rho_2$ becomes weakly dominated by $\rho = \rho_3$ and $\rho = \rho^{in}$, this is a degeneration. Similarly, if condition (C.1) binds, then $\lambda_4 = \lambda_3$ and strategy $\rho = \rho_3$ becomes weakly dominated by $\rho = \rho_4$ and $\rho = \rho_2$, this is also degeneration.

Suppose that (C.1) holds, then $\lambda_4 > \lambda_3$. This means that $\rho = \rho_3$ is strictly dominated by $\rho = \rho_4$ for all $\Delta\psi \geq \lambda_4 > \lambda_3$, see lemma 7. Also, from lemma 8, $\rho = \rho_3$ is strictly dominated by $\rho = \rho_2$ for all $\Delta\psi < \lambda_4$. Thus, $\rho_3$ is a strictly dominated strategy either by $\rho_4$ or $\rho_2$ when $\lambda_4 > \lambda_3$, and therefore we need to compare them in order to determine the complete equilibrium strategy. This is one case to follow up.

The other case to follow up occurs when both (C.1) and (C.2) fail. Then $\lambda_4 \in (\lambda_1, \lambda_2)$. And it follows that $\rho = \rho_2$ is strictly dominated by $\rho = \rho^{in}$ for all $\Delta\psi \leq \lambda_4 < \lambda_2$, see lemma 6. Moreover, from lemma 8, $\rho = \rho_2$ is strictly dominated by $\rho = \rho_3$ for all $\Delta\psi > \lambda_4$. Thus, $\rho_2$ is a strictly dominated strategy either by $\rho^{in}$ or $\rho_3$ when $\lambda_4 \in (\lambda_1, \lambda_2)$, and therefore we need to compare them in order to determine the complete equilibrium strategy.

First, we proceed to find out what happens when (C.1) holds, i.e., $\lambda_4 > \lambda_3$. Lemma 9 provides the optimal policy between $\rho_4$ and $\rho_2$.

**Lemma 9.** Suppose $\lambda_4 > \lambda_3$ and let $\Delta\psi = \lambda_5$ given by $\bar{V}_{1e}(\rho_4) = \bar{V}_{1c}(\rho_2)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_4, & \Delta\psi \geq \lambda_5, \\ \rho_2, & \Delta\psi \leq \lambda_5. \end{cases}$$

In addition, $\lambda_5 > \lambda_2$ when $\phi_2 \in (0, 2/5 - \theta]$ and (C.1) holds.
See appendix C.1.1 for the proof.

It follows from lemmas 6-9 that when (C.1) holds, $\lambda_1 < \lambda_2 < \lambda_5$, and thus the feasible equilibrium strategies, $\rho_1$, $\rho^{in}$, $\rho_2$, and $\rho_4$ are mutually exclusive for different $\Delta\psi$. This categorises another complete equilibrium. The last complete equilibrium occurs when both (C.1) and (C.2) fail. Lemma 10 provides the optimal policy between $\rho_3$ and $\rho^{in}$.

**Lemma 10.** Suppose that $\lambda_4 \in (\lambda_1, \lambda_2)$ and let $\Delta\psi = \lambda_6$ given by $\bar{V}_{1d}(\rho_3) = \bar{V}_{1b}(\rho^{in})$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_3, & \Delta\psi \geq \lambda_6, \\ \rho^{in}, & \Delta\psi \leq \lambda_6. \end{cases}$$

In addition $\lambda_6 \in (\lambda_4, \lambda_2)$ when both (C.1) and (C.2) fail.
See appendix C.1.1 for the proof.

It follows from lemmas 6-10 that when (C.2) fails, $\lambda_1 < \lambda_6 < \lambda_3$. This implies that all feasible strategies, namely, $\rho_1$, $\rho^{in}$, $\rho_3$, and $\rho_4$ occur in distinct $\Delta\psi$ region. The following proposition summarises the unique first stage equilibria actions.

**Proposition 4.** Let $\rho^*$ denote the equilibrium solution for the home country's price subgame when the TAC is exogenous. In addition, let $\xi = 1$ and $\phi_2 \in (0, 2/5 - \theta]$ with $\theta < 2/5$. Then, there exist three mutually exclusive equilibria that depend on the market and cost parameters as follows:

$$\rho^* = \begin{cases} \rho^1, & \text{(C.1) holds,} \\ \rho^2, & \text{(C.1) fails and (C.2) fails,} \\ \rho^3, & \text{(C.1) fails and (C.2) holds.} \end{cases}$$

See appendix C.3 for the specification of $\rho^1$, $\rho^2$ and $\rho^3$.

Proposition 4 provides a complete characterisation of the home country's optimal strategy in the entire parameter space, i.e., $\alpha \times \beta_1 \times \beta_2 \times \psi_1 \times \psi_2 \in (0,1) \times \mathbb{R}^4_+$, when the foreign agent is welfare-maximiser and the market efficiency parameter is very low. The market parameters determine whether conditions (C.1) and (C.2) hold or not, and thus the home country's distinct equilibrium options. The distinct equilibrium price is then prescribed by the relative cost efficiency of the firms.

So far we have derived the optimal pricing policy merely by comparing the various strategies with each other, and have disregarded the fact that they shall guarantee a non-negative payoff for the home country, i.e., $\bar{V}_1(\rho^*) \geq 0$. Unlike the foreign agent, whose outside option occurs at a distinct strategy, $u_2 = 0$,[9] the home country does not have a zero-payoff pricing strategy by definition. Even if the home country does not exploit the resource, i.e., $u_1 = 0$, the price $\rho = \rho_4$ that it has to charge in order to induce $u_2 = 1$ does not ensure a non-negative payoff. This is true for all other pricing policies. Therefore, we need to evaluate the home country's objective at all equilibrium prices and find the thresholds or bounds that make it zero. Since we have used the cost parameters to distinguish between the different optimality regions, we derive the zero-welfare bounds with respect to them. This yields a continuous cost frontier in the $\psi_2 \times \psi_1$ parameter space, above which it is too costly to exploit the resource for both firms. Details regarding the derivation of the cost frontier are given in appendix C.4 for this case only. In what follows, we briefly summarise the results for all remaining cases and then present the complete solution in subsection 3.5.3. All details can be found in the respective appendices.

#### 3.5.1.2 Low market efficiency

In case the foreign agent is welfare-maximiser and the foreign market efficiency parameter is low, i.e.,$\phi_2 \in (\max(0, 2/5 - \theta), 1/2]$, the sequential equilibrium outcome in the foreign agent's quota and the firms' sales subgames is given by $u_2^2(\rho)$ and $\mathbf{q}^1(u_2)$, respectively, see propositions 1 and 2. Substituting in $\bar{V}_1$ yields the following piecewise function:

$$\bar{V}_1(\rho, u_2^2(\rho), \mathbf{q}^1(u_2^2(\rho))) = \begin{cases} \bar{V}_{1a} = \bar{V}_1(\rho, 0, {}_2\mathbf{q}_{\text{bp}}), & \rho \geq \rho_1, \\ \bar{V}_{1b} = \bar{V}_1(\rho, u_2^{in}(\rho), {}_2\mathbf{q}_{\text{bp}}), & \rho_2 \leq \rho \leq \rho_1, \\ \bar{V}_{1c} = \bar{V}_1(\rho, \phi_2, {}_2\mathbf{q}_{\text{bp}}), & \rho_5 \leq \rho \leq \rho_2, \\ \bar{V}_{1e} = \bar{V}_1(\rho, 1, {}_2\mathbf{q}_{\text{pb}}), & \rho \leq \rho_5, \end{cases} \tag{3.13}$$

which is discontinuous at $\rho_5$. Compared to the very low efficiency case, all remaining sub-functions are the same, sub-function $\bar{V}_{1d}$ no longer exists since $u_2 = 1 - \phi_2$ is a strictly dominated strategy in the foreign agent's quota subgame. Moreover, price thresholds $\rho_3$ and $\rho_4$ are replaced by $\rho_5$. Starting from top, the curvature is constant, concave, upwards linear, and upwards linear. Therefore, the equilibrium candidates are $\rho_1$, $\rho^{in}$, $\rho_2$, and $\rho_5$. Because the process of identifying all unique equilibria is similar to the previous case, we have placed all details on appendix C.1.2 and proceed with proposition 5 that summarises the solution.

**Proposition 5.** Let $\rho^*$ denote the equilibrium solution for the home country's price subgame when the TAC is exogenous. In addition, let $\xi = 1$ and $\phi_2 \in (\max(0, 2/5 - \theta), 1/2]$. Then, there exist two mutually exclusive equilibria that depend on the market and cost parameters as follows:

$$\rho^* = \begin{cases} \rho^4, & \text{(C.3) holds,} \\ \rho^5, & \text{(C.3) fails.} \end{cases}$$

See appendices C.1.2 and C.3 for the specification of (C.3), and $\rho^4$ and $\rho^5$, respectively.

[9]The foreign agent's outside option is zero when $\xi = 0$, or equal to its consumer surplus when $\xi = 1$.

#### 3.5.1.3 Medium market efficiency

In case the foreign agent is welfare-maximiser and the foreign market efficiency parameter is medium, i.e., $\phi_2 \in (1/2, 2/3)$, the sequential equilibrium outcome in the foreign agent's quota and the firms' sales subgames is given by $u_2^4(\rho)$ and $\mathbf{q}^2(u_2)$, respectively, see proposition 1 and 3. Substituting in $\bar{V}_1$ yields the following piecewise multi-valued function:

$$\bar{V}_1(\rho, u_2^4(\rho), \mathbf{q}^2(u_2^4(\rho))) = \begin{cases} \bar{V}_{1a} = \bar{V}_1(\rho, 0, {}_2\mathbf{q}_{\text{bp}}), & \rho \geq \rho_1, \\ \bar{V}_{1b} = \bar{V}_1(\rho, u_2^{in}(\rho), {}_2\mathbf{q}_{\text{bp}}), & \rho_7 \leq \rho \leq \rho_1, \\ \bar{V}_{1c} = \bar{V}_1(\rho, 2-3\phi_2, {}_2\mathbf{q}_{\text{bp}}), & \rho_8 \leq \rho \leq \rho_7, \\ \bar{V}_1(\rho_8, u_2, {}_2\mathbf{q}_{\text{pp}}), \forall u_2 \in [2-3\phi_2, 3\phi_2 - 1], & \rho = \rho_8, \\ \bar{V}_{1d} = \bar{V}_1(\rho, 3\phi_2 - 1, {}_2\mathbf{q}_{\text{pb}}), & \rho_9 \leq \rho \leq \rho_8, \\ \bar{V}_{1e} = \bar{V}_1(\rho, 1, {}_2\mathbf{q}_{\text{pb}}), & \rho \leq \rho_9, \end{cases} \tag{3.14}$$

which is discontinuous at $\rho_9$, and at $\rho_8$ its value is associated with multiple elements, namely all $u_2$ in the $[2-3\phi_2, 3\phi_2-1]$ region. Compared to the very low and low efficiency cases, i.e., $\phi_2 \leq 1/2$, sub-functions $\bar{V}_{1a}$, $\bar{V}_{1b}$ and $\bar{V}_{1e}$ remain the same, whereas $\bar{V}_{1c}$ and $\bar{V}_{1d}$ differ since they are evaluated at $u_2 = 2 - 3\phi_2$ and $u_2 = 3\phi_2 - 1$, respectively, but their curvature remains the same. The five equilibrium candidates are $\rho_1$, $\rho^{in}$, $\rho_7$, $\rho_8$, and $\rho_9$. As before, we proceed with proposition 6 that summarises the solution. For additional details see appendix C.1.4.

**Proposition 6.** Let $\rho^*$ denote the equilibrium solution for the home country's price subgame when the TAC is exogenous. In addition, let $\xi = 1$ and $\phi_2 \in (1/2, 2/3)$. Then, there exist two mutually exclusive equilibria that depend on the market and cost parameters as follows:

$$\rho^* = \begin{cases} \rho^6, & \text{(C.4) holds}, \\ \rho^7, & \text{(C.4) fails}. \end{cases}$$

See appendices C.1.4 and C.3 for the specification of (C.4), and $\rho^6$ and $\rho^7$, respectively.

#### 3.5.1.4 High market efficiency

In case the foreign market efficiency parameter is high, i.e., $\phi_2 \geq 2/3$, the sequential equilibrium outcome is independent of the foreign agent's type and is given by $u_2^5(\rho)$ and $\mathbf{q}^3(u_2)$, see propositions 1 and 3. Substituting in $\bar{V}_1$ yields the following piecewise multi-valued function:

$$\bar{V}_1(\rho, u_2^5(\rho), \mathbf{q}^3(u_2^5(\rho))) = \begin{cases} \dfrac{\beta_2 - 1}{\beta_2} - \psi_1, & \rho > \rho_8, \\ (1-u_2)\left(\dfrac{\beta_1 - 1}{\beta_2} - \psi_1\right) + \rho u_2, \ \forall u_2 \in [0,1], & \rho = \rho_8, \\ \rho, & \rho < \rho_8, \end{cases} \tag{3.15}$$

where $\rho_8 = (\beta_2 - 1)/\beta_2 - \psi_2$. Figure 3.5 (f) plots the objective when $\phi_2 \geq 2/3$. The multi-valued property of the objective does not exist under cost symmetry. Otherwise, at $\rho = \rho_8$ the foreign agent is indifferent between all $u_2 \in [0,1]$. The home country prefers to keep the entire quota for its firm when it has a cost advantage, i.e., $\psi_1 < \psi_2$, otherwise it prefers to sell it at the higher possible price, which is $\rho_8$. Assuming that the foreign agent buys accordingly, the optimal strategy is given by

$$\rho^* = \rho^8 = \begin{cases} \rho_8, & \psi_1 \geq \psi_2, \\ \rho \in \mathbb{R} : \rho > \rho_8, & \psi_1 \leq \psi_2. \end{cases}$$

### 3.5.2 Profit-maximiser foreign agent

#### 3.5.2.1 Low market efficiency

In case the foreign agent is profit-maximiser and the foreign market efficiency parameter is low, i.e., $\phi_2 \in (0, 2/7]$, the sequential equilibrium outcome in the foreign agent's quota and the firms' sales subgames is given by $u_2^2(\rho)$ and $\mathbf{q}^1(u_2)$, respectively, see proposition 1 and 2. Substituting in $\bar{V}_1$ yields the following piecewise function:

$$\bar{V}_1(\rho, u_2^2(\rho), \mathbf{q}^1(u_2^2(\rho))) = \begin{cases} \bar{V}_{1a} = \bar{V}_1(\rho, 0, {}_2\mathbf{q}_{\text{bp}}), & \rho \geq \rho_1, \\ \bar{V}_{1b} = \bar{V}_1(\rho, u_2^{in}(\rho), {}_2\mathbf{q}_{\text{bp}}), & \rho_2 \leq \rho \leq \rho_1, \\ \bar{V}_{1c} = \bar{V}_1(\rho, \phi_2, {}_2\mathbf{q}_{\text{bp}}), & \rho_5 \leq \rho \leq \rho_2, \\ \bar{V}_{1e} = \bar{V}_1(\rho, 1, {}_2\mathbf{q}_{\text{pb}}), & \rho \leq \rho_5, \end{cases} \tag{3.16}$$

which is discontinuous at $\rho_5$ and is similar to the objective when the foreign agent is welfare-maximiser and the market efficiency parameter is low, see subsection 3.5.1.2. Their difference lies in the magnitude of the price thresholds and $\bar{V}_{1b}$, which depend on $\xi$. The curvature remains the same, since $\bar{V}_{1b}$ is concave.[10] The four equilibrium candidates are $\rho_1$, $\rho^{in}$, $\rho_2$ and $\rho_5$. All solution details can be found in appendix C.2.1, proposition 7 summarises the equilibrium outcomes.

**Proposition 7.** Let $\rho^*$ denote the equilibrium solution for the home country's price subgame when the TAC is exogenous. In addition, let $\xi = 0$ and $\phi_2 \in (0, 2/7]$. Then, there exist two mutually exclusive equilibria that depend on the market and cost parameters as follows:

$$\rho^* = \begin{cases} \rho^9, & \text{(C.5) holds,} \\ \rho^{10}, & \text{(C.5) fails.} \end{cases}$$

See appendices C.2.1 and C.3 for the specification of (C.5), and $\rho^9$ and $\rho^{10}$.

#### 3.5.2.2 Medium market efficiency

In case the foreign agent is profit-maximiser and the foreign market efficiency parameter is medium, i.e., $\phi_2 \in (2/7, 2/3)$, the sequential equilibrium outcome in the foreign agent's quota and the firms' sales subgame is given by $u_2^3(\rho)$ and $\mathbf{q}^1(u_2)$, respectively, see propositions 1, 2 and 3. Substituting in $\bar{V}_1$ yields the following piecewise function:

$$\bar{V}_1(\rho, u_2^3(\rho), \mathbf{q}^1(u_2^3(\rho))) = \begin{cases} \bar{V}_{1a} = \bar{V}_1(\rho, 0, {}_2\mathbf{q}_{\text{bp}}), & \rho \geq \rho_1, \\ \bar{V}_{1b} = \bar{V}_1(\rho, u_2^{in}(\rho), {}_2\mathbf{q}_{\text{bp}}), & \rho_6 \leq \rho \leq \rho_1, \\ \bar{V}_{1e} = \bar{V}_1(\rho, 1, {}_2\mathbf{q}_{\text{pb}}), & \rho \leq \rho_6, \end{cases} \tag{3.17}$$

which is discontinuous at $\rho_6$. The top branch is constant, the middle is concave, and the lower is linear and increasing. The three equilibrium candidates are $\rho_1$, $\rho^{in}$ and $\rho_6$. All solutions details can be found in appendix C.2.3, proposition 8 summaries our findings.

**Proposition 8.** Let $\rho^*$ denote the equilibrium solution for the home country's price subgame when the TAC is exogenous. In addition, let $\xi = 0$ and $\phi_2 \in (2/7, 2/3)$. Then, there exist two mutually exclusive equilibria that depend on the market and cost parameters as follows:

$$\rho^* = \begin{cases} \rho^{11}, & \text{(C.6) holds,} \\ \rho^{12}, & \text{(C.6) fails.} \end{cases}$$

See appendices C.2.3 and C.3 for the specificaiton of (C.6), and $\rho^{11}$ and $\rho^{12}$.

[10] $\bar{V}_{1b}'' = -\beta_2(5\alpha\beta_2 + 6\beta_1)/(4\beta_1) < 0$, thus the function remains concave when $\xi = 0$.

### 3.5.3 Complete solution

The complete solution of the home country's quota price subgame follows from propositions 4 to 8 and is given by the following piecewise continuous functions:

$$\rho^*|_{\xi=1} = \begin{cases} \rho^1, & \phi_2 \in (0, \max(0, 2/5 - \theta)] & \text{and (C.1) H,} \\ \rho^2, & \phi_2 \in (0, \max(0, 2/5 - \theta)] & \text{and (C.1) F and (C.2) F,} \\ \rho^3, & \phi_2 \in (0, \max(0, 2/5 - \theta)] & \text{and (C.1) F and (C.2) H,} \\ \rho^4, & \phi_2 \in (\max(0, 2/5 - \theta), 1/2] & \text{and (C.3) H,} \\ \rho^5, & \phi_2 \in (\max(0, 2/5 - \theta), 1/2] & \text{and (C.3) F,} \\ \rho^6, & \phi_2 \in (1/2, 2/3) & \text{and (C.4) F,} \\ \rho^7, & \phi_2 \in (1/2, 2/3) & \text{and (C.4) F,} \\ \rho^8, & \phi_2 \in [2/3, \infty), \end{cases} \quad \rho^*|_{\xi=0} = \begin{cases} \rho^9, & \phi_2 \in (0, 2/7] & \text{and (C.5) H,} \\ \rho^{10}, & \phi_2 \in (0, 2/7] & \text{and (C.5) F,} \\ \rho^{11}, & \phi_2 \in (2/7, 2/3) & \text{and (C.6) H,} \\ \rho^{12}, & \phi_2 \in (2/7, 2/3) & \text{and (C.6) F,} \\ \rho^8, & \phi_2 \in [2/3, \infty), \end{cases}$$

where H stands for holds and F for fails. Figure 3.6 depicts all possible equilibria that can occur in the home country's price subgame, and thus gives a complete picture of the outcome for the entire game when the TAC is exogenous. The type of the foreign agent and the market parameters determine the set of optimal pricing policies, which we have grouped according to the firms cost of fishing in $\rho^m$, $m = 1, \ldots, 12$.[11] For each equilibrium $\rho^m$ there exist regions in the cost parameter space where different prices are optimal. The outer curve represents the cost frontier, above which it is not worth for both the foreign agent and the home country to have their firms exploit the resource. The dashed line is the 45-degree symmetry line.

First, we consider the cases where the foreign agent is welfare-maximiser and the market efficiency parameter is very low. Then, the optimal pricing regions are given by subfigures (a), (b) and (c) depending on whether conditions (C.1) and (C.2) hold or not. Because $\lambda_1$ is negative and $\lambda_2$ and $\lambda_6$ are strictly positive, the optimal pricing strategy in all subcases under cost symmetry is given by $\rho^{in}$ with the home firm getting the majority of quotas since $1 - u_2^{in} \geq 1 - \phi_2 > 1/2$.[12,13] If the home firm's cost is less than $\psi_2 + \lambda_1 < \psi_2$, then it is optimal for the home country to price at $\rho_1$ and induce the foreign agent to buy zero quotas. On the other hand, when the home firm's cost exceeds $\psi_2 + \lambda_5$ or $\psi_2 + \lambda_3$, it is optimal to price at $\rho_4$ and induce the foreign agent to buy the entire TAC. For cost differences between $\lambda_5$ and $\lambda_1$, or, $\lambda_3$ and $\lambda_1$ the foreign agent buys a percentage of the TAC and both firms are active. Based on the relevant magnitude of the home country's pricing and the foreign agent's quota purchasing strategies, we infer that the higher the cost disadvantage the home firm has, the lower the quota price the home country charges, and thus the higher the amount of quotas the foreign agent purchases.

The pricing regions in all remaining cases where the foreign agent is welfare-maximiser follow more or less the same pattern. Subfigures (d) and (e) illustrate the low efficiency cases, (f) and (g) the medium efficiency cases, and (h) the high efficiency case. Note that the threshold $\lambda_9$ in the medium efficiency cases is plotted as positive but it can also be negative. The optimal strategy under cost symmetry in this case depends on the sign of $\lambda_9$ and is either $\rho^{in}$ or $\rho_7$. Either way, the foreign agent purchases a positive amount of quotas and both firms are active in equilibrium. For the case of high market efficiency, which is identical when the foreign agent is profit-maximiser, the home country is indifferent between keeping the TAC at home or selling it to the foreign agent when both firms have the same cost. In case of cost asymmetry, the optimal pricing policy

[11] The market parameters affect the size but not the shape of the equilibrium regions.

[12] See lemmas 6 for the signs of $\lambda_1$ and $\lambda_2$. The sign of $\lambda_6$ follows from lemma 10 and the fact that $\lambda_4 > 0$ when $\phi_2 < 1/2$.

[13] Strategy $u_2^{in}$ is the inner solution of a concave function bounded in the $[0, \phi_2]$ region, thus $u_2^{in} \leq \phi_2$.

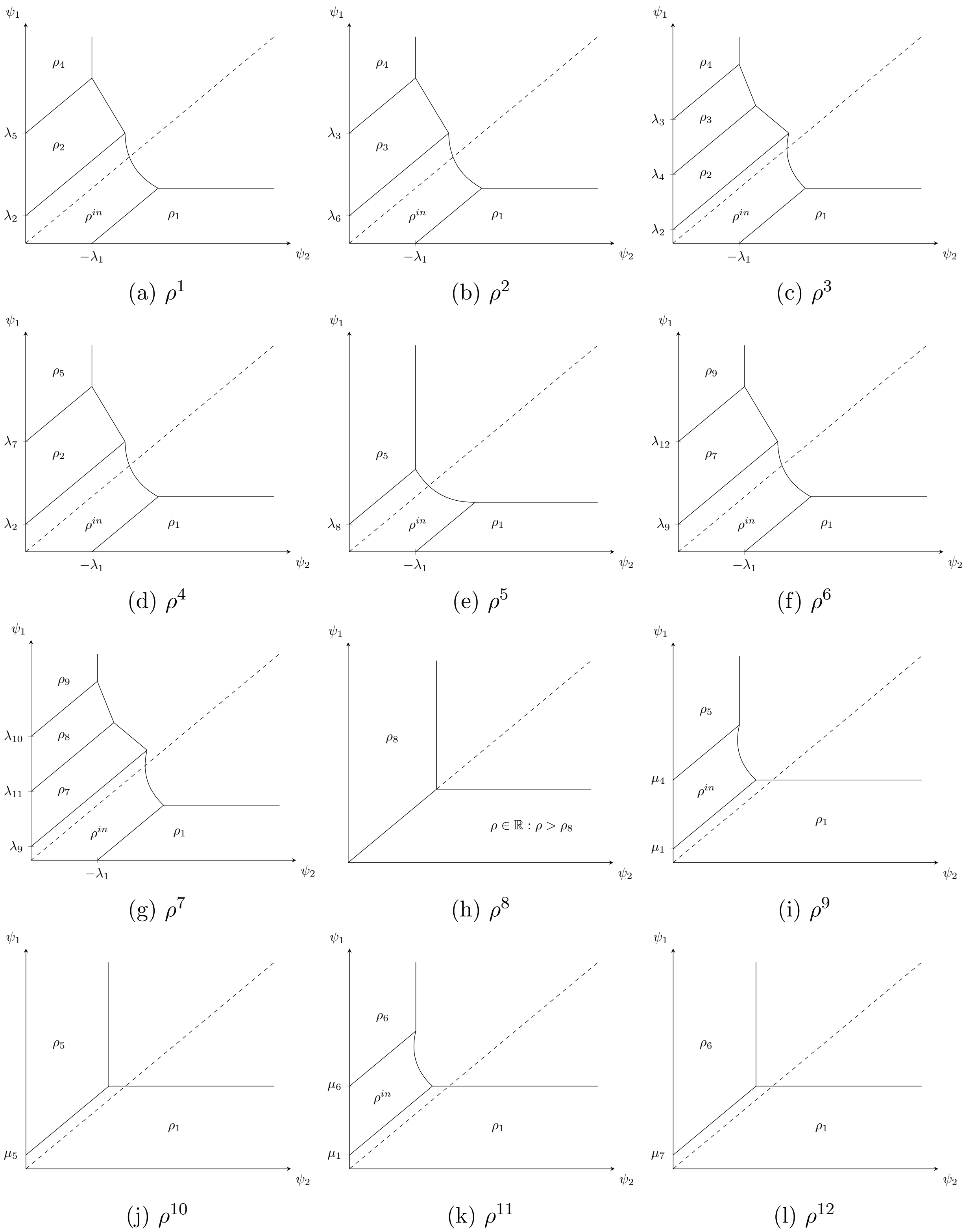
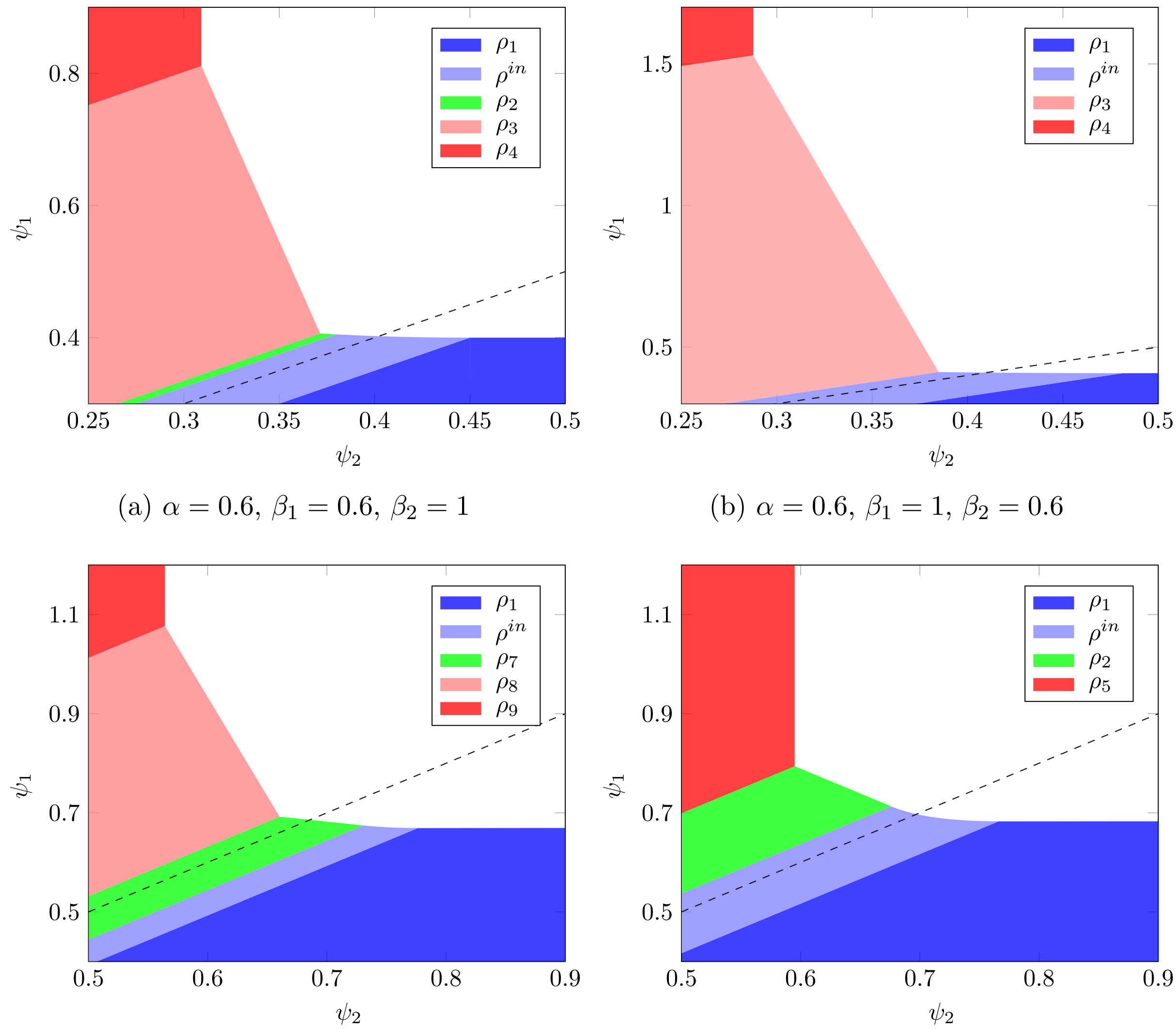



Figure 3.6. Home country's optimal pricing policies in the entire parameter space, i.e., $\alpha \times \beta_1 \times \beta_2 \times \psi_1 \times \psi_2 \times \xi \in (0,1) \times \mathbb{R}_+^4 \times \{0,1\}$. The market parameters and the type of the foreign agent determine the distinct equilibrium $\rho^m$, with $m = 1, \dots, 12$. The optimal pricing policy is inferred by the cost parameters. The dashed line is the 45-degree symmetry line. The magnitude of the market parameters affect the size but not the shape of the equilibrium regions.




(a) $\alpha = 0.6$, $\beta_1 = 0.6$, $\beta_2 = 1$

(b) $\alpha = 0.6$, $\beta_1 = 1$, $\beta_2 = 0.6$

(c) $\alpha = 0.4$, $\beta_1 = 3$, $\beta_2 = 3$

(d) $\alpha = 0.5$, $\beta_1 = 3$, $\beta_2 = 3$

Figure 3.7. Home country's optimal pricing policies in the cost parameter space for different market parameters. The dashed line is the 45-degree line. The foreign agent is assumed to be welfare-maximiser.

is such that the most efficient firm takes the entire TAC, see subfigure (h). It is worth mentioning that in the case of high market efficiency, strategic interaction between firms ceases to exist because only one remains active.

For the case of a profit-maximiser foreign agent, the home country's equilibrium price regions are depicted in subfigures (i) and (j) when market efficiency is low, and in (k) and (l) when it is medium. The interesting thing here is the fact that entry into the fishery for the foreign firm is deterred under cost symmetry in all cases since $\mu_1$, $\mu_5$ and $\mu_7$ are strictly positive. The reason why this happens is that the consumer surplus, which is a strictly positive quantity when the foreign market is the preferred one, irrespective of the foreign agent's action, is disregarded in its optimisation procedure. This decreases its outside option, namely the share of the foreign consumer surplus attributed to sales by the home firm, to zero, and thus shifts the regions where the foreign agent affords a positive quota share above the 45-degree symmetry line. In order to balance the fact that the consumer surplus is disregarded, the foreign firm has to be more efficient in order for the foreign agent to accept a price that induces a positive quota share.

In order to investigate how the magnitude of the market parameters influences the size of the price regions, we plot the corresponding equilibria in figure 3.7 for some parameter vectors when the foreign agent is welfare-maximiser. In particular, we distinguish between cases where the market saturation parameters vary in subfigures (a) and (b), and cases where the relative maximum price varies in subfigures (c) and (d). The different price regions are illustrated in shades of blue. The white region represents the area in the cost-space where nothing happens, i.e., the resource remains unexploited.

The respective equilibria in (a) and (b) are $\rho^3$ and $\rho^2$. The difference lies in the market saturation quantities where in (a) the foreign market is able to absorb the entire quota, i.e., $\beta_2 = 1$, whereas in (b) the home market can do so. A significant change in the price regions occurs in the area where price $\rho_3$ is optimal, which expands vertically and nearly doubles when $\beta_1 = 1$, see subfigure (b). This means that it is now optimal for the home country to grant its firm a positive share of quotas for nearly double the cost disadvantage. In addition, strategy $\rho_2$ is only optimal when $\beta_2 = 1$.

Moving on to (c) and (d), the respective equilibria are $\rho^7$ and $\rho^4$. Keep in mind that because $\phi_2$ is equal to one-half when $\alpha = 0.5$ and $\beta_2 = 3$, the following price strategies are equal: $\rho_7 = \rho_2$ and $\rho_9 = \rho_5$, and thus the price areas are comparable. Compared to (c), strategy $\rho_8$ is no longer optimal in (d) and the area of $\rho_9$ expands vertically implying that it is optimal for the home country to sell the entire TAC to the foreign agent even though the home firm may be more efficient. In addition, the optimal pricing policy under cost symmetry switches from $\rho^{in}$ when $\alpha = 0.4$ to $\rho_2$ when $\alpha = 0.5$.

## 3.6 Endogenous TAC

In this section we drop the assumption of exogenous TAC and address the problem of optimal fishing within the context of the game theoretic framework we have just presented. The purpose here is not to dwell into details regarding the type of the fishery and its biology but rather to illustrate how our model can be incorporated into the standard bioeconomic framework. Therefore, we focus on the simplest possible case, namely that of sustainable fisheries management, and show how this influences the optimal price regions. This means that the home country's additional optimisation problem is static. It should be noted though that the strategic framework developed here can also be embodied within a dynamic setting since its sophistication lies solely in the complexity of the objective function, which is autonomous as it is typically assumed in most dynamic fishery models.

For the biological model we follow the standard framework used in single-species fisheries management (Clark, 2010) where the evolution of the stock biomass is described by the following dynamic equation:

$$\frac{dx}{dt} = G(x) - U. \tag{3.18}$$

Function $G(x)$ describes the net natural growth of the resource for a given stock biomass level, $x$, and $U$ is the total harvest, which is the TAC within our context. To set the TAC in a sustainable manner means that for any stock level $x$ the TAC is equal to the natural growth function, i.e., $U = G(x)$. This is equivalent to saying that the home country decides on a sustainable stock biomass level that maximises its welfare, while taking into consideration how this influences the outcome in all subsequent subgames, namely, the optimal price, the optimal foreign agent' quota, and the optimal sale strategies.

The home country's optimisation problem can be described by maximising the following implicit function:

$$V_1\big(x, p^*(x), U_2^*(\cdot), \mathbf{Q}^*(U_2^*(\cdot))\big), \tag{3.19}$$

for $x \geq 0$, where $U_2^*(\cdot) \equiv U_2^*(x, p^*(x))$. The arguments $p^*$, $U_2^*$ and $\mathbf{Q}^*$ are the optimal equilibria strategies of all subsequent subgames scaled back to the initial units. The reason is that in the exogenous case the TAC has been scaled to unity for convenience. Because the optimal equilibria are piecewise functions of the stock biomass, the above implicit function is also piecewise, see appendix D for its complete specification.

To scale back all equilibria and relevant quantities to the initial units the transformations described in section 3.2.4 are applied in reverse. For example, $p^* = a_2\rho^*$, $U_2^* = Uu_2^*$ and $Q^* = Uq^*$. The unit of the market efficiency parameter is as in $q_i$. Thus, when expressed in the initial units it becomes:

$$\varPhi_i \equiv U\phi_i = \frac{b_i}{3}\left(1 - \frac{a_j}{a_i}\right), \quad \forall i, j = 1, 2 : i \neq j. \tag{3.20}$$

The units of all price and cost thresholds follow from $\rho$ and $\psi_i$ respectively, thus $p_i = a_2\rho_i$ for all $i = 1, \ldots, 9$, $\varLambda_i = a_2\lambda_i$ for all $i = 1, \ldots, 12$, and $M_i = a_2\mu_i$ for all $i = 1, \ldots, 7$. Keep in mind that the cost thresholds are stock dependent, i.e., $\varLambda_i \equiv \varLambda_i(x)$ and $M_i \equiv M_i(x)$. This is because the cost parameters are stock dependent, and this dependency becomes "visible" when the TAC is endogenous.

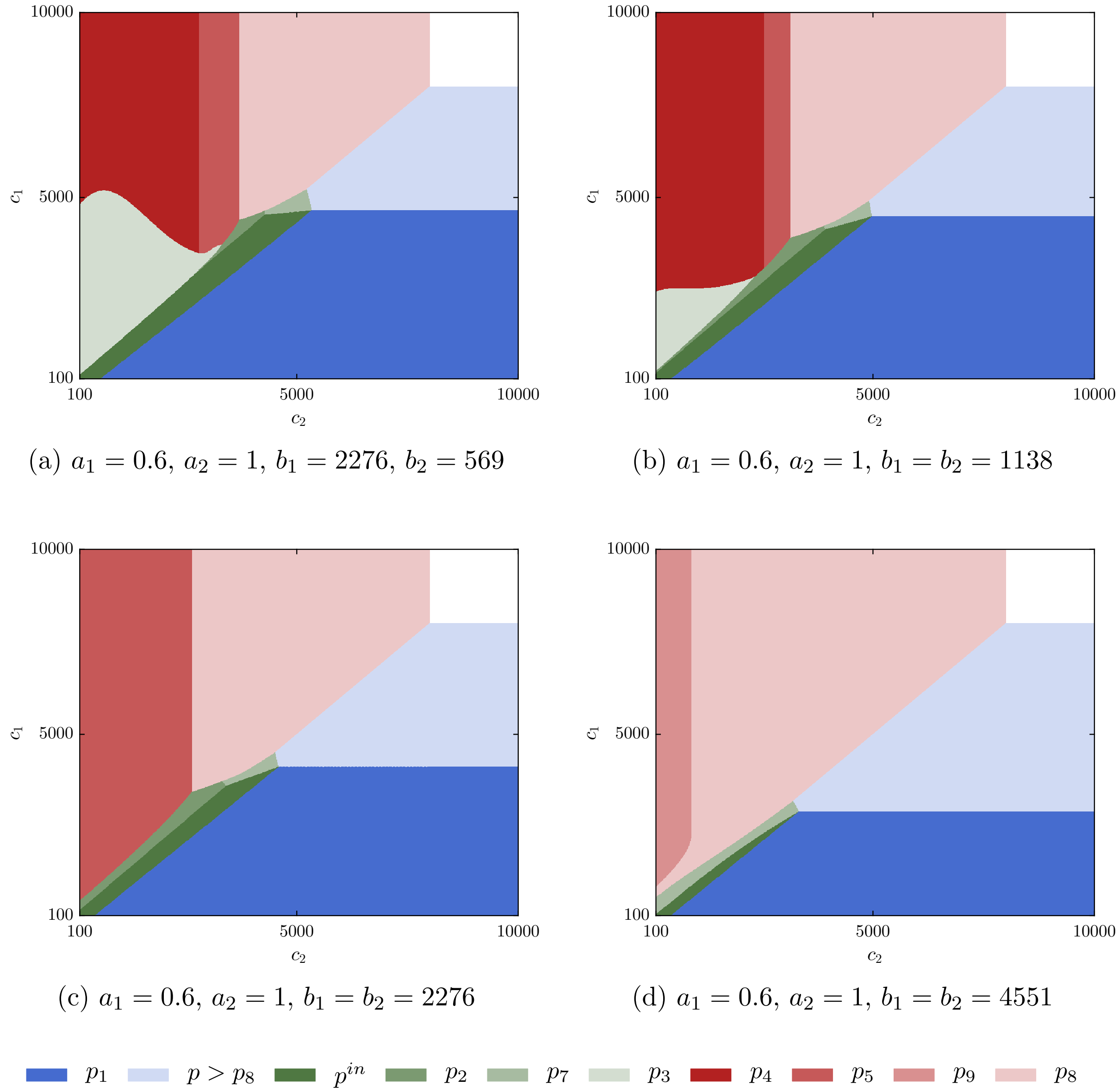
Because of the complexity of the objective, we solve it numerically for different market parameters in a predefined cost parameter space, i.e., $c_1 \times c_2 \in [100, 10000]^2$, in order to show how the areas of the different price policies change. For the growth function, we assume a modified logistic growth skewed to the left, which is given by

$$G(x) = rx^2\left(1 - \frac{x}{k}\right), \tag{3.21}$$

where parameters $r$ and $k$ are the respective intrinsic growth rate and carrying capacity of the resource. The maximum sustainable yield (MSY) biomass level occurs at $x_{\mathrm{MSY}} = 2k/3$ and the TAC at that level is given by $U_{\mathrm{MSY}} = 4rk^2/27$.

In the numerical examples the foreign agent is assumed to be welfare-maximiser, i.e., $\xi = 1$, and the biological and economic parameters are fixated as follows. The growth parameters are set to $r = 12 \times 10^{-5}$ and $k = 8000$, and the maximum prices to $a_1 = 0.6$ and $a_2 = 1$. For the market saturation parameters $b_1$ and $b_2$, we consider the following four scenarios based on $U_{\mathrm{MSY}}$: i) $b_1 = 2U_{\mathrm{MSY}}$, $b_2 = U_{\mathrm{MSY}}/2$, ii) $b_1 = b_2 = U_{\mathrm{MSY}}$, iii) $b_1 = b_2 = 2U_{\mathrm{MSY}}$, and iv) $b_1 = b_2 = 4U_{\mathrm{MSY}}$. By changing the market saturation parameters we allow for changes in the market efficiency parameter $\varPhi_2$, which is increasing across the four scenarios. The first combination creates a crossing point in the inverse demand functions. This means that there exists a quantity level that yields the same price in both markets and therefore any quantity below it makes the foreign market more attractive, whereas any quantity above it makes the home market more attractive. In the remaining scenarios, the foreign inverse demand always exceeds the inverse demand in the home country. Note that although the market efficiency parameter is fixed for a specific scenario, the designations very low, low, medium, and high that we have applied in the exogenous TAC analysis are no longer fixed but they depend on the magnitude of the TAC.

Figure 3.8 shows the equilibrium price regions when the TAC is endogenous for all numerical scenarios. In total there are ten distinct pricing strategies that we have grouped




(a) $a_1 = 0.6$, $a_2 = 1$, $b_1 = 2276$, $b_2 = 569$

(b) $a_1 = 0.6$, $a_2 = 1$, $b_1 = b_2 = 1138$

(c) $a_1 = 0.6$, $a_2 = 1$, $b_1 = b_2 = 2276$

(d) $a_1 = 0.6$, $a_2 = 1$, $b_1 = b_2 = 4551$

Figure 3.8. Home country's optimal price regions in the cost parameter space for different market realisations when the TAC is endogenous. The foreign agent is welfare-maximiser.

in table 3.4 based on the TAC allocation outcome. Note that price strategy $p_8$ induces the foreign agent to buy either the entire TAC, when $\Phi_2 \geq 2U/3$, or a percentage of it, otherwise. In contrast to the exogenous TAC where the maximum number of distinct pricing strategies for a given market specification is five, see figures 3.6 (c) and (g), almost all pricing strategic can become feasible when the TAC is endogenous. The reason why this is happening is that when the TAC is fixed, it is only profitable to operate in a limited subspace of the cost-space that occurs below the cost frontier. Allowing for the TAC to be endogenous means that we allow for adjustments, both positive and negative, within the cost-space, which increases the pricing options the home country has for any market realisation. In addition to this increase in flexibility, the home country is able to reshape the equilibrium price regions since they now depend on both the optimal TAC and biomass level. The most profound change in our examples occurs in figure 3.8 (a) between price strategies $p_3$ and $p_4$ where the indifference cost threshold is given by a

Table 3.4. Home country's price strategies grouped according to the TAC allocation outcome, when the foreign agent is welfare-maximiser. Note that the TAC allocation outcome is the same irrespective of whether the TAC is exogenous or endogenous.

| TAC allocation outcome | Price strategies | Reference colour in Fig. 3.8 |
|---|---|---|
| Home firm gets the entire TAC | $p_1; \forall p : p > p_8$ | Shades of blue |
| Foreign firm gets the entire TAC | $p_4; p_5; p_8; p_9$ | Shades of red |
| Firms share the TAC | $p^{in}; p_2; p_3; p_7; p_8$ | Shades of green |

concave shaped bound. The corresponding bound when the TAC is exogenous is given by $\psi_2 + \lambda_3$, see figure 3.6 (c), which is linear since $\lambda_3$ is constant. This is no longer true when the TAC is endogenous since $\Lambda_3(x)$ is a polynomial function of $x$ of degree seven.

Regarding the optimal price strategies, starting from figures 3.8 (a) and (b) all strategies but $p_9$ are feasible, however the size of their region differs. There is a significant reduction in the area of $p_3$, where now it is optimal to mostly price according to $p_4$. In figure 3.8 (c) strategies $p_3$ and $p_4$ are no longer optimal. This is because the market efficiency parameter is now given by $\Phi_2 = 303.46$, which always exceeds the $\max(0, K(x^*))$ threshold that makes the two strategies suboptimal.[14] Compared to the exogenous case this occurs when $\phi_2 > \max(0, 2/5 - \theta)$, i.e., whenever the market efficiency parameter is not very low, see figures 3.6 (d) to (h). Finally, in figure 3.8 (d) $\Phi_2 = 606.8$ and only a handful of strategies are now optimal, which corresponds to the exogenous cases of medium and high market efficiency, i.e., $\phi_2 \geq 1/2$. Price $p_9$ now becomes optimal, and the most prevailing strategies are $p_1$, $p > p_8$ and $p_8$, which imply that the either the home or the foreign firm gets the entire TAC.

To sum up, the equilibrium price options in the case where the TAC is endogenous have significantly increased, which implies that there is more flexibility in the decision-making process of the home country. Regarding how the TAC is allocated between the firms, the red shaded areas represent the cases where the foreign firm gets everything, the blue shaded areas the cases where the home firm gets everything, and the green shaded areas the cases where both firms are active. As the foreign market efficiency parameter increases, the areas in the cost-space where both firms are active are getting confined around the 45-degree symmetry line. For very-low/low market efficiency levels, the home firm is active despite the fact that it has a significantly higher cost compared to the foreign firm, e.g. area $p_3$ in figure 3.8 (a).

## 3.7 Conclusion

In this paper we introduce a framework in an attempt to understand and quantify the basis upon which fisheries agreements are being drawn up. The framework is based on a game theoretic model that captures, among other things, how much it is worth for a coastal state to give access to its fishery resources to foreign fleets. In order to focus on the strategic interaction between the players, the problems of how much to fish and who should fish are decoupled and dealt separately. In particular, we assume that for any fixed period of time the TAC is known and categorise all possible strategic outcomes according to the foreign agent's type, and the market and cost parameters. After all outcomes are

[14] The definition of $K(x)$ is given by $U(2/5 - \theta)$ or $2G(x)/5 - (a_2 - a_1)b_1/(10a_1)$.

identified, we numerically optimise over them in order to determine the optimal TAC level for the case of a sustainable exploited single-species fishery.

The game where the TAC is exogenous consists of three sequential subgames: a quota pricing subgame, a quota purchasing subgame, and a sales subgames. For the case of a welfare-maximiser foreign agent, both firms are active under cost symmetry, whereas entry for the foreign firm is deterred when the foreign agent is profit-maximiser. This happens because the consumer surplus in the foreign market, which is strictly positive in our context, is disregarded in the foreign agent's optimisation procedure when it acts on behalf of a foreign firm instead of a foreign country. Under cost asymmetry, the home country has multiple pricing strategies at its disposal depending on the foreign market's efficiency level and the firms' fishing costs. Based on the relevant magnitude of the home country's pricing and the foreign agent's quota purchasing strategies, we infer that the higher the cost disadvantage the home firm has, the lower the quota price the home country charges, and thus the higher the amount of quotas the foreign agent purchases.

Compared to the exogenous case, when the TAC is endogenous, the home country has more flexibility with respect to its pricing options for given market realisations. Though in both cases, the number of options depends on the foreign market efficiency parameter, and decreases as it increases. For medium and high market efficiency levels the prevailing price strategies in the cost-space induce a "bang-bang" type of behaviour especially when there are significant fishing cost differences between firms. This means that either the home or the foreign firm gets the entire TAC. Finally, the equilibrium price regions are reshaped, which reflects the dependency between their bounds and the optimal TAC and stock biomass level.

A natural extension of the current research is to apply the proposed framework to specific fisheries and compare the results with those from existing agreements. For example, the SFPAs conducted between the EU and non-EU countries, which allow the EU fleet to fish in the signatory countries' EEZ in exchange for financial and sectoral support.


## Acknowledgements

The authors are grateful to Ondřej Osička for help with some of the proofs. Financial support from the Norwegian Research Council through the MESSAGE Project (Grant No. 255530/E40) is gratefully acknowledged.


# Appendices

## A Firms' sales subgame

Let $z_i(q_j)$, with $i, j = 1, 2 : i \neq j$, denote the inner solution of the firms optimisation problems (3.4) and (3.5):

$$\frac{\partial \bar{R}_1}{\partial q_1} = 0 \Leftrightarrow z_1(q_2) = \frac{2u_1\beta_1 - \alpha\beta_2 u_2 - (1-\alpha)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)} + \frac{q_2}{2}$$
$$\frac{\partial \bar{R}_2}{\partial q_2} = 0 \Leftrightarrow z_2(q_1) = \frac{2\alpha\beta_2 u_2 - \beta_1 u_1 + (1-\alpha)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)} + \frac{q_1}{2}.$$

Moreover, let $\underline{q_j}$ and $\bar{q_j}$, with $j = 1, 2$, be the thresholds that bound the inner solution within the feasible region, i.e., $\bar{q_j} : z_i(\bar{q_j}) = u_i$ and $\underline{q_j} : z_i(\underline{q_j}) = 0$

$$\underline{q_1} = \frac{\beta_1 u_1 - 2\alpha\beta_2 u_2 - (1-\alpha)\beta_1\beta_2}{\alpha\beta_2 + \beta_1}, \qquad \bar{q_1} = \frac{\beta_1(u_1 + 2u_2) - (1-\alpha)\beta_1\beta_2}{\alpha\beta_2 + \beta_1},$$
$$\underline{q_2} = \frac{\alpha\beta_2 u_2 - 2\beta_1 u_1 + (1-\alpha)\beta_1\beta_2}{\alpha\beta_2 + \beta_1}, \qquad \bar{q_2} = \frac{\beta_2(\alpha u_2 + 2\alpha u_1) + (1-\alpha)\beta_1\beta_2}{\alpha\beta_2 + \beta_1}.$$

Given the structure of the best response functions, see equation (3.6), there are nine potential equilibria pairs $(q_1^*, q_2^*)$ that can occur in the sales subgame. In what follows, we show which pairs are feasible in equilibrium, derive necessary conditions for their existence, and show that they are mutually exclusive.

## A.1 Proof of proposition 1

**1. Suppose that** $(u_1, u_2)$ **is an equilibrium.** Then, from (3.6), the following must hold: $q_1^* = u_1 \geq \bar{q_1}$ and $q_2^* = u_2 \geq \bar{q_2}$. Let $u_1 \geq f_1(u_2) : u_1 \geq \bar{q_1}$ and $u_1 \leq f_2(u_2) : u_2 \geq \bar{q_2}$. Expressions $f_1$ and $f_2$ are given by

$$f_1(u_2) = \frac{(\alpha-1)\beta_1}{\alpha} + \frac{2\beta_1}{\alpha\beta_2}u_2, \quad f_2(u_2) = \frac{(\alpha-1)\beta_1}{2\alpha} + \frac{\beta_1}{2\alpha\beta_2}u_2.$$

The pair $(u_1, u_2)$ is an equilibrium if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq f_1(u_2) \wedge u_1 \leq f_2(u_2)\}$ is a non-empty set. For any given combination of $\alpha$, $\beta_1$, and $\beta_2$ one can verify that the above set is empty, thus $(u_1, u_2)$ cannot be an equilibrium.

**2. Suppose that** $(0, 0)$ **is an equilibrium.** Then, from (3.6), the following must hold: $q_1^* = 0 \leq \underline{q_1}$ and $q_2^* = 0 \leq \underline{q_2}$. Let $u_1 \geq f_3(u_2) : 0 \leq \underline{q_1}$ and $u_1 \leq f_4(u_2) : 0 \leq \underline{q_2}$. Expressions $f_3$ and $f_4$ are given by

$$f_3(u_2) = (1-\alpha)\beta_2 + \frac{2\alpha\beta_2}{\beta_1}u_2, \quad f_4(u_2) = \frac{(1-\alpha)\beta_2}{2} + \frac{\alpha\beta_2}{2\beta_1}u_2.$$

The pair $(0, 0)$ is an equilibrium if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq f_3(u_2) \wedge u_1 \leq f_4(u_2)\}$ is a non-empty set. For any given combination of $\alpha$, $\beta_1$, and $\beta_2$ one can verify that the above set is empty, thus $(0, 0)$ cannot be an equilibrium.

**3. Suppose that** $(z_1(z_2(q_1)), z_2(z_1(q_2)))$ **is an equilibrium.** The equilibrium solution is given by

$$q_1^* = \frac{\beta_1(3u_1 - (1-\alpha)\beta_2)}{3(\alpha\beta_2 + \beta_1)}, \quad q_2^* = \frac{\beta_2(3\alpha u_2 + (1-\alpha)\beta_1)}{3(\alpha\beta_2 + \beta_1)}.$$

Then, from (3.6), the following must hold: $\underline{q_1} \leq q_1^* \leq \bar{q_1}$ and $\underline{q_2} \leq q_2^* \leq \bar{q_2}$. Let $u_2 \geq f_5 : q_1^* \geq \underline{q_1}$, $u_2 \geq f_6 : q_1^* \leq \bar{q_1}$, $u_1 \geq f_7 : q_2^* \geq \underline{q_2}$, and $u_1 \geq f_8 : q_2^* \leq \bar{q_2}$. Expressions $f_5$, $f_6$, $f_7$, and $f_8$ are given by

$$f_5 = f_8 = \phi_1 = \frac{(\alpha-1)\beta_1}{3\alpha}, \quad f_6 = f_7 = \phi_2 = \frac{(1-\alpha)\beta_2}{3}.$$

Let market two be the primary market, i.e., $\alpha \in (0, 1)$, then $\phi_1$ becomes negative and the equilibrium exists if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq \phi_2 \wedge u_2 \geq \phi_2 \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true whenever $\phi_2 \leq 1/2$ and $u_2 \in [\phi_2, 1 - \phi_2]$.

Let market one be the primary market, i.e., $\alpha > 1$, then $\phi_2$ becomes negative and the equilibrium exists if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq \phi_1 \wedge u_2 \geq \phi_1 \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true if $\phi_1 \leq 1/2$ and $u_2 \in [\phi_1, 1 - \phi_1]$.

Let both markets have the same maximum price, i.e., $\alpha = 1$, then $\phi_1 = \phi_2 = 0$ and the equilibrium exists for all $u_1, u_2 \in [0,1]^2$. The equilibrium strategies become $(q_1^*, q_2^*) = (u_1\beta_1/(\beta_1 + \beta_2), u_2\beta_2/(\beta_1 + \beta_2))$.

**4. Suppose that $(0, u_2)$ is an equilibrium.** Then, from (3.6), the following must hold: $0 \geq \bar{q}_1$ and $u_2 \leq \underline{q}_2$. Let $u_1 \leq f_9(u_2) : 0 \geq \bar{q}_1$ and $u_1 \leq f_{10}(u_2) : u_2 \leq \underline{q}_2$. Expressions $f_9$ and $f_{10}$ are given by

$$f_9(u_2) = (1-\alpha)\beta_2 - 2u_2, \quad f_{10}(u_2) = \frac{(1-\alpha)\beta_2}{2} - \frac{1}{2}u_2.$$

The pair $(0, u_2)$ is an equilibrium if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \leq f_9(u_2) \wedge u_1 \leq f_{10}(u_2) \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true when $\alpha \in (0,1)$ and, $\phi_2 \in [1/2, 2/3]$ and $u_2 \in [2 - 3\phi_2, 3\phi_2 - 1]$, or, $\phi_2 \geq 2/3$ and $u_2 \in [0,1]$. And follows from the following: the intersection of $f_9$ and $f_{10}$ occurs at $u_2 = \phi_2$, the intersection of $f_{10}$ and $1 - u_2$ occurs at $u_2 = 2 - 3\phi_2$, and the intersection of $f_9$ and $1 - u_2$ occurs at $u_2 = 3\phi_2 - 1$.

**5. Suppose that $(0, z_2(0))$ is an equilibrium.** The equilibrium solution is given by

$$q_1^* = 0, \quad q_2^* = \frac{2\alpha\beta_2 u_2 - \beta_1 u_1 + (1-\alpha)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}.$$

Then, from (3.6), the following must hold: $\underline{q}_1 \leq q_1^* \leq \bar{q}_2$ and $q_2^* \leq \underline{q}_2$. Let $u_1 \leq f_{11}(u_2) : q_1^* \geq \underline{q}_1$, $u_1 \geq f_{12}(u_2) : q_1^* \leq \bar{q}_1$, and $u_1 \leq f_{13} : q_2^* \leq \underline{q}_2$. Expressions $f_{11}$, $f_{12}$, and $f_{13}$ are given by

$$f_{11}(u_2) = f_3(u_2), \quad f_{12}(u_2) = f_9(u_2), \quad f_{13} = \phi_2.$$

The equilibrium solution exists if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq f_{12}(u_2) \wedge u_1 \leq f_{13} \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true when $\alpha \in (0,1)$, and, $\phi_2 \in [1/2, 2/3]$ and $u_2 \in [3\phi_2 - 1, 1]$, or, $\phi_2 \in [0, 1/2]$ and $u_2 \in [1 - \phi_2, 1]$. And follows from the following: the constant terms in $f_{11}$ and $f_{12}$ are equivalent to $3\phi_2$, and $u_1 \leq f_{11}$ is automatically satisfied whenever $0 \leq u_1 \leq f_{13}$ is true since $f_{11}$ is upward-slopping.

**6. Suppose that $(z_1(u_2), u_2)$ is an equilibrium.** The equilibrium solution is given by

$$q_1^* = \frac{\beta_1(2u_1 + u_2) - (1-\alpha)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}, \quad q_2^* = u_2.$$

Then, from (3.6), the following must hold: $q_1^* \geq \bar{q}_1$ and $\underline{q}_2 \leq q_2^* \leq \bar{q}_2$. Let $u_2 \leq f_{14} : q_1^* \geq \bar{q}_1$, $u_1 \geq f_{15}(u_2) : q_2^* \geq \underline{q}_2$ and $u_1 \geq f_{16}(u_2) : q_2^* \leq \bar{q}_2$. Expressions $f_{14}$, $f_{15}$ and $f_{16}$ are given by

$$f_{14} = \phi_2, \quad f_{15}(u_2) = f_{10}(u_2), \quad f_{16}(u_2) = f_2(u_2).$$

The equilibrium solution exists if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq f_{15}(u_2) \wedge u_2 \leq f_{14} \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true when $\alpha \in (0,1)$, and, $\phi_2 \in [1/2, 2/3]$ and $u_2 \in [0, 2 - 3\phi_2]$, or, $\phi_2 \in [0, 1/2]$ and $u_2 \in [0, \phi_2]$. And follows from the following: the intersection of $f_{15}$ and $f_{16}$ occurs at $u_2 = 3\phi_2$, and $u_1 \geq f_{16}$ is automatically satisfied whenever $0 \leq u_2 \leq f_{14}$ is true since $f_{16}$ is upward-slopping.

**7. Suppose that** $(u_1, 0)$ **is an equilibrium.** Then, from (3.6), the following must hold: $u_1 \leq \underline{q}_1$ and $0 \geq \bar{q}_2$. Let $u_1 \leq f_{17}(u_2) : u_1 \leq \underline{q}_1$ and $u_1 \leq f_{18}(u_2) : 0 \geq \bar{q}_2$. Expressions $f_{17}$ and $f_{18}$ are given by

$$f_{17}(u_2) = \frac{(\alpha - 1)\beta_1}{\alpha} - 2u_2, \quad f_{18}(u_2) = \frac{(\alpha - 1)\beta_1}{2\alpha} - \frac{1}{2}u_2.$$

The pair $(u_1, 0)$ is an equilibrium if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \leq f_{17}(u_2) \wedge u_1 \leq f_{18}(u_2) \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true when $\alpha > 1$, and, $\phi_1 \in [1/2, 2/3]$ and $u_2 \in [2 - 3\phi_1, 3\phi_1 - 1]$, or, $\phi_1 \geq 2/3$ and $u_2 \in [0, 1]$. And follows from the following: the intersection of which occurs at $u_2 = \phi_1$. In addition, the intersection of $f_{18}$ and $1 - u_2$ occurs at $u_2 = 2 - 3\phi_1$, and the intersection of $f_{17}$ and $1 - u_2$ occurs at $u_2 = 3\phi_1 - 1$.

**8. Suppose that** $(u_1, z_2(u_1))$ **is an equilibrium.** The equilibrium solution is given by

$$q_1^* = u_1, \quad q_2^* = \frac{\alpha\beta_2(u_1 + 2u_2) - (\alpha - 1)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}.$$

Then, from (3.6), the following must hold: $\underline{q}_1 \leq q_1^* \leq \bar{q}_1$ and $q_2^* \geq \bar{q}_2$. Let $u_1 \geq f_{19}(u_2) : q_1^* \geq \underline{q}_1$, $u_1 \leq f_{20}(u_2) : q_1^* \leq \bar{q}_1$ and $u_1 \leq f_{21}$. Expressions $f_{19}$, $f_{20}$ and $f_{21}$ are given by

$$f_{19}(u_2) = f_{17}(u_2), \quad f_{20}(u_2) = f_1(u_2), \quad f_{21} = \phi_1.$$

The equilibrium solution exists if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq f_{19}(u_2) \wedge u_1 \leq f_{21} \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true when $\alpha > 1$, and $\phi_1 \in [1/2, 2/3]$ and $u_2 \in [3\phi_1 - 1, 1]$, or, $\phi_1 \in [0, 1/2]$ and $u_2 \in [1 - \phi_1, 1]$. And follows from the following: the constant terms in $f_{19}$ and $f_{20}$ are equivalent to $3\phi_1$, and $u_1 \leq f_{20}$ is automatically satisfied whenever $0 \leq u_1 \leq f_{21}$ is true since $f_{20}$ is upward-slopping.

**9. Suppose that** $(z_1(0), 0)$ **is an equilibrium.** The equilibrium solution is given by

$$q_1^* = \frac{2\beta_1 u_1 - \alpha\beta_2 u_2 + (\alpha - 1)\beta_1\beta_2}{2(\alpha\beta_2 + \beta_1)}, \quad q_2^* = 0.$$

Then, from (3.6), the following must hold: $q_1^* \leq \underline{q}_1$ and $\underline{q}_2 \leq q_2^* \leq \bar{q}_2$. Let $u_2 \leq f_{22} : q_1^* \leq \underline{q}_1$, $u_1 \geq f_{23}(u_2) : q_2^* \geq \underline{q}_2$ and $u_1 \geq f_{24}(u_2) : q_2^* \leq \bar{q}_2$. Expressions $f_{22}$, $f_{23}$ and $f_{24}$ are given by

$$f_{22} = \phi_1, \quad f_{23}(u_2) = f_4(u_2), \quad f_{24}(u_2) = f_{18}(u_2).$$

The equilibrium solution exists if $\{u_1 \times u_2 \in [0,1]^2 : u_1 \geq f_{24}(u_2) \wedge u_2 \leq f_{22} \wedge u_1 + u_2 = 1\}$ is a non-empty set. This is true when $\alpha > 1$, and, $\phi_1 \in [1/2, 2/3]$ and $u_2 \in [0, 2 - 3\phi_1]$, or, $\phi_1 \in [0, 1/2]$ and $u_2 \in [0, \phi_1]$. And follows from the following: the intersection of $f_{23}$ and $f_{24}$ occurs at $u_2 = 3\phi_1$, and $u_1 \geq f_{23}$ is automatically satisfied whenever $0 \leq u_2 \leq f_{22}$ is true since $f_{23}$ is upward-slopping.

# B Foreign agent's quota purchasing subgame

## B.1 Low market efficiency: Proofs

Price thresholds and parameter expressions used in the proofs are summarised here.

$$\rho_1 = \frac{k_1}{k_2} - \psi_2, \quad \rho_2 = \frac{k_1 - \phi_2}{k_2} - \psi_2, \quad \rho_3 = \frac{\alpha(\beta_1 + \beta_2 - 1)}{\alpha\beta_2 + \beta_1} - \psi_2$$

$$\rho_4 = \frac{k_3}{\phi_2} - \psi_2, \quad \rho_5 = \frac{k_4}{1 - \phi_2} - \psi_2, \quad \rho_6 = \frac{\sqrt{k_6^2 - 4k_5k_7} - k_6}{2k_5} - \psi_2$$

where

$$k_1 = \frac{\beta_2(4(\alpha\beta_2+\beta_1)^2-(2\alpha\beta_2+(2-\xi)\beta_1)(\beta_1(1-\alpha)+2\alpha))}{\beta_1(4\alpha\beta_2+(4-\xi)\beta_1)}, \quad k_2 = \frac{4\beta_2(\alpha\beta_2+\beta_1)^2}{\beta_1(4\alpha\beta_2+(4-\xi)\beta_1)},$$

$$k_3 = \frac{\phi_2(5\beta_1(1-\alpha)+12\alpha(\beta_1+\beta_2-1))}{12(\alpha\beta_2+\beta_1)} - \frac{\phi_2\beta_1(7\beta_1(1-\alpha)+12\alpha)\xi}{24(\alpha\beta_2+\beta_1)^2},$$

$$k_4 = \frac{5\phi_2\beta_1(1-\alpha)+12(1-\phi_2)\alpha(\beta_1+\beta_2-1)}{12(\alpha\beta_2+\beta_1)} - \frac{\phi_2\beta_1(7\beta_1(1-\alpha)+12\alpha)\xi}{24(\alpha\beta_2+\beta_1)^2},$$

$$k_5 = 4\beta_2(\alpha\beta_2+\beta_1)^2, \quad k_6 = 4(\alpha\beta_2+\beta_1)((2-\beta_2)(\alpha\beta_2+\beta_1)-\alpha\beta_2(\beta_1+\beta_2)),$$

$$k_7 = 4\alpha((\beta_2-1)(\alpha\beta_2+\beta_1)-\beta_1)(\beta_1+\beta_2-1)-3\phi_2(1-\alpha)\beta_1^2.$$

**Proof of lemma 1.** The inner policy is given by $u_2^{in}(\rho) = k_1 - k_2(\rho+\psi_2)$ with $k_2 > 0$. Let $\rho_1 = k_1/k_2 - \psi_2 : u_2^{in}(\rho) = 0$ and $\rho_2 = (k_1-\phi_2)/k_2 : u_2^{in}(\rho) = \phi_2$, with $\rho_2 < \rho_1$ since $\phi_2 > 0$. The optimal policy follows from the concavity of $\bar{V}_{2c}$. **Q.E.D.**

**Proof of lemma 2.** The slope of $\bar{V}_{2b}$ is given by $\bar{V}'_{2b} = \alpha(\beta_1+\beta_2-1)/(\alpha\beta_2+\beta_1)-\rho-\psi_2$. Let $\rho_3 = \alpha(\beta_1+\beta_2-1)/(\alpha\beta_2+\beta_1)-\psi_2 : \bar{V}'_{2b} = 0$. The slope is positive for $\rho < \rho_3$ and negative for $\rho > \rho_3$. The optimal policy follows from the linearity of $\bar{V}_{2b}$. **Q.E.D.**

Suppose that $\rho_3 \geq \rho_2$. Then,

$$\rho_3 - \rho_2 \geq 0 \Leftrightarrow \frac{\beta_1(\alpha-1)(\alpha\beta_2+\beta_1)+(2(\alpha-1)\beta_1^2-3\alpha\beta_1)\xi}{6(\alpha\beta_2+\beta_1)^2} \geq 0,$$

which is not true since $\alpha \in (0,1)$. Thus, $\rho_3 < \rho_2$. **Q.E.D.**

**Proof of lemma 3.** The objective in the $[1-\phi_2, 1]$ region is convex. The foreign agent is indifferent between $u_2 = 1-\phi_2$ and $u_2 = 1$ if and only if $\bar{V}_{2a}(1-\phi_2) - \bar{V}_{2a}(1) = 0 \Leftrightarrow \phi_2(\rho+\psi_2) - k_3 = 0$. Let $\rho_4 = k_3/\phi_2 - \psi_2 : \bar{V}_{2a}(1-\phi_2) - \bar{V}_{2a}(1) = 0$. Since $\phi_2 > 0$, $u_2^* = 1-\phi_2$ for all $\rho \geq \rho_4$, and $u_2^* = 1$ for all $\rho \leq \rho_4$. **Q.E.D.**

First, we compare $\rho_4$ and $\rho_3$. Their difference is given by

$$\rho_4 - \rho_3 = \frac{\mathrm{A}_1 - \xi\mathrm{A}_2}{24(\alpha\beta_2+\beta_1)^2}\beta_1,$$

where $\mathrm{A}_1 = 10(1-\alpha)(\alpha\beta_2+\beta_1) > 0$ and $\mathrm{A}_2 = 7(1-\alpha)\beta_1 + 12\alpha > 0$, since $\alpha \in (0,1)$. It follows that $\rho_4 > \rho_3$ when $\xi = 0$. If $\xi = 1$, then the sign follows from $\mathrm{A}_1 - \mathrm{A}_2$, which can be both positive and negative in the region of interest, i.e., $\phi_2 \in (0, 1/2]$ and $\beta_1 + \beta_2 \geq 1$. One can verify this by plotting the inequalities in the $\beta_1 \times \beta_2$ space. Thus, $\rho_4 \leq \rho_3$ when $(1-\alpha)(10\alpha\beta_2+3\beta_1)-12\alpha \leq 0$ or $\phi_2 \leq 2/5 - \beta_1(1-\alpha)/(10\alpha)$, otherwise $\rho_4 > \rho_3$.

Since $\rho_4$ can exceed $\rho_3$, we need to compare it with $\rho_2$. Their difference is given by

$$\rho_4 - \rho_2 = \frac{\mathrm{A}_3 - \xi\mathrm{A}_4}{8(\alpha\beta_2+\beta_1)^2}\beta_1,$$

where $\mathrm{A}_3 = 2(1-\alpha)(\alpha\beta_2+\beta_1) > 0$ and $\mathrm{A}_4 = 5(1-\alpha)\beta_1 + 8\alpha > 0$, since $\alpha \in (0,1)$. It follows that $\rho_4 > \rho_2$ when $\xi = 0$. If $\xi = 1$, then $\mathrm{A}_3 - \mathrm{A}_4$ is negative for all $\phi_2 \in (0, 1/2]$. Suppose that $\mathrm{A}_3 - \mathrm{A}_4 \geq 0 \Leftrightarrow 6\alpha\phi_2 - 3(1-\alpha)\beta_1 - 8\alpha \geq 0 \Leftrightarrow \phi_2 \geq 4/3 + (1-\alpha)\beta_1/(2\alpha) > 1$, which is a contradiction since $\phi_2$ cannot exceed one-half. Thus, $\rho_4 \in (\rho_3, \rho_2)$ when $\xi = 1$ and $\phi_2 \in (2/5 - \beta_1(1-\alpha)/(10\alpha), 1/2]$.

Since $\rho_4$ exceeds $\rho_2$ when $\xi = 0$, we need to compare it with $\rho_1$. Their difference is:

$$\rho_4 - \rho_1 = \frac{-\mathrm{A}_3}{8(\alpha\beta_2 + \beta_1)^2}\beta_1,$$

which is negative since $\mathrm{A}_3 > 0$. Thus, $\rho_4 \in (\rho_2, \rho_1)$ when $\xi = 0$. **Q.E.D.**

**Proof of lemma 4.** The foreign agent is indifferent between $u_2 = \phi_2$ and $u_2 = 1$ if and only if $\bar{V}_{2c}(\phi_2) - \bar{V}_{2a}(1) = 0 \Leftrightarrow (1 - \phi_2)(\rho + \psi_2) - k_4 = 0$. Let $\rho_5 = k_4/(1 - \phi_2) - \psi_2$ : $\bar{V}_{2c}(\phi_2) - \bar{V}_{2a}(1) = 0$. Since $1 - \phi_2 > 0$, $u_2^* = \phi_2$ for all $\rho \geq \rho_5$, and $u_2^* = 1$ for all $\rho \leq \rho_5$.

First, we compare $\rho_5$ and $\rho_3$. Their difference is given by

$$\rho_5 - \rho_3 = \frac{\mathrm{A}_1 - \xi\mathrm{A}_2}{24(\alpha\beta_2 + \beta_1)^2}\beta_1 \frac{\phi_2}{1 - \phi_2} = (\rho_4 - \rho_3)\frac{\phi_2}{1 - \phi_2},$$

which is strictly positive when $\rho_4 > \rho_3$, which is a prerequisite. $\mathrm{A}_1$ and $\mathrm{A}_2$ are defined in the previous proof. Next, we compare $\rho_5$ and $\rho_4$. Their difference is given by

$$\rho_5 - \rho_4 = \frac{\mathrm{A}_1 - \xi\mathrm{A}_2}{24(\alpha\beta_2 + \beta_1)^2}\beta_1 \frac{2\phi_2 - 1}{1 - \phi_2} = (\rho_4 - \rho_3)\frac{2\phi_2 - 1}{1 - \phi_2},$$

which is strictly negative for all $\phi_2 \in (0, 1/2)$ and zero if $\phi_2 = 1/2$. Thus, $\rho_5 \in (\rho_3, \rho_4]$ for all $\phi_2 \in (0, 1/2]$ and both $\xi = \{0, 1\}$. If $\xi = 1$, no further comparisons are needed since $\rho_4 < \rho_2$. If $\xi = 0$, $\rho_4 > \rho_2$ and two more comparisons are needed. First, we compare $\rho_5$ and $\rho_2$. Their difference when $\xi = 0$ is given by

$$\rho_5 - \rho_2 = \frac{(1 - \alpha)\beta_1(7\phi_2 - 2)}{12(\alpha\beta_2 + \beta_1)(1 - \phi_2)},$$

which is negative or zero for all $\phi_2 \in (0, 2/7]$ and strictly positive for all $\phi_2 \in (2/7, 1/2]$. Finally, we compare $\rho_5$ and $\rho_1$. Their difference when $\xi = 0$ is given by

$$\rho_5 - \rho_1 = \frac{(1 - \alpha)\beta_1(11\phi_2 - 6)}{12(\alpha\beta_2 + \beta_1)^2(1 - \phi_2)},$$

which is strictly negative for all $\phi_2 \in (0, 1/2]$. Thus, when $\xi = 0$, $\rho_5 \in (\rho_3, \rho_2]$ for all $\phi_2 \in (0, 2/7]$, and $\rho_5 \in (\rho_2, \rho_1)$ for all $\phi_2 \in (2/7, 1/2]$. **Q.E.D.**

**Proof of lemma 5.** Let $\xi = 0$. The foreign agent is indifferent between $u_2 = u_2^{in}$ and $u_2 = 1$ if and only if $\bar{V}_{2c}(u_2^{in}) - \bar{V}_{2a}(1) = 0 \Leftrightarrow k_5\rho^2 + (2k_5\psi_2 + k_6)\rho + k_5\psi_2^2 + k_6\psi_2 + k_7 = 0$. The discriminant $\Delta = (2k_5\psi_2 + k_6)^2 - 4(k_5^2\psi_2^2 + k_5k_6\psi_2 + k_5k_7)$ simplifies to $\Delta = k_6^2 - 4k_5k_7$ or $32\beta_1^2(\alpha\beta_2 + \beta_1)^2(1 + (1 - 3\phi_2)^2)$ which is always positive. Thus, the prices that make the foreign agent indifferent between the two strategies are $\rho_6 = (\pm\sqrt{k_6^2 - 4k_5k_7} - k_6)/(2k_5) - \psi_2$. From the two prices we keep the greater one, which is the only one that can exceed $\rho_2$, which as we show below must be satisfied. The difference between the zero-discriminant root, $-k_6/(2k_5) - \psi_2$, and $\rho_2$ is given by $\beta_1(\phi_2 - 1)/(\beta_2(\alpha\beta_2 + \beta_1))$, which is negative for all $\phi_2 \in (0, 1/2]$. The direction of the optimal policy follows from the fact the quadratic function is convex and that $\rho_6$ is the greater of the two roots. Thus, for all $\rho < \rho_6$, $\bar{V}_{2c}(u_2^{in}) < \bar{V}_{2a}(1)$, and for all $\rho > \rho_6$, $\bar{V}_{2c}(u_2^{in}) > \bar{V}_{2a}(1)$.

Suppose that $\rho_6 \leq \rho_2$. Then, for any $\rho \in [\rho_6, \rho_2]$ the following are true:

- From the first part of lemma 5, $\bar{V}_{2c}(u_2^{in}) \geq \bar{V}_{2a}(1)$ for all $\rho \in [\rho_6, \rho_2]$.
- From lemma 1, $\bar{V}_{2c}(\phi_2) \geq \bar{V}_{2c}(u_2^{in})$ for all $\rho \in [\rho_6, \rho_2]$.
- From lemma 4, $\bar{V}_{2a}(1) > \bar{V}_{2c}(\phi_2)$ for all $\rho \in [\rho_6, \rho_2]$, since $\rho_5 > \rho_2$.

Thus,

$$\bar{V}_{2a}(1) > \bar{V}_{2c}(\phi_2) \geq \bar{V}_{2c}(u_2^{in}) \geq \bar{V}_{2a}(1), \quad \forall \rho \in [\rho_6, \rho_2],$$

which is a contradiction, and therefore, $\rho_6 > \rho_2$.

Suppose that $\rho_6 \geq \rho_5$. Then, for any $\rho \in [\rho_5, \rho_6]$ the following are true:

- From the first part of lemma 5, $\bar{V}_{2a}(1) \geq \bar{V}_{2c}(u_2^{in})$ for all $\rho \in [\rho_5, \rho_6]$.
- From lemma 1, $\bar{V}_{2c}(u_2^{in}) > \bar{V}_{2c}(\phi_2)$ for all $\rho \in [\rho_5, \rho_6]$, since $\rho_5 > \rho_2$.
- From lemma 4, $\bar{V}_{2c}(\phi_2) \geq \bar{V}_{2a}(1)$ for all $\rho \in [\rho_5, \rho_6]$.

Thus,

$$\bar{V}_{2c}(\phi_2) \geq \bar{V}_{2a}(1) \geq \bar{V}_{2c}(u_2^{in}) > \bar{V}_{2c}(\phi_2), \quad \forall \rho \in [\rho_5, \rho_6],$$

which is a contradiction, and therefore, $\rho_6 < \rho_5$. Thus, $\rho_6 \in (\rho_2, \rho_5)$. **Q.E.D.**

## B.2 Medium and high market efficiency: Detailed description

For the case of medium market efficiency the five equilibrium candidates are one, $3\phi_2 - 1$, $2 - 3\phi_2$, $u_2^{in}(\rho)$, and zero. Lemmas 11, 12 and 13 provide the optimal policy in the respective concave, linear and convex parts of the objective, see Eq. (3.10). For all explicit expressions used in this subsection see appendix B.3.

**Lemma 11.** Let $\rho_1$ and $\rho_7$ given by $u_2^{in}(\rho) = 0$ and $u_2^{in}(\rho) = 2 - 3\phi_2$, respectively, be quota price thresholds. Then, the optimal policy $u_2^*$ in the $[0, 2 - 3\phi_2]$ region is given by

$$u_2^*(\rho) = \min(\max(0, u_2^{in}), 2 - 3\phi_2) = \begin{cases} 0, & \rho \geq \rho_1, \\ u_2^{in}, & \rho_7 \leq \rho \leq \rho_1, \\ 2 - 3\phi_2, & \rho \leq \rho_7. \end{cases}$$

In addition, $\rho_1 > \rho_7$ for all $\phi_2 \in [1/2, 2/3)$, and $\rho_1 = \rho_7$ when $\phi_2 = 2/3$.[15]
See appendix B.3 for the proof.

**Lemma 12.** Let $\rho_8$ given by $\bar{V}'_{2d}(u_2) = 0$ be a quota price threshold. Then, the optimal policy $u_2^*$ in the $[2 - 3\phi_2, 3\phi_2 - 2]$ region is given by

$$u_2^*(\rho) = \begin{cases} 2 - 3\phi_2, & \rho > \rho_8, \\ u_2 \in [2 - 3\phi_2, 3\phi_2 - 1], & \rho = \rho_8, \\ 3\phi_2 - 1, & \rho < \rho_8. \end{cases}$$

The relative position of $\rho_8$ depends on $\xi$ as follows:
a. Let $\xi = 1$ and $\phi_2 \in [1/2, 2/3]$. Then, $\rho_8 < \rho_7$.
b. Let $\xi = 0$. Then, $\rho_8 \in (\rho_7, \rho_1)$ for all $\phi_2 \in [1/2, 2/3)$, and $\rho_1 = \rho_7 = \rho_8$ when $\phi_2 = 2/3$.
See appendix B.3 for the proof.

**Lemma 13.** Let $\rho_9$ given by $\bar{V}_{2a}(3\phi_2 - 1) = \bar{V}_{2a}(1)$ be a quota price threshold. Then, the optimal policy $u_2^*$ is given by

$$u_2^*(\rho) = \begin{cases} 3\phi_2 - 1, & \rho \geq \rho_9, \\ 1, & \rho \leq \rho_9. \end{cases}$$

The relative position of $\rho_9$ depends on $\xi$ as follows:
a. Let $\xi = 1$ and $\phi_2 \in [1/2, 2/3)$. Then, $\rho_9 < \rho_8$.
b. Let $\xi = 0$ and $\phi_2 \in [1/2, 2/3)$. Then, $\rho_9 \in (\rho_8, \rho_1)$.[16]
See appendix B.3 for the proof.

[15] Notice that $\rho_7 = \rho_2$ when $\phi_2 = 1/2$.
[16] Notice that $\rho_9 = \rho_4 = \rho_5$ when $\phi_2 = 1/2$.

Suppose that $\xi = 1$ and $\phi_2 \in [1/2, 2/3)$. It then follows from lemmas 11-13 that $\rho_1 > \rho_7 > \rho_8 > \rho_9$. This implies that all equilibria candidates occur in distinct $\rho$ regions. Therefore, a complete equilibrium exists. Note that at $\phi_2 = 2/3$, candidates $u_2^{in}$, $2 - 3\phi_2$ are equal to zero, and candidate $3\phi_2 - 1$ is equal to one, and thus price threshold $\rho_8$ becomes the switching price between the two strategies.

Suppose that $\xi = 0$ and $\phi_2 \in [1/2, 2/3)$. It then follows from lemmas 11-13 that $\rho_1 > \rho_9 > \rho_8 > \rho_7$. This implies that strategies $u_2 = 3\phi_2 - 1$ and $u_2 = 2 - 3\phi_2$ are strictly dominated by $u_2 = u_2^{in}$ for all $\rho \geq \rho_8$ since $\rho_8 > \rho_7$, and by $u_2 = 1$ for all $\rho \leq \rho_8$ since $\rho_8 < \rho_9$. Thus, we need to compare candidates $u_2^{in}$ and 1 in order to determine the complete equilibrium strategy. Because $\bar{V}_{2c}$ and $\bar{V}_{2a}$ are specified as in the low market efficiency case the indifference price threshold is equivalent to $\rho_6$, which is defined in lemma 5, its relevant position however now differs since $\phi_2 \geq 1/2$. Lemma 14 provides the optimal policy for completeness.

**Lemma 14.** Suppose that $\xi = 0$ and let $\rho_6$ given by $\bar{V}_{2c}(u_2^{in}) = \bar{V}_{2a}(1)$ be a quota price threshold. Then the optimal policy $u_2^*$ is given by

$$u_2^*(\rho) = \begin{cases} u_2^{in}, & \rho \geq \rho_6, \\ 1, & \rho \leq \rho_6. \end{cases}$$

In addition, $\rho_6 \in (\rho_7, \rho_9)$ for all $\phi_2 \in [1/2, 2/3)$.
See appendix B.3 for the proof.

Thus, when $\xi = 0$, it follows from lemmas 11-14 that $\rho_1 > \rho_6$ for all $\phi_2 \in [1/2, 2/3)$ and the feasible equilibrium strategies, namely, zero, one, and $u_2^{in}$ are mutually exclusive. Note that at $\phi_2 = 2/3$, $u_2^{in} = 0$, and thus $\rho_8$ becomes the switching threshold between strategies zero and one.

## B.3 Medium and high market efficiency: Proofs

Price thresholds and parameter expressions used in the proofs are summarised here.

$$\rho_1 = \frac{k_1}{k_2} - \psi_2, \quad \rho_7 = \frac{k_1 - (2 - 3\phi_2)}{k_2} - \psi_2, \quad \rho_8 = \frac{\beta_2 - 1}{\beta_2} - \psi_2,$$
$$\rho_9 = \frac{k_8}{2 - 3\phi_2} - \psi_2, \quad \rho_6 = \frac{\sqrt{k_6^2 - 4k_5k_7} - k_6}{2k_5} - \psi_2,$$

where

$$k_8 = \frac{5\phi_2\beta_1(1-\alpha) + 12(2-3\phi_2)\alpha(\beta_1+\beta_2-1)}{12(\alpha\beta_2+\beta_1)} - \frac{\beta_1(8\phi_2^2 - 6\phi_2 + 1)}{\beta_2(\alpha\beta_2+\beta_1)} - \frac{(2-3\phi_2)\beta_1((3+2/\phi_2)\beta_1(1-\alpha)+12\alpha)\xi}{24(\alpha\beta_2+\beta_1)^2}$$

For the explicit expressions of $k_1$, $k_2$, $k_5$, $k_6$ and $k_7$ see appendix B.1.

**Proof of lemma 11** The inner policy is given by $u_2^{in}(\rho) = k_1 - k_2(\rho + \psi_2)$ with $k_2 > 0$. Let $\rho_1 = k_1/k_2 - \psi_2 : u_2^{in}(\rho) = 0$ and $\rho_7 = (k_1 - (2 - 3\phi_2))/k_2 : u_2^{in}(\rho) = 2 - 3\phi_2$, with $\rho_7 \leq \rho_1$ since $2 - 3\phi_2 \geq 0$. The optimal policy follows from the concavity of $\bar{V}_{2c}$. **Q.E.D.**

**Proof of lemma 12.** The slope of $\bar{V}_{2d}$ is given by $\bar{V}_{2d}' = (\beta_2 - 1)/\beta_2 - \rho - \psi_2$. Let $\rho_8 = (\beta_2 - 1)/\beta_2 - \psi_2 : \bar{V}_{2b}' = 0$. The slope is positive for $\rho < \rho_8$ and negative for $\rho > \rho_8$. The optimal policy follows from the linearity of $\bar{V}_{2d}$.

The difference between $\rho_8$ and $\rho_7$ is given by

$$\rho_8 - \rho_7 = \frac{\beta_1(2 - 3\phi_2 - \xi)}{2\beta_2(\alpha\beta_2 + \beta_1)},$$

which in the region of interest, i.e., $\phi_2 \in [1/2, 2/3]$, is greater or equal to zero when $\xi = 0$, and negative when $\xi = 1$. Since $\rho_8 \geq \rho_7$ when $\xi = 0$, we need to compare it with $\rho_1$. Their difference is given by

$$\rho_8 - \rho_1 = \frac{\beta_1(3\phi_2 - 2)(\alpha\beta_2 + \beta_1)}{2\beta_2(\alpha\beta_2 + \beta_1)^2},$$

which is less or equal to zero in the region of interest. To sum up, $\rho_8 < \rho_7$ when $\xi = 1$ and $\phi_2 \in [1/2, 2/3]$. Moreover, $\rho_8 \in (\rho_7, \rho_1)$ when $\xi = 0$ and $\phi_2 \in [1/2, 2/3)$, and $\rho_1 = \rho_2 = \rho_3$ when $\xi = 0$ and $\phi_2 = 2/3$. **Q.E.D.**

**Proof of lemma 13.** The objective in the $[3\phi_2 - 1, 1]$ region is convex. The foreign agent is indifferent between $u_2 = 3\phi_2 - 1$ and $u_2 = 1$ if and only if $\bar{V}_{2a}(3\phi_2 - 1) - \bar{V}_{2a}(1) = 0 \Leftrightarrow (2 - 3\phi_2)(\rho + \psi_2) - k_8 = 0$. Let $\rho_9 = k_8/(2 - 3\phi_2) - \psi_2 : \bar{V}_{2a}(3\phi_2 - 1) - \bar{V}_{2a}(1) = 0$. At $\phi_2 = 2/3$, $\rho_4$ is not defined since $2 - 3\phi_2$ becomes zero, which causes no trouble since the convex function is absorbed by the linear at that point and the two equilibria coincide. For any $\phi_2 \in [1/2, 2/3)$, $2 - 3\phi_2$ is strictly positive and the optimal policy is given by $u_2^* = 3\phi_2 - 1$ for all $\rho \geq \rho_4$, and $u_2^* = 1$ for all $\rho \leq \rho_4$.

First, we compare $\rho_9$ and $\rho_8$. Their difference when $\xi = 1$ is given by

$$\rho_9 - \rho_8 = -\left(\alpha\beta_2 + \beta_1\frac{9\phi_2 - 2}{6\phi_2}\right)\frac{\beta_1\phi_2}{8\beta_2(\alpha\beta_2 + \beta_1)^2}$$

which is negative since $(9\phi_2 - 2)/(6\phi_2) > 0$ for all $\phi_2 \in [1/2, 2/3)$. Thus, $\rho_9 < \rho_8$. Their difference when $\xi = 0$ is given by

$$\rho_9 - \rho_8 = \frac{(2 - 3\phi_2)\beta_1}{4\beta_2(\alpha\beta_2 + \beta_1)},$$

which is strictly positive since $2 - 3\phi_2 > 0$ for all $\phi_2 \in [1/2, 2/3)$ implying that $\rho_9 > \rho_8$. Finally, we compare $\rho_9$ and $\rho_1$ when $\xi = 0$. Their difference is given by

$$\rho_9 - \rho_1 = -\frac{\beta_1(2 - 3\phi_2)}{4\beta_2(\alpha\beta_2 + \beta_1)},$$

which is negative since $2 - 3\phi_2 > 0$ for all $\phi_2 \in [1/2, 2/3)$. To sum up, $\rho_9 < \rho_8$ when $\xi = 1$ and $\phi_2 \in [1/2, 2/3)$. Moreover, $\rho_9 \in (\rho_8, \rho_1)$ when $\xi = 0$ and $\phi_2 \in [1/2, 2/3)$. **Q.E.D.**

**Proof of lemma 14.** Because $\bar{V}_{2c}$ and $\bar{V}_{2a}$ are the same as the case when $\phi_2 \in (0, 1/2]$, the price threshold and the direction of the optimal policy follows from lemma 5. Here we verify that the greater root is the "right" one, and show that $\rho_6 \in (\rho_7, \rho_9)$. First, the difference between the zero-discriminant root and $\rho_7$ is given by $\beta_1(1-3\phi_2)/(\beta_2(\alpha\beta_2+\beta_1))$, which is negative for all $\phi_2 \in [1/2, 2/3)$. A reminder before moving on, $\rho_1 > \rho_9 > \rho_8 > \rho_7$ when $\xi = 0$.

Suppose that $\rho_6 \leq \rho_7$. Then, for any $\rho \in [\rho_6, \rho_7]$ the following are true:

- From the first part of lemma 14, $\bar{V}_{2c}(u_2^{in}) \geq \bar{V}_{2a}(1)$ for all $\rho \in [\rho_6, \rho_7]$.
- From lemma 11, $\bar{V}_{2c}(2-3\phi_2) \geq \bar{V}_{2c}(u_2^{in})$ for all $\rho \in [\rho_6, \rho_7]$.
- From lemma 12, $\bar{V}_{2a}(3\phi_2 - 1) > \bar{V}_{2c}(2-3\phi_2)$ for all $\rho \in [\rho_6, \rho_7]$, since $\rho_8 > \rho_7$.
- From lemma 13, $\bar{V}_{2a}(1) > \bar{V}_{2a}(3\phi_2 - 1)$ for all $\rho \in [\rho_6, \rho_7]$, since $\rho_9 > \rho_7$.

Thus,

$$\bar{V}_{2a}(1) > \bar{V}_{2a}(3\phi_2 - 1) > \bar{V}_{2c}(2-3\phi_2) \geq \bar{V}_{2c}(u_2^{in}) \geq \bar{V}_{2a}(1), \quad \forall \rho \in [\rho_6, \rho_7],$$

which is a contradiction, and therefore, $\rho_6 > \rho_7$.

Suppose that $\rho_6 \geq \rho_9$. Then, for any $\rho \in [\rho_9, \rho_6]$ the following are true:

- From the first part of lemma 14, $\bar{V}_{2a}(1) \geq \bar{V}_{2c}(u_2^{in})$ for all $\rho \in [\rho_9, \rho_6]$.
- From lemma 11, $\bar{V}_{2c}(u_2^{in}) > \bar{V}_{2c}(2-3\phi_2)$ for all $\rho \in [\rho_9, \rho_6]$, since $\rho_9 > \rho_7$.
- From lemma 12, $\bar{V}_{2c}(2-3\phi_2) > \bar{V}_{2a}(3\phi_2 - 1)$ for all $\rho \in [\rho_9, \rho_6]$, since $\rho_8 < \rho_9$.
- From lemma 13, $\bar{V}_{2a}(3\phi_2 - 1) \geq \bar{V}_{2a}(1)$ for all $\rho \in [\rho_9, \rho_6]$,

Thus,

$$\bar{V}_{2c}(u_2^{in}) > \bar{V}_{2c}(2-3\phi_2) > \bar{V}_{2a}(3\phi_2 - 1) \geq \bar{V}_{2a}(1) \geq \bar{V}_{2c}(u_2^{in}), \quad \forall \rho \in [\rho_9, \rho_6],$$

which is a contradiction, and therefore, $\rho_6 < \rho_9$. Thus, $\rho_6 \in (\rho_7, \rho_9)$. **Q.E.D.**

## B.4 Specification of the foreign agent's quota equilibria

The foreign agent's equilibria functions are:

$$u_2^1(\rho) = \begin{cases} 0, & \rho \geq \rho_1, \\ u_2^{in}(\rho), & \rho_2 \leq \rho \leq \rho_1, \\ \phi_2, & \rho_3 \leq \rho \leq \rho_2, \\ u_2 \in [\phi_2, 1-\phi_2], & \rho = \rho_3, \\ 1-\phi_2, & \rho_4 \leq \rho \leq \rho_3, \\ 1, & \rho \leq \rho_4, \end{cases} \qquad u_2^4(\rho) = \begin{cases} 0, & \rho \geq \rho_1, \\ u_2^{in}(\rho), & \rho_7 \leq \rho \leq \rho_1, \\ 2-3\phi_2, & \rho_8 \leq \rho \leq \rho_7, \\ u_2 \in [2-3\phi_2, 3\phi_2 - 1], & \rho = \rho_8, \\ 3\phi_2 - 1, & \rho_9 \leq \rho \leq \rho_8, \\ 1, & \rho \leq \rho_9, \end{cases}$$

$$u_2^2(\rho) = \begin{cases} 0, & \rho \geq \rho_1, \\ u_2^{in}(\rho), & \rho_2 \leq \rho \leq \rho_1, \\ \phi_2, & \rho_5 \leq \rho \leq \rho_2, \\ 1, & \rho \leq \rho_5, \end{cases} \quad u_2^3(\rho) = \begin{cases} 0, & \rho \geq \rho_1, \\ u_2^{in}(\rho), & \rho_6 \leq \rho \leq \rho_1, \\ 1, & \rho \leq \rho_6, \end{cases} \quad u_2^5(\rho) = \begin{cases} 0, & \rho \geq \rho_8, \\ u_2 \in [0,1], & \rho = \rho_8, \\ 1, & \rho \leq \rho_8. \end{cases}$$

# C Home country's price subgame

## C.1 Welfare-maximiser foreign agent

### C.1.1 Very low market efficiency: Proofs

Cost difference thresholds and parameter expressions used below are summarised here.

$$\lambda_1 = \frac{l_2}{l_1}, \quad \lambda_2 = \frac{l_3}{l_1}, \quad \lambda_3 = \frac{l_4}{\phi_2}, \quad \lambda_4 = \frac{l_5}{1-2\phi_2}, \quad \lambda_5 = \frac{l_6}{1-\phi_2}, \quad \lambda_6 = -\frac{l_8 + \sqrt{l_8^2 - 4l_7 l_9}}{2l_7}$$

where

$$l_1 = \frac{4\alpha\beta_2 + 3\beta_1}{5\alpha\beta_2 + 4\beta_1}, \quad l_2 = \frac{(\alpha-1)\beta_1(4\alpha\beta_2 + 3\beta_1)}{4(\alpha\beta_2+\beta_1)(5\alpha\beta_2+4\beta_1)}, \quad l_3 = \frac{(1-\alpha)\beta_1(2\alpha\beta_2+\beta_1)(4\alpha\beta_2+3\beta_1)}{12(\alpha\beta_2+\beta_1)^2(5\alpha\beta_2+4\beta_1)},$$

$$l_4 = \frac{\beta_1((1-\alpha)(8\phi_2-3)\beta_1 + 3\alpha((15\phi_2-14)\phi_2+4))}{24(\alpha\beta_2+\beta_1)^2}, \quad l_5 = \frac{\beta_1\phi_2((1-\alpha)(\alpha\beta_2+3\beta_1)+3\alpha)}{6(\alpha\beta_2+\beta_1)^2},$$

$$l_6 = \frac{\beta_1(\beta_1(1-\alpha)(20\phi_2-3) + 3\alpha(19\phi_2^2-10\phi_2+4))}{24(\alpha\beta_2+\beta_1)^2}, \quad l_8 = \frac{(8\phi_2-5)\alpha\beta_2 + (7\phi_2-4)\beta_1}{5\alpha\beta_2+4\beta_1},$$

$$l_7 = \frac{2\beta_2(\alpha\beta_2+\beta_1)^2}{\beta_1(5\alpha\beta_2+4\beta_1)}, \quad l_9 = \frac{\beta_1^2(49(1-\alpha)\phi_2\beta_1^2 + 12\alpha(\alpha\beta_2(6\phi_2^2+5\phi_2) + 4\beta_1(5\phi_2^2+\phi_2)))}{24\beta_1(\alpha\beta_2+\beta_1)^2(5\alpha\beta_2+4\beta_1)}.$$

**Proof of lemma 6.** The differences between the two bounds $\rho_1, \rho_2$ and the inner policy $\rho^{in}$ are given by $\rho_1 - \rho^{in} = (\psi_1 - \psi_2)l_1 - l_2$ and $\rho_2 - \rho^{in} = (\psi_1 - \psi_2)l_1 - l_3$ with $l_1 > 0$, $l_2 < 0$ and $l_3 > 0$ for all $\alpha \in (0,1)$. Let $\Delta\psi \equiv \lambda_1 = l_2/l_1 : \rho_1 = \rho^{in}$ and $\Delta\psi \equiv \lambda_2 = l_3/l_1 : \rho_2 = \rho^{in}$. The optimal policy follows from the concavity of $\bar{V}_{1b}$. In addition, $\lambda_1 < 0$ and $\lambda_2 > 0$. **Q.E.D.**

**Proof of lemma 7.** The home country is indifferent between $\rho = \rho_4$ and $\rho = \rho_3$ if and only if $\bar{V}_{1d}(\rho_3) - \bar{V}_{1e}(\rho_4) = 0 \Leftrightarrow l_4 - \phi_2(\psi_1 - \psi_2) = 0$. Let $\Delta\psi \equiv \lambda_3 = l_4/\phi_2 : \bar{V}_{1d}(\rho_3) - \bar{V}_{1e}(\rho_4) = 0$. Since $\phi_2 > 0$, $\rho^* = \rho_3$ for all $\Delta\psi \leq \lambda_3$, and $\rho^* = \rho_4$ for all $\Delta\psi \geq \lambda_3$.

Suppose that $\lambda_3 \leq \lambda_2$. Then,

$$\lambda_3 - \lambda_2 \leq 0 \Leftrightarrow \frac{\beta_1(\alpha(\phi_2(11\phi_2-14)+4) - \beta_1(1-\alpha)(1-2\phi_2))}{8(\alpha\beta_2+\beta_1)^2\phi_2} \leq 0$$

Our conjecture is true if and only if $\mathrm{B}_1 = \alpha(\phi_2(11\phi_2-14)+4) - \beta_1(1-\alpha)(1-2\phi_2) \leq 0$. First, we know that $\phi_2 \in (0, 2/5-\theta]$, where $\theta = \beta_1(1-\alpha)/(10\alpha) > 0$ since $\alpha \in (0,1)$. This implies that $\phi_2$ must be strictly less than 2/5. Next, we build $\mathrm{B}_1$ from $\phi_2 \leq 2/5-\theta$, which we re-write as: $-\beta_1(1-\alpha) \geq 2\alpha(5\phi_2-2)$. Multiplying both sides with $1-2\phi_2 > 0$, and adding $\alpha\phi_2(11\phi_2-14)+4)$ yields $\mathrm{B}_1 \geq \alpha\phi_2(4-9\phi_2)$. For all $\phi_2 \leq 2/5$, $4-9\phi_2 > 0$ implying that $\mathrm{B}_1 > 0$. The conjecture is not true and therefore $\lambda_3 > \lambda_2$. **Q.E.D.**

**Proof of lemma 8.** The home country is indifferent between $\rho = \rho_3$ and $\rho = \rho_2$ if and only if $\bar{V}_{1c}(\rho_2) - \bar{V}_{1d}(\rho_3) = 0 \Leftrightarrow l_5 + (2\phi_2-1)(\psi_1-\psi_2) = 0$. Let $\Delta\psi \equiv \lambda_4 = l_5/(1-2\phi_2) : \bar{V}_{1c}(\rho_2) - \bar{V}_{1d}(\rho_3) = 0$. Note that in the region of interest $\phi_2 < 2/5 < 1/2$, thus $\lambda_4$ is always defined. Since $1-2\phi_2 > 0$, $\rho^* = \rho_2$ for all $\Delta\psi \leq \lambda_4$, and $\rho^* = \rho_3$ for all $\Delta\psi \geq \lambda_4$.

First, we compare $\lambda_4$ and $\lambda_3$. Their difference is given by

$$\lambda_4 - \lambda_3 = \frac{\alpha\beta_1(\beta_1\mathrm{B}_2/(\alpha\beta_2) + \mathrm{B}_3)}{8(\alpha\beta_2+\beta_1)^2(1-2\phi_2)\phi_2},$$

where $\mathrm{B}_2 = 28\phi_2^3 - 14\phi_2^2 + 3\phi_2$ and $\mathrm{B}_3 = 34\phi_2^3 - 39\phi_2^2 + 22\phi_2 - 4$. $\mathrm{B}_2 > 0$, since its only real root occurs at $\phi_2 = 0$, which is outside the region of interest. Moreover, $\mathrm{B}_3$ has one real root at $\phi_2 \approx 0.3$. Thus, $\mathrm{B}_3 < 0$ for all $\phi_2 \in (0, 0.3)$, and $\mathrm{B}_3 > 0$ for all $\phi_2 > 0.3$. Since the denominator is strictly positive, the sign is determined by the numerator and specifically by $\beta_1\mathrm{B}_2/(\alpha\beta_2) + \mathrm{B}_3$, which can be both positive and negative in the region of interest, i.e., when $\phi_2 \in (0, 2/5-\theta]$. The numerator is strictly positive, if $\beta_1/(\alpha\beta_2) > -\mathrm{B}_3/\mathrm{B}_2$, implying that $\lambda_4 > \lambda_3$. And, negative or zero otherwise, implying that $\lambda_4 \leq \lambda_3$.

We illustrate this claim using two numerical examples. Consider the parameter vectors $\Theta_1 = [\alpha = 0.6, \beta_1 = 1, \beta_2 = 2.4]$ and $\Theta_2 = [\alpha = 0.8, \beta_1 = 1, \beta_2 = 2.4]$. $\phi_2$ takes the values of $0.32 < 0.33$ and $0.16 < 0.375$ respectively, but $\lambda_4 > \lambda_3$ at $\Theta_1$ and vice-versa at $\Theta_2$.

If (C.1) fails, $\lambda_4 \leq \lambda_3$ and we need to compare $\lambda_4$ and $\lambda_2$. Their difference is given by

$$\lambda_4 - \lambda_2 = \frac{\alpha\beta_1(\beta_1(8\phi_2^2 - \phi_2)/(\alpha\beta_2) + 6\phi_2^2)}{4(\alpha\beta_2 + \beta_1)^2(1 - 2\phi_2)}$$

The roots of $8\phi_2^2 - \phi_2$ occur at $\phi_2 = 0$ and $\phi_2 = 1/8$. This implies that for all $\phi_2 \in (0, 1/8]$ its value is negative and for all $\phi_2 > 1/8$ its value positive. The sign is determined by $\beta_1(8\phi_2^2 - \phi_2)/(\alpha\beta_2) + 6\phi_2^2$, which can be both positive and negative in the region of interest, i.e., when $\phi_2 \in (0, 2/5 - \theta]$ and (C.1) fails. Thus, $\lambda_4 \in [\lambda_2, \lambda_3)$, when $\beta_1(8\phi_2^2 - \phi_2)/(\alpha\beta_2) + 6\phi_2^2 \geq 0$, and $\lambda_4 < \lambda_2$ otherwise.

We illustrate this claim using two numerical examples. Consider the parameter vectors $\Theta_2 = [\alpha = 0.8, \beta_1 = 1, \beta_2 = 2.4]$ and $\Theta_3 = [\alpha = 0.95, \beta_1 = 1, \beta_2 = 2.4]$. Condition (C.1) fails for both vectors, $\phi_2$ takes the values of $0.16 < 0.375$ and $0.04 < 0.395$, respectively, but $\lambda_4 \geq \lambda_2$ at $\Theta_2$ and vice-versa at $\Theta_3$.

If (C.2) fails, $\lambda_4 < \lambda_2$ and we need to compare, $\lambda_4$ and $\lambda_1$. Their difference is:

$$\lambda_4 - \lambda_1 = \frac{\beta_1(1-\alpha)(4\alpha\beta_2(1-\phi_2) + \alpha\beta_2 + 3\beta_1)}{12(\alpha\beta_2 + \beta_1)^2(1 - 2\phi_2)}$$

which is strictly positive for all $\phi_2 \in (0, 2/5 - \theta] \subset (0, 1/2)$, thus $\lambda_4 > \lambda_1$. **Q.E.D.**

**Proof of lemma 9.** The home country is indifferent between $\rho = \rho_4$ and $\rho = \rho_2$ if and only if $\bar{V}_{1c}(\rho_2) - \bar{V}_{1e}(\rho_4) = 0 \Leftrightarrow l_6 - (1 - \phi_2)(\psi_1 - \psi_2) = 0$. Let $\Delta\psi \equiv \lambda_5 = l_6/(1 - \phi_2)$ : $\bar{V}_{1c}(\rho_2) - \bar{V}_{1e}(\rho_4) = 0$. Since $1 - \phi_2 > 0$, $\rho^* = \rho_2$ for all $\Delta\psi \leq \lambda_5$, and $\rho^* = \rho_4$ for all $\Delta\psi \geq \lambda_5$.

The difference between $\lambda_5$ and $\lambda_2$ is given by

$$\lambda_5 - \lambda_2 = \frac{\alpha\beta_1(\beta_1 \mathrm{B}_4/(\alpha\beta_2) + \mathrm{B}_5)}{8(\alpha\beta_2 + \beta_1)^2(1 - \phi_2)}$$

where $\mathrm{B}_4 = 22\phi_2^2 - 5\phi_2$ and $\mathrm{B}_5 = 23\phi_2^2 - 14\phi_2 + 4$. $\mathrm{B}_5 > 0$, since its discriminant is negative and at zero its value is positive. The roots of $\mathrm{B}_4$ are 0 and 5/22, thus for all $\phi_2 \in (0, 5/22)$ it is negative and for all $\phi_2 \geq 5/22$ it is positive. The sign is determined by the numerator, which is positive when $\mathrm{B}_4 \geq 0$, but may be negative, when $\mathrm{B}_4 < 0$.

Let $\mathrm{B}_4$ be negative and suppose that $\beta_1 \mathrm{B}_4/(\alpha\beta_2) + \mathrm{B}_5 \leq 0$, which we re-write as $\beta_1/(\alpha\beta_2) \geq -\mathrm{B}_5/\mathrm{B}_4$. One can verify that the intersection of the above inequality with the region of interest is empty. The region of interest occurs when condition (C.1) holds and $\phi_2 \in (0, 5/22) \cap (0, 2/5 - \theta]$. Thus, $\lambda_5 > \lambda_2$.
Hint: re-write $\phi_2 \leq 2/5 - \theta$ as $\beta_1/(\alpha\beta_2) \leq 4/(3\phi_2) - 10/3$. **Q.E.D.**

**Proof of lemma 10.** The home country is indifferent between $\rho = \rho_3$ and $\rho = \rho^{in}$ if and only if $\bar{V}_{1b}(\rho^{in}) - \bar{V}_{1d}(\rho_3) = 0 \Leftrightarrow l_7(\psi_1 - \psi_2)^2 + l_8(\psi_1 - \psi_2) + l_9 = 0$. The discriminant $\Delta = l_8^2 - 4l_7l_9$ simplifies to $(\alpha\beta_2(8\phi_2^2 - 20\phi_2 + 5) + 2\beta_1(2 - 7\phi_2))/(5\alpha\beta_2 + 4\beta_1)$, which is always positive in the region of interest, i.e., when both conditions (C.1) and (C.2) fail. Thus, the cost differences that make the home country indifferent between $\rho = \rho_3$ and $\rho = \rho^{in}$ are $\lambda_6 = (\pm\sqrt{l_8^2 - 4l_7l_9} - l_8)/(2l_7)$. From the two roots we keep the lower one, which is the one that satisfies $\lambda_6 < \lambda_2$, which as we show below must be satisfied. The difference between the zero-discriminant root, $-l_8/(2l_7)$, and $\lambda_2$ is given by $\beta_1(5\alpha\beta_2 + 4\beta_1)(1 - 2\phi_2)/(4\beta_2(\alpha\beta_2 + \beta_1)^2$, which is positive for all $\phi_2 \in (0, 1/2]$. The direction of the optimal policy follows from the fact that the quadratic function is convex and that $\lambda_6$ is the lower of the two roots. Thus, $\rho^* = \rho^{in}$ for all $\Delta\psi \leq \lambda_6$, and $\rho^* = \rho_4$ for all $\Delta\psi \geq \lambda_6$.

Suppose that $\lambda_6 \leq \lambda_4$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\lambda_6, \lambda_4]$ the following are true:
- From the first part of lemma 10, $\bar{V}_{1d}(\rho_3) \geq \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\lambda_6, \lambda_4]$.
- From lemma 6, $\bar{V}_{1b}(\rho^{in}) > \bar{V}_{1c}(\rho_2)$ for all $\Delta\psi \in [\lambda_6, \lambda_4]$, since $\lambda_4 < \lambda_2$.
- From lemma 8, $\bar{V}_{1c}(\rho_2) \geq \bar{V}_{1d}(\rho_3)$ for all $\Delta\psi \in [\lambda_6, \lambda_4]$.

Thus,
$$\bar{V}_{1b}(\rho^{in}) > \bar{V}_{1c}(\rho_2) \geq \bar{V}_{1d}(\rho_3) \geq \bar{V}_{1b}(\rho^{in}), \quad \forall\Delta\psi \in [\lambda_6, \lambda_4],$$
which is a contradiction, and therefore, $\lambda_6 > \lambda_4$.

Suppose that $\lambda_6 \geq \lambda_2$. Then, for any $\Delta\psi \in [\lambda_2, \lambda_6]$ the following are true:
- From the first part of lemma 10, $\bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1d}(\rho_3)$ for all $\Delta\psi \in [\lambda_2, \lambda_6]$.
- From lemma 6,$\bar{V}_{1c}(\rho_2) \geq \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\lambda_2, \lambda_6]$.
- From lemma 8, $\bar{V}_{1d}(\rho_3) > \bar{V}_{1c}(\rho_2)$ for all $\Delta\psi \in [\lambda_2, \lambda_6]$, since $\lambda_4 < \lambda_2$.

Thus,
$$\bar{V}_{1d}(\rho_3) > \bar{V}_{1c}(\rho_2) \geq \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1d}(\rho_3), \quad \forall\Delta\psi \in [\lambda_2, \lambda_6],$$
which is a contradiction, and therefore, $\lambda_6 < \lambda_2$. Thus, $\lambda_6 \in (\lambda_4, \lambda_2)$. **Q.E.D.**

### C.1.2 Low market efficiency: Details

For the case of low market efficiency the four equilibrium candidates are $\rho_1$, $\rho^{in}$, $\rho_2$, and $\rho_5$. The following lemmas provide the solution that maximises Eq. (3.13). In the $[\rho_2, \rho_1]$ interval the objective is similar to the very low efficiency case and the optimal policy is thus given by lemma 6. Next, we compare the welfare generated at $\rho_5$ and $\rho_2$, which are local optimal of $\bar{V}_{1e}$ and $\bar{V}_{1c}$. If $\bar{V}_{1e}(\rho_5)$ exceeds $\bar{V}_{1c}(\rho_2)$, the preferred strategy is $\rho_5$ and the home country prefers the foreign agent to buy everything. Otherwise, the preferred strategy is $\rho_2$ and the foreign agent buys $u_2 = \phi_2$. Lemma 15 summarises.

**Lemma 15.** Let $\Delta\psi = \lambda_7$ given by $\bar{V}_{1e}(\rho_5) = \bar{V}_{1c}(\rho_2)$ be a cost difference threshold.[17] Then, the optimal policy $\rho^*$ is given by
$$\rho^* = \begin{cases} \rho_5, & \Delta\psi \geq \lambda_7, \\ \rho_2, & \Delta\psi \leq \lambda_7. \end{cases}$$
Let $\alpha\phi_2(17-23\phi_2)-(22\phi_2^2-21\phi_2+2)\beta_1/\beta_2 \geq 0$ be condition (C.3). The relative position of $\lambda_7$ depends on (C.3) as follows:
a. Let condition (C.3) to hold. Then, $\lambda_7 \geq \lambda_2$.
b. Let condition (C.3) to fail. Then, $\lambda_7 \in (\lambda_1, \lambda_2)$.
See appendix C.1.3 for the proof.

Suppose that (C.3) holds, then it follows from lemmas 6 and 15 that $\lambda_1 < \lambda_2 \leq \lambda_7$. This implies that strategies $\rho_1$, $\rho^{in}$, $\rho_2$ and $\rho_5$ occur in distinct $\Delta\psi$ regions and thus a complete equilibrium exists. If condition (C.3) binds, then $\lambda_7 = \lambda_2$ and strategy $\rho = \rho_2$ becomes weakly dominated by $\rho = \rho_5$ and $\rho = \rho^{in}$, this is a degeneration.

Suppose that (C.3) fails, i.e., $\lambda_7 < \lambda_2$. Then, $\rho = \rho_2$ is strictly dominated by $\rho = \rho^{in}$ for all $\Delta\psi \leq \lambda_7 < \lambda_2$, see lemma 6. Also, from lemma 15, $\rho = \rho_2$ is strictly dominated by $\rho = \rho_5$ for all $\Delta\psi > \lambda_7$. This implies that we need to compare strategies $\rho_5$ and $\rho_2$. Lemma 16 provides the optimal policy.

[17]For explicit definitions of the cost difference thresholds associated with this case see appendix C.1.3.

**Lemma 16.** Suppose that $\lambda_7 \in (\lambda_1, \lambda_2)$ and let $\Delta\psi = \lambda_8$ given by $\bar{V}_{1e}(\rho_5) = \bar{V}_{1b}(\rho^{in})$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_5, & \Delta\psi \geq \lambda_8, \\ \rho^{in}, & \Delta\psi \leq \lambda_8. \end{cases}$$

In addition, $\lambda_8 \in (\lambda_7, \lambda_2)$ when (C.3) fails.
See appendix C.1.3 for the proof.

It then follows from lemmas 6, 15 and 16, that $\lambda_8 > \lambda_1$, when $\lambda_7 \in (\lambda_1, \lambda_2)$. This implies that strategies $\rho_1$, $\rho^{in}$ and $\rho_5$ are mutually exclusive.

### C.1.3 Low market efficiency: Proofs

Cost difference thresholds and parameter expressions used below are summarised here.

$$\lambda_7 = \frac{l_{10}}{1-\phi_2}, \quad \lambda_8 = -\frac{l_{12} + \sqrt{l_{12}^2 - 4l_{11}l_{13}}}{2l_{11}}$$

where

$$l_{10} = \frac{\beta_1\phi_2((17-20\phi_2)(1-\alpha)\beta_1 - 3\alpha(19\phi_2^2 - 9\phi_2 - 4))}{24(\alpha\beta_2 + \beta_1)^2(1-\phi_2)},$$

$$l_{11} = \frac{2\beta_2(\alpha\beta_2+\beta_1)^2}{\beta_1(5\alpha\beta_2 + 4\beta_1)}, \quad l_{12} = \frac{(3\phi_2 - 5)\alpha\beta_2 + (3\phi_2 - 4)\beta_1}{5\alpha\beta_2 + 4\beta_1},$$

$$l_{13} = \frac{\beta_1\phi_2\big((23-27\phi_2)(1-\alpha)\beta_1^2 - (180\phi_2^2 - 125\phi_2 - 16)\alpha\beta_1 - (99\phi_2^2 - 49\phi_2 - 20)\alpha^2\beta_2\big)}{8(5\alpha\beta_2 + 4\beta_1)(\alpha\beta_2+\beta_1)^2(1-\phi_2)}$$

**Proof of lemma 15.** The home country is indifferent between $\rho = \rho_5$ and $\rho = \rho_2$ if and only if $\bar{V}_{1c}(\rho_2) - \bar{V}_{1e}(\rho_5) = 0 \Leftrightarrow l_{10} - (1-\phi_2)(\psi_1 - \psi_2)$. Let $\Delta\psi \equiv \lambda_7 = l_{10}/(1-\phi_2)$ : $\bar{V}_{1c}(\rho_2) - \bar{V}_{1e}(\rho_5) = 0$. Since $1 - \phi_2 > 0$, $\rho^* = \rho_2$ for all $\Delta\psi \leq \lambda_7$, and $\rho^* = \rho_5$ for all $\Delta\psi \geq \lambda_7$.

First, we compare $\lambda_7$ and $\lambda_2$. Their difference is given by

$$\lambda_7 - \lambda_2 = \frac{\beta_1\phi_2(\alpha\phi_2(17-23\phi_2) - (22\phi_2^2 - 21\phi_2 + 2)\beta_1/\beta_2)}{8(\alpha\beta_2+\beta_1)^2(1-\phi_2)^2},$$

which can be both positive and negative in the region of interest, i.e.,
$\phi_2 \in (\max(0, 2/5-\theta, 1/2]$. Thus, if $\alpha\phi_2(17-23\phi_2) - (22\phi_2^2 - 21\phi_2 + 2)\beta_1/\beta_2 \geq 0$, then $\lambda_7 \geq \lambda_2$, otherwise, $\lambda_7 < \lambda_2$.

We illustrate this claim using two numerical examples. Consider the parameter vectors $\Theta_4 = [\alpha = 0.5, \beta_1 = 4, \beta_2 = 1]$ and $\Theta_5 = [\alpha = 0.5, \beta_1 = 4, \beta_2 = 0.5]$. Parameter $\phi_2$ takes the values of $0.167 \in (0, 1/2]$ and $0.083 \in (0, 1/2]$, respectively, but $\lambda_7 > \lambda_2$ at $\Theta_4$ and vice-versa at $\Theta_5$.

If (C.3) fails, $\lambda_7 < \lambda_2$ and we need to compare $\lambda_7$ and $\lambda_1$. Their difference is given by

$$\lambda_7 - \lambda_1 = -\frac{\beta_1\phi_2(\alpha\beta_2(13\phi_2^2 + 3\phi_2 - 10) + \beta_1(14\phi_2^2 - 5\phi_2 - 6))}{8\beta_2(\alpha\beta_2+\beta_1)^2(1-\phi_2)^2},$$

which is positive for all $\phi_2 \in (\max(0, 2/5-\theta), 1/2] \subseteq (0, 1/2]$, and thus $\lambda_7 > \lambda_1$. **Q.E.D.**

**Proof of lemma 16.** The home country is indifferent between $\rho = \rho_5$ and $\rho = \rho^{in}$ if and only if $\bar{V}_{1b}(\rho^{in}) - \bar{V}_{1e}(\rho_5) = 0 \Leftrightarrow l_{11}(\psi_1 - \psi_2)^2 + l_{12}(\psi_1 - \psi_2) + l_{13} = 0$. The discriminant $\Delta = l_{12}^2 - 4l_{11}l_{13}$ simplifies to $((18\phi_2^3 - 2\phi_2^2 - 15\phi_2 + 5)\alpha\beta_2 + (18\phi_2^3 - 9\phi_2^2 - 10\phi_2 + 4)\beta_1)/((1 - \phi_2)(5\alpha\beta_2 + 4\beta_1))$, which is always positive in the region of interest, i.e., when condition (C.3) fails. Thus, the cost differences that make the home country indifferent between $\rho = \rho_5$ and $\rho = \rho^{in}$ are $\lambda_8 = (\pm\sqrt{l_{12}^2 - 4l_{11}l_{13}} - l_{12})/(2l_{11})$. From the two roots we keep the lower one, which is the only one that satisfies $\lambda_8 < \lambda_2$, which as we shall show next must be satisfied. The difference between the zero-discriminant root, $-l_{12}/(2l_{11})$, and $\lambda_2$ is given by $(1 - \phi_2)(15\alpha\beta_2 + 12\beta_1)\beta_1/(12(\alpha\beta_2 + \beta_1)^2\beta_2)$, which is strictly positive for all $\phi_2 \in (0, 1/2]$. The direction of the optimal policy follows from the fact that the quadratic function is convex and that $\lambda_8$ is the lower of the two roots. Thus, $\rho^* = \rho^{in}$ for all $\Delta\psi \leq \lambda_8$, and $\rho^* = \rho_5$ for all $\Delta\psi \geq \lambda_8$.

Suppose that $\lambda_8 \leq \lambda_7$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\lambda_8, \lambda_7]$ the following are true:
- From the first part of lemma 16, $\bar{V}_{1e}(\rho_5) \geq \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\lambda_8, \lambda_7]$.
- From lemma 15, $\bar{V}_{1c}(\rho_2) \geq \bar{V}_{1e}(\rho_5)$ for all $\Delta\psi \in [\lambda_8, \lambda_7]$.
- From lemma 6, $\bar{V}_{1b}(\rho^{in}) > \bar{V}_{1c}(\rho_2)$ for all $\Delta\psi \in [\lambda_8, \lambda_7]$, since $\lambda_7 < \lambda_2$.

Thus,
$$\bar{V}_{1b}(\rho^{in}) > \bar{V}_{1c}(\rho_2) \geq \bar{V}_{1e}(\rho_5) \geq \bar{V}_{1b}(\rho^{in}), \quad \forall \Delta\psi \in [\lambda_8, \lambda_7],$$
which is a contradiction, and therefore, $\lambda_8 > \lambda_7$.

Suppose that $\lambda_8 \geq \lambda_2$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\lambda_2, \lambda_8]$ the following are true:
- From the first part of lemma 16, $\bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_5)$ for all $\Delta\psi \in [\lambda_2, \lambda_8]$.
- From lemma 15, $\bar{V}_{1e}(\rho_5) > \bar{V}_{1c}(\rho_2)$ for all $\Delta\psi \in [\lambda_2, \lambda_8]$, since $\lambda_7 < \lambda_2$.
- From lemma 6, $\bar{V}_{1c}(\rho_2) \geq \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\lambda_2, \lambda_8]$.

Thus,
$$\bar{V}_{1e}(\rho_5) > \bar{V}_{1c}(\rho_2) \geq \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_5), \quad \forall \Delta\psi \in [\lambda_2, \lambda_8],$$
which is a contradiction, and therefore, $\lambda_8 < \lambda_2$. Thus, $\lambda_8 \in (\lambda_7, \lambda_2)$. **Q.E.D.**

### C.1.4 Medium market efficiency: Details

For the case of medium market efficiency the five equilibrium candidates are $\rho_1$, $\rho^{in}$, $\rho_7$, $\rho_8$, and $\rho_9$. The following lemmas provide the solution that maximises Eq. (3.14). First, we compare the candidates in the concave branch, i.e., $[\rho_7, \rho_1]$ subregion. Lemma 17 provides the optimal policy.

**Lemma 17.** Let $\Delta\psi = \lambda_1$ and $\Delta\psi = \lambda_9$ given by $\rho^{in} = \rho_1$ and $\rho^{in} = \rho_7$, respectively, be cost difference thresholds.[18] Then, the optimal policy $\rho^*$ in the $[\rho_7, \rho_1]$ region is

$$\rho^* = \min(\max(\rho_7, \rho^{in}), \rho_1) = \begin{cases} \rho_7, & \Delta\psi \geq \lambda_9, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_9, \\ \rho_1, & \Delta\psi \leq \lambda_1. \end{cases}$$

In addition, $\lambda_1 < 0$, $\lambda_9 \lesseqgtr 0$, and $\lambda_1 < \lambda_9$ for all $\phi_2 \in (1/2, 2/3)$.
See appendix C.1.5 for the proof.

Since $\lambda_9 \lesseqgtr 0$, there exist market conditions such that $\lambda_9 < 0$ where the foreign agent buys the positive quantities, $u_2^{in}(\rho^{in})$ and $2 - 3\phi_2$ despite the fact that it has a

[18]For explicit definitions of the cost difference thresholds associated with this case see appendix C.1.5.

cost disadvantage. For efficiency levels less than $\lambda_1$, the home country charges $\rho_1$ which induces the foreign agent to buy zero quotas. Next, we compare the value at $\rho_9$ and $\rho_8$, which are local optimal of $\bar{V}_{1e}$ and $\bar{V}_{1d}$ respectively. Lemma 18 provides the preferred strategy. In addition, lemma 19 provides the optimal policy between $\rho_8$ and $\rho_7$, which are local optima of $\bar{V}_{1d}$ and $\bar{V}_{1c}$ respectively.

**Lemma 18.** Let $\Delta\psi = \lambda_{10}$ given by $\bar{V}_{1e}(\rho_9) = \bar{V}_{1d}(\rho_8)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_9, & \Delta\psi \geq \lambda_{10}, \\ \rho_8, & \Delta\psi \leq \lambda_{10}. \end{cases}$$

In addition, $\lambda_{10} > 0$ and $\lambda_{10} > \lambda_9$ for all $\phi_2 \in (1/2, 2/3)$.
See appendix C.1.5 for the proof.

**Lemma 19.** Let $\Delta\psi = \lambda_{11}$ given by $\bar{V}_{1d}(\rho_8) = \bar{V}_{1c}(\rho_7)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_8, & \Delta\psi \geq \lambda_{11}, \\ \rho_7, & \Delta\psi \leq \lambda_{11}. \end{cases}$$

Let $\alpha\beta_2(162\phi_2^3 - 315\phi_2^2 + 174\phi_2 - 28) + \beta_1(108\phi_2^3 - 234\phi_2^2 + 135\phi_2 - 22) > 0$ be condition (C.4). The relative position of $\lambda_{11}$ depends on (C.4) as follows:
a. Let condition (C.4) to hold. Then, $\lambda_{11} > \lambda_{10}$.
b. Let condition (C.4) to fail. Then $\lambda_{11} \in (\lambda_9, \lambda_{10}]$.
See appendix C.1.5 for the proof.

Suppose (C.4) fails, then it follows from lemmas 17-19 that $\lambda_1 < \lambda_9 < \lambda_{11} \leq \lambda_{10}$. This implies that all strategies occur in distinct $\Delta\psi$ region and a thus complete equilibrium exists. If condition (C.4) binds, then $\lambda_{11} = \lambda_{10}$ and strategy $\rho = \rho_8$ becomes weakly dominated by $\rho_9$ and $\rho_7$, this is a degeneration.

Suppose that (C.4) holds, then $\lambda_{11} > \lambda_{10}$ and strategy $\rho = \rho_8$ is strictly dominated by $\rho = \rho_9$ for all $\Delta\psi \geq \lambda_{11} > \lambda_{10}$, see lemma 18, and by $\rho = \rho_7$ for all $\Delta\psi < \lambda_{11}$, see lemma 19. This means that we need to compare strategies $\rho_9$ and $\rho_7$. The next lemma provides the optimal policy.

**Lemma 20.** Let $\Delta\psi = \lambda_{12}$ given by $\bar{V}_{1e}(\rho_9) = \bar{V}_{1c}(\rho_7)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_9, & \Delta\psi \geq \lambda_{12}, \\ \rho_7, & \Delta\psi \leq \lambda_{12}. \end{cases}$$

In addition, $\lambda_{12} > 0$ and $\lambda_{12} > \lambda_9$ for all $\phi_2 \in (1/2, 2/3)$.
See appendix C.1.5 for the proof.

Thus, when $\lambda_{11} > \lambda_{10}$, it follows from lemmas 17-20 that $\lambda_1 < \lambda_9 < \lambda_{12}$. This implies that strategies $\rho_1$, $\rho^{in}$, $\rho_7$ and $\rho_9$ are mutually exclusive for different $\Delta\psi$ levels, and thus another complete equilibrium exists.

### C.1.5 Medium market efficiency: Proofs

Cost difference thresholds and parameter expressions used below are summarised here.

$$\lambda_1 = \frac{l_2}{l_1}, \quad \lambda_9 = \frac{l_{14}}{l_1}, \quad \lambda_{10} = \frac{l_{15}}{2-3\phi_2}, \quad \lambda_{11} = \frac{l_{16}}{3(2\phi_2-1)}, \quad \lambda_{12} = \frac{l_{17}}{3\phi_2-1},$$

where

$$l_{14} = \frac{\beta_1(4\alpha\beta_2+3\beta_1)((2-3\phi_2)(5\alpha\beta_2+4\beta_1)-3\phi_2(\alpha\beta_2+\beta_1))}{4\beta_2(\alpha\beta_2+\beta_1)^2(5\alpha\beta_2+4\beta_1)},$$
$$l_{15} = -\frac{\beta_1(\alpha\beta_2(9\phi_2^2-18\phi_2+4)+\beta_1(2-9\phi_2))}{8\beta_2(\alpha\beta_2+\beta_1)^2}, \quad l_{16} = -\frac{\beta_1(9\phi_2^2-9\phi_2+2)}{2\beta_2(\alpha\beta_2+\beta_1)},$$
$$l_{17} = -\frac{\beta_1(3\alpha\beta_2(15\phi_2^2-18\phi_2+4)+\beta_1(36\phi_2^2-45\phi_2+10))}{8\beta_2(\alpha\beta_2+\beta_1)^2}.$$

For the implicit expressions of $l_1$ and $l_2$ see appendix C.1.1.

**Proof of lemma 17.** The differences between the two bounds $\rho_1$, $\rho_7$ and the inner policy $\rho^{in}$ are given by $\rho_1-\rho^{in} = (\psi_1-\psi_2)l_1-l_2$ and $\rho_7-\rho^{in} = (\psi_1-\psi_2)l_1-l_{14}$ with $l_1>0$, $l_2<0$ and $l_{14} \lesseqgtr 0$. Let $\Delta\psi \equiv \lambda_1 = l_2/l_1 : \rho_1 = \rho^{in}$ and $\Delta\psi \equiv= \lambda_9 = l_1/l_{14} : \rho_7 = \rho^{in}$. The optimal policy follows from the concavity of $\bar{V}_{1b}$. In addition, $\lambda_1 < 0$ and $\lambda_9 \lesseqgtr 0$, with $\lambda_1 < \lambda_9$. Their difference is given by

$$\lambda_1 - \lambda_2 = -\frac{\beta_1(2-3\phi_2)(5\alpha\beta_2+4\beta_1)}{4\beta_2(\alpha\beta_2+\beta_1)^2},$$

which is negative for all $\phi_2 \in (1/2, 2/3)$. **Q.E.D.**

**Proof of lemma 18.** The home country is indifferent between $\rho=\rho_9$ and $\rho=\rho_8$ if and only if $\bar{V}_{1d}(\rho_8)-\bar{V}_{1e}(\rho_9) = 0 \Leftrightarrow l_{15}-(2-3\phi_2)(\psi_1-\psi_2) = 0$. Let $\Delta\psi \equiv \lambda_{10} = l_{15}/(2-3\phi_2) : \bar{V}_{1d}(\rho_8)-\bar{V}_{1e}(\rho_9) = 0$. Since $2-3\phi_2 > 0$, $\rho^* = \rho_8$ for all $\Delta\psi \leq \lambda_{10}$, and $\rho^* = \rho_9$ for all $\Delta\psi \geq \lambda_{10}$.

Threshold $\lambda_{10}$ is strictly positive since $l_{15} > 0$ for all $\phi_2 \in (1/2, 2/3)$. Moreover, the difference between $\lambda_{10}$ and $\lambda_9$ is given by

$$\lambda_{10}-\lambda_9 = -\frac{\beta_1(\alpha\beta_2(117\phi_2^2-150\phi_2+44)+\beta_1(90\phi_2^2-117\phi_2+34))}{8\beta_2(\alpha\beta_2+\beta_1)^2(2-3\phi_2)}$$

For all $\phi_2 \in (1/2, 2/3)$, both $117\phi_2^2-150\phi_2+44$ and $90\phi_2^2-117\phi_2+34$ are negative. Thus, $\lambda_{10} > \lambda_9$. **Q.E.D.**

**Proof of lemma 19.** The home country is indifferent between $\rho=\rho_8$ and $\rho=\rho_7$ if and only if $\bar{V}_{1c}(\rho_7)-\bar{V}_{1d}(\rho_8) = 0 \Leftrightarrow l_{16}-3(2\phi_2-1)(\psi_1-\psi_2) = 0$. Let $\Delta\psi \equiv \lambda_{11} = l_{16}/(3(2\phi_2-1)) : \bar{V}_{1c}(\rho_7)-\bar{V}_{1d}(\rho_8) = 0$. Since $2\phi_2-1 > 0$ for all $\phi_2 \in (1/2, 2/3)$, $\rho^* = \rho_7$ for all $\Delta\psi \leq \lambda_{11}$, and $\rho^* = \rho_8$ for all $\Delta\psi \geq \lambda_{11}$.

First, we compare $\lambda_{11}$ and $\lambda_{10}$. Their difference is given by

$$\lambda_{11}-\lambda_{10} = \frac{\beta_1(\alpha\beta_2(162\phi_2^3-315\phi_2^2+174\phi_2-28)+\beta_1(108\phi_2^3-234\phi_2^2+135\phi_2-22)}{24\beta_2(\alpha\beta_2+\beta_1)^2(2\phi_2-1)(2-3\phi_2)}$$

For all $\phi_2 \in (1/2, 2/3)$ the numerator can be both negative and positive. Thus, $\lambda_{11} > \lambda_{10}$ when $\alpha\beta_2(162\phi_2^3-315\phi_2^2+174\phi_2-28)+\beta_1(108\phi_2^3-234\phi_2^2+135\phi_2-22) > 0$, otherwise $\lambda_{11} \leq \lambda_{10}$.

We illustrate this claim using two numerical examples. Consider the parameter vectors $\Theta_6 = [\alpha = 0.3, \beta_1 = 1, \beta_2 = 2.2]$ and $\Theta_7 = [\alpha = 0.3, \beta_1 = 1, \beta_2 = 2.5]$, $\phi_2$ takes the values of 0.513 and 0.583 respectively, but $\lambda_{11} > \lambda_{10}$ at $\Theta_6$ and vice-versa at $\Theta_7$.

If (C.4) fails, $\lambda_{11} \leq \lambda_{10}$ and we need to compare $\lambda_{11}$ and $\lambda_9$. Their difference is given by

$$\lambda_{11} - \lambda_9 = \frac{\beta_1(\alpha\beta_2(90\phi_2^2 - 96\phi_2 + 26) + \beta_1(72\phi_2^2 - 75\phi_2 + 20))}{12\beta_2(\alpha\beta_2 + \beta_1)^2(2\phi_2 - 1)},$$

which is strictly positive for all $\phi_2 \in (1/2, 2/3)$. Thus $\lambda_{11} > \lambda_9$. **Q.E.D.**

**Proof of lemma 20.** The home country is indifferent between $\rho = \rho_9$ and $\rho = \rho_7$ if and only if $\bar{V}_{1c}(\rho_7) - \bar{V}_{1e}(\rho_9) = 0 \Leftrightarrow l_{17} - (3\phi_2 - 1)(\psi_1 - \psi_2)$. Let $\Delta\psi \equiv \lambda_{12} = l_{17}/(3\phi_2 - 1) : \bar{V}_{1c}(\rho_7) - \bar{V}_{1e}(\rho_9) = 0$, with $l_{17} > 0$ for all $\phi_2 \in (1/2, 2/3)$. Since $3\phi_2 - 1 > 0$, $\rho^* = \rho_2$ for all $\Delta\psi \leq \lambda_{12}$, and $\rho^* = \rho_9$ for all $\Delta\psi \geq \lambda_{12}$.

The difference between $\lambda_{12}$ and $\lambda_9$ is given by

$$\lambda_{12} - \lambda_9 = \frac{\beta_1(\alpha\beta_2(63\phi_2^2 - 42\phi_2 + 8) + \beta_1(54\phi_2^2 - 33\phi_2 + 6))}{8\beta_2(\alpha\beta_2 + \beta_1)^2(3\phi_2 - 1)},$$

which is strictly positive for all $\phi_2 \in (1/2, 2/3)$. Thus $\lambda_{12} > \lambda_9$. **Q.E.D.**

## C.2 Profit-maximiser foreign agent

### C.2.1 Low market efficiency: Details

For the case of low market efficiency when the foreign agent is profit-maximiser the four equilibrium candidates are $\rho_1$, $\rho^{in}$, $\rho_2$ and $\rho_5$. The following lemmas provide the solution that maximises Eq. (3.16). We use $\mu_i$ to denote the cost difference thresholds when $\xi = 0$. We start with the concave part.

**Lemma 21.** Let $\Delta\psi = \mu_1$ and $\Delta\psi = \mu_2$ given by $\rho^{in} = \rho_1$ and $\rho^{in} = \rho_2$, respectively, be cost difference thresholds.[19] Then, the optimal policy $\rho^*$ in the $[\rho_2, \rho_1]$ region is given by

$$\rho^* = \min(\max(\rho_2, \rho^{in}), \rho_1) = \begin{cases} \rho_2, & \Delta\psi \geq \mu_2, \\ \rho^{in}, & \mu_1 \leq \Delta\psi \leq \mu_2, \\ \rho_1, & \Delta\psi \leq \mu_1. \end{cases}$$

In addition, $\mu_1 > 0$ and $\mu_2 > 0$ for all $\phi_2 \in (0, 2/7]$ with $\mu_2 > \mu_1$.
See appendix C.2.2 for the proof.

Lemma 21 is the equivalent of lemma 6 when the foreign agent is profit-maximiser. Since $\mu_1 > 0$, the home country induces the foreign agent to buy nothing even if its firm has a cost disadvantage. In order for the foreign firm to buy some quotas, its cost advantage has to exceed $|\mu_1|$. Next, we compare the welfare generated at $\rho_5$ and $\rho_2$. The following lemma provides the optimal policy.

**Lemma 22.** Let $\Delta\psi = \mu_3$ given by $\bar{V}_{1e}(\rho_5) = \bar{V}_{1c}(\rho_2)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_5, & \Delta\psi \geq \mu_3, \\ \rho_2, & \Delta\psi \leq \mu_3. \end{cases}$$

[19] For explicit definitions of the cost difference thresholds associated with this case see appendix C.2.2.

In addition, $\mu_3 < \mu_2$ for all $\phi_2 \in (0, 2/7]$.
See appendix C.2.2 for the proof.

Since $\mu_3$ is less than $\mu_2$, $\rho_2$ is strictly dominated by $\rho = \rho^{in}$ for all $\Delta\psi \leq \mu_3 < \mu_2$, see lemma 21, and by $\rho = \rho_5$ for all $\Delta\psi > \mu_3$, see lemma 22. Thus, we proceed to the comparison of $\rho_5$ and $\rho^{in}$. Lemma 23 gives the optimal policy.

**Lemma 23.** Let $\Delta\psi = \mu_4$ given by $\bar{V}_{1e}(\rho_5) = \bar{V}_{1b}(\rho^{in})$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_5, & \Delta\psi \geq \mu_4, \\ \rho^{in}, & \Delta\psi \leq \mu_4. \end{cases}$$

Let $1 - \sqrt{\dfrac{(3\alpha\beta_2 + 2\beta_1)(6\phi_2^3 - \phi_2^2 - 5\phi_2 + 3) + \alpha\beta_2\phi_2^2 + 2\beta_1\phi_2(3\phi_2^2 - 3\phi_2 + 2))}{(1-\phi_2)(5\alpha\beta_2 + 6\beta_1)}} \geq 0$ be condition (C.5). The relative position of $\mu_4$ depends on (C.5) as follows:
a. Let condition (C.5) to hold. Then $\mu_4 \in [\mu_1, \mu_2)$.
b. Let condition (C.5) to fail. Then, $\mu_4 < \mu_1$.
See appendix C.2.2 for the proof.

Suppose that (C.5) holds, then it follows from lemmas 21-23 that $\mu_1 \leq \mu_4 < \mu_2$, and strategies $\rho_1$, $\rho^{in}$ and $\rho_5$ are mutually exclusive, and thus a complete equilibrium exists. If condition (C.5) binds, then $\mu_4 = \mu_1$ and $\rho^{in}$ becomes a weakly dominated strategy, this is a degeneration.

Suppose that (C.5) fails, then $\mu_4 < \mu_1$ and $\rho = \rho^{in}$ is strictly dominated by $\rho = \rho_1$ for all $\Delta\psi \leq \mu_4 < \mu_1$ and by $\rho = \rho_5$ for all $\Delta\psi > \mu_4$, see lemmas 21 and 23. Then, we need to compare $\rho = \rho_5$ and $\rho = \rho_1$.

**Lemma 24.** Suppose that $\mu_4 < \mu_1$ and let $\Delta\psi = \mu_5$ given by $\bar{V}_{1e}(\rho_5) = \bar{V}_{1a}(\rho_1)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_5, & \Delta\psi \geq \mu_5, \\ \rho_1, & \Delta\psi \leq \mu_5. \end{cases}$$

In addition, $\mu_5 \in (\mu_4, \mu_1)$. See appendix C.2.2 for the proof.

Thus, when (C.5) fails, it follows from lemmas 21-23 that both $\rho_2$ and $\rho^{in}$ are strictly dominated, and that strategies $\rho_1$ and $\rho_5$ occur in distinct $\Delta\psi$ regions, another complete equilibrium.

### C.2.2 Low market efficiency: Proofs

Cost difference thresholds and parameter expressions used below are summarised here.

$$\mu_1 = \frac{m_2}{m_1}, \quad \mu_2 = \frac{m_3}{m_1}, \quad \mu_3 = \frac{m_4}{1-\phi_2}, \quad \mu_4 = -\frac{m_6 + \sqrt{m_6^2 - 4m_5m_7}}{2m_5}, \quad \mu_5 = m_8$$

where

$$m_1 = \frac{4(\alpha\beta_2+\beta_1)}{5\alpha\beta_2+6\beta_1}, \quad m_2 = \frac{\alpha\beta_1(2-3\phi_2)}{(\alpha\beta_2+\beta_1)(5\alpha\beta_2+6\beta_1)}, \quad m_3 = \frac{2\beta_1((1-\alpha)(\alpha\beta_2+3\beta_1)+3\alpha)}{(\alpha\beta_2+\beta_1)(5\alpha\beta_2+6\beta_1)},$$

$$m_4 = \frac{\beta_1\phi_2(2\phi_2(1-6\phi_2)\beta_1/\beta_2-\alpha(19\phi_2^2-13\phi_2+4))}{8(1-\phi_2)(\alpha\beta_2+\beta_1)^2}, \quad m_5 = \frac{2\beta_2(\alpha\beta_2+\beta_1)^2}{\beta_1(5\alpha\beta_2+6\beta_1)},$$

$$m_6 = -\frac{\alpha\beta_2(3\phi_2^2-10\phi_2+7)+6\beta_1(1-\phi_2)}{(1-\phi_2)(5\alpha\beta_2+6\beta_1)}, \quad m_8 = \frac{(1-\alpha)\beta_1(4-9\phi_2)\phi_2}{12(1-\phi_2)(\alpha\beta_2+\beta_1)},$$

$$m_7 = \frac{(12\beta_1/\beta_2+22\alpha)(4-9\phi_2)\phi_2^2\beta_1^2-(99\phi_2^3-61\phi_2^2+16\phi_2-4)\alpha^2\beta_2\beta_1}{8(1-\phi_2)(\alpha\beta_2+\beta_1)^2(5\alpha\beta_2+6\beta_1)}.$$

**Proof of lemma 21.** The differences between the two bounds $\rho_1$, $\rho_2$ and the inner policy $\rho^{in}$ are given by $\rho_1-\rho^{in}=(\psi_1-\psi_2)m_1-m_2$ and $\rho_2-\rho^{in}=(\psi_1-\psi_2)m_1-m_3$ with $m_1>0$, $m_2>0$, and $m_3>0$. Let $\Delta\psi\equiv\mu_1=m_2/m_1:\rho_1=\rho^{in}$ and $\Delta\psi\equiv\mu_2=m_3/m_1:\rho_2=\rho^{in}$, with $\mu_1>0$, $\mu_2>0$ and $\mu_2>\mu_1$ for all $\phi_2\in(0,2/7]$. The optimal policy follows from the concavity of $\bar{V}_{1b}$. **Q.E.D.**

**Proof of lemma 22.** The home country is indifferent between $\rho=\rho_5$ and $\rho=\rho_2$ if and only if $\bar{V}_{1c}(\rho_2)-\bar{V}_{1e}(\rho_5)=0 \Leftrightarrow m_4-(1-\phi_2)(\psi_1-\psi_2)=0$. Let $\Delta\psi\equiv\mu_3=m_4/(1-\phi_2):\bar{V}_{1c}(\rho_2)-\bar{V}_{1e}(\rho_5)=0$. Since $1-\phi_2>0$, $\rho^*=\rho_2$ for all $\Delta\psi\leq\mu_3$, and $\rho^*=\rho_5$ for all $\Delta\psi\geq\mu_3$.

Suppose that $\mu_3\geq\mu_2$. Then,

$$\mu_3-\mu_2\geq 0 \Leftrightarrow -\frac{\beta_1(3\alpha\mathrm{C}_1+2(1-\alpha)\beta_1\mathrm{C}_2)}{24(\alpha\beta_2+\beta_1)^2\mathrm{C}_3}\geq 0,$$

where $\mathrm{C}_1=23\phi_2^3-17\phi_2^2+4$, $\mathrm{C}_2=12\phi_2^2-13\phi_2+6$, and $\mathrm{C}_3=\phi_2^2-2\phi_2+1$. This is a contradiction, since all $\mathrm{C}_1$, $\mathrm{C}_2$ and $\mathrm{C}_3$ are strictly positive when $\phi_2\in(0,2/7]$. Thus, $\mu_3<\mu_2$ for all $\phi_2\in(0,1/2]$. **Q.E.D.**

**Proof of lemma 23.** The home country is indifferent between $\rho=\rho_5$ and $\rho=\rho^{in}$ if and only if $\bar{V}_{1b}(\rho^{in})-\bar{V}_{1e}(\rho_5)=0\Leftrightarrow m_5(\psi_1-\psi_2)^2+m_6(\psi_1-\psi_2)+m_7=0$. The discriminant $\Delta=m_6^2-4m_5m_7$ simplifies to $((3\alpha\beta_2+2\beta_1)(6\phi_2^3-\phi_2^2-5\phi_2+3)+\alpha\beta_2\phi_2^2+2\beta_1\phi_2(3\phi_2^2-3\phi_2+2))/((1-\phi_2)(5\alpha\beta_2+6\beta_1))$, which is strictly positive for all $\phi_2\in(0,1/2]$. Thus, the cost differences that make the home country indifferent between $\rho=\rho_5$ and $\rho=\rho^{in}$ are $\mu_4=(\pm\sqrt{m_6^2-4m_5m_7}-m_6)/(2m_5)$. From the two roots we keep the lower one, which is the only one that satisfies $\mu_4<\mu_2$, which as we show below must be satisfied. The difference between the zero-discriminant root, $-m_6/(2m_5)$, and $\mu_2$ is given by $(15\alpha\beta_2+18\beta_1)(1-\phi_2)\beta_1/(12(\alpha\beta_2+\beta_1)^2\beta_2)$, which is positive for all $\phi_2\in(0,1/2]$. The direction of the optimal policy follows from the fact that the quadratic function is convex and that $\mu_4$ is the lower of the two roots. Thus, $\rho^*=\rho^{in}$ for all $\Delta\psi\leq\mu_4$, and $\rho^*=\rho_5$ for all $\Delta\psi\geq\mu_4$.

Suppose that $\mu_4\geq\mu_2$. Then, for any $\Delta\psi=\psi_1-\psi_2\in[\mu_2,\mu_4]$ the following are true:

- From the first part of lemma 23, $\bar{V}_{1b}(\rho^{in})\geq\bar{V}_{1e}(\rho_5)$ for all $\Delta\psi\in[\mu_2,\mu_4]$.
- From lemma 21, $\bar{V}_{1c}(\rho_2)\geq\bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi\in[\mu_2,\mu_4]$.
- From lemma 22, $\bar{V}_{1e}(\rho_5)>\bar{V}_{1c}(\rho_2)$ for all $\Delta\psi\in[\mu_2,\mu_4]$, since $\mu_3<\mu_2$.

Thus,

$$\bar{V}_{1e}(\rho_5) > \bar{V}_{1c}(\rho_2) \geq \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_5), \quad \forall \Delta\psi \in [\mu_2, \mu_4],$$

which is a contradiction, and therefore, $\mu_4 < \mu_2$.

The difference between $\mu_4$ and $\mu_1$ is: $\mu_4 - \mu_1 = \mathrm{C}_4 \dfrac{\beta_1(5\alpha\beta_2 + 6\beta_1)}{4\beta_2(\alpha\beta_2 + \beta_1)^2}$, where

$$\mathrm{C}_4 = 1 - \sqrt{\frac{(3\alpha\beta_2 + 2\beta_1)(6\phi_2^3 - \phi_2^2 - 5\phi_2 + 3) + \alpha\beta_2\phi_2^2 + 2\beta_1\phi_2(3\phi_2^2 - 3\phi_2 + 2))}{(1 - \phi_2)(5\alpha\beta_2 + 6\beta_1)}}.$$

$\mathrm{C}_4$ can be both positive and negative in the region of interest, i.e., $\phi_2 \in (0, 2/7]$. This implies that $\mu_4 \in [\mu_1, \mu_2)$ if $\mathrm{C}_4 \geq 0$, and $\mu_4 < \mu_1$ otherwise.

We illustrate this claim using two numerical examples. Consider the parameter vectors $\Theta_6 = [\alpha = 0.15, \beta_1 = 1, \beta_2 = 0.5]$ and $\Theta_7 = [\alpha = 0.15, \beta_1 = 1, \beta_2 = 1]$, $\phi_2$ takes the values of 0.142 and 0.283 respectively (the 2/7 bound is approximately 0.2857), but $\mu_4 < \mu_1$ at $\Theta_6$ and vice-versa at $\Theta_7$. **Q.E.D.**

**Proof of lemma 24.** The home country is indifferent between $\rho = \rho_5$ and $\rho = \rho_1$ if and only if $\bar{V}_{1a}(\rho_1) - \bar{V}_{1e}(\rho_5) = 0 \Leftrightarrow m_8 - (\psi_1 - \psi_2) = 0$. Let $\Delta\psi \equiv \mu_5 = m_8 : \bar{V}_{1a}(\rho_1) - \bar{V}_{1e}(\rho_5) = 0$. It follows that $\rho^* = \rho_1$ for all $\Delta\psi \leq \mu_5$, and $\rho^* = \rho_5$ for all $\Delta\psi \geq \mu_5$.

Suppose that $\mu_5 \leq \mu_4$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\mu_5, \mu_4]$ The following are true:

- From the first part of lemma 24, $\bar{V}_{1e}(\rho_5) \geq \bar{V}_{1a}(\rho_1)$ for all $\Delta\psi \in [\mu_5, \mu_4]$.
- From lemma 21, $\bar{V}_{1a}(\rho_1) > \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\mu_5, \mu_4]$, since $\mu_4 < \mu_1$.
- From lemma 23, $\bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_5)$ for all $\Delta\psi \in [\mu_5, \mu_4]$.

Thus,

$$\bar{V}_{1a}(\rho_1) > \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_5) \geq \bar{V}_{1a}(\rho_1), \quad \forall \Delta\psi \in [\mu_5, \mu_4]$$

which is a contradiction, and therefore $\mu_5 > \mu_4$.

Suppose that $\mu_5 \geq \mu_1$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\mu_1, \mu_5]$ The following are true:

- From the first part of lemma 24, $\bar{V}_{1a}(\rho_1) \geq \bar{V}_{1e}(\rho_5)$ for all $\Delta\psi \in [\mu_1, \mu_5]$.
- From lemma 21, $\bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1a}(\rho_1)$ for all $\Delta\psi \in [\mu_1, \mu_5]$.
- From lemma 23, $\bar{V}_{1e}(\rho_5) > \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\mu_1, \mu_5]$, since $\mu_4 < \mu_1$.

Thus,

$$\bar{V}_{1e}(\rho_5) > \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1a}(\rho_1) \geq \bar{V}_{1e}(\rho_5), \quad \forall \Delta\psi \in [\mu_1, \mu_5]$$

which is a contradiction, and therefore $\mu_5 < \mu_1$. Thus, $\mu_5 \in (\mu_4, \mu_1)$. **Q.E.D.**

### C.2.3 Medium market efficiency: Details

For the case of medium market efficiency when the foreign agent is profit-maximiser the four equilibrium candidates are $\rho_1$, $\rho^{in}$ and $\rho_6$. The following lemmas provide the solution that maximises Eq. (3.17). The optimal policy in the concave part of the objective is given in lemma 21. That is, $\rho_1$ is preferred for all $\Delta\psi \leq \mu_1$, and $\rho^{in}$ is preferred for all $\Delta\psi \geq \mu_1$. Next, we compare $\rho_6$ and $\rho^{in}$, the following lemma provides the optimal policy.

**Lemma 25.** Let $\Delta\psi = \mu_6$ given by $\bar{V}_{1e}(\rho_6) = \bar{V}_{1b}(\rho^{in})$ be a cost difference threshold.[20] Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_6, & \Delta\psi \geq \mu_6, \\ \rho^{in}, & \Delta\psi \leq \mu_6. \end{cases}$$

[20] For explicit definitions of the cost difference thresholds associated with this case see appendix C.2.4.

Let $1-\sqrt{\frac{(4\sqrt{2}\sqrt{9\phi_2^2-6\phi_2+2}+1-18\phi_2^2+6\phi_2)(\alpha\beta_2+\beta_1)+3\beta_1(2\phi_2-1)}{5\alpha\beta_2+6\beta_1}} \geq 0$ be condition (C.6). The relative position of $\mu_6$ depends on the market parameters as follows:
a. Let condition (C.6) to hold. Then, $\mu_6 \geq \mu_1$.
b. Let condition (C.6) to fail. Then, $\mu_6 < \mu_1$.
See appendix C.2.4 for the proof.

If $\mu_6 \geq \mu_1$, strategies $\rho_1$, $\rho^{in}$ and $\rho_6$ are mutually exclusive, thus a complete equilibrium exists. If condition (C.6) binds, then $\mu_6 = \mu_1$, and $\rho^{in}$ becomes a weakly dominated strategy, this is a degeneration. Otherwise, $\rho = \rho^{in}$ is strictly dominated by $\rho = \rho_1$ for all $\Delta\psi \leq \mu_6 < \mu_1$, and by $\rho = \rho_6$ for all $\Delta\psi > \mu_6$, see lemmas 21 and 25. Thus, we proceed to compare $\rho_1$ and $\rho_6$.

**Lemma 26.** Suppose that $\mu_6 < \mu_1$ and let $\Delta\psi = \mu_7$ given by $\bar{V}_{1e}(\rho_6) = \bar{V}_{1a}(\rho_1)$ be a cost difference threshold. Then, the optimal policy $\rho^*$ is given by

$$\rho^* = \begin{cases} \rho_6, & \Delta\psi \geq \mu_7, \\ \rho_1, & \Delta\psi \leq \mu_7. \end{cases}$$

In addition, $\mu_7 \in (\mu_6, \mu_1)$.
See appendix C.2.4 for the proof.

When (C.6) fails, it follows from lemmas 21, 25 and 26 that strategies $\rho_1$ and $\rho_6$ occur in distinct $\Delta\psi$ regions, another complete equilibrium.

### C.2.4 Medium market efficiency: Proofs

Cost difference thresholds and parameter expressions used below are summarised here.

$$\mu_6 = \frac{m_{10} - \sqrt{m_{10}^2 - 4m_9(m_{11}+m_{12})}}{2m_9}, \quad \mu_7 = m_{13},$$

where

$$m_9 = \frac{2\beta_2(\alpha\beta_2+\beta_1)^2}{\beta_1(5\alpha\beta_2+6\beta_1)}, \quad m_{10} = \frac{\alpha\beta_2(7-3\phi_2)+6\beta_1}{5\alpha\beta_2+6\beta_1},$$
$$m_{11} = \frac{\beta_1((9\phi_2^2-6\phi_2+2)(\beta_1^2+11(\alpha\beta_2+\beta_1)^2)+22(\alpha\beta_2+\beta_1)^2-6\alpha^2\beta_2^2\phi_2+2\beta_1^2)}{8\beta_2(5\alpha\beta_2+6\beta_1)(\alpha\beta_2+\beta_1)^2},$$
$$m_{12} = -\frac{\beta_1\sqrt{2}\sqrt{9\phi_2^2-6\phi_2+2}}{2\beta_2(\alpha\beta_2+\beta_1)}, \quad m_{13} = \frac{\beta_1(9\phi_2^2-6\phi_2+4)}{4\beta_2(\alpha\beta_2+\beta_1)} + m_{12}$$

**Proof of lemma 25.** The home country is indifferent between $\rho = \rho_6$ and $\rho = \rho^{in}$ if and only if $\bar{V}_{1b}(\rho^{in}) - \bar{V}_{1e}(\rho_6) = 0 \Leftrightarrow m_9(\psi_1-\psi_2)^2 - m_{10}(\psi_1-\psi_2) + m_{11} + m_{12} = 0$. The discriminant $\Delta = m_{10}^2 - 4m_9(m_{11}+m_{12})$ simplifies to $\big((4\sqrt{2}\sqrt{9\phi_2^2-6\phi_2+2}+1-18\phi_2^2+6\phi_2)(\alpha\beta_2+\beta_1)+3\beta_1(2\phi_2-1)\big)/\big(5\alpha\beta_2+6\beta_1\big)$, which is strictly positive for all $\phi_2 \in (0, 2/3]$. Thus, the cost differences that make the home country indifferent between $\rho = \rho_6$ and $\rho = \rho^{in}$ are $\mu_6 = (m_{10} \pm \sqrt{m_{10}^2 - 4m_9(m_{11}+m_{12})})/(2m_9)$. From the two roots we keep the lower one because the zero-discriminant root solution, $m_{10}/(2m_9)$, exceeds the cost difference threshold, $\mu_2$, where $\rho^{in}$ equals $\rho_6$, making $\rho^{in}$ infeasible and thus non-comparable. Their difference is given by $m_{10}/(2m_9) - \mu_2 = \sqrt{2(9\phi_2^2-6\phi_2+2)}\beta_1(5\alpha\beta_2+6\beta_1)/(8(\alpha\beta_2+\beta_1)^2\beta_2)$, which is always positive. The direction of the optimal policy follows from the fact that the quadratic function is convex and that $\mu_6$ is the lower of the two roots. Thus, $\rho^* = \rho^{in}$ for all $\Delta\psi \leq \mu_6$, and $\rho^* = \rho_6$ for all $\Delta\psi \geq \mu_6$.

The difference between $\mu_6$ and $\mu_1$ is: $\mu_6 - \mu_1 = \mathrm{C}_5 \dfrac{\beta_1(5\alpha\beta_2 + 6\beta_1)}{4\beta_2(\alpha\beta_2 + \beta_1)^2}$, where

$$\mathrm{C}_5 = 1 - \sqrt{\frac{(4\sqrt{2}\sqrt{9\phi_2^2 - 6\phi_2 + 2} + 1 - 18\phi_2^2 + 6\phi_2)(\alpha\beta_2 + \beta_1) + 3\beta_1(2\phi_2 - 1)}{5\alpha\beta_2 + 6\beta_1}}.$$

$\mathrm{C}_5$ can be both positive and negative in the region of interest, i.e., $\phi_2 \in (2/7, 2/3]$. This implies that $\mu_6 \geq \mu_1$ if $\mathrm{C}_4 \geq 0$, and $\mu_6 < \mu_1$ otherwise.

We illustrate this claim using two numerical examples. Consider the parameter vectors $\Theta_8 = [\alpha = 0.4, \beta_1 = 2, \beta_2 = 2]$ and $\Theta_9 = [\alpha = 0.4, \beta_1 = 4, \beta_2 = 2]$, $\phi_2$ takes the value of 0.4 in both cases (the 2/7 bound is approximately 0.2857), but $\mu_6 < \mu_1$ at $\Theta_8$ and vice-versa at $\Theta_9$. **Q.E.D.**

**Proof of lemma 26.** The home country is indifferent between $\rho = \rho_6$ and $\rho = \rho_1$ if and only if $\bar{V}_{1a}(\rho_1) - \bar{V}_{1e}(\rho_6) = 0$. Let $\Delta\psi \equiv \mu_7 = m_{13} : \bar{V}_{1a}(\rho_1) - \bar{V}_{1e}(\rho_6) = 0$. It follows that $\rho^* = \rho_1$ for all $\Delta\psi \leq \mu_7$, and $\rho^* = \rho_6$ for all $\Delta\psi \geq \mu_7$.

Suppose that $\mu_7 \leq \mu_6$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\mu_7, \mu_6]$ The following are true:
- From the first part of lemma 26, $\bar{V}_{1e}(\rho_6) \geq \bar{V}_{1a}(\rho_1)$ for all $\Delta\psi \in [\mu_7, \mu_6]$.
- From lemma 21, $\bar{V}_{1a}(\rho_1) > \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\mu_7, \mu_6]$, since $\mu_6 < \mu_1$.
- From lemma 25, $\bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_6)$ for all $\Delta\psi \in [\mu_7, \mu_6]$.

Thus,
$$\bar{V}_{1a}(\rho_1) > \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1e}(\rho_6) \geq \bar{V}_{1a}(\rho_1), \quad \forall\Delta\psi \in [\mu_7, \mu_6],$$
which is a contradiction, and therefore $\mu_7 > \mu_6$.

Suppose that $\mu_7 \geq \mu_1$. Then, for any $\Delta\psi = \psi_1 - \psi_2 \in [\mu_1, \mu_7]$ The following are true:
- From the first part of lemma 26, $\bar{V}_{1a}(\rho_1) \geq \bar{V}_{1e}(\rho_6)$ for all $\Delta\psi \in [\mu_1, \mu_7]$.
- From lemma 21, $\bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1a}(\rho_1)$ for all $\Delta\psi \in [\mu_1, \mu_7]$.
- From lemma 25, $\bar{V}_{1e}(\rho_6) > \bar{V}_{1b}(\rho^{in})$ for all $\Delta\psi \in [\mu_1, \mu_7]$, since $\mu_6 < \mu_1$.

Thus,
$$\bar{V}_{1e}(\rho_6) > \bar{V}_{1b}(\rho^{in}) \geq \bar{V}_{1a}(\rho_1) \geq \bar{V}_{1e}(\rho_6), \quad \forall\Delta\psi \in [\mu_1, \mu_7],$$
which is a contradiction, and therefore $\mu_7 < \mu_1$. Thus, $\mu_7 \in (\mu_6, \mu_1)$. **Q.E.D.**

## C.3 Specification of the home country's price equilibria

The home country's equilibria functions are:

$$\rho^1 = \begin{cases} \rho_4, & \Delta\psi \geq \lambda_5, \\ \rho_2, & \lambda_2 \leq \Delta\psi \leq \lambda_5, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_2, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases} \quad \rho^2 = \begin{cases} \rho_4, & \Delta\psi \geq \lambda_3, \\ \rho_3, & \lambda_6 \leq \Delta\psi \leq \lambda_3, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_6, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases} \quad \rho^3 = \begin{cases} \rho_4, & \Delta\psi \geq \lambda_3, \\ \rho_3, & \lambda_4 \leq \Delta\psi \leq \lambda_3, \\ \rho_2, & \lambda_2 \leq \Delta\psi \leq \lambda_4, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_2, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases}$$

$$\rho^4 = \begin{cases} \rho_5, & \Delta\psi \geq \lambda_7, \\ \rho_2, & \lambda_2 \leq \Delta\psi \leq \lambda_7, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_2, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases} \quad \rho^5 = \begin{cases} \rho_5, & \Delta\psi \geq \lambda_8, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_8, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases} \quad \rho^6 = \begin{cases} \rho_9, & \Delta\psi \geq \lambda_{12}, \\ \rho_7, & \lambda_9 \leq \Delta\psi \leq \lambda_{12}, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_9, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases}$$

$$\rho^7 = \begin{cases} \rho_9, & \Delta\psi \geq \lambda_{10}, \\ \rho_8, & \lambda_{11} \leq \Delta\psi \leq \lambda_{10}, \\ \rho_7, & \lambda_9 \leq \Delta\psi \leq \lambda_{11}, \\ \rho^{in}, & \lambda_1 \leq \Delta\psi \leq \lambda_9, \\ \rho_1, & \Delta\psi \leq \lambda_1, \end{cases} \quad \rho^8 = \begin{cases} \rho_8, & \psi_1 \geq \psi_2, \\ \rho \in \mathbb{R} : \rho > \rho_8, & \psi_1 \leq \psi_2, \end{cases} \quad \rho^9 = \begin{cases} \rho_5, & \Delta\psi \geq \mu_4, \\ \rho^{in}, & \mu_1 \leq \Delta\psi \leq \mu_4, \\ \rho_1, & \Delta\psi \leq \mu_1, \end{cases}$$

$$\rho^{10} = \begin{cases} \rho_5, & \Delta\psi \geq \mu_5, \\ \rho_1, & \Delta\psi \leq \mu_5, \end{cases} \quad \rho^{11} = \begin{cases} \rho_6, & \Delta\psi \geq \mu_6, \\ \rho^{in}, & \mu_1 \leq \Delta\psi \leq \mu_6, \\ \rho_1, & \Delta\psi \leq \mu_1, \end{cases} \quad \rho^{12} = \begin{cases} \rho_6, & \Delta\psi \geq \mu_7, \\ \rho_1, & \Delta\psi \leq \mu_7. \end{cases}$$

## C.4 Derivation of the cost frontier

In this appendix subsection, we show how to derive the cost frontier when the foreign agent is welfare-maximiser, i.e., $\xi = 1$, and the foreign market efficiency parameter is very low, i.e., $\phi_2 \in (0, \max(0, 2/5 - \theta)$.

Let $\rho = \rho_1$ be the optimal pricing policy. Solving $\bar{V}_{1a}(\rho_1) = 0$ with respect to $\psi_1$ yields

$$\bar{\psi}_1 = \frac{\beta_1(18\phi_2^2\beta_1/\beta_2 + \alpha(27\phi_2^2 - 12\phi_2 + 4)) + 8\alpha(\beta_1 + \beta_2 - 1)(\alpha\beta_2 + \beta_1)}{8(\alpha\beta_2 + \beta_1)^2} > 0,$$

which is the break even cost level when the home firm exploits the resource alone.

Let $\rho = \rho_4$ be the optimal pricing policy. Solving $\bar{V}_{1e} = 0$ with respect to $\psi_2$ yields

$$\bar{\psi}_2 = \frac{\beta_1\phi_2(3\beta_1/\beta_2 + 9\phi_2\alpha) + 8\alpha((\beta_1 + \beta_2 - 1)\alpha\beta_2 + (\beta_1 + \beta_2 - 1 - \phi_2/4)\beta_1)}{8(\alpha\beta_2 + \beta_1)^2} > 0,$$

which is the break even cost level when the foreign firm exploits the resource alone.

Let $\rho = \rho_2$ be the optimal pricing policy. Solving $\bar{V}_{1c} = 0$ with respect to $\psi_1$ yields

$$\psi_1 = \frac{\beta_1(5\phi_2^2\beta_1/\beta_2 + \alpha(7\phi_2^2 - 3\phi_2 + 1)) + 2\alpha(\beta_1 + \beta_2 - 1)(\alpha\beta_2 + \beta_1)}{2(\alpha\beta_2 + \beta_1)^2(1 - \phi_2)} - \frac{\phi_2}{1 - \phi_2}\psi_2,$$

which is a downwards slopping function of $\psi_2$ and represents the break even cost levels where both firms exploit the resource according to $u_2 = \phi_2$ and $u_1 = 1 - \phi_2$. The constant is strictly positive for all $\phi_2 \in (0, 1/2]$.

Let $\rho = \rho_3$ be the optimal pricing policy. Solving $\bar{V}_{1d} = 0$ with respect to $\psi_1$ yields

$$\psi_1 = \frac{\beta_1(4(2\phi_2^2\beta_1/\beta_2 + \alpha(6\phi_2^2 - 4\phi 2 + 1)) + 8\alpha(\beta_1 + \beta_2 - 1)(\alpha\beta_2 + \beta_1)}{8(\alpha\beta_2 + \beta_1)^2\phi_2} - \frac{1 - \phi_2}{\phi_2}\psi_2,$$

which is a downwards slopping function of $\psi_2$ and represents the break even cost levels where both firms exploit the resource according to $u_2 = 1 - \phi_2$ and $u_1 = \phi_2$. The constant is strictly positive for all $\phi_2 \in (0, 1/2]$.

Let $\rho = \rho^{in}$ be the optimal pricing policy. Solving $\bar{V}_{1b} = 0$ with respect to $\psi_1$ yields

$$\psi_1 = \frac{2\mathrm{B}_6\psi_2 - \mathrm{B}_7 - \sqrt{\mathrm{B}_7^2 - 4\mathrm{B}_6(\mathrm{B}_8 - \psi_2)}}{2\mathrm{B}_6}$$

which is the lower of the two roots of the quadratic expression given by $\bar{V}_{2a}(\rho^{in}) = \mathrm{B}_6\psi_1^2 + (\mathrm{B}_7 - 2\mathrm{B}_6\psi_2)\psi_1 + \mathrm{B}_6\psi_2^2 - (1 + \mathrm{B}_7)\psi_2 + \mathrm{B}_8$, where

$$\mathrm{B}_6 = \frac{2(\alpha\beta_2 + \beta_1)^2\beta_2}{\beta_1(5\alpha\beta_2 + 4\beta_1)}, \quad \mathrm{B}_7 = \frac{\beta_2(1 - \alpha)(\alpha\beta_2 + \beta_1)}{(5\alpha\beta_2 + 4\beta_1)} - 1,$$

$$\mathrm{B}_8 = (\beta_1^2(81\phi_2^2\beta_1^2/(4\beta_2) + \alpha(6\beta_1\phi_2^2 + (3\alpha\beta_2 + 4\beta_1)(12\phi_2^2 + 3\phi_2 - 1))) + \alpha\beta_1(\alpha\beta_2 + \beta_1)(8\beta_1(\alpha\beta_2 + \beta_1) + 10\alpha\beta_2(\beta_1 + \beta_2 - 1)))/(2\beta_1(5\alpha\beta_2 + 4\beta_1)(\alpha\beta_2 + \beta_1)^2).$$

We pick the lower of the roots because its the only one that, when plotted on the $\psi_2 \times \psi_1$ space, crosses the region where $\rho^{in}$ is feasible. This is defined by curves $\psi_2 + \lambda_1$ and $\psi_2 + \lambda_2$ when $\rho_2$ is not strictly dominated (see figure 3.6 (a) and (c)), or by $\psi_2 + \lambda_1$ and $\psi_2 + \lambda_6$ when $\rho_2$ is strictly dominated (see 3.6 (b)).

Existence of the roots can be verified by showing that the zero-discriminant root is always on the left of the $\psi_2 + \lambda_2$ curve. This takes care of both cases since $\lambda_6 < \lambda_2$ (see lemma 10) and $\psi_2 + \lambda_6$ is always on the right of $\psi_2 + \lambda_2$. The zero-discriminant root is given by $\psi_2 - \mathrm{B}_7/(2\mathrm{B}_6)$, its difference with $\psi_2 - \lambda_2$ is given by $\beta_1(1-\phi_2)(5\alpha\beta_2 + 4\beta_1)/(4(\alpha\beta_2+\beta_1)^2\beta_2)$, which is always positive when $\phi_2 \in (0, 1/2]$.

Next, we show by contradiction why we pick the lower root. Suppose that the the higher is the correct one. Then,

$$\frac{2\mathrm{B}_6\psi_2 - \mathrm{B}_7 + \sqrt{\mathrm{B}_7^2 - 4\mathrm{B}_6(\mathrm{B}_8 - \psi_2)}}{2\mathrm{B}_6} = \psi_2 + \lambda_2 \Leftrightarrow$$
$$\sqrt{\mathrm{B}_7^2 - 4\mathrm{B}_6(\mathrm{B}_8 - \psi_2)} = 2\mathrm{B}_6\lambda_2 + \mathrm{B}_7 = \phi_2 - 1 < 0, \quad \forall \phi_2 \in (0, 1/2],$$

which is a contradiction because the square root must be positive since the two roots exist in the feasible region.

On a final remark, it is straightforward to check that the cost frontier is continuous at the intersections of the different curves.

# D Specification of $V_1$ when the TAC is endogenous

In case the TAC is endogenous, the home country's objective is given by the following piecewise functions, which depend on the foreign agent's type,

$$V_1|_{\xi=1} = \begin{cases} V_1(x, p^1, U_2^1(p^1), \mathbf{Q}^1(U_2^1(p^1))), & \Phi_2 \in (0, \max(0, K(x)] & \text{and } \Lambda_4(x) > \Lambda_3(x), \\ V_1(x, p^2, U_2^1(p^2), \mathbf{Q}^1(U_2^1(p^2))), & \Phi_2 \in (0, \max(0, K(x)] & \text{and } \Lambda_4(x) < \Lambda_2(x), \\ V_1(x, p^3, U_2^1(p^3), \mathbf{Q}^1(U_2^1(p^3))), & \Phi_2 \in (0, \max(0, K(x)] & \text{and } \Lambda_2(x) \le \Lambda_4(x) \le \Lambda_3(x), \\ V_1(x, p^4, U_2^2(p^4), \mathbf{Q}^1(U_2^2(p^4))), & \Phi_2 \in (\max(0, K(x)), G(x)/2] & \text{and } \Lambda_7(x) \ge \Lambda_2(x), \\ V_1(x, p^5, U_2^2(p^5), \mathbf{Q}^1(U_2^2(p^5))), & \Phi_2 \in (\max(0, K(x)), G(x)/2] & \text{and } \Lambda_7(x) < \Lambda_2(x), \\ V_1(x, p^6, U_2^4(p^6), \mathbf{Q}^2(U_2^4(p^6))), & \Phi_2 \in (G(x)/2, 2G(x)/3) & \text{and } \Lambda_{11}(x) > \Lambda_{10}(x), \\ V_1(x, p^7, U_2^4(p^7), \mathbf{Q}^2(U_2^4(p^7))), & \Phi_2 \in (G(x)/2, 2G(x)/3) & \text{and } \Lambda_{11}(x) \le \Lambda_{10}(x), \\ V_1(x, p^8, U_2^5(p^8), \mathbf{Q}^3(U_2^5(p^8))), & \Phi_2 \ge 2G(x)/3, & \end{cases}$$

$$V_1|_{\xi=0} = \begin{cases} V_1(x, p^9, U_2^2(p^9), Q^1(U_2^2(p^9))), & \Phi_2 \in (0, 2G(x)/7] & \text{and } M_4(x) \ge M_1(x), \\ V_1(x, p^{10}, U_2^2(p^{10}), Q^1(U_2^2(p^{10}))), & \Phi_2 \in (0, 2G(x)/7] & \text{and } M_4(x) < M_1(x), \\ V_1(x, p^{11}, U_2^3(p^{11}), Q^2(U_2^3(p^{11}))), & \Phi_2 \in (2G(x)/7, 2G(x)/3) & \text{and } M_6(x) \ge M_1(x), \\ V_1(x, p^{12}, U_2^3(p^{12}), Q^2(U_2^3(p^{12}))), & \Phi_2 \in (2G(x)/7, 2G(x)/3) & \text{and } M_6(x) < M_1(x), \\ V_1(x, p^8, U_2^5(p^8), Q^3(U_2^5(p^8))), & \Phi_2 \ge 2G(x)/3, & \end{cases}$$

where $K(x) \equiv U(2/5 - \theta) = 2G(x)/5 - (a_2 - a_1)b_1/(10a_1)$. The dependencies between $p^m$, $U_2^l(p^m)$ and $\mathbf{Q}^k(U_2^l(p^m))$ follow from propositions 1 to 8.

# References

Clark, C. W. (1973). Profit Maximization and the Extinction of Animal Species, *The Journal of Political Economy*, Vol. 81, 950-961.

Clark, C. W. (2010). *Mathematical Bioeconomics: The Mathematics of Conservation*, Wiley, New Jersey.

Clark, C. W. and Munro, G. R. (1975). "The economics of fishing and modern capital theory: a simplified approach", *Journal of environmental economics and management*, Vol. 2, 92-106.

Cullberg, M. and Lövin, I. (2009). *To Draw the Line: EU fisheries Agreements in West Africa*, Stockholm: Swedish Society for Nature Conservation.

European Commission (2017). *EU sustainable fisheries partnership agreements*, Directorate-General for Maritime Affairs and Fisheries (European Commission), ISBN: 978-92-79-65420-6. Available at: `https://op.europa.eu/en/publication-detail/-/publication/c8b5d962-0d38-11e7-8a35-01aa75ed71a1/language-en/format-PDF/source-37907030`.

Gorez, B. (2006). *The Future of ACP-EU Fisheries Relations: Towards more Sustainability and Improved Social and Economic Well-being for ACP Coastal Communities*, Technical Centre for Agricultural and Rural Cooperation (CTA), Wageningen.

Hannesson, R. (1983). "Optimal harvesting of ecologically interdependent fish species", *Journal of Environmental Economics and Management*, Vol. 10, 329-345.

Heredia, J. M. S. and Oanta, G. A. (2015). The Sustainable Fisheries Partnership Agreements of the European Union and the Objectives of the Common Fisheries Policy: Fisheries and/or Development?, *Spanish Yearbook of International Law*, Vol. 19, 61-85.

Kaczynski, V. M. and Fluharty, D. L. (2002). European policies in West Africa: who benefits from fisheries agreements? *Marine Policy*, Vol. 26, 75-93.

Maroto, J. M., Moran, M., Sandal, L. K. and Steinshamn, S. I. (2012). "Potential collapse in fisheries with increasing returns and stock-dependent costs", *Marine Resource Economics*, Vol. 27, 43-63.

Nagel, P. and Gray, T. (2012). Is the EU's Fisheries Partnership Agreement (FPA) with Mauritania a genuine partnership or exploitation by the EU?, *Ocean & coastal management*, Vol. 56, 26-34.

Okafor-Yarwood, I. and Belhabib, D. (2019). The duplicity of the European Union Common Fisheries Policy in third countries: Evidence from the Gulf of Guinea, *Ocean & Coastal Management*, Vol. 184, 104953.

Sandal, L. K. and Steinshamn, S. I. (1997). "A feedback model for the optimal management of renewable natural capital stocks", *Canadian Journal of Fisheries and Aquatic Sciences*, Vol. 54, 2475-2482.

United Nations (1982). *United Nations Convention on the Law of the Sea*, UN DOC. A/Conf.62/122.