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How do Alternative Asset Classes Affect Performance of Traditional Stock & Bond Portfolios?

An Empirical Analysis of Strategic Asset Allocation and Risk Management through Business Cycles

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Abstract

The main scope of this thesis is to examine how alternative asset classes affect performance of traditional stock and bond portfolios. We will employ financial engineering and quantitative analytics to construct the most optimal portfolio of asset classes from 1928 – 2020 and investigate the diversification effect of alternative asset classes. The methodology to find an optimal portfolio follows the prominent mean-variance framework of Harry Markowitz, which determines portfolio weights to maximize the return-to-risk ratio. The analyzed asset classes are U.S. equity, government bonds, corporate bonds, gold, real estate, commodities, options strategies and factor exposure towards Fama-French' SMB, HML and UMD portfolios.

The purpose of utilizing 93 years of data is to construct a portfolio which performs in every stage of the business cycle. It should handle inflation and deflation, rising and falling interest rates, as well as market booms and crashes. Quadratic optimization suggests an allocation of government bonds, corporate bonds, real estate, UMD factor exposure and a covered call option strategy. This combination leads to a significant improvement of risk management, where the risk exposure is halved compared to a traditional stock and bond portfolio without influencing returns. The optimal portfolio achieves an annual alpha of 3.40% compared to our benchmark, and Sharpe ratio increases from 0.397 to 0.835. As the risk-adjusted return is significantly improved, will our research suggest that including alternative asset classes enhances portfolio quality.

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1. Introduction

A common practice in investing is to construct a risky portfolio of stocks and bonds. We see evidence of this strategy all the way back to the 1940s, when Benjamin Graham wrote the well-recognized book *The Intelligent Investor*. He claimed an optimal portfolio should consist of high-grade bonds and leading common stocks, with an allocation of minimum 25% in each. This is a widespread practice today for mutual, pension and sovereign wealth funds, as well as professional investors (NBIM, 2020) (Storebrand, 2021). The procedure is furthermore well-documented in leading textbooks on portfolio theory and asset management, which is the curriculum in business schools around the globe (LSE, 2020) (Wharton, 2021) (NHH, 2021).

Combining asset classes to achieve diversification is based on the acknowledged paper "Portfolio Selection" by Markowitz (1952). The entrance of various assets will reduce idiosyncratic risk and should hence improve portfolio quality, and this is the only free lunch in investing according to Harry Markowitz (Forbes, 2021). An optimal risky portfolio will either have the highest possible expected return for any given risk, or the lowest risk for any given expected return. The presumption is that diversification is achievable by combining assets which behave differently. If some securities perform badly, others should perform well. A portfolio's assets should therefore be uncorrelated, or even better, negatively correlated.

Stocks and bonds have historically had periods of both high and low correlation, which means the diversification effect varies over time. This gives rise to a two-folded problem which is further addressed in this paper. Firstly, holding a traditional stock and bond portfolio will not protect an investor in all periods. A portfolio of two assets which constantly perform in dissimilar periods should hence provide better diversification. Secondly, adding more uncorrelated assets should improve the overall performance and enhance portfolio quality. Asset classes perform differently in each stage of the business cycle, and a broader allocation might lead to a portfolio better prepared for future investment environments. We want to investigate the following and test whether alternative asset classes improve the quality of a risky portfolio through every business cycle over the past 93 years. Our main research question is accordingly: how do alternative asset classes affect performance of traditional stock and bond portfolios through business cycles?

We will hence construct a portfolio including alternative asset classes based on performance from January 1928 to December 2020. Due to the length of the analyzed time series, it can be conveyed that an optimal portfolio should perform in every period and no matter the stage of the business cycle. The impact will be measured by comparing our constructed portfolio to a traditional 60/40 stock and bond portfolio.

2. Background

This chapter will issue necessary prerequisites before constructing and analyzing portfolios. Section 2.1 will focus on investment formalities, where a mandate will be further explained. We will then continue by reviewing U.S. macroeconomic factors such as interest rate levels and inflation, before presenting a quick summary of the U.S. investment environment from 1928 to 2020. All asset classes are either extracted from the United States or denominated in U.S. dollars to make it comparable with each other. Thereafter, we will finalize this chapter by introducing each applied asset class and assess its key characteristics.

2.1 The Investor Process

Every investor should assess their own investment profile and appropriate mandate before entering the market. They should describe their own willingness and capacity towards risk, as well as their strategy to achieve returns. Risk capacity concerns an investor's investment horizon, liquidity needs, future income expectations and liabilities, while risk willingness relates to his risk tolerance. This paper does not go into further depth to apprise utility functions, because different investors have different investment universes and interprets risk differently. We are therefore simplifying the matter in question by assuming that investors have the same investment universe, and that the optimal risky portfolio should be equal for all investors independent on risk aversion (Treynor, 1962) (Sharpe, 1964) (Lintner, 1965) (Mossin, 1966).

An investment mandate is the agreement between the investor and asset manager on how the fund should be managed. In our case, it should clarify the desire of diversification, where well-performing assets in both expansions and recessions should be combined. Due to our relatively long accumulation of data, dating back to 1928 are we considering a long investment horizon. Positions are long only, besides from indirectly short exposure to equity factors and options strategies. These assets, strategies and factor portfolios are considered well-known, highly tradable, and easily accessible through trading platforms, and will be further described in section 2.3, *presentation of asset classes*.

Our desired risky portfolio is actively managed with a monthly rebalancing policy. When considering rebalancing, it is evident to point out potential transaction costs it would accommodate, even though this is relevant for the benchmark portfolio as well. Transaction costs change over time, dependent on rebalancing frequency and amount for each transaction, as well as the degree of active management. We will not take this further into account when evaluating portfolio performance.

Because we are interested in the effect of strategic asset allocation and not market timing or security selection, will a traditional 60/40 stock and bond portfolio be used as the reference when evaluating the impact of alternative asset classes. This implies a benchmark with quite different characteristics, and our results will explain differences in performance due to asset allocation.

2.2 Business Cycles and Time Periods

We start this section by separating the 93-year time series into five parts. These are naturally divided by the activity level and other macroeconomic factors such as interest rates and inflation rates. We have the secular decline from 1928 – 1945, secular growth from 1946 – 1964, secular stagnation from 1965 – 1981, the boom from 1982 – 2007 and another secular decline from 2008 – 2020. The intention for using data which dates to 1928 is to provide the longest feasible time series containing reliable and easily accessible data. Additionally, we can offset the recency bias from the remarkable equity and bond performance since the 1980s and prevent anomalies from yielding biased results. This makes the inducement of selecting the asset classes also based on recognition, accessibility, and tradability for any investor. In the extent of measuring historical performance, it is necessary to understand the underlying business cycles and what effect it has on asset allocation.

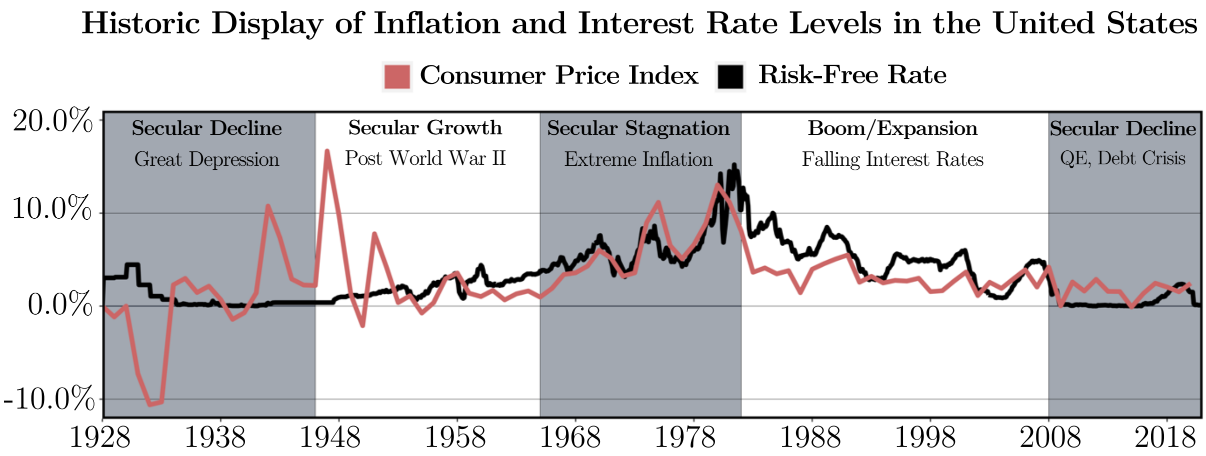

Figure 1 – Inflation and interest rate levels in the United States, 1928 - 2020.

During times of stagflation and poor prospects, central banks may lower interest rates, provide quantitative easing, and print money to stimulate the economy (CFI, 2021). In periods of growth however, higher labor rates and promising expectations may result in higher interest rates levels due to the fear of higher inflation (Norges Bank, 2003, 2009). Monetary policy and especially interest rates will hence indirectly affect inflation, exchange rates, GDP growth and unemployment rates. The relation between interest rates and inflation is as follows:

$$\textit{Nominal interest rate} = (1 + \textit{real rate})\ (1 + \textit{inflation rate}) - 1 \tag{1}$$

Hence, changes in either expected inflation or expected real rate will change the expected interest rate. We thus advance with a short summary of the macroeconomic development in all five periods.

1928 – 1945: Secular Decline

The first period of secular decline emerged from the Wall Street crash in 1929, which contributed to the following great depression in the 1930s (Davis, 2018). The market had just undergone a rapid expansion where market participants issued bonds and increased their debt to pour cash into the Hoover bull market, forcing the inevitable market bubble to crack. This resulted in several years of high unemployment and homeless rates, a 90% stock market value loss, and years of deflation. Interest rates remained low, while spending increased as the preparation of World War II regained GDP growth and led to a 9.9%

inflation peak in 1941. The relatively sharp decline in interest rates early in this period made bonds outperform most other assets.

1946 – 1964: Secular Growth

As for the next period, the economy was recovering, and interest rates remained low. The post war optimism was prominent as unemployment rates remained low as well, and GDP growth and newborns boomed. Equity-linked assets such as real estate, private equity and ordinary equity performed well, while bonds made a zero excess return due to low and flat bond yields. Gold experienced low returns because of the Gold Reserve Act of 1934, which forced gold to follow the U.S. dollar evolution (Federal Reserve, 2013).

1965 – 1981: Secular Stagnation

At the beginning of the third period, GDP was declining and unemployment rates rising (U.S. Bureau of Economic Analysis, 2021) (U.S. Bureau of Labor Statistics, 2021). It was a start of a mild recession, which made the Nixon administration introduce new fiscal policies in 1971 which ended contradictory (Office of the Historian, 2021). Conflicting expansionary and contractionary fiscal policies, as well as the oil embargo in 1973 expanded the recession and created high levels of inflation. Within monetary policy, the Federal Reserve was taking the U.S. dollar off the gold standard, which essentially made gold prices rise from \$40 to \$666 per ounce in a decade (Federal Reserve, 2021a). This was followed by a rapid expansion which further increased inflation and forced the Federal Reserve to raise its interest rates to almost 20%, followed by a record high 30-year mortgage rate above 18% (Federal Reserve, 2021b) (Freddie Mac, 2021). Contractionary fiscal policy made all equity-linked assets except Fama French' equity factors yield negatively, while non-equity-linked assets yielded positive. The rapidly increasing inflation resulting to its absolute peak in 1979, is known for a period of stagflation (Macrotrends, 2021).

1982 – 2007: Boom/Expansion

As the previous recession had ended, interest rates and inflation were steadily decreasing. This was followed by years with relatively high GDP growth, resulting in high return on bonds and equities throughout the period. Rapid expansions in both equity and bond markets resulted in three large market crashes: Black Monday in 1987, dot-com bubble in 2000-2002 and the large financial crisis in 2008 (Siiber, 2008) (CFI, 2021).

2008 – 2020: Secular Decline

The period from 2008 and until today are unparallel from other periods. We are experiencing low inflation and quantitative easing, combined with low interest rates and historically high levels of debt. This has resulted in high excess returns on equity-linked assets and gold. Yield curves have recently been inverted, and it seems to be consensus that the historically low interest rates will remain low in the coming years (Peter G. Peterson Foundation, 2021).

2.3 Presentation of Asset Classes

We will now continue by presenting relevant asset classes and their characteristics[1]. As already stated, are these well-known, highly tradable and with a long track record. We want to take a closer look at U.S. equities, government bonds, corporate bonds, real estate, gold, commodities, options strategies, and factor exposure towards SMB, HML and UMD. Because we are already familiar with the historical macroeconomic environment, we are interested in asset classes which handle either optimism and growth, or stagnation and low economic activity. An optimal portfolio should perform in all periods, and hence, it should consist of asset classes which perform in all types of investment environments.

Traditional Portfolio Allocation

We start by introducing equity and bonds, which are the two most common asset classes in investing. Equity returns might come from both dividends and price appreciation and is often considered an estimation of the overall economic activity. Company valuations are a result of future cash flows, which means equity are in some extent secured for inflation. However, when inflation rise rapidly, we might experience weak performance and depreciation of values. Equities are also sensitive to interest rate changes because a company's cash flow is directly linked to interest expenses.

Bonds are also sensitive to macroeconomic factors, where both corporate and government bonds share an inverse relationship to interest rate levels (Bodie, Kane, & Marcus, 2018). Bonds yield below market rates when interest rates rise, and vice versa. Thus, bonds become

[1] A chart of asset class performance from 1928 – 2020 is displayed in Appendix II.

a less favorable asset to hold when interest rates are close to zero[2]. This explains the poor performance of U.S. Treasuries and corporate bonds before the interest rate peak in 1981, and why bonds have outperformed other assets after the interest rate peak.

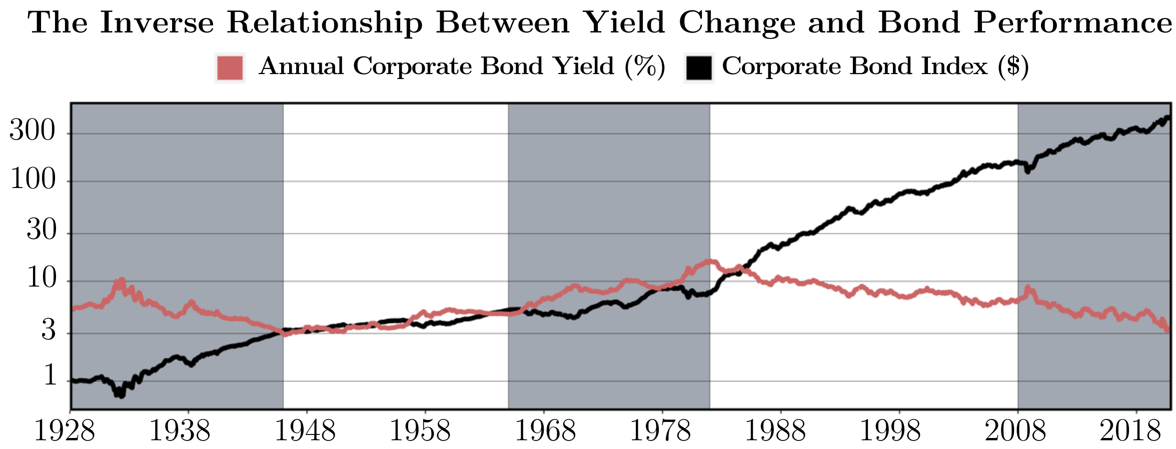

Figure 2 – The inverse relationship between yield change and bond performance from 1928 - 2020. Declining bond yields leads to increasing bond returns and vice versa. Corporate bond index presents the accumulation from 1$ invested in January 1928 to December 2020.

Commodities

While a traditional 60/40 stock and bond portfolio performs well in expansions and when interest rates decline, will it struggle in periods of stagnation, inflation, and increasing interest rates. A more sufficient portfolio should be able to hedge against these occurrences, and commodities follow the price movement in the market and will level out fluctuations in inflation (Skiadopoulos, 2012) (Schroders, 2021). It is also a countercyclical asset class which makes a profit when traditional assets underperform. Intuitively, including commodities to a portfolio could hence reduce the overall variance.

Gold

A second alternative asset class is gold, a material with returns solely based on price appreciation and not dividends. It has historically been used as a measure of a country's guarantee to print money and has been heavily involved in political interests. The Gold Reserve Act of 1934 by President Roosevelt forced gold to follow the U.S. dollar evolution until the 1970s (Federal Reserve, 2021a). Due to its limited supply, universally acceptance,

[2] If we assume that negative interest rate levels are limited. The European Central Bank e.g., have not lowered the deposit facility key interest rate of -0.50% despite the Covid-19 situation, which indicates a threshold level for negative interest rates.

historical perceived value, and recognition as a safety reserve for federal banks, it has become an asset to prefer when currencies disrupt, and in periods where other assets experience downward performance (Gürgün & Ünalmıs, 2014) (Beckmann, Berger, & Czudaj, 2014) (Baur & Lucey, 2010). As in times where inflation soar and fiat currency[3] loses its purchasing power, gold prices tend to be more accurately priced according to general cost of living. Research of both Beckmann & Czudaj (2013) and Van Hoang, Lahiani, & Heller (2016) finds that gold has hedging abilities against inflation in the short run, whilst there is lack of significance supporting this in the longer run. It has also protected against deflation under the great depression and the great financial crisis and is hence requested when U.S. dollars devaluate (Capie, Mills, & Wood, 2005) (Reboredo, 2013). Considering these abilities and a zero to negative correlation to traditional asset classes, the overall risk should be reduced by the entrance of gold.

Real Estate

U.S. real estate has some of the same characteristics as commodities. When GDP soar, demand for real estate increases and rents go higher, which leads to a rise in capital values (Forbes, 2019). Thus, it exists a somewhat linear relationship between real estate purchasing power and inflationary pressure. This makes real estate an inflation protecting asset, and due to its low correlation with bonds and equities it can enhance portfolio quality (Case, Wachter, & Worley, 2017). The disadvantage for real estate is high transaction and maintenance costs, as well as a relatively illiquid market.

Fama French Factors

The three Fama French factors SMB[4], HML[5] and UMD[6] are originally not separate asset classes, but portfolios of securities within the equity domain (French, 2021). The overall characteristics are equal to traditional equity, while there are some minor differences dependent on its factor exposure. SMB portfolios are exposed to small market cap companies, HML to high book-to-market companies and UMD to well-performing stocks. This is asset factors which is used to explain outperformance tendencies relative to the

[3] Fiat Currency is money issued by governments and is not backed by any physical asset.
[4] SMB = Small Minus Big
[5] HML = High Minus Low
[6] UMD = Momentum

market and might thus indicate allocation tilt. Calculations and factor exposure on each of the three investment portfolios will be further explained in chapter 4.

Option Strategies

Additionally, options strategies are incorporated as a proxy towards the complex alternative asset spectrum of hedge funds. These are essential to evaluate due to its large capitalization size and liquidity volume in the market and could provide a great hedge because of their independence towards traditional asset performance. The following strategies are long straddle, married put and covered call options strategies, which aim to gain returns based on movements on their underlying asset. Each strategy is a result of put and call options on the S&P 500 stock index, and their behavior and construction are explained in section 4.2.

3. Literature Review

This chapter will address prior research on alternative asset classes and asset management to put our work into context of other studies. Alternative asset classes might seem like a new phenomenon due to the entrance of advanced financial instruments over the last decades, however some alternative assets have been around for over a century without causing attention. The main explanation might be that Exchange-Traded Funds (ETFs) and Exchange-Traded Notes (ETNs) have increased the investment opportunities for an average investor with limited capital and knowledge (Abner, 2016). Greenbaum (2006) explain this by indicating that alternative asset classes are associated with exclusivity, large transaction costs, low liquidity, and physical inconveniences. Accordingly, they have not been relevant for most investors until lately, which might also explain its low research coverage.

Brinson, Hood, and Beebower (1995) discovered that investment policy is the key factor of asset management. Their study revealed that 93.6% of a portfolio's variation over time can be explained by strategic asset allocation. In other words, asset class selection and weight allocation explain almost all the ups and downs in a portfolio, while market timing and security selection will only explain a few percent.

Ibbotson and Kaplan (2000) brought this subject even further by clearing up misunderstandings in the interpretation of Brinson et al. (1995). Ibbotson and Kaplan (2000) agreed that over 90% of a portfolio's variation over time can be explained by its benchmark, while they further analyzed other potential contributions. They proved that 40% of the variation in returns between two different portfolios are explained by strategic asset allocation. Hence, differences in performance between portfolios is mostly explained by market timing and security selection, not strategic asset allocation. Ibbotson and Kaplan (2000) also analyzed strategic asset allocation's contribution to portfolio returns, and these results were even more striking. It turns out that 99% of the return level in a pension fund and 104% in a mutual fund come from strategic asset allocation. In other words, strategic asset allocation is the most important part of the portfolio construction process because it explains returns, while market timing and security selection will only differentiate portfolios from each other.

Other studies discuss the relevance and results of adding one extra asset class to a portfolio. Lamm (1998) tested the effect of replacing U.S. Treasury bills by Treasury Inflation Protected Securities (TIPS). His research revealed that T-bills and TIPS behave similarly when inflation is stable, except for the extra inflation premium on TIPS. When inflation is rising, TIPS outperform T-bills, while its reduced excess return in decreasing inflation is partly covered by enhanced diversification. Hence, Lamm (1998) suggested to replace T-bills by TIPS to manage inflation in a more sufficient way[7].

Small, Smith, & Small (2012) discussed the inclusion of diamonds and pointed out its outperformance in terms of Sharpe ratio, Treynor ratio and maximum loss from December 2001 to December 2011 against the S&P 500 and MSCI World Index. The correlation coefficients were 0.061 and 0.045 respectively, which demonstrate its diversification qualities. Hence, they concluded that diamonds might improve the risk-adjusted performance in a portfolio. Erb and Harvey (2006) on the other hand, analyzed the strategic and tactical opportunities of commodity futures for investors. Their findings revealed that an average commodity futures contract does not have an excess return distinguishable from zero, while a portfolio of futures contracts, however, might achieve an equity-like performance because of stronger diversification. Chudy and Cubbage (2020) investigated forest investments as a financial asset class, and the advantage of including it to a traditional investment portfolio. Their findings suggested that forest land investments, either individually or pooled, have negative correlation towards the equity market and is a strong hedge against inflation. They do not go into specifics of improvement in terms of e.g., Sharpe ratio, but insinuate that the improvement should be significant.

Bekkers, Doeswijk, & Lam (2009) included several alternative asset classes in their search for an optimal portfolio. They argued that adding just one more class will lead to a sub-optimal portfolio, and hence criticized the work in Lamm (1998) and Erb and Harvey (2006). By implementing the methodology from Markowitz (1952), they determined

[7] Note that this paper was written in 1998, and that the Federal Reserve transformed their monetary policy in 2012 to a two percent inflation target (Federal Reserve, 2012).

weights for a portfolio with the highest possible Sharpe ratio. By including real estate, commodities, and high yield bonds to a traditional allocation, they increased the portfolio Sharpe ratio from 0.346 to 0.396. Our research question is close to the work of Bekkers et al. (2009), but we utilize a much larger time series and consider different types of asset classes as well. At last, we will also touch the work of Dzikevičius and Vetrov (2012), where they evaluate alternative asset classes' performance in different stages of the business cycle.

4. Data and Methodology

This chapter presents essential inputs and methods to construct an optimal risky portfolio. The first section concerns our data set, where we make a detailed review of the source for each asset class. We will then pursue by explaining options strategies which is used to replicate hedge funds. Lastly, the mean-variance framework by Harry Markowitz is presented before its implementation in chapter 5. We are using the R software for statistical computation and graphics.

4.1 Data Sources

The following section will explain how asset class data is collected and computed for further use in our analysis. The relevant asset classes are U.S. equity, government bonds, corporate bonds, gold, commodities, options strategies, and factor exposure towards Fama-French' SMB, HML and UMD portfolios.

Equity

The traditional S&P 500 Index from Standard & Poor's is used to capture value creation in the U.S. equity market. It contains 500 large companies listed on stock exchanges in the United States, which makes it a broad measure of U.S. equities in all sectors. It was not created until February 1957, and we have hence used a 90-stock composite index backtested by S&P for the period from 1928 to 1957. Returns are calculated as the change in index values, corrected for dividends paid to shareholders[8]. Data is available in the Bloomberg Terminal, as well as Robert Shiller's database on the homepage of Yale School of Management (Bloomberg L.P., 2021) (Yale School of Management, 2021). These databases include the S&P 90-stock index on the extended S&P 500 data set.

Government Bonds

10-year U.S. Treasury bonds are used for representing long-term government bonds. Monthly observations of bond yields are available in Robert Shiller's database at Yale

[8] U.S. equities are hence represented by the total return index, while the non-dividend index is used for options strategies.

School of Management, where observations are cross validated with the Federal Reserve Bank of St. Louis. However, we are interested in monthly bond returns, not monthly yields. We have thus performed a recalculation using Aswarth Damodaran's calculation procedure (Damodaran, 2021). This is done by using the promised coupon yield at the end of the prior period, followed by controlling for interest rate changes

$$\text{Bond Return} = \text{Yield}_{t\text{-}1} + \left(\left(\text{Yield}_{\ t\text{-}1} * \left(\frac{1 - (1+ \text{Yield}_t\ )^{-n}}{1}\right) + \frac{1}{(1 + \text{Yield}_{\ t}\ )^n}\right) - 1\right). \qquad (2)$$

Corporate Bonds

Our proximation for corporate bonds is Moody's Corporate BAA Yield data. This is medium investment grade bonds in the United States with remaining maturity close to 30 years, and no less than 20 years. Data has been retrieved from the Bloomberg Terminal[9], and Damodaran's transformation procedure is applied to transform yield data into monthly returns.

Commodities and Gold

Commodity data are obtained from the Economic Research department at Federal Reserve Bank of St. Louis, constructed by the US Bureau of Labor Statistics. We use the Producer Price Index by All Commodities as an estimate for commodity prices. This is a broad index of U.S. commodities within farm products, processed foods, and industrial commodities such as metals and petroleum products. Gold is downloaded separately and will be treated as an own investment alternative in this thesis. These returns are retrieved from the Bloomberg Terminal[10] and is already denominated in dollars, meaning there is no need for further adjustments.

Real Estate

Real estate data from private U.S. homes can be downloaded from Robert J. Shiller's database at Yale School of Management. It dates to 1890 and is assembled from several sources for each period. Housing prices from 1890 to 1933 are retrieved from *Capital Formation in Residential Real Estate* by Grebler, Blank, & Winnick (1956), published by

[9] Corporate bonds, Bloomberg Ticker: MOODCBAA
[10] Gold, Bloomberg Ticker: XAU Curncy

Princeton University and National Bureau of Economic Research. They collected data from 22 U.S. cities to find the median price of each month.

Next, students at Yale University collected median home prices from Chicago, Los Angeles, New Orleans, New York, and Washington D.C. from 1934 to 1953 by reviewing old newspapers. In the period from 1953 to 1975, house prices were collected by the Bureau of Labor Statistics in the United States. Lastly, prices from 1975 to 2020 are represented by the S&P/Case-Shiller U.S. National Home Price Index. This was originally created by Case Shiller Weiss but are now presented by CoreLogic.

Fama-French Factors

The three Fama-French factors SMB, HML and UMD are downloaded from Wharton Research Data Services, which is originally obtained from Kenneth French' own calculations. His findings are based on common stocks at the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and Nasdaq. Each factor return is calculated as the average of several stock portfolios, where you go long in preferable qualities and short in opposite qualities (French, 2021). Each factor calculation is presented underneath:

SMB = ⅓ (Small Value + Small Neutral + Small Growth) - ⅓ (Big Value + Big Neutral + Big Growth),
HML = ½ (Small Value + Big Value) - ½ (Small Growth + Big Growth),
UMD = ½ (Small High + Big High) - ½ (Small Low + Big Low).

Other

A 3-month U.S. Treasury bill is used as an estimate of the risk-free rate. Data is accessible at the Federal Reserve Bank of St. Louis' database, and cross validated by Damodaran's data sets. It measures the 3-month yield in the secondary U.S. market, and has been transformed to monthly returns using equation 2. Inflation is estimated using the Consumer Price Index by U.S. Bureau of Statistics. It is accessible in Robert Shiller's database at Yale School of Management and covers price movements in U.S. consumer goods from 1928 to 2020.

The last component of our data set is the CBOE Volatility Index (VIX) created by Chicago Board Options Exchange. It measures implied volatility on the S&P 500 Index from 1990 to 2020 using Black and Scholes' option pricing theory and is retrieved from Wharton

Research Data Services. Our hedge fund strategies are constructed by options, and we are hence dependent on S&P 500 volatility to calculate prices. This is not possible to obtain from 1928 to 1990 and must hence be estimated using multiple regressions on VIX from 1990 to 2020. Methods to estimate implied volatility and our options strategies is explained thoroughly in section *4.2.1, Options Strategies.*

Descriptive Statistics

Table 1 displays an overview of the analyzed asset classes, including the three options strategies which will be reviewed in the next section. Total return includes the nominal risk-free rate, which has been 3.28% in annual average over the 93-year period. Standard deviation and Sharpe ratio are based on excess returns, with the intuition that the risk-free rate should be completely risk free. To make asset class returns more comparable, is a theoretical leverage to achieve 15% volatility and its respective leveraged return included. This implies an investor will either hold some of his funds in T-bills or borrow by issuing T-bills himself. Additionally, maximum drawdown for each asset class over the 93-year period is calculated, as well as the value at risk, which illustrates the maximum expected loss for the next month with a 99% confidence level[11].

Summary Statistics of all Asset Classes from 1928 - 2020

1928 - 2020	Equity	Gov. Bonds	Corporate Bonds	Gold	Real Estate	Comm.	SMB Factor	HML Factor	UMD Factor	Long Straddle	Covered Call	Married Put
Total Return	9.3%	4.9%	6.9%	4.9%	3.9%	2.7%	10.5%	12.3%	15.6%	3.8%	9.8%	1.6%
Inflation-adj. TR	6.4%	2.0%	4.0%	2.0%	1.0%	-0.2%	7.6%	9.4%	12.7%	0.9%	6.9%	-1.3%
Excess Return	6.1%	1.6%	3.6%	1.6%	0.7%	-0.6%	7.2%	9.0%	12.3%	0.5%	6.6%	-1.7%
Standard deviation	19.3%	5.1%	7.8%	14.7%	3.9%	3.7%	24.4%	24.1%	19.9%	13.4%	13.6%	10.6%
Sharpe Ratio	0.32	0.32	0.46	0.11	0.14	-0.16	0.30	0.38	0.62	0.04	0.48	-0.16
Leverage, 15% vol	0.78	2.92	1.93	1.02	3.09	4.10	0.61	0.62	0.75	1.12	1.10	1.42
TR at 15% vol	5.1%	5.1%	7.3%	2.0%	2.4%	-2.1%	4.8%	6.0%	9.6%	1.0%	7.6%	-2.0%
Max drawdown	88.1%	14.6%	37.2%	71.1%	27.6%	39.7%	89.9%	92.6%	81.9%	61.1%	79.7%	68.4%
Value at Risk	-22.2%	-4.0%	-12.8%	-14.8%	-4.5%	-4.2%	-26.9%	-36.1%	-19.9%	-9.9%	-21.9%	-5.1%

Table 1 – Summary statistics of all asset classes. Excess returns are adjusted by the average nominal risk-free rate of 3.28%. CPI of 2.91%. Leverage displays the theoretical position which has 15% volatility, TR at 15% vol is equivalent to Total Return and is adjusted for inflation and 15% volatility. Max drawdown shows the maximum drawdown each asset has endured over the 93-year sample period, while value at risk (VaR) is set to a 99% confidence level.

[11] Calculations will be further explained and analyzed in chapter 6. Formulas are attached in Appendix I.

4.2 Methodology

This section concentrates on relevant methodologies and models for answering our research question. The first part presents a model to obtain put and call prices, before three options strategy models are implemented to finalize our data set. Thereafter, we will present Harry Markowitz' mean-variance framework which determines asset classes weights in an optimal risky portfolio.

4.2.1 Options Strategies

Our hedge fund replication is based on traditional put and call option strategies on S&P 500 backtested to January 1928. We have computed long straddle, covered call and married put options strategies, and will treat them separately in the mean-variance framework. Before we go into details of each strategy, we will go through the procedure of obtaining historical option prices. S&P 500 options were not tradable before the 1990s, which means we cannot obtain true pricing data for the whole period (Historical Options Data, 2021). However, this can be estimated using the acknowledged pricing formula of Black and Scholes and a multiple regression model to estimate volatility. Black and Scholes employ the current stock price, strike price, risk-free rate, time to expiration and volatility of the underlying asset to compute options prices. The true S&P 500 volatility is unknown from 1928 to 1990 but can be estimated by exploiting the relationship between the implied volatility and S&P 500 market conditions in the period 1990 to 2020. Implied volatility is measured in the CBOE Volatility Index (VIX) from 1990 to 2020 and captures the market expectations for the next 30 days. This time frame forms our training set which is utilized to estimate values of VIX from 1928 to 2020.

Replicating the CBOE Volatility Index

CBOE use standard S&P 500 options with expiration within 23 and 37 days to calculate VIX, and our multiple regression model should hence capture similar information. We thus need a model which utilizes current information on market conditions to explain expectations. Various standard deviation measures and moving averages are used to find causal effects.

Six explanatory variables are log-transformed and differentiated to develop a reliable model without noise or overfitting. Transformation stabilizes variance, and our results does not indicate any seasonality. Residuals appear to have zero mean and constant variance, while the distribution of residuals are slightly skewed to the left[12]. While residuals seem to be white noise, will a Breuch-Godfrey test imply some autocorrelation, which means our model would struggle to produce exceedingly precise confidence intervals in its predictions. The regression output below reveals our preferred model, where 10, 25 and 60-days standard deviations and moving averages explain the current volatility level in the market. There are 7789 trading days in the replication period, and six explanatory variables are hence assessing the relationship between current information and expectations.

Regression Output on VIX, Model with 6 Explanatory Variables

lm (VIX ~ sd10 + sd25 + sd60 + ma10 + ma 25 + ma60)

	Estimate	Std. Error	t value	Pr (>|t|)	
(Intercept)	7.20	0.09	83.49	<2e-16	***
sd10	250.39	14.33	17.47	<2e-16	***
sd25	399.30	20.22	19.75	<2e-16	***
sd60	603.22	14.49	41.64	<2e-16	***
ma10	-516.21	15.76	-32.75	<2e-16	***
ma25	-332.86	31.07	-10.71	<2e-16	***
ma60	529.97	42.73	12.40	<2e-16	***

*Table 2 – Panel A: Regression output from 6 explanatory variables where all regressors are significant at a > 99% confidence level. '***', '**', '*', indicates significance at the 99%, 95% and 90% level, respectively.*
sd = standard deviation, ma = moving average.
Residual standard error: 3.059 on 7722 degrees of freedom

Multiple R-squared:	Adjusted R-squared:	F-statistic:	p-value:
0.86	0.86	7852 on 6 and 7722 DF	< 2.2e-16

Panel B: Result from the 6 variable regression model.

All explanatory variables are statistically significant at a > 99% confidence level, and the model explains 86% of the variation in VIX from 1990 to 2020. Significant coefficients and white noise residuals justify our model, and it will hence be applied to replicate S&P 500 volatility for all 23 383 trading days from January 1928 to December 2020. Figure 3 and 4

[12] Residual plots are included in Appendix I.

displays estimations and real values in both the replication period and the overall period.

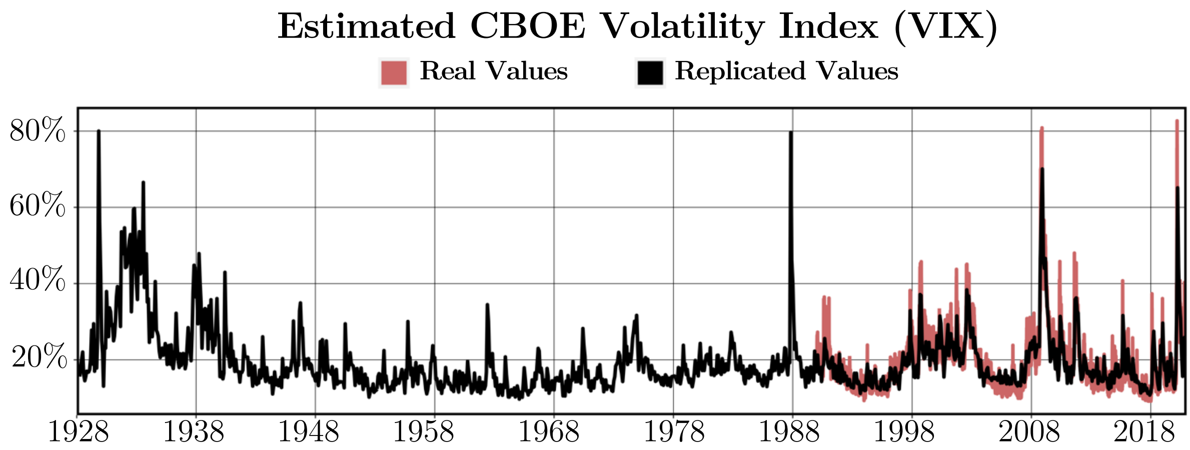

Figure 3 – Full sample of S&P 500 Volatility Index (VIX), 1928 - 2020.

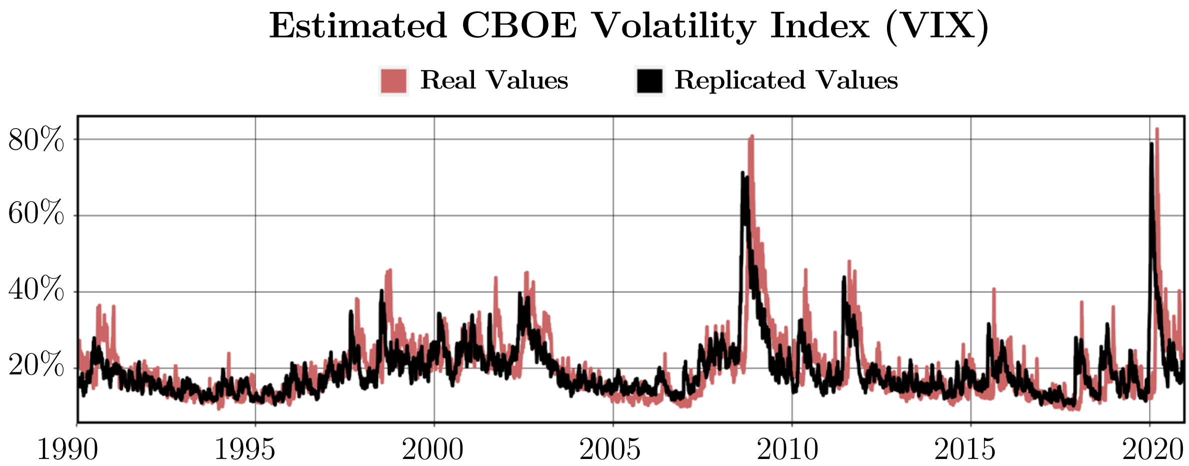

Figure 4 – Test sample of S&P 500 Volatility Index (VIX), 1990 - 2020.

Implementation of Black and Scholes

We have now obtained all necessary inputs to calculate historical put and call prices using Black and Scholes. Covered calls and married puts are based on at-the-money options, while a long straddle strategy can be in-the-money or out-of-the-money. We will now present a sample of options prices, using at-the-money options where the strike price is equal to the current stock price. We are using 1116 observations in our options strategies, where one observation equals one month of the time series.

Sample of Option Price Calculations on S&P 500							
Date	Stock Price	Strike	1M Risk-Free	Time to Expiration	Volatility	Call Price	Put Price
1929-01-02	$25.74	$25.74	0.253%	1 Month	16.8%	$0.501	$0.496
1929-02-01	$25.59	$25.59	0.260%	1 Month	17.5%	$0.517	$0.512
1929-03-01	$25.53	$25.53	0.259%	1 Month	20.0%	$0.592	$0.586
1929-04-01	$25.94	$25.94	0.259%	1 Month	27.8%	$0.833	$0.827

Table 3 – A four-period excerpt sample of Black and Scholes input data for S&P 500.

Long Straddle

In a long straddle options strategy, you buy put and call options with the same strike price and maturity simultaneously. The illustration below reveals the payoff pattern of a long straddle, where you are dependent on large stock movements which exceed the level of total option premiums paid at each month to make a return (Natenberg, 2015).

Long Straddle Payoff Illustration

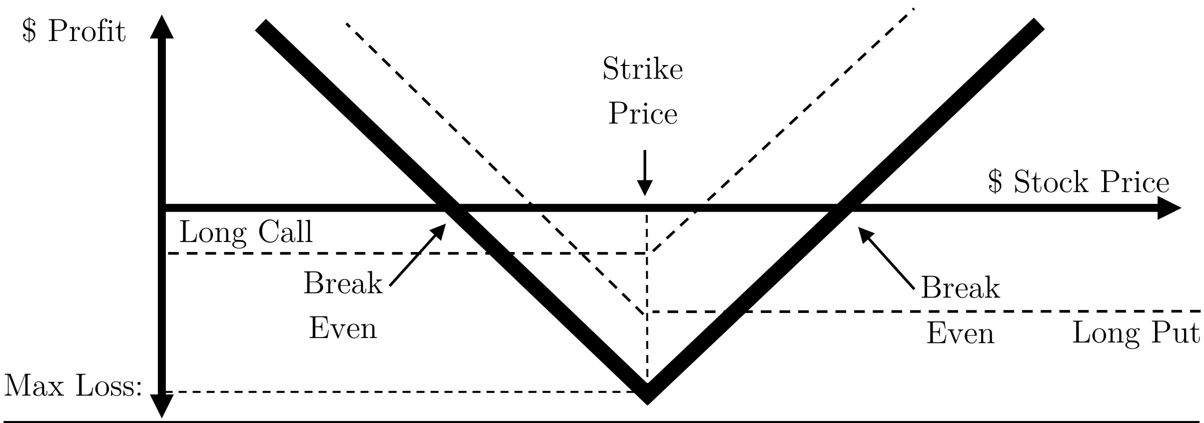

Figure 5 – Long straddle illustration.

If the stock is stationary till expiration and you buy at-the-money options, you would lose 100% of your stake in one month. This can be avoided by always placing 90% of your portfolio in U.S. Treasury bills, and 10% of your portfolio in a long straddle strategy. You can also adjust the strike price to find the most optimal solution for your data set in retrospect. A strike price of 90% of the current stock price was the most optimal for our data set, which means we are investing in out-of-the-money put options and in-the-money call options.

A sample of long straddle calculations are displayed below, where we can see a solid call payoff of $2.42 in February 1929 is not enough to cover the large call premium of $2.59

that was paid in January 1929. It turns out the stock movements are too weak in a broad index over 30 days, and that a long straddle strategy is not suitable. We achieved an annualized return of 0.5% and a standard deviation of 13.37% with the most optimal strike price. By investing in at-the-money options, the annualized return would be significantly negative.

Sample of Long Straddle Calculations

Date	Stock Price	Strike	Call Price	Put Price	Call Payoff	Put Payoff	Total Payoff	Excess Return
1929-01-02	$25.74	$23.17	$2.59	$0.006	$3.83	$0	$1.22	4.47%
1929-02-01	$25.59	$23.03	$2.57	$0.008	$2.42	$0	-$0.17	-0.87%
1929-03-01	$25.53	$22.98	$2.58	$0.019	$2.50	$0	-$0.11	-0.74%
1929-04-01	$25.94	$23.35	$2.69	$0.087	$2.96	$0	$0.18	0.37%

Table 4 – Sample of the first four long straddle calculations. Excess returns after 0.5% monthly transaction costs[13].

Covered Call

In a covered call strategy, you go long in an asset while you write a call option on the same underlying asset. If the stock is purchased simultaneously with the option writing, it is called a buy-write transaction. The call payoff will eliminate the stock payoff, and your prime source of income is the option premium and stock dividends. It is suitable when the market is moving relatively flat, and you do not expect any strong movements in the underlying asset. Hence, it is a neutral strategy and should not be held if you expect large movements in the short term.

[13] A monthly trading cost estimate of 0.5% is incorporated to our options strategies due to the frequent need of rebalancing with 30-days contracts. The majority proceed from explicit costs such as broker commissions and platform fees, while implicit costs are assumed to be low because of high liquidity in S&P 500 options (Foucalt, Pagano, & Röell, 2013).

Covered Call Payoff Illustration

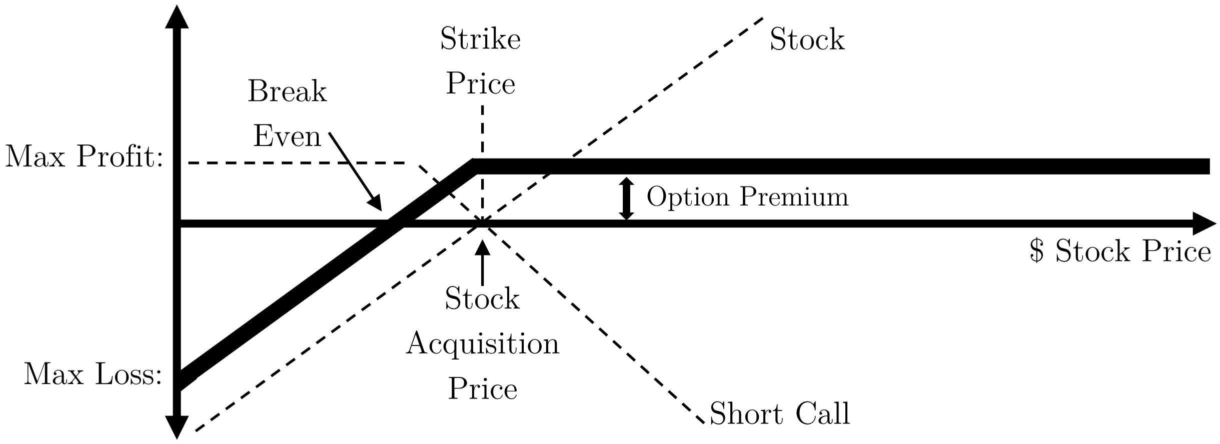

Figure 6 – Covered call illustration.

Covered call calculations are presented below, where the total payoff of $0.37 in February 1929 comes from a call option premium gain of $0.52 less the $0.15 depreciation of S&P 500. Even though we have written call options ourselves, will no investor exercise when the payoff is negative. The downside is not secured, but option premiums generate a steady stream of income each month.

Sample of Covered Call Calculations

Date	Stock Price	Strike	Call Price	Call Payoff	Stock Payoff	Total Payoff	Excess Return
1929-01-02	$25.74	$25.74	$0.50	$1.39	$1.39	$0.50	1.70%
1929-02-01	$25.59	$25.59	$0.52	$0.00	-$0.15	$0.37	1.21%
1929-03-01	$25.53	$25.53	$0.59	$0.00	-$0.06	$0.53	1.82%
1929-04-01	$25.94	$25.94	$0.83	$0.41	$0.41	$0.83	0.29%

Table 5 – Sample of the first four covered call calculations. Excess return after 0.5% monthly transaction costs.

Married Put

A married put strategy has similarities to covered calls, but you will now hold a long position in a stock and purchase a put option on the same underlying stock. The downside is limited to the put option premium, while the upside is unlimited because you will not exercise your option if it has negative payoff. The potential profit will be lower than by holding a simple stock, because the put option premium is constantly charged for limiting downside.

Married Put Payoff Illustration

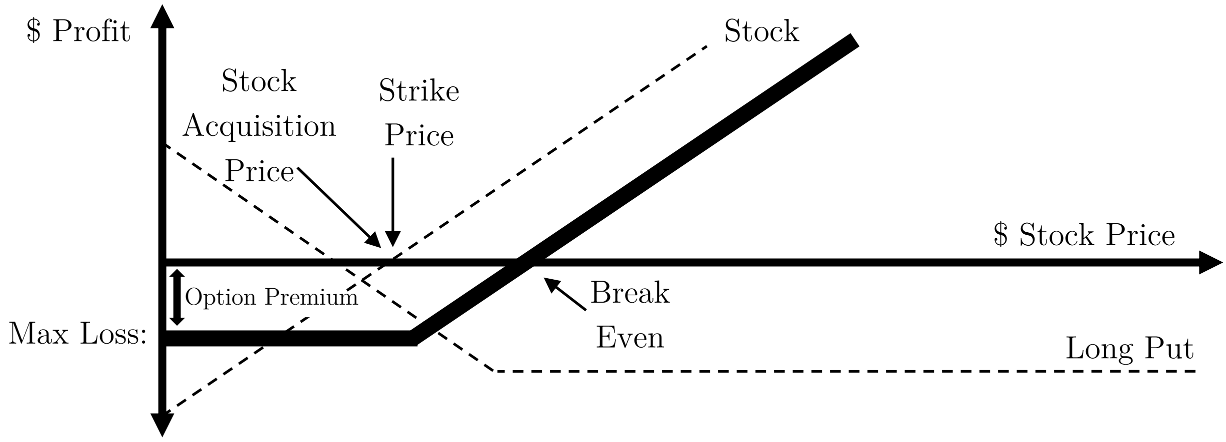

Figure 7 – Married put illustration.

A sample of married put calculations are presented underneath, and we can discover that an investor might experience negative payoffs even though S&P 500 are going up. In April 1929 for example, the S&P 500 increased by 0.41 points, not enough to cover the put option premium of $0.83. It turns out that a married put strategy does not perform well because of weak S&P 500 movements each month, just as with long straddle options.

Sample of Married Put Calculations

Date	Stock Price	Strike	Put Price	Put Payoff	Stock Payoff	Total Payoff	Excess Return
1929-01-01	$25.74	$25.74	$0.50	$0.00	$1.39	$0.89	3.19%
1929-02-01	$25.59	$25.59	$0.51	$0.15	-$0.15	-$0.51	-2.31%
1929-03-01	$25.53	$25.53	$0.59	$0.06	-$0.06	-$0.59	-2.59%
1929-04-01	$25.94	$25.94	$0.83	$0.00	$0.41	-$0.42	-1.87%

Table 6 – Sample of the first four married put calculations. Excess return after 0.5% monthly transaction costs.

4.2.2 Mean-Variance Framework

All three options strategies are now computed to finalize our data set, and we will advance to the core methodology of this thesis. A mean-variance model will compute the tangent portfolio from twelve asset classes using Harry Markowitz' framework and quadratic optimization. This portfolio lies on the efficient frontier and the capital allocation line and will hence have the highest possible Sharpe ratio in our investment universe. Markowitz' work from 1952 shows how to compute returns and standard deviations in a portfolio with "n" number of assets. The framework is presented underneath, where a portfolio's expected return and variance is derived. The expected return is a result of two multiplied vectors of portfolio weights and separate expected returns for each asset class,

$$E[R_P] = \mu_P = [w_1\ w_2\ w_3\ \dots\ w_n] \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_n \end{bmatrix} = \sum_{j=1}^{n} w_j\ E(R_j). \qquad (3)$$

A portfolio's variance is slightly more complicated because of the concept of diversification. The combined risk is calculated by multiplying a vector of weights by the covariance matrix and then a transposed vector of the same weights. In our case, we are calculating a portfolio's variance by organizing 12 asset class weights, multiply by a covariance matrix of 12x12, before multiplying with a transposed vector of the same 12 weights

$$Var[R_P] = \sigma_P^2 = [w_1\ w_2\ w_3\ \dots\ w_n] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \dots & \sigma_{2n} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \dots & \sigma_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \dots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i\ x_j\ \sigma_{ij}. \qquad (4)$$

The PortfolioAnalytics package in R can be used to solve quadratic optimization problems, while also fulfilling given requirements from the investment mandate. It is developed by professors and quantitative analysts and is one of the most well-recognized packages within finance in R[14]. Appropriate objectives and constraints must be defined before running the optimizer, where we want to maximize Sharpe ratio in a portfolio which is always fully invested without any short selling[15]. Maximum weight constraints for each asset can also be controlled, although this is dependent on the necessity given our output. Assets allocating less than 5% is not accounted for, meaning larger positions will cover up the tiniest fractions. This is avoiding minor positions which do not have a significant impact on performance, except for unnecessary costs and inconveniences.

The ROI method within PortfolioAnalytics will be utilized to achieve the best return-to-risk ratio (Peterson & Carl, 2018). ROI is short for R Optimization Infrastructure and will find the most suitable solver given its inputs, constraints, and objectives (Turlach, 2021).

[14] Brian G. Peterson, Peter Carl, Kris Boudt, Ross Bennett, Hezky Varon, Guy Yollin and Douglas Martin.

[15] Although our model does not allow short selling, will options strategies and equity factor exposure imply indirectly short positions.

Because portfolio optimization problems are quadratic, will ROI select the acknowledged Quadprog solver based on Goldfarb and Idnani (1983) for its computation[16].

[16] The quadratic optimization problem is denoted min ($-d^T b + 1/2 b^T D b$) with constraints $A^T b \geq b_0$. There is no linear term in our problem, which implies $d = 0$. Variable b equals the weight vector, while variable D represents the covariance matrix.

5. Constructing the Optimal Risky Portfolio

The mean-variance framework from section 4.2.2 will now be utilized to construct an optimal risky portfolio. Employing realized returns implies backward-looking findings, and we will hence present the most optimal risky portfolio from 1928 to 2020. A completely retrospective approach would normally not be preferable because security performance is not repeatable in the future. However, we are analyzing asset classes and not individual securities. While a security's value creation and return are dependent on a non-repeatable life cycle, are asset classes dependent on the fundamental macroeconomic environment. These conditions will to some extent repeat itself, and a retrospective approach will hence be suitable when allocating asset classes[17].

Realized Excess Return

Equity	Gov. Bonds	Corporate Bonds	Gold	Real Estate	Comm.	SMB Factor	HML Factor	UMD Factor	Long Straddle	Covered Call	Married Put
6.07%	1.63%	3.61%	1.58%	0.67%	-0.60%	7.24%	9.04%	12.29%	0.54%	6.56%	-1.65%

Table 7 – Realized annual excess returns for each asset class from 1928 to 2020.

The quadratic optimizer in R implements the mean-variance framework to calculate portfolio return and variance. The utilized return vector is presented above, which is multiplied with a given portfolio combination to find return. Variance will be computed by including the covariance matrix below, which is based on monthly movements from 1928 to 2020, presented in annual terms.

[17] See evidence in figure 20 – *Portfolio performance, different optimization periods.*

Covariance Matrix

1928 - 2020	Equity	Gov. Bonds	Corporate Bonds	Gold	Real Estate	Comm.	SMB Factor	HML Factor	UMD Factor	Married Put	Covered Call	Long Straddle
Equity	0.0371	0.0007	0.0044	0.0009	0.0004	0.0006	0.0048	0.0055	0.0033	0.0170	0.0223	0.0203
Gov. Bonds	0.0007	0.0026	0.0020	0.0006	-0.0001	-0.0003	-0.0014	-0.0010	-0.0001	0.0004	0.0004	0.0005
Corp. Bonds	0.0044	0.0020	0.0060	0.0016	0.0000	0.0003	0.0062	0.0074	0.0027	0.0017	0.0027	0.0020
Gold	0.0009	0.0006	0.0016	0.0215	0.0000	0.0004	-0.0003	-0.0009	0.0009	0.0003	0.0009	0.0001
Real Estate	0.0004	-0.0001	0.0000	0.0000	0.0024	0.0002	0.0011	0.0011	0.0005	0.0003	0.0001	0.0004
Commodities	0.0006	-0.0003	0.0003	0.0004	0.0002	0.0013	0.0020	0.0022	0.0012	0.0002	0.0005	0.0002
SMB Factor	0.0048	-0.0014	0.0062	-0.0003	0.0011	0.0020	0.0597	0.0476	0.0310	0.0033	0.0019	0.0037
HML Factor	0.0055	-0.0010	0.0074	-0.0009	0.0011	0.0022	0.0476	0.0580	0.0258	0.0037	0.0016	0.0043
UMD Factor	0.0033	-0.0001	0.0027	0.0009	0.0005	0.0012	0.0310	0.0258	0.0397	0.0031	0.0007	0.0029
Married Put	0.0170	0.0004	0.0017	0.0003	0.0003	0.0002	0.0033	0.0037	0.0031	0.0112	0.0070	0.0113
Covered Call	0.0223	0.0004	0.0027	0.0019	0.0001	0.0005	0.0019	0.0016	0.0007	0.0070	0.0185	0.0088
Long Straddle	0.0203	0.0005	0.0020	0.0001	0.0004	0.0002	0.0037	0.0043	0.0029	0.0113	0.0088	0.0179

Table 8 – Annualized covariance matrix, 1928 – 2020.

The portfolio optimization output with all assets is extracted underneath, including both weights and performance measures. There are in total five asset classes which form the most optimal return-to-risk ratio, with a relatively moderate allocation distribution. Government bonds, corporate bonds, real estate, covered calls, and momentum exposure are all representing slightly different risk exposures, which implies improved diversification.

Mean-Variance Optimization 1: All Assets

Optimal Weights:

Equity	Government Bonds	Corporate Bonds	Gold	Real Estate	Comm.	SMB Factor	HML Factor	UMD Factor	Long Straddle	Covered Call	Married Put
0.0%	29.7%	14.0%	0.0%	12.8 %	0.0%	0.0%	0.0%	21.2%	0.0%	22.3%	0.0%

StdDev (monthly): 1.70%	StdDev (annual): 5.88%
Mean (excess, monthly): 0.41%	Mean (excess, annual): 4.91%
Constraints: Long only, fully invested, weights = 0.00 or (0.05 – 1.00)	

Table 9 – Portfolio optimization, all assets.

At first glance, it could be surprising to discover an U.S. equity weight of zero percent. Financial literature suggests otherwise, and it might seem like a contradiction. However, we are investing 43.5% in U.S. equities indirectly through other asset classes. A covered call strategy includes a long position in S&P 500 (22.3%), while a position in UMD (21.2%)

gives exposure to equities on NYSE, AMEX, and Nasdaq. This optimization avoids allocations between 0% and 5%[18], and could also be tuned to restrict larger fractions to force better risk allocation. No asset class is massively exposed in the suggested allocation, and a maximum weight constraint turns out to be unnecessary. Optimization 1 reveals annual excess returns of 4.91%, implying a Sharpe ratio of 0.835.

Even though it was possible to invest in options and equity factor exposures back in 1928, the options market was over the counter, and empirical evidence from Fama-French was yet to be published. We could thus run the same optimization algorithm as in *Optimization 1,* except equity factor exposure from Fama-French and options strategies. This would result in six asset classes which was widely known back in 1928: U.S. equity, government bonds, corporate bonds, gold, real estate, and commodities. Using the same methodology and constraints, while also adding a maximum weight constraint of 30%, results in the portfolio displayed in table 11.

Mean-Variance Optimization 2: Six Assets

Optimal Weights:

Equity	Government Bonds	Corporate Bonds	Gold	Real Estate	Commodities
10.41 %	30.00 %	30.00 %	3.23 %	26.36 %	0.00 %
StdDev (monthly): 1.32 %			StdDev (annual): 4.56 %		
Mean (excess, monthly): 0.19 %			Mean (excess, annual): 2.34 %		
Constraints: Long only, fully invested, weights = 0.00 – 0.30					

Table 10 – Portfolio optimization, six assets.

Gold and commodities still appear as unfavorable assets in a mean-variance situation, while strong absolute return levels of traditional equity are ignored due to a risk-adjusted return focus. Fixed income and equity-linked securities form the most optimal combination, and the two mean variance allocations are presented below.

[18] A non-constraint portfolio is displayed in Appendix II.

Mean-Variance Optimization 1: All assets	Mean-Variance Optimization 2: Six Assets

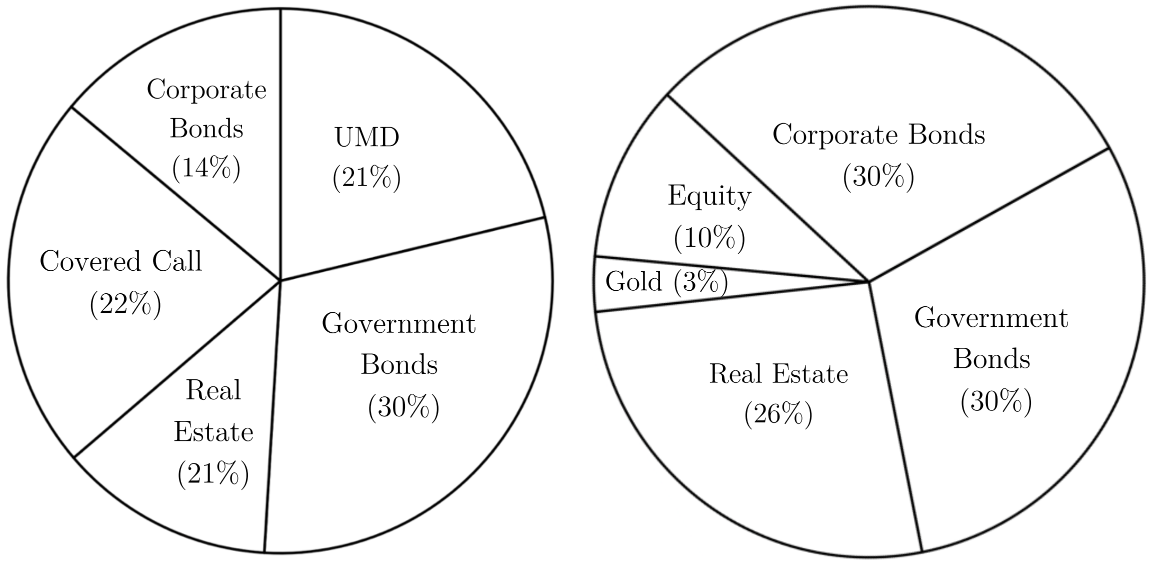

Figure 8 – Mean-variance allocations.

When comparing performance between our two suggested portfolios and the traditional 60/40 stock and bond portfolio, we discover a much higher standard deviation with traditional allocation. Implementing more assets seem to create diversification, where the tangent portfolio in a full sample mean-variance optimization yields approximately the same level of return with a much lower risk. The six-asset portfolio has also improved its risk-adjusted return, increasing the Sharpe ratio from 0.397 to 0.512 after the entrance of alternative asset classes. This gives the impression of being an efficient improvement, although it might not be a favorable portfolio for investors. Reducing the weight of traditional equity has improved the return-to-risk ratio at the expense of absolute levels of excess returns. While including assets to maximize Sharpe ratio seems correct, is it still important to have adequate returns to meet your liabilities. Poor excess returns force us to omit this portfolio combination, which means our analysis will only contain traditional stock and bond portfolios and the tangent portfolio from the mean-variance optimization of all assets.

Summary Statistics			
	Classic 60/40	Tangent Portfolio, 6 assets	Tangent Portfolio, all assets
Total return adj. for inflation	5.45%	2.70%	5.27%
Arithmetic excess return	5.09%	2.34%	4.91%
Geometric excess return	4.27%	2.24%	4.74%
Standard deviation	12.81%	4.56%	5.88%
Arithmetic Sharpe ratio	0.397	0.512	0.835
Geometric Sharpe ratio	0.333	0.490	0.806

Table 11 – Summary statistics of different portfolios.

6. Portfolio Analysis

The optimal combination of asset classes in a risky portfolio is now determined, and we have acquired an appropriate foundation to answer our research question. We will advance by analyzing the tangent portfolio and compare our findings to a traditional stock and bond portfolio. The results will give an answer to what effect alternative asset classes might have in a traditional portfolio. The first section, 6.1. in this chapter will investigate correlations between assets, while different performance measurements will be applied in section 6.2.

6.1 Correlation

The concept of diversification was enlightened within the introduction to this paper, where uncorrelated assets reduce portfolio variance without necessarily influence returns. Our tangent portfolio proves this because some asset classes perform well when others perform poorly. To investigate why our optimal portfolio yields half the fluctuations of a classic 60/40 portfolio, we will analyze correlations between asset classes.

One of the main reasons for adding several asset classes in a portfolio is because correlation changes over time (Fabozzi & Markowitz, 2011). It might go from negative to positive in only a few months, and the whole diversification effect would disappear. An increased amount of asset classes will hence decrease the possibility of experiencing strong correlation in a portfolio. Varying correlation also implies that full sample measures[19] will not explain the complete co-movements between assets, and we will hence analyze rolling correlation of 60 observations, equivalent to five years. This timeframe provides a reliable foundation, while also capturing large parts of the business cycle.

The first rolling correlation plot reveals the behavior of U.S. corporate bonds and U.S. Treasury bonds to U.S. equities. It captures the diversification effect within a traditional 60/40 portfolio, by either using corporate or government bonds. The striking discovery is how vulnerable traditional portfolios have been, with a consistent positive correlation for

[19] A full sample correlation matrix is displayed in Appendix I.

nearly 40 years between the 1960s and 2000s. There is also a 0.6 correlation peak observed just before the financial crisis in 2008, which impeded its hedging effect.

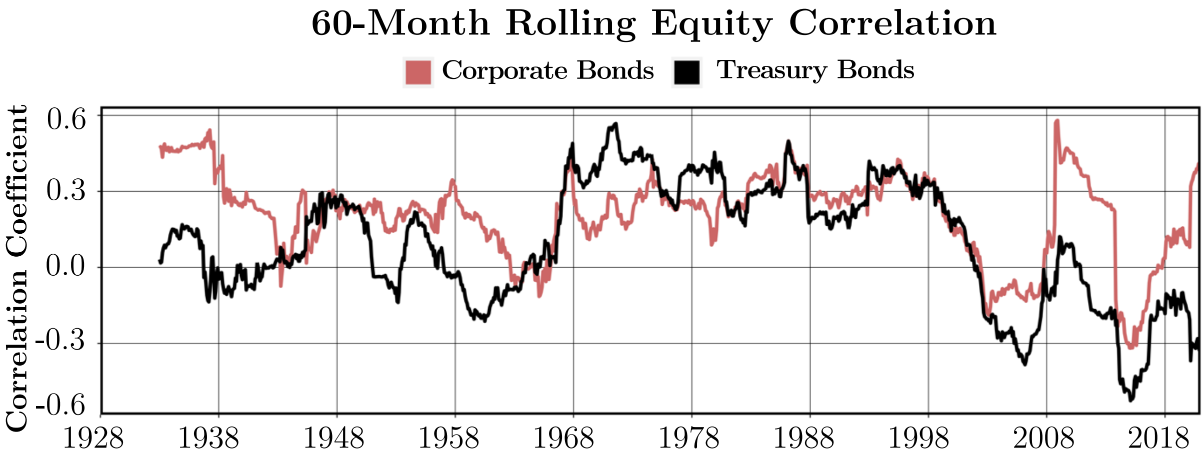

Figure 9 – 60-month rolling correlation against U.S. equites.

It could be tempting to hold high-correlated assets in expansions and when the cost of credit is low, simply because of its excellent performance. An investor might even unknowingly sacrifice a balanced portfolio in favor of risky and correlated assets. The true risk might not be visible until the activity level drops, which is aligned with the famous claim that the only thing that increase in crises is the correlation. Hence, correlation should be as low as possible before a crash occurs to prevent large losses. The correlation behavior is perfectly illustrated by the financial crisis in 2008.

The same rolling correlation method has further been applied when analyzing correlation in our tangent portfolio. Zero-weight asset classes have been excluded, meaning we only focus on real estate, government bonds, corporate bonds, covered calls and UMD factor exposure. The S&P 500 is no longer a pure investment alternative and is replaced by covered calls as reference for the 60-month rolling correlations.

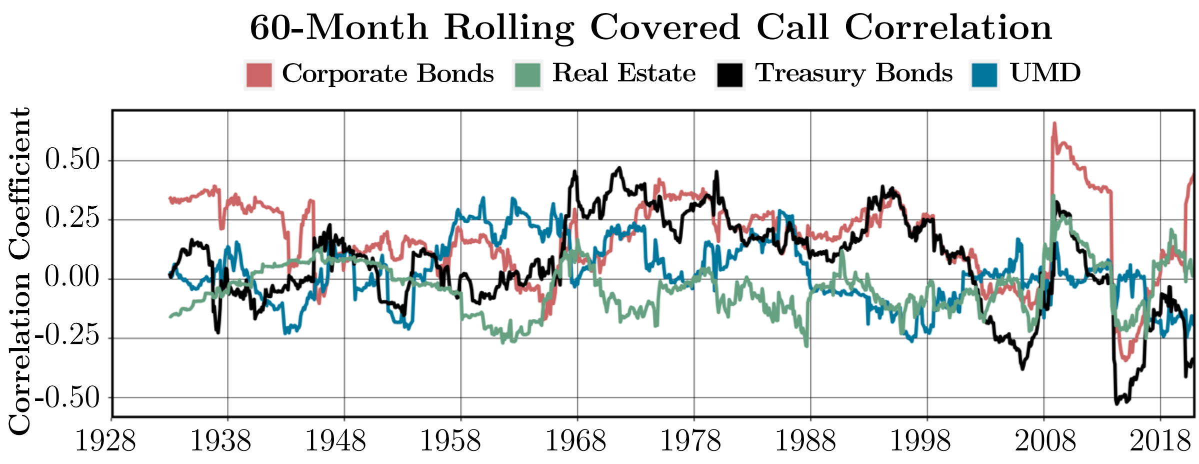

Figure 10 – 60-month rolling correlations against covered calls.

Figure 10 reveals the massive improvement in diversification over time between the tangent portfolio and the 60/40 equity-bond portfolio. Each asset class in our optimal portfolio has periods of positive correlation, although there are always other assets with negative correlation at the same time. This makes the portfolio able to substantially outperform the traditional portfolio, mostly due to its better hedge in periods of recessions and periods of stagnation. Its distribution is more well-balanced, and the total correlation is much lower.

6.2 Performance

We have discovered that the implementation of alternative asset classes in a portfolio has improved the effect of diversification compared to a traditional stock and bond portfolio. While this is great, it does not provide enough insight to explicitly denote its effect on performance. To measure the effect of diversification on portfolio performance, this section will investigate downside risk, risk-adjusted return, active management, and performance in different periods.

6.2.1 Downside Risk

There are several methods to assess downside risk. The most common approach is to measure the standard deviation, even though it does not distinguish positive from negative movements around its mean. Additionally, it is a result of past information despite historical movements might be irrelevant. To analyze downside risk in this chapter, we will therefore focus on five different measurements: portfolio drawdown, value at risk (VaR), skewness, kurtosis and Sortino ratio.

Figure 11 displays drawdown for both portfolios, where historical losses are assessed in a time series. It reveals larger drawdowns for a traditional portfolio, where the tangent portfolio outperforms every year. The worst period for both portfolios were the market crash in 1929, which reached its trough in 1933. The capital in the tangent portfolio was reduced by 44%, significantly less than the traditional portfolio which suffered a loss of 75%. The effect of less correlated assets and improved diversification is easily interpretable by this visualization. The relatively large difference in drawdown implies that the tangent portfolio is more robust in recessions and market crashes.

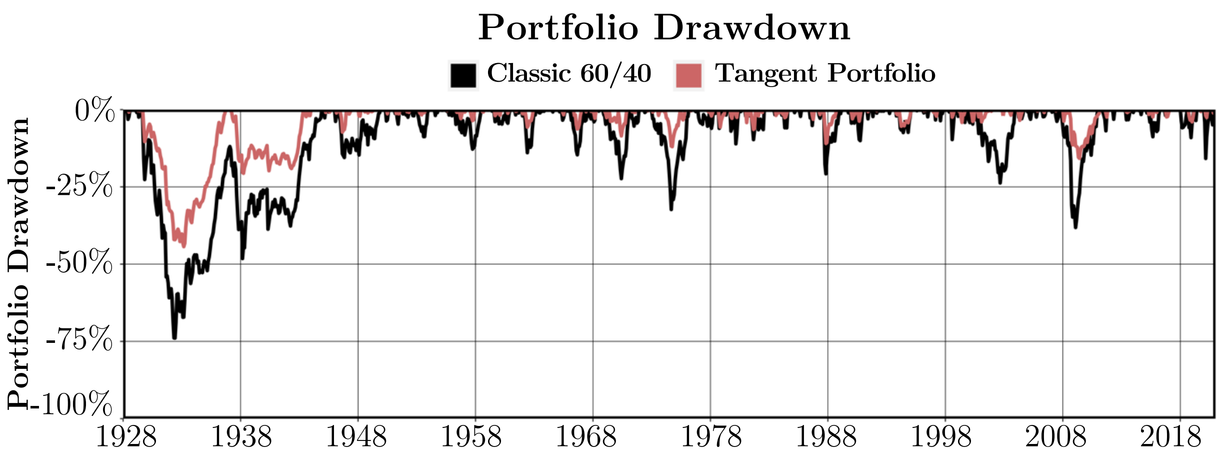

Figure 11 – Portfolio drawdown.

Another method of assessing downside risk is to analyze the distribution of returns. This is important, because standard deviation and hence the Sharpe ratio is dependent on normality to capture a full risk-return picture. Analyzing distribution of returns can therefore reveal characteristics which are not captured by traditional measures. Distribution of returns is explicitly displayed in figure 12, where the magnitude of observations is positive. There is also a large difference between the portfolios, where our optimal portfolio has a much higher frequency around its mean.

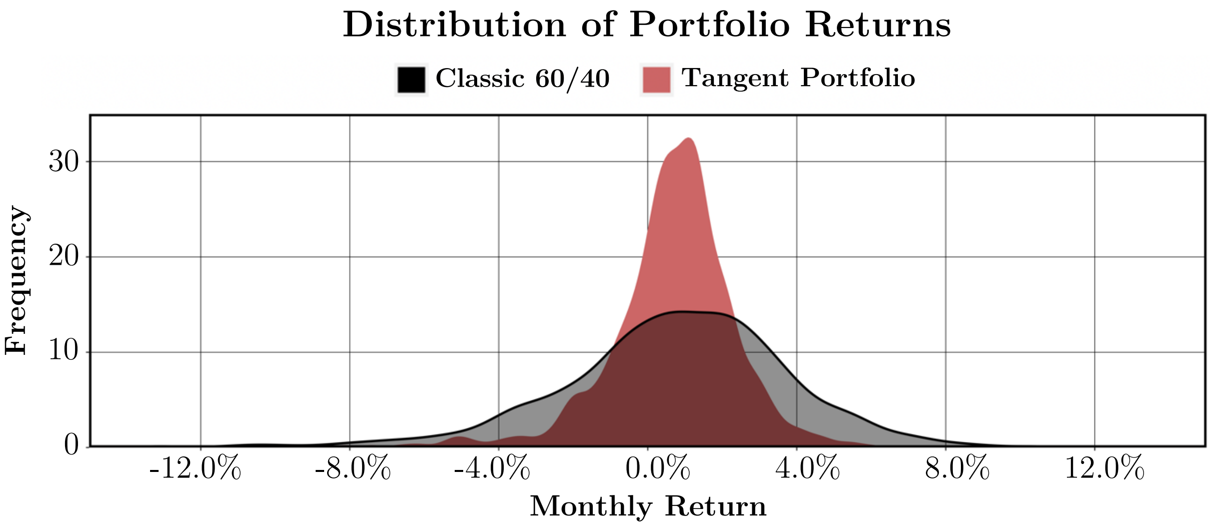

Figure 12 – Distribution of portfolio returns.

While a chart provides a quick insight on distribution of returns, will a statistical approach be necessary to achieve accurate and reliable measurements of our results. Table 13 summarizes results from analyzing Value at Risk (VaR), skewness, kurtosis and the Sortino ratio of each portfolio[20].

Value at Risk calculates the maximum loss of a portfolio over a given period with a specified degree of certainty. It is derived from the distribution of returns assessing the magnitude of left tail risk. Our calculations are based on 1 month with a 90%, 95% and 99% confidence level, and reveals a higher left tail risk for traditional portfolios. For example, with a 99% level of confidence, we will not expect a loss greater than 16.83% in one month for the traditional 60/40 portfolio. For the tangent portfolio, however, the expected loss would be restrained to 5.79% at the same level of confidence.

When investigating returns, it is also essential to assess if the distribution is skewed. Skewness is a statistical measure of asymmetry and will evaluate extreme values at each tail. A positively skewed distribution implies that positive extreme returns are dominating negative extreme returns and vice versa. By computing the skewness in R, we discover negative skewness for both portfolios. This implies that the magnitude of extreme returns is negative, and that the standard deviation underestimates the true level of risk (Bodie,

[20] Formulas are attached in Appendix I.

Kane, & Marcus, 2018). While this is the case for both portfolios, it is more severe for the tangent portfolio given its higher value of skewness.

Kurtosis is another distribution measure which assess the likelihood of extreme values by evaluating fat tails. Given an overweight of extreme negative values in our portfolios, it is salient to investigate the kurtosis as well. If a distribution has fat tails, a model will underestimate the probability of extreme values when assuming normality. A normally distributed sample will have a kurtosis of 3, while our findings reveal higher kurtosis and hence a leptokurtic distribution. This is characterized by fatter tails which implies higher probability of extreme values for each portfolio, especially the traditional stock and bond portfolio.

The last considered measure of downside risk is Sortino ratio, which is a risk-adjusted measurement with similarities to Sharpe ratio. The only difference is that the standard deviation term is replaced by lower partial standard deviation (LPSD). By only measuring deviations on negative returns, the Sortino ratio will not punish a portfolio for its positive movements. Our findings are as expected, where the tangent portfolio has more than double the return-to-risk ratio of a normal stock and bond portfolio.

Downside Risk Measures

	Classic 60/40	Tangent Portfolio
VaR, 90%	-1.77%	-1.24%
VaR, 95%	-5.14%	-2.44%
VaR, 99%	-16.83%	-5.79%
Skewness	-0.52	-0.88
Kurtosis	12.05	7.24
Sortino Ratio	0.164	0.367

Table 12 – Risk measurements.

6.2.2 Risk-Adjusted Return

Risk-adjusted return and portfolio performance can be analyzed and visualized in multiple ways. The classic 60/40 portfolio yielded an arithmetic Sharpe ratio of 0.397, while the tangent portfolio massively outperformed by yielding 0.835. This means that our optimal portfolio can be levered up to match the risk of a traditional portfolio and achieve excess returns. We are then allowing for equal comparisons, which is essential when evaluating portfolios.

Another method to illustrate the diversification effect is to draw a risk-reward chart and realize the efficient frontier. This line represents the most efficient portfolio given an exact value of risk or return. A risk-reward chart visualizes this relation by displaying excess returns on the Y-axis and standard deviation on the X-axis. Figure 13 includes all asset classes from our data set, in addition to the tangent portfolio and the traditional 60/40 stock and bond portfolio. Additionally, it contains one million simulated portfolios, where the darker data represents a more efficient portfolio. Accordingly, the efficient frontier is formed by the darkest data points, where the Sharpe ratio is above 0.75.

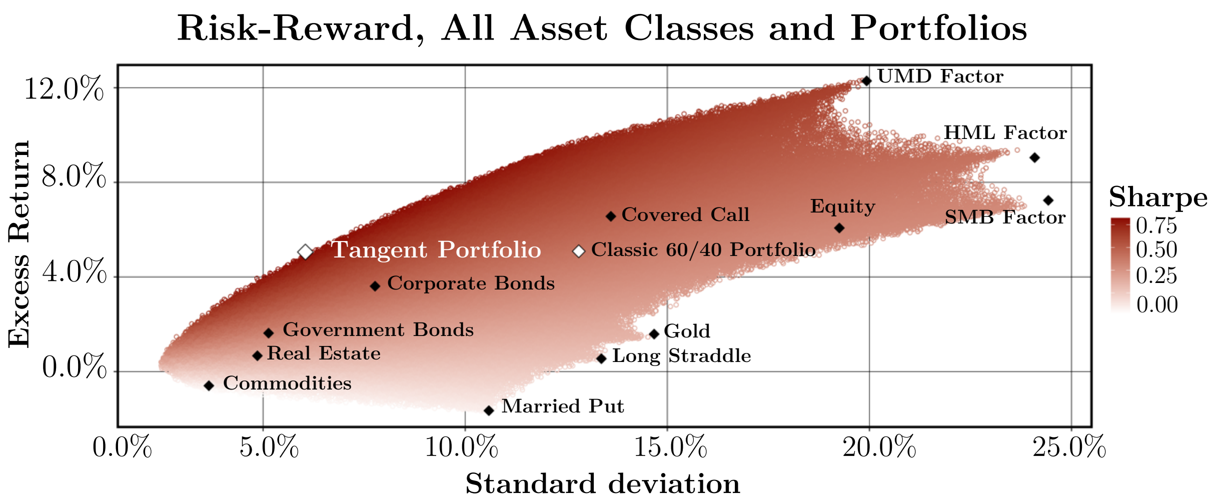

Figure 13 – Risk-reward chart.

By interpreting figure 13, we discover large differences in portfolio quality. The optimal portfolio will ultimately change as more asset classes are applied to this model. In this constructed investment universe, the risk-return ratio of a traditional 60/40 portfolio is poor and heavily inefficient, and thus no rational investor would hold it. The tangent portfolio lies on the efficient frontier and will tangent the capital allocation line (CAL) if you draw it from the origin.

To advance, we will perform a more in-depth analysis and comparison between portfolio returns and value creation. Table 12 has already shown arithmetic returns, revealing that a classic 60/40 stock and bond portfolio yields the highest unadjusted return with 5.45%

annually, while our optimal portfolio yields slightly less with 5.30%. However, arithmetic returns are not 100% representative[21]. If a portfolio suffers a loss of 50% and thereafter rise by 100%, arithmetic returns claim the total return to be 25%, even though the real return is 0%.

Plotting realized returns of both portfolios from 1928 to 2020 reveals the outperformance of the tangent portfolio against the traditional portfolio due to its limited downside. Investing $1 in a 60/40 stock and bond portfolio in January 1928 would accumulate to $72 by December 2020, while $113 would be gained by the tangent portfolio after adjusting for inflation[22]. A traditional 60/40 portfolio needs larger returns to recover from its larger losses, and this is not reflected in arithmetic return values.

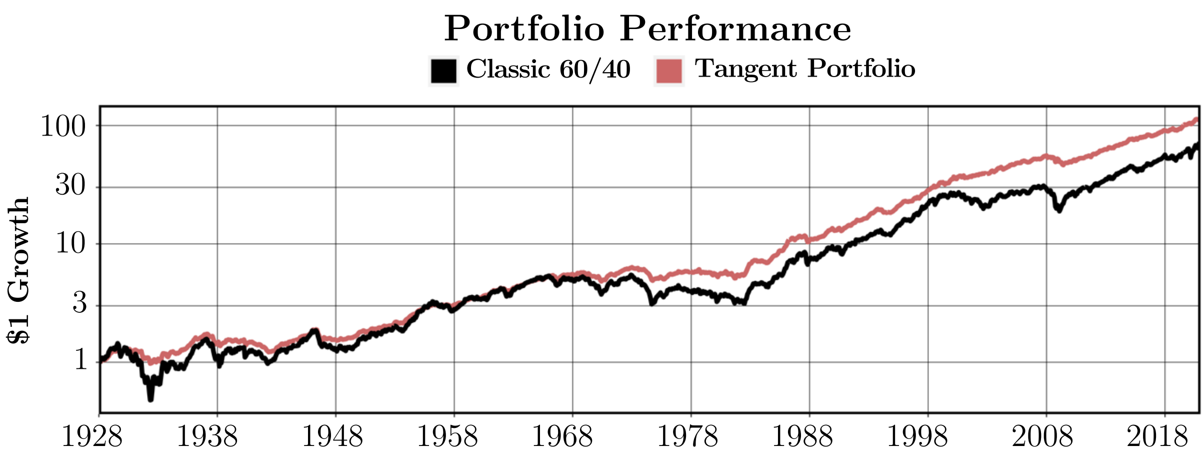

Figure 14 – Portfolio performance, adjusted by inflation.

However, while figure 14 shows two portfolios which seems rather similar, should we be careful to compare without controlling for risk. We have therefore created a new chart adjusted to 15% volatility. This implies holding a partial cash position if the portfolio is volatile and leveraging the position at the risk-free interest rate if the portfolio is less volatile than 15% annually[23]. It might not be feasible in practice if the volatility is extremely low or the capital very high, but it does illustrate the risk-return ratio of different investment alternatives. Each portfolio starts with $1 in January 1928, and the best in December 2020

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[21] While an arithmetic approach is preferred for expected returns in the future, will a geometric computation describe past performance in a more sufficient way. $R_{Arithmetic} = R_{Geometric} + ½$ variance.

[22] Tables of end values for all relevant asset classes and portfolios are included in Appendix II.

[23] Borrowing at the risk-free rate will be infeasible, which implies a less steeper Capital Allocation Line in practice when your portfolio exceeds 100% investing. Credit premiums are not universal for investors and will hence not be considered when leveraging positions in our thesis.

is the portfolio with the best geometric Sharpe ratio. The log-scale might cover up the large difference between our portfolios, where $1 in 1928 compound to $121 for a traditional portfolio and $58 860 for our optimal portfolio.

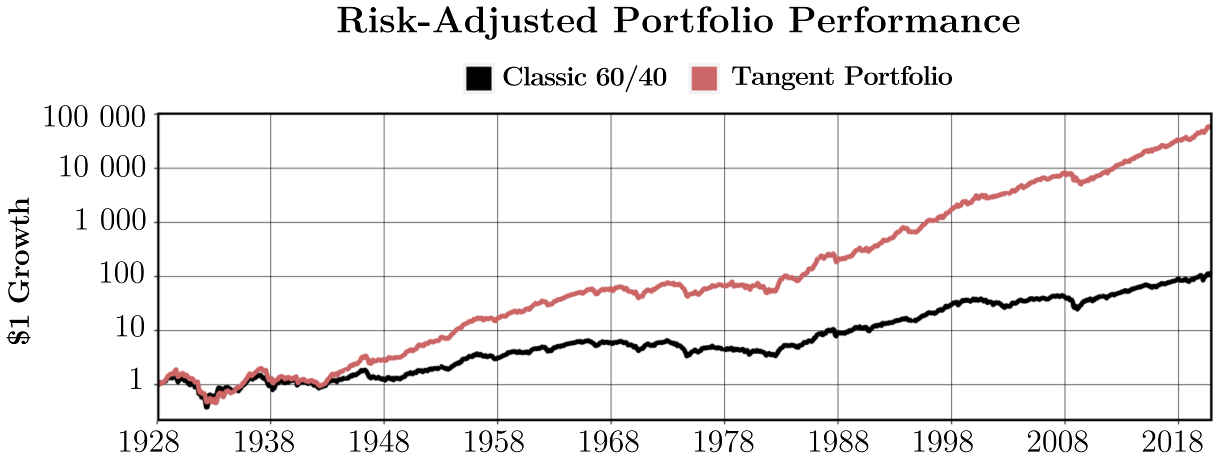

Figure 15 – Portfolio performance, adjusted by inflation and to 15% annual volatility.

The same methodology is applied to each relevant asset class, where a UMD factor portfolio outperforms all other asset classes massively. This is very much in line with their geometric Sharpe ratios presented in table 1 in chapter 4.1. Leverage and the concept of compounding make extreme differences over 1116 months, where a $1 UMD portfolio in January 1928 evolves to $2560 in December 2020 adjusted for inflation, while traditional equity accumulates to $39.

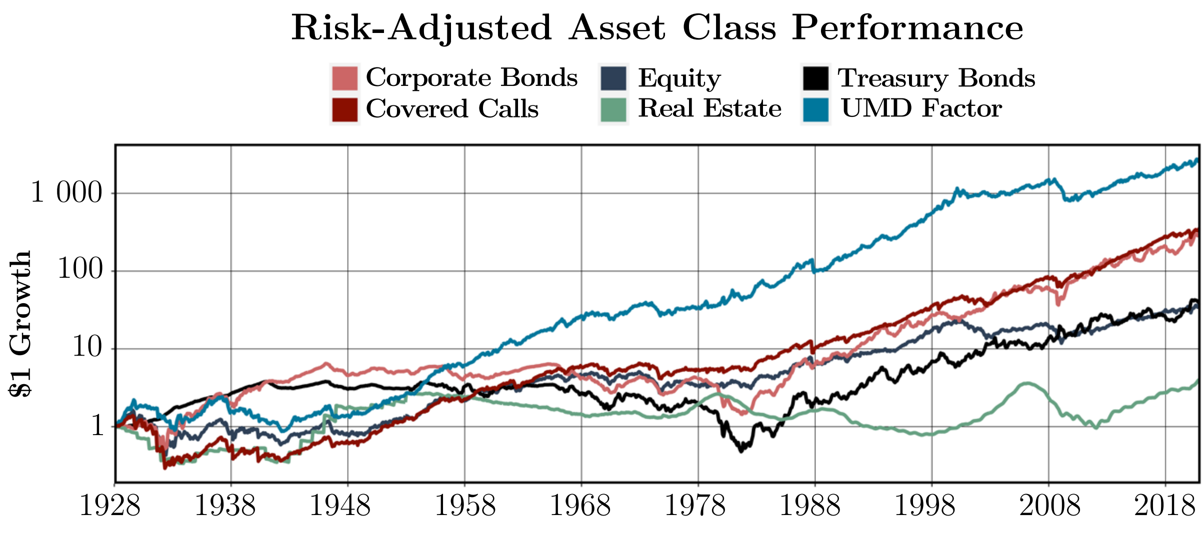

Figure 16 – Asset class performance, adjusted by inflation and to 15% annual volatility.

6.2.3 Active Management Performance

Another approach to evaluate our portfolio is to separate risk and return relatively to its benchmark. Because we want to emphasize the effect of including alternative asset classes, rather than the effect of excluding them, we will treat the 60/40 stock and bond portfolio as the benchmark in this section only[24]. Originally would the tangent portfolio form the market of our twelve-asset universe[25], but this perspective would focus on the 60/40 portfolio's disadvantages, not the tangent portfolio's strengths. Active management measures such as abnormal returns are hence a measure relatively to our benchmark portfolio, and not the true overall market. Table 14 contains evaluation measures of active management, and it reveals a complex and two-sided answer on its value contribution.

The tangent portfolio achieves a negative active return of 0.18% annually, which is derived from the pure difference in arithmetic returns. The active return standard deviation of 10.17%, denoted as tracking error, hence forms a negative information ratio (IR). While this is not preferable for an asset manager which aims to beat the market, it is not that representative if the key objective is risk management. IR is thus not considering the overall risk exposure, and other measures should be considered before we conclude on active management's value contribution.

When evaluating the portfolio's active return-to-risk contribution, we discover strong evidence of active management quality. An annual alpha of 3.40% implies returns which cannot be explained by its benchmark. Abnormal returns arrive at the expense of idiosyncratic risk, which can be avoided by holding the overall market (Falkenstein, 2009). Idiosyncratic risk of 4.42% annually leads to an appraisal ratio (AR) of 0.77. This is a solid ratio, especially compared to the benchmark Sharpe ratio of 0.397. It implies that the tangent portfolio produces 0.77 units of active return per unit of risk. Hence, active management improves the risk-adjusted return massively, even though it comes at the

[24] We are originally assuming an investment universe of twelve asset classes in this thesis, meaning the tangent portfolio is the market in a CAPM world. In section 6.2.3, however, we use a different angle by assuming the 60/40 portfolio to be the market, only to evaluate an asset manager's contribution when changing investment policy from 60/40 to our tangent portfolio. Alternatively, we could illustrate the same point by measuring the negative active management contribution of a traditional 60/40 portfolio using the exact same calculation methods, but we find this angle more appropriate.

[25] Capital Asset Pricing Model (CAPM), by Treynor (1961, 1962), Sharpe (1964), Lintner (1965) and Mossin (1966).

expense of absolute return levels. This is also captured by M2, which measures the excess return of the tangent portfolio adjusted to the same level of risk as its benchmark. Derived from their Sharpe ratios, we discover an additional 5.72% risk-adjusted excess return, which is in line with our existing findings on risk-adjustments and leveraging.

Active Management Measures

	Benchmark: 60/40 Portfolio	Tangent Portfolio
Excess return	5.09%	4.91%
Standard deviation	12.81%	5.82%
Beta	1	0.296
Active return		-0.18%
Alpha		3.40%
Tracking error		10.17%
Residual risk		4.42%
M2		5.72%
Appraisal ratio (AR)		0.77
Information ratio (IR)		-0.02

Table 13 – Active management measures[26] from 1928 – 2020. We are using the 60/40 portfolio as a benchmark, and all performance measures is relative to benchmark performance. An alpha of 3.40% is hence not representative in other contexts where the market portfolio differs from this benchmark.

Table 14 suggests an asset manager will improve the risk-adjusted return, although the arithmetic active return is slightly negative. It might be a small cost relative to the massive improvement of risk, especially by considering a geometric active return of positive 0.47%. An annual alpha of 3.40% must be recognized as high and should be tested for statistical significance before we conclude on whether active management provides value. A simple t-test can assess the reliability in a sufficient way, where the appraisal ratio is multiplied by the square root of years of observations to find the t-value. This provides definite information on reliability, based on the number of observations and its level of outperformance,

[26] Formulas are attached in Appendix I.

$$t = \frac{\alpha_P}{\sigma(e_P) / \sqrt{n}} = AR\sqrt{n} = 0.77 * \sqrt{93} = 7.43. \tag{5}$$

A t-table with infinite degrees of freedom will then be employed to find our p-value, which tells us the probability of getting an appraisal ratio of 0.77 if the true value is not positive

$$1 - N(7.43) = 5.43e\text{-}14 \tag{6}$$

In other words, there is only a 0.0000000000000543% chance of getting this result after 93 years if the true value of alpha is not positive. Hence, there is an extremely low probability of a type 1 error, and the true value of alpha is probably positive.

6.2.4 Performance in Different Time Periods

To bring our analysis to completion we will investigate portfolio performance in different time periods. As introduced in chapter 2, will macroeconomic factors and externalities influence value creation for all asset classes, and unanticipated crises could result in value destruction and difficult times for investors. It is close to impossible to predict the future state of the economy with a high degree of certainty, which is why an optimal risky portfolio should perform in all stages of the business cycle. The 93-year time frame substantiates this and allows us to make unbiased assumptions and compute a portfolio which performs well in every period.

Portfolio performance from January 1928 to December 2020 can be analyzed by separating the time series into five parts based on underlying macroeconomic factors. We can then compare value creation through time by treating each period separately as an index. Figure 17 and 18 illustrate how a traditional 60/40 stock and bond portfolio and our optimal portfolio perform without controlling for risk. Each period starts with $1, meaning each figure consist of 5 indices starting in 1928, 1946, 1965, 1982 and 2008. These index values are also presented in actual terms for both portfolios and asset classes in table 15.

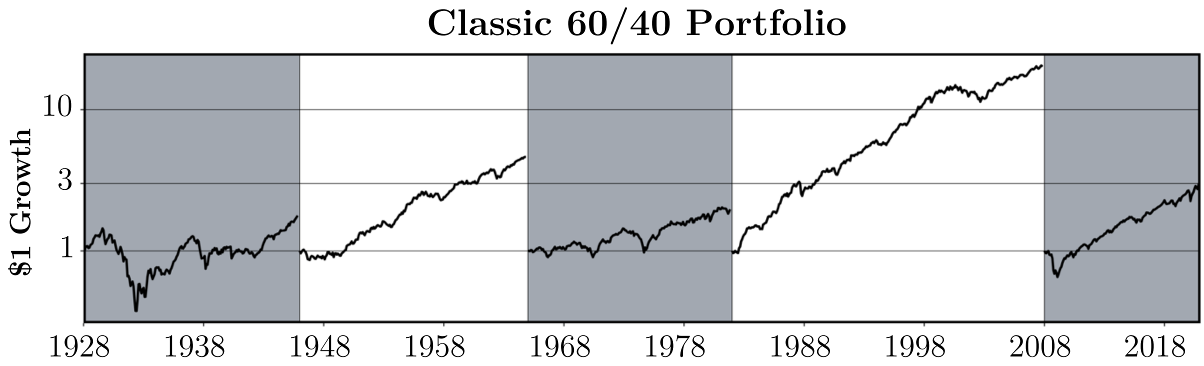

Figure 17 – Index of a classic 60/40 stock and bond portfolio, starting at $1 in each marked period.

Our two charts are remarkably similar, even though the tangent portfolio outperforms if we look more closely. This is not very surprising, given the almost identical arithmetic returns. We can also discover larger fluctuations in the stock and bond portfolio, which is expected based on the uncovering regarding risk exposure. Further, it is worth noting how both portfolios performed extremely well from the early 1980s to the financial crisis in 2008, mainly due to their large allocation towards equity linked asset classes.

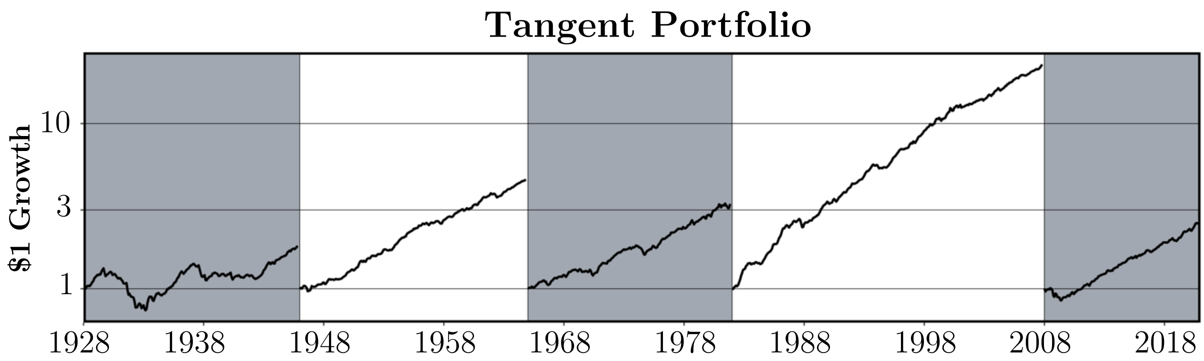

Figure 18 – Index of the tangent portfolio, starting at $1 in each marked period.

The fourth period from 1982 to 2007 is characterized by several factors, particularly constantly decreasing interest rate levels, GDP growth and the entrance of the baby boomer generation in the financial markets. While an increasing amount of funds in the stock markets provided large returns for shareholders, would the concept of duration make it favorable for bond holders as well. This implies that we should not expect the same value creation today in the short-term, because the current interest rate level is close to zero. Thus, figure 19 displays the unparallel value creation in corporate bonds in the fourth period.

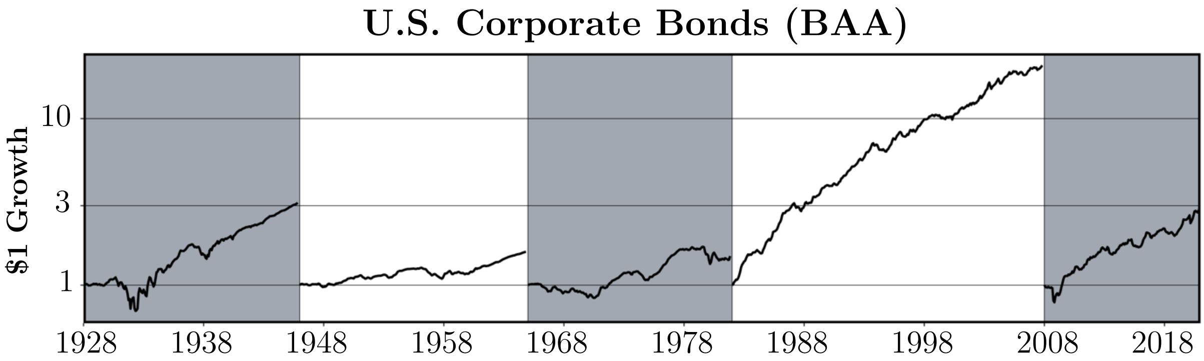

Figure 19 – Index of U.S. corporate bonds, starting at $1 in each marked period.

A summary of value creation for all relevant asset classes in each of the five periods are displayed in table 15 underneath. Each period starts with $1 and evolves to its displayed end value after adjusting for inflation.

Sample of Index Values, 5-Period Chart

	Equity	Government Bonds	Corporate Bonds	Real Estate	UMD Factor	Covered Call	Classic 60/40	Tangent Portfolio
01/01/1928	1	1	1	1	1	1	1	1
01/12/1945	0,94	1,8	3,16	1,18	1,67	0,83	1,84	1,87
01/01/1946	1	1	1	1	1	1	1	1
01/12/1964	9,01	1,51	1,59	1,72	38,11	9,68	4,66	4,65
01/01/1965	1	1	1	1	1	1	1	1
01/12/1981	2,25	1,76	1,5	3,02	10,45	3,42	2	3,31
01/01/1982	1	1	1	1	1	1	1	1
01/12/2007	17,45	10,34	10,32	3,53	162,35	27,39	20,01	22,84
01/01/2008	1	1	1	1	1	1	1	1
01/12/2020	2,97	1,69	2,87	1,35	2,45	4,74	3,02	2,89

Table 14 – Sample of index values, 5-period chart. Shows the same value creation as in figure 17, 18 and 19. All values in U.S. dollars.

Table 16 summarizes and presents asset class and portfolio performance for each period from 1928 to 2020. It is of interest to notice the remarkable outperformance of the tangent portfolio in terms of alpha and Sharpe ratio in every period, implying that our portfolio handles all investment environments in a convincing way. We can also discover that the main source of alpha comes from UMD factor exposure, which has strong performance in all periods.

Performance in Different Time Periods

	Equity	Government Bonds	Corporate Bonds	Real Estate	UMD Factor	Covered Call	Classic 60/40	Tangent Portfolio
1928 – 2020:								
Total Return	9.35%	4.90%	6.89%	3.95%	15.56%	9.84%	8.36%	8.19%
Inflation adj. TR	6.43%	1.99%	3.97%	1.03%	12.65%	6.92%	5.45%	5.27%
Excess Return	6.07%	1.63%	3.61%	0.67%	12.29%	6.56%	5.09%	4.91%
Standard deviation	19.26%	5.13%	7.77%	4.86%	19.93%	13.60%	12.81%	5.82%
Sharpe Ratio	0.315	0.317	0.465	0.138	0.617	0.482	0.397	0.835
TR at 15% vol.	5.09%	5.11%	7.33%	2.43%	9.61%	7.60%	6.32%	13.01%
Max Drawdown	88.05%	14.63%	37.17%	27.62%	81.94%	79.68%	73.96%	44.33%
Value at Risk	-22.22%	-4.01%	-12.76%	-4.52%	-19.93%	-21.93%	-16.70%	-5.48%
Alpha	-1.33%	1.14%	1.99%	0.55%	11.37%	2.08%	0.00%	3.40%
1928 – 1945:								
Total Return	4.55%	3.27%	7.04%	1.16%	6.52%	1.91%	5.55%	3.80%
Inflation adj. TR	3.95%	2.67%	6.44%	0.56%	5.93%	1.31%	4.95%	3.20%
Excess Return	3.59%	2.31%	6.08%	0.20%	5.56%	0.95%	4.59%	2.84%
Standard deviation	30.81%	0.71%	11.55%	7.23%	26.69%	22.72%	20.71%	8.10%
Sharpe Ratio	0.116	3.259	0.527	0.027	0.208	0.042	0.221	0.350
TR at 15% vol.	2.11%	49.24%	8.26%	0.77%	3.49%	0.99%	3.68%	5.61%
Max Drawdown	88.05%	2.30%	37.17%	26.77%	81.94%	79.68%	73.96%	44.33%
Value at Risk	-28.91%	-0.23%	-12.34%	-10.69%	-24.28%	-25.66%	-20.75%	-7.56%
Alpha	-3.09%	2.30%	4.64%	0.21%	4.81%	-3.20%	0.00%	1.62%
1946 – 1964:								
Total Return	12.99%	2.21%	2.59%	4.12%	20.42%	12.58%	8.83%	8.37%
Inflation adj. TR	11.39%	0.61%	0.99%	2.53%	18.82%	10.99%	7.23%	6.78%
Excess Return	11.03%	0.25%	0.63%	2.17%	18.46%	10.62%	6.87%	6.42%
Standard deviation	14.33%	2.46%	2.80%	7.31%	14.93%	9.58%	8.84%	4.14%
Sharpe Ratio	0.770	0.102	0.225	0.297	1.236	1.109	0.777	1.550
TR at 15% vol.	11.91%	1.90%	3.73%	4.81%	18.91%	16.99%	12.02%	23.62%
Max Drawdown	24.33%	8.95%	14.08%	1.93%	22.62%	16.49%	15.50%	7.28%
Value at Risk	-9.47%	-1.99%	-1.96%	-1.66%	-11.51%	-7.92%	-5.77%	-3.29%
Alpha	0.30%	-0.29%	-0.45%	1.18%	15.57%	4.35%	0.00%	4.06%
1965 – 1981:								
Total Return	6.14%	3.56%	2.59%	6.56%	15.74%	7.87%	4.72%	7.23%
Inflation adj. TR	0.09%	-2.49%	-3.46%	0.51%	9.68%	1.82%	-1.33%	1.17%
Excess Return	-0.27%	-2.85%	-3.83%	0.15%	9.32%	1.45%	-1.69%	0.81%
Standard deviation	15.46%	6.60%	6.01%	1.58%	19.55%	10.27%	10.14%	5.60%
Sharpe Ratio	-0.018	-0.432	-0.636	0.093	0.477	0.142	-0.167	0.145
TR at 15% vol.	0.10%	-6.12%	-9.18%	1.76%	7.51%	2.49%	-2.14%	2.54%
Max Drawdown	45.96%	14.63%	21.20%	0.71%	33.35%	23.59%	32.33%	11.99%
Value at Risk	-10.80%	-5.37%	-5.31%	-0.39%	-15.33%	-8.97%	-7.27%	-3.49%
Alpha	2.21%	-2.33%	-3.32%	0.19%	9.71%	2.91%	0.00%	1.44%

Performance in Different Time Periods								
	Equity	Government Bonds	Corporate Bonds	Real Estate	UMD Factor	Covered Call	Classic 60/40	Tangent Portfolio
1928 – 2007:								
Total Return	12.15%	9.10%	11.79%	4.88%	21.38%	13.36%	12.01%	12.17%
Inflation adj. TR	7.38%	4.33%	7.02%	0.10%	16.60%	8.58%	7.24%	7.40%
Excess Return	7.02%	3.97%	6.66%	-0.26%	16.24%	8.22%	6.87%	7.04%
Standard deviation	15.17%	6.39%	6.56%	1.83%	18.26%	10.20%	9.92%	5.28%
Sharpe Ratio	0.463	0.622	1.015	-0.141	0.889	0.806	0.893	1.331
Trat 15% vol.	7.30%	9.68%	15.59%	-1.75%	13.70%	12.45%	10.74%	20.33%
Max Drawdown	47.16%	10.79%	10.96%	6.13%	41.20%	25.78%	23.73%	10.94%
Value at Risk	-14.66%	-3.89%	-3.84%	-0.75%	-22.55%	-16.54%	-8.77%	-3.42%
Alpha	-3.01%	2.17%	4.51%	-0.34%	14.95%	2.51%	0.00%	4.72%
2008 – 2020:								
Total Return	9.25%	4.46%	8.76%	2.25%	9.14%	12.35%	9.05%	7.28%
Inflation adj. TR	9.02%	4.22%	8.53%	2.02%	8.90%	12.12%	8.82%	7.05%
Excess Return	8.65%	3.86%	8.16%	1.66%	8.54%	11.75%	8.46%	6.69%
Standard deviation	16.31%	6.12%	9.65%	2.92%	18.93%	11.07%	11.48%	5.20%
Sharpe Ratio	0.531	0.631	0.846	0.586	0.451	1.062	0.737	1.285
TR at 15% vol.	8.32%	9.83%	13.06%	8.89%	7.13%	16.29%	11.42%	19.64%
Max Drawdown	50.80%	9.29%	21.15%	22.90%	58.94%	24.85%	36.58%	15.78%
Value at Risk	-13.63%	-3.43%	-11.82%	-1.98%	-15.75%	-10.88%	-12.29%	-5.11%
Alpha	-2.78%	3.99%	4.17%	1.40%	9.22%	4.55%	0.00%	4.79%

Table 15 – Summary Statistics in different Time Periods. Value at risk at a 99% confidence level. TR at 15% vol. is inflation adjusted. Alpha is relative to the 60/40 portfolio.

7. Discussion

A thorough analysis revealed remarkable strengths of including alternative asset classes in a risky portfolio, and we should further discuss its relevance for investors and asset managers. We will assess its feasibility and the interaction between past performance and future expectancy, and then compare our findings to previous research. At last, we will discuss potential limitations of our investment strategy and implementation for market participants.

7.1 Future Expectancy

The inevitable question for investors and asset managers is whether our discoveries are repeatable or not. Outperformance in the past will never be a guarantee of outperformance in the future, but a few key elements could indicate its probability. Most of all, we have constructed a portfolio which performs in all stages of the business cycle, no matter current monetary policy or activity level. Over one thousand months of observations reveal uncorrelated assets, and a few assets will most probably always generate returns no matter the state of the economy. This is the immense strength of collecting 93 years of asset class data, because their return is dependent on recurring macroeconomic factors and not unpredictable company-specific events.

To visualize the effect of a balanced and well-prepared portfolio, are comparable portfolios with different optimization periods displayed in figure 20. A 10-year optimization portfolio is based on the most optimal portfolio over the past 10 years and will be held for the next 10 years. Hence, the most optimal combination from 1930 – 1940 will be held from 1940 – 1950 and so on. The same methodology is also computed with 5-year intervals, where we could expect a decent performance because it exploits the most attractive asset classes in each period. However, shifting business cycles and changing investment environments make these portfolios underperform against the 93-year mean-variance optimized portfolio. Timing the market is close to impossible, and it seems like a portfolio prepared for all economic scenarios provides a much higher reward than trying to exploit existing asset class trends. These findings are very much in line with the research and investigation in Ibbotson and Kaplan (2000), which was summarized in chapter 3, Literature Review.

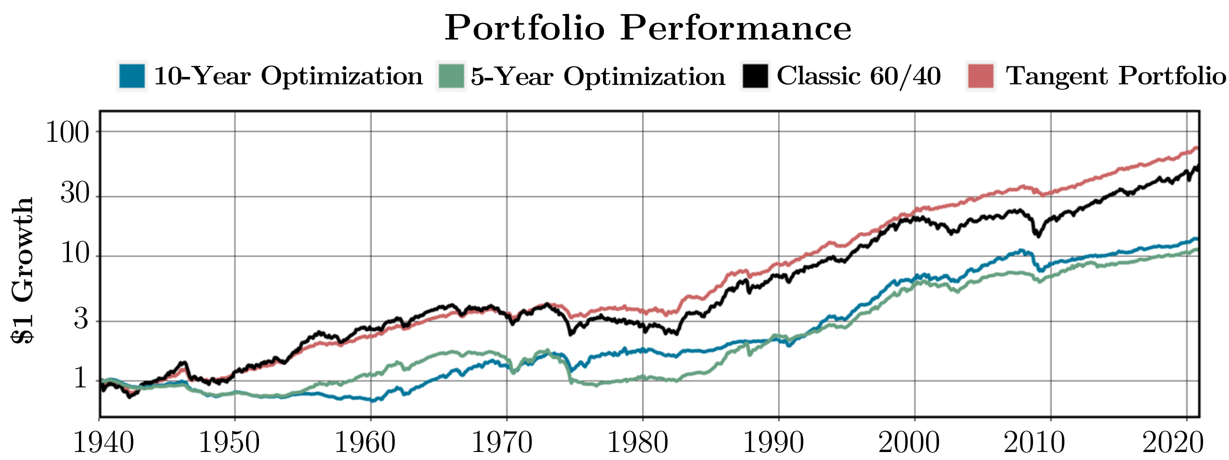

Figure 20 – Portfolio performance, different optimization periods.

It is also worth questioning the likelihood of repetition in asset class performance. The 25-year bull market displayed in figure 17, 18 and 19, from 1982 to 2007 must be considered as atypical compared to the other periods. This was a period where all equity linked assets performed extremely well due to favorable conditions caused in the previous period. This might be something that investors should not expect to occur again, at least not in the short term. The collected data also reveals an exceptional high Sharpe ratio of U.S. Treasury bonds in the first period of our data set, with almost only positive returns and gradually devaluating interest rates from 1928 to 1952. This affects the overall risk interpretation of Treasury bonds, where 24 years of our dataset is assumed to bear no risk in terms of an annual standard deviation of 0.006. Expecting such results in the future would be very unlikely, and we could end up with a fairer result by adjusting the overall Sharpe ratio. By replacing the average performance from 1928 to 1952 by the weighted mean from the initial 1928 to 2020 period, our estimates could be more representative. This would affect Sharpe ratio of Treasury bonds to descend from 0.317 to 0.280, a 11.5% decrease.

Strong portfolio returns and a massive improvement in Sharpe ratio in the tangent portfolio is partly due to the extreme performance of UMD portfolios over the past century. A return-to-risk ratio of 0.62 is striking, and by excluding this investment opportunity we reduce the portfolio´s Sharpe ratio from 0.835 to 0.690. Our original portfolio strategy assumes UMD exposure to be just as profitable in the future, but there is evidence which suggests abnormal returns to fade when research and monitoring increase (The Economist Group Limited, 2016). Accordingly, asset factor exposures are also affected by market trends. We cannot explicitly highlight UMD exposure as the ultimate factor, but rather insinuate that it has performed well in the past, and that it would be a profitable investment even if abnormal returns are reduced.

The expectancy of future benefits is also dependent on the investment profile. An investor will for example possess a higher risk of not meeting unexpected liabilities by investing in a traditional stock and bond portfolio. Large drawdowns and recovery periods increase the probability of liquidity problems and insolvency, which is a difficult effect to measure in a general term. The investment horizon should also be considered, where a disciplined investor without liabilities and an infinite investment horizon could accept larger drawdowns and variance without much concern. His effect of adding alternative asset classes in a portfolio would be minor, even though our calculations suggest a marginal return increasement due to the avoidance of huge losses. The effect would probably be most noticeable when leveraging the tangent portfolio to match the risk of a traditional stock and bond portfolio. By assuming an investor could borrow at the risk-free rate, would he in retrospect gain an average of 11.05% per year, compared to a traditional stock and bond portfolio of 5.27%. However, a gap of almost 6 percent annually would be reduced in practice due to credit premiums when borrowing funds.

We are using scenarios when measuring risks, e.g., max drawdown and value at risk. Since risk is the tradeoff for expected return, are we not considering scenarios and probability distributions for predictions of future return expectations. We are only considering historical return levels on different asset classes and analyzing what $1 in 1928 would accumulate to in different portfolios. We are thus not trying to predict the future, but rather use econometrics to point out that the tangent portfolio constructed with alternative asset classes in this paper significantly outperforms the traditional 60/40 stock and bond portfolio. Due to the huge improvement in reward-to-risk ratio, its diversification effect and the size of the dataset covering multiple business cycles, it indicates further outperformance as well.

7.2 Comparison of Discoveries

In this section, we will compare our discoveries to existing literature to assess our contribution to this field of research. Alternative asset classes as a pure investment alternative are a relatively unexplored area, and it is both interesting and valuable to put our results into context of other papers.

Our literature review summarized relevant studies and revealed that various economists suggest that a risky portfolio should contain alternative asset classes. These papers were mostly conducted in a singular form, where only one alternative asset class was included to a traditional stock and bond portfolio to e.g., measure its effect of being an inflationary hedge, or to increase performance in terms of Sharpe ratio. As the results point out in this paper, only measuring Sharpe ratio does not provide enough evidence and information to recognize if one portfolio or combination of assets is better than another. It has also given evidence towards questioning the extent use of commodities and gold, as the first is an exceptional hedge against inflation, although it yields negative excess returns[27]. The latter provides a relatively high trade-off between risk and return, which makes it less favourable to hold in our portfolio optimization. The findings of Small et al. (2012) on diamonds, and Chudy and Cubbage (2020) on forest investments, indicates that singular asset classes within the commodity domain could improve portfolio quality even though the asset class as a whole yield a low return output.

While the studies above investigate the impact of only one alternative asset class, are Bekkers et al. (2009) investigating multiple alternative asset classes in the same portfolio. Although they utilize a different timeframe and have other proxies for the same asset classes, will they still be considered the most equally comparable research paper. When considering allocation and performance, we discover the sensitivity of a mean-variance optimization to its input variables. Our work differentiates in terms of asset classes and length of data series, and the results are thus quite different. The optimal portfolio of Bekkers et al. (2009) contains U.S. high yield, real estate, and commodities, together with common stocks and bonds. A visualization of portfolio allocations is presented underneath:

[27] See table 1 in section 4.1, also in line with the findings in Erb and Harvey (2006).

Our Mean-Variance Optimization:	Bekkers et al. (2009) Optimization:

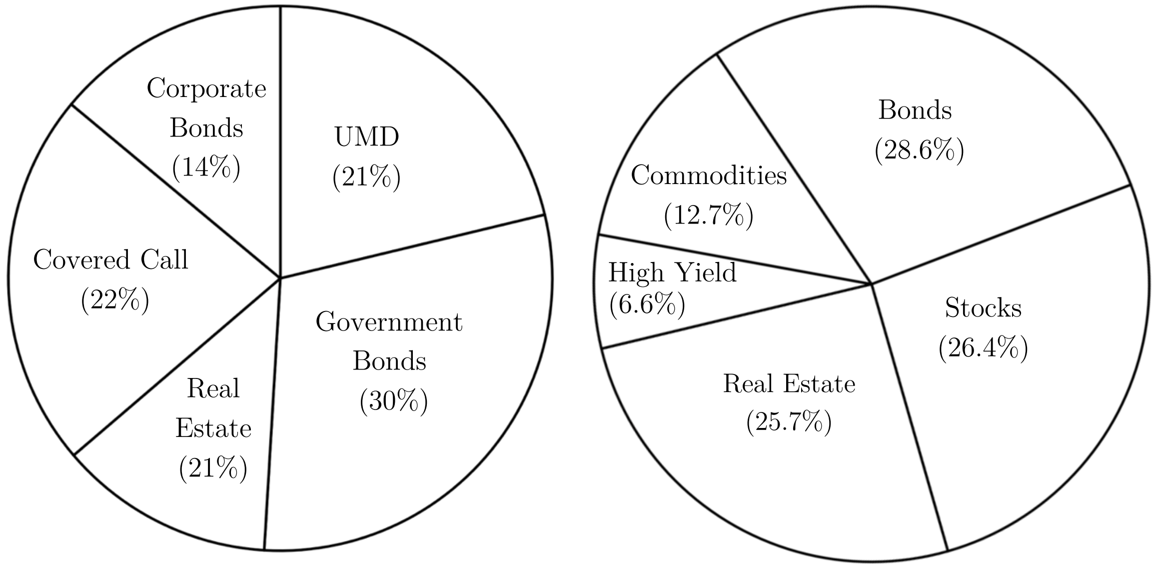

Figure 21 – Portfolio allocations.

The inclusion of alternative asset classes in Bekkers et al. (2009) increased Sharpe ratio from 0.346 to 0.396, compared to our improvement from 0.397 to 0.835. Their poor improvement can partly be explained by a 12.7% position in commodities, which provides diversification at the expense of zero excess returns. Our preferred alternative asset classes provide better diversification yielding a better return-to-risk-ratio due to higher excess returns. Bekkers et al. (2009) did not include specific options strategies either, but rather the performance of HRFI Fund of Funds Composite Index as a representation of hedge funds. This investment alternative yielded an arithmetic Sharpe ratio of only 0.16 compared to covered calls of 0.482. There is also a large difference in equity, where they invest in equity with similar performance to our S&P 500 estimate, whereas we prefer the outperforming strategy of UMD factor exposure.

The likelihood of improving a portfolio when including alternative asset classes seems apparent, although the impact is dependent on available investment alternatives. Bekkers et al. (2009) proved that even underperforming assets could enhance portfolio quality, while we demonstrated the immense gain when exploiting the diversification effect more efficiently. The methodology and intuition in both papers are very much the same, although the impacts are remarkably different. The largest difference in methodology and execution might be our dedication towards business cycles and length of the evaluated time series. Our objective has been to construct a portfolio which performs in every stage of the cycle, and we are hence consistent by using the 93-year time frame for all asset data. Bekkers et al. (2009) on the other hand, assembled risk premiums which they found appropriate and made their own return and risk estimates based on assessments of others. This led to inconsistency within data, where e.g., bond performance is extracted from 1900 – 2008, and hedge fund from 1990 – 2008. Hence, their portfolio does not reflect or capture performance in corresponding periods, whereas our allocation strategy suits the investment environment for each business cycle that has occurred after the beginning of collecting financial data.

7.3 Limitations

We will now advance by assessing limitations in our investment strategy to determine its feasibility and potential weaknesses. An unequal investment universe will for example make our allocation impossible or non-optimal for some investors, while others should be capable of replicating our portfolio and make a profit. Investors must be able to expose themselves to government bonds, corporate bonds, real estate, covered calls and the UMD factor if our investment strategy should be feasible. There is no doubt about the strong accessibility and liquidity of government and corporate bonds given its track record in asset allocation, while the latter three are perhaps more questionable.

The most intuitive method to replicate real estate would be to buy physical properties. Many have created wealth in the real estate market, but it requires capital, credit score, time and organizing. Large transaction costs and illiquid market could also make it unprofitable in a small portfolio. However, we could follow the value creation in real estate while avoiding the disadvantages of physical properties by investing in an ETF. There are tradable ETFs on the market which aim to track broad real estate indices such as the Dow Jones Global Select Real Estate Securities Index or the GRP Global 100 Index (Morningstar, 2021) (Nordnet, 2021). Hence, real estate is a feasible investment opportunity for investors, no matter wealth, knowledge, or experience.

Investing in covered calls could be challenging for an investor with limited knowledge on financial instruments, because of the process of writing call options. However, it should not cause major problems for experienced investors and asset managers. For private households it should be equally applicable, while they might pursue market professionals for exposure to this options strategy. The portfolio replication will be finalized by exposure to the momentum factor. Kenneth French' original method of frequently going long and short in stocks could also be challenging and too complex for an average investor, and thus can ETFs with a momentum strategy exposure be a relevant alternative. There are several tradable products which follow the value creation of momentum indices, for example the well-recognized MSCI USA Momentum Index (iShares, 2021). Professional asset managers can create their own UMD strategy, although the arrival of ETFs makes this unnecessary.

Our suggested portfolio allocation turns out to be very much feasible in practice, even though private households might favor a professional asset manager to achieve covered call exposure. This leads us to the next topic of this section because asset managers must follow their mandate when investing in financial instruments. Most mutual and pension funds are restricted to only hold stocks and bonds, which means asset managers are not allowed to invest in covered calls for their clients.

Hence, we are dependent on a specialized investment mandate to make our portfolio strategy feasible. The asset manager needs permission to invest in financial derivatives, and we are hence interested in a typical hedge fund structure which provides freedom to invest in all possible financial instruments. Hedge funds are usually associated with billionaire investors and complex products, and they are also heavily regulated in the United States by the SEC (CFA Institute, 2021). Restrictions are fortunately softer in certain countries, where several Norwegian hedge fund managers have registered their fund in Luxembourg to avoid strict requirements for their investors (Pareto Asset Management, 2021) (Sissener AS, 2021). Hence, the optimal risky portfolio should be accessible for private households, and it should thus be feasible to invest in our suggested portfolio.

However, there might also be limitations regarding methodology and procedure of constructing our optimal risky portfolio. We assume an investment universe of twelve asset classes, whereas this field could be extended with new asset classes or exposure towards other markets around the world. U.S. data were the only option which fulfilled our requirements of reliability and accessibility back to the 1920s, whereas the MSCI World Index for example was constructed in 1969. Our discoveries could also be impacted by the entrance of other asset classes such as U.S. high yield, private equity, or cryptocurrency, even though our twelve preferred asset classes should be adequate to emphasize the diversification effect of alternative asset classes.

The mean-variance model used to compute portfolio results is as earlier described a quadratic optimization model. Its objective is to maximize Sharpe ratio, while having non-acceptance constrains towards short sale and pre-set maximum and minimum weights, where the maximum weight is not applicable. Among researchers and mathematicians within finance, there has been developed multiple other models to optimize portfolio performance over the last two decades. These models are to a higher extent trying to utilize multi-objective models to optimize multiple objective functions at once. Kalayci, Ertenlice, & Akbay (2019) review a comprehensive collection of 175 research articles on deterministic models and applications for mean-variance optimization models. They find that a multi-objective model is better suited to comprehend real-world cases. Whereas solving single optimization problems is set to find a universal optimal solution, multi-objective models postulate several optimal solutions, alternatively known as Pareto optimal solutions. Related to our research, we are not including frictions, e.g., transaction costs or taxes. In other words, we are avoiding the complexity of including several conflicting objectives such as minimum lots of investments, ensuring that the amount invested in an asset is multiples of the minimum transaction size. We are thus neither implementing sector capitalization or turnover constrains. Based on these assumptions we are avoiding dominance-based approaches in our optimization model. This could be alternated for instance by including rebalancing costs to reveal a more real-world approach. On the contrary would this be an ambiguous number since there is no reliable information on historical transaction costs within different asset classes and sectors, and the effect of large transactions opposed to smaller. This traditional view on portfolio optimization makes it easier comparable to the work of others. This is partly the reason for also excluding the possibility of short positions, and transaction cost even though it is applied in the option strategies. The paper could also consider other models such as the framework of risk parity, but this would rather be to construct a portfolio which would be less exposed to the maximum drawdown risk. It would thus not be able to compute a portfolio that would yield an equally high Sharpe ratio as the tangent portfolio.

8. Conclusion

The research and discoveries in this paper clearly demonstrate that alternative asset classes would improve portfolio quality over the past 93 years. The magnitude of improvements can be explained by lower and better distributed correlation coefficients among asset classes through our time series. Improved risk management exploits the diversification effect more efficiently, which implies reduced variance and a less vulnerable portfolio.

The quadratic optimization problem suggests an optimal risky portfolio of government bonds, corporate bonds, real estate, covered calls, and UMD factor exposure. This portfolio experiences approximately half the variance and drawdown of a traditional 60/40 stock and bond portfolio, as well as a slightly better geometric return. Risk-adjusted return measurements illustrates the difference even more clearly, where the tangent portfolio outperforms its benchmark in terms of Sharpe ratio, Sortino ratio, value at risk, and appraisal ratio for every period from January 1928 to December 2020. An annual alpha of 3.40% against the classic 60/40 portfolio will also reveal a massive outperformance in terms of abnormal returns over 1116 months. Hence, our research suggests that including alternative asset classes will improve portfolio quality in terms of risk-adjusted return measurements. The exact impact on the portfolio, however, is dependent on investors' ability to exploit the improvement in diversification.

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Appendices

Appendix I: Statistics

Following in this Appendix are some explanations and formulas for the assessments and methods which is used in this paper. The correlation matrix in Appendix Table 1 is based on all 12 assets classes with 1116 observations from January 1928 to December 2020 for each asset class:

Correlation Matrix

1928 - 2020	Equity	Gov. bonds	Corporate Bonds	Gold	Real Estate	Comm.	SMB Factor	HML Factor	UMD Factor	Married Put	Covered Call	Long Straddle
Equity	1.00	0.07	0.29	0.03	0.04	0.09	0.10	0.12	0.08	0.84	0.85	0.79
Gov. Bonds	0.07	1.00	0.49	0.08	-0.02	-0.16	-0.11	-0.08	-0.01	0.07	0.05	0.07
Corp. Bonds	0.29	0.49	1.00	0.14	0.00	0.09	0.32	0.39	0.18	0.21	0.25	0.20
Gold	0.03	0.08	0.14	1.00	0.01	0.08	-0.01	-0.02	0.03	0.02	0.04	0.01
Real Estate	0.04	-0.02	0.00	0.01	1.00	0.13	0.09	0.09	0.05	0.05	0.02	0.07
Commodities	0.09	-0.16	0.09	0.08	0.13	1.00	0.23	0.25	0.16	0.05	0.10	0.05
SMB Factor	0.10	-0.11	0.32	-0.01	0.09	0.23	1.00	0.81	0.64	0.13	0.06	0.11
HML Factor	0.12	-0.08	0.39	-0.02	0.09	0.25	0.81	1.00	0.54	0.15	0.05	0.13
UMD Factor	0.08	-0.01	0.18	0.03	0.05	0.16	0.64	0.54	1.00	0.15	0.03	0.11
Married Put	0.84	0.07	0.21	0.02	0.05	0.05	0.13	0.15	0.15	1.00	0.48	0.80
Covered Call	0.85	0.05	0.25	0.04	0.02	0.10	0.06	0.05	0.03	0.48	1.00	0.48
Long Straddle	0.79	0.07	0.20	0.01	0.07	0.05	0.11	0.13	0.11	0.80	0.48	1.00

Appendix Table 1 – Correlation matrix of all asset classes, 1928 - 2020.

Appendix Table 1 makes the overall diversification effect between two asset classes easily interpretable. It reveals high correlation between equity and options strategies, and between different Fama-French equity factors. This is one of the main reasons for why well-performing asset classes such as equity, SMB or HML factor exposure is not preferred in our suggested portfolio.

Value at Risk has been computed for a 90%, 95% and 99% confidence interval, and is introduced for asset classes in section 4.1, and portfolios in section 6.2.1. VaR is computed with the PerformanceAnalytics package in R, which is based on the following,

$$\text{VaR} = \text{E(Portfolio Return) - (Z-score of the conf. interval * Portfolio Std. Dev.)} \quad (1)$$

Skewness was analyzed in chapter 6.2.1. as a measure of asymmetry in returns. The formula is presented underneath, which shows the average of the cubed deviation from its mean, over the cubed standard deviation of the sample.

$$\text{Skew} = \text{Average}\left[\frac{(R - \bar{R})^3}{\hat{\sigma}^3}\right] \quad (2)$$

The formula of kurtosis is quite similar, but we are now capturing the effect of the fourth moment of the distribution. As explained in chapter 6.2.1 will kurtosis measure the degree of fat tails, where a level of 3 imply normality. Some professors and researchers are hence including a subtraction of 3 in its formula, which means we should expect a zero value in a perfect distribution. Our R calculations do not include this subtraction term, and we will hence treat the level of 3 as a sign of normality.

$$\text{Kurtosis} = \text{Average}\left[\frac{(R - \bar{R})^4}{\hat{\sigma}^4}\right] \quad (3)$$

$$\text{Active Return} = R_A = R_P - R_B \quad (4)$$

$$M^2 = (SR_P - SR_B)\ \sigma_B \quad (5)$$

$$\text{Alpha} = \alpha_P = R_P - (R_F + \beta_P(R_B - R_F)) \quad (6)$$

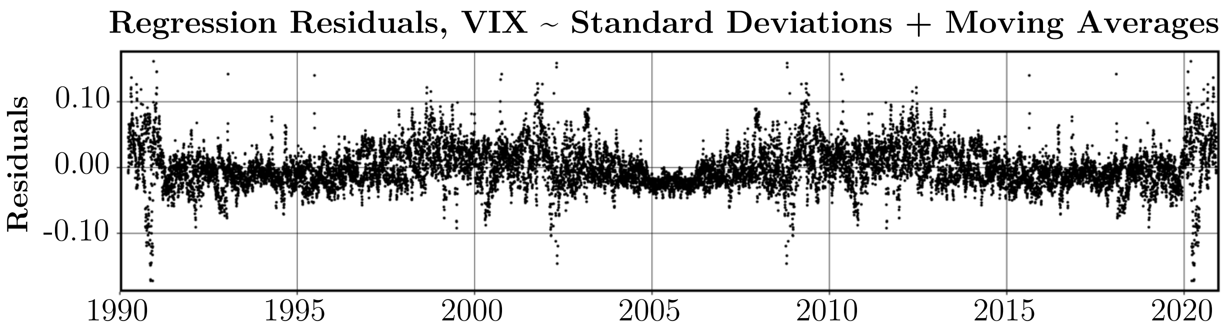
$$\text{Residual Risk} = \sigma(e_P) = \sqrt{\left(\sigma_P^2 - \beta_P^2\ \sigma_B^2\right)} \quad (7)$$

$$\text{Tracking Error} = TE_P = \sigma(R_P - R_B) = \sigma\ (\text{Active Return}) \quad (8)$$

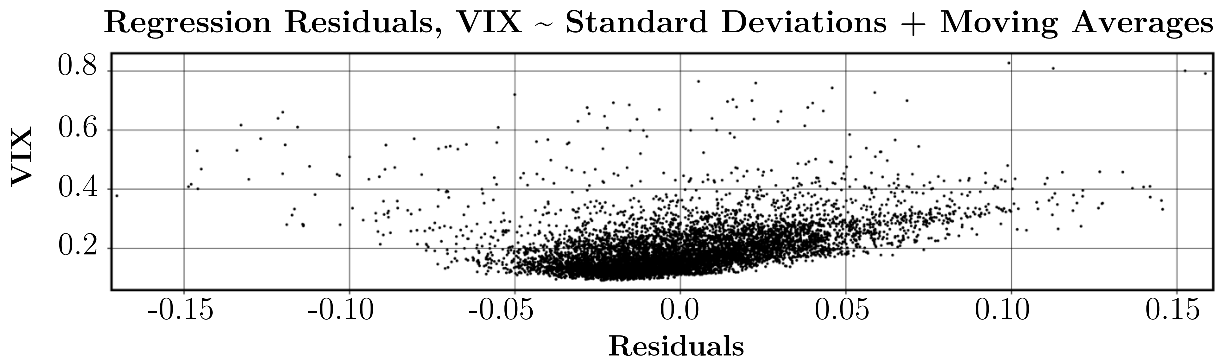
$$\text{Appraisal Ratio} = AR_P = \frac{\alpha_P}{\sigma(e_P)} \quad (9)$$

$$\text{Information Ratio} = IR_P = \frac{\alpha_P + (\beta_P - \beta_B)(R_B - R_F)}{\sqrt{\sigma(e_P)^2 + (\beta_P - \beta_B)^2\ \sigma_B^2}} = \frac{\text{Active Return}}{\text{TE}} \quad (10)$$

Appendix Figure 1 presents residuals from the regression model of VIX and 6 explanatory variables. Residuals are information not captured by the regression intercept or coefficients, and we want them to be as small and random as possible. We cannot interpret a clear pattern over the 30-year period, and a mean of -1.14e-15 implies that almost all the data is explained in our model. The residuals seem to behave random, and they are hence white noise.

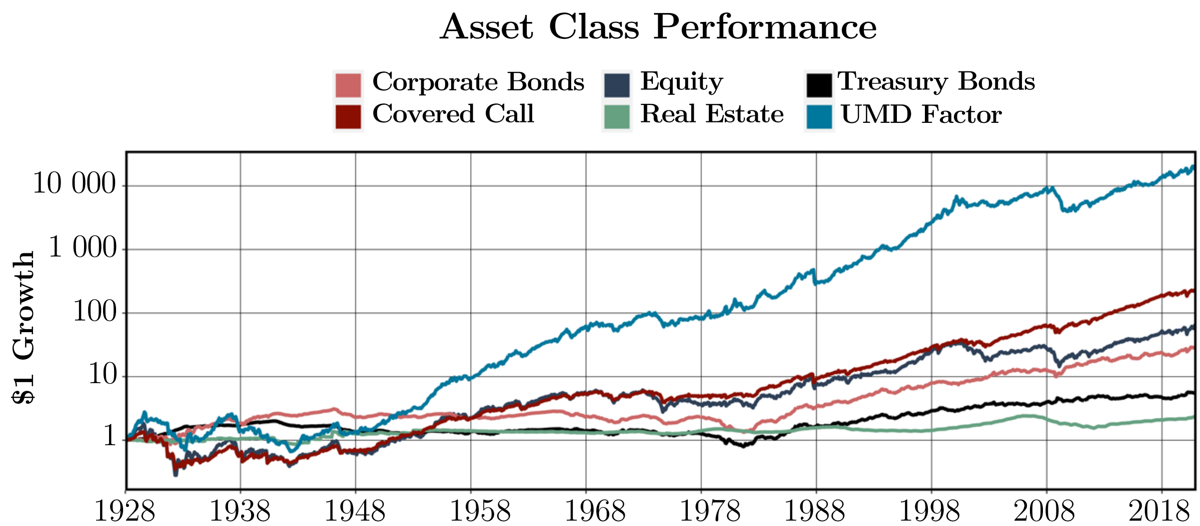


Appendix Figure 1 - Residual plot.



Appendix Figure 2 - Residual plot.

Appendix II: Performance



Appendix Figure 3 - Inflation-adjusted asset class performance. Not adjusted to risk.

The mean-variance optimization in Appendix Table 2 is a supplementary model to our existing optimizations in chapter 5. It displays the effect of no weight constrains, and the difference is marginal as expressed earlier. An allocation of 1.4% in gold and 0.8% to HML factor exposure make the portfolio yield 1 basis point better than our preferred model, although this improvement would diminish by transaction costs and rebalancing. The fractions are so small that it could also be interpreted as a contradiction to the investment mandate, whereas we are searching for long positions with stable and significant performance in different states of the business cycle. We are thus not taking this model into further consideration due to both inconveniences with management, and for the investors understanding of how we create value.

Mean-Variance Optimization 3

Optimal Weights:

Equity	Gov. Bonds	Corporate Bonds	Gold	Real Estate	Comm.	SMB Factor	HML Factor	UMD Factor	Long Straddle	Covered Call	Married Put
0.0%	30.4%	12.3%	1.4%	12.4 %	0.0%	0.0%	0.8%	20.6%	0.0%	22.1%	0.0%

StdDev (monthly): 1.74%	StdDev (annual): 6.04%
Mean (excess, monthly): 0.42%	Mean (excess, annual): 5.05%

Constraints: Long only, fully invested

Appendix Table 2 – Mean-variance optimization 3, no weight constraints.

Appendix Table 3 and 4 are sample indices of how much 1$ accumulate from January 1928 to December 2020 after adjusting for inflation. They represent the same value creation as Appendix Figure 3 and Figure 16, where a log-transformation and huge differences in value creation made it challenging to interpret specific values. Our tables reveal a massive outperformance from both UMD factor exposure and covered calls when not adjusting for volatility, while when accounting for risk, the tangent portfolio is clearly a better option.

Sample of Index Values, unadjusted to risk, adjusted for inflation								
	Equity	Government Bonds	Corporate Bonds	Real Estate	UMD Factor	Covered Call	Classic 60/40	Tangent Portfolio
1928-01-01	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---
2020-09-01	59	5.60	29	2.29	20 413	223	66	113
2020-10-01	57	5.54	28	2.32	20 264	223	65	113
2020-11-01	63	5.51	29	2.35	18 626	229	69	112
2020-12-01	67	5.48	30	2.35	18 556	239	72	113

Appendix Table 3 – Sample of index values on asset classes and portfolios, not adjusted to risk. All values in U.S. dollars.

Sample of Index Values, adjusted to 15% risk and inflation								
	Equity	Government Bonds	Corporate Bonds	Real Estate	UMD Factor	Covered Call	Classic 60/40	Tangent Portfolio
1928-01-01	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---
2020-09-01	35	42	291	3.70	2 749	333	109	59 207
2020-10-01	34	41	286	3.85	2 733	332	106	58 560
2020-11-01	37	40	300	3.98	2 567	343	115	57 471
2020-12-01	39	39	315	3.98	2 560	358	121	58 839

Appendix Table 4 – Sample of index values on asset classes and portfolios, adjusted to risk. All values in U.S. dollars.