CALLABLE RISKY PERPETUAL DEBT WITH PROTECTION PERIOD

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ABSTRACT. Issuances in the USD 260 Bn global market of *perpetual* risky debt are often motivated by capital requirements for financial institutions. We analyze callable risky perpetual debt emphasizing an initial protection ('grace') period before the debt may be called. The total market value of debt including the call option is expressed as a portfolio of perpetual debt and barrier options with a time dependent barrier. We also analyze how an issuer's optimal bankruptcy decision is affected by the existence of the call option by using closed-form approximations. The model quantifies the increased coupon and the decreased initial bankruptcy level caused by the embedded option. Examples indicate that our closed form model produces reasonably precise coupon rates compared to numerical solutions. The credit-spread produced by our model is in a realistic order of magnitude compared to market data.

1. Introduction

Perpetual debt securities seldom turn out to be particularly long-lived - in spite of their *ex ante* infinite horizon. This contractual horizon gives the securities a, using regulatory language, *permanence*, which is crucial when banks and other financial institutions are allowed to include them as regulatory required risk capital. However, the contracting parties, the issuing institution and the investors in the securities, typically appreciate financing flexibility and may thus prefer a more tractable finite horizon. These apparently conflicting objectives are resolved by embedding such perpetual securities, almost without exceptions, with an issuer's call-option, facilitating a finite realized horizon T.

In a companion paper, Chapter 3 of Mjøs (2007), we develop a closed form valuation model for perpetual debt securities including this option. This model allows both for calibration of coupon rate and calculation of optimal bankruptcy asset level. In the current paper we analyze how a finitely lived option embedded in the perpetual security impacts coupon rates and bankruptcy in the period before the option expires. Market practise indicates that issuers typically pay a fixed credit margin on top of a market reference interest rate, and are thus not directly exposed to the nominal interest rate levels. Although we use a constant interest rate model, our model produces a credit spread which reflects the risk of bankruptcy due to the volatility of the cashflow(EBIT)-process. This approach disregards potential correlation effects between a company's EBIT process and general interest rates dynamics.

Market	Outstanding amount (USD Bn)	Tier I (%)	Upper Tier II (%)	Non-financials' share(%)
U.S. dollar	82.8	75	17	8
Euro	105.6	43	16	41
Sterling	71.4	29	57	13
Total	259.8	50	28	23

TABLE 1. Global outstanding volumes of large perpetual debt capital securities end-2005. Amounts and relative shares are calculated from Lehman Brothers' index of debt- and debt-like risk capital securities. Percentages relate to share of each market. Source: Lehman Brothers.

Table 1 provides an overview of the estimated volume of outstanding large issues of perpetual debt securities as of end-2005 from Pomper and Varma (2005). This table shows that the total issuance is evenly spread across the main markets, but that there are large regional variations. The largest category, denoted 'Tier I', refers to securities that

qualify as the highest quality risk capital for issuing financial institutions[1]. 'Upper Tier II'-capital is also perpetual but less risky and within the subordinated debt-category for financial institutions. The latter dominates the GBP Sterling market whilst utilities and other non-financial issuers have a large share of the Euro-market. The globally accepted principles for capital adequacy and classification of risk capital were specified by Bank for International Settlements (BIS) in 1988[2]. National variations in regulations, tax and capital markets explain most of the differences.

Mapondera and Bossert (2005) include 50 large European banks in their research universe and show that amongst those, the volume of new perpetual securities equals 28% of the volume of newly issued senior market debt for the years 2000 - 2005. This category is split between Tier I capital, representing 20%-points and Upper Tier II capital covering the remaining 8%-points. The report also lists all individual new issues of new perpetual securities by these banks during 2004/2005 and all of them have a deferred issuer's call option, typically exercisable 10 years from date of issue. King (2007) studies empirically the implicit value of embedded options in various U.S. agency bonds, their levels and relationship to maturity and interest rates. He finds that embedded call options in bonds issued by Federal Home Loan Mortgage Corporation on average represents 3.9 % of the bond values in the call protection period of up to 10 years.

The embedded calls are contractually *American* in that they may be exercised at any coupon-paying dates after time T, typically year 10. The coupon rate is also typically stepped-up by 75-150 bp at the first call date. In this paper we model the call feature as a *European* option. This assumption means that the call may only be exercised at the first possible exercise date. Market practice indicates that all issuers exercise them at the first possible date see, e.g., Ineke, Guillard, and Mareels (2003) who state: 'To our knowledge, there has only ever been one instance of an issuer not calling a bond and allowing it to step-up and this was actually done unintentionally'. This practice indicates that the additional value of the contractual American property of further postponing the exercise date beyond time T has a low, if any, value. In any case, European option values are lower bounds for American option values. In practice these contracts are equipped with additional features such as a coupon rate step-up to further incentivize exercise at the earliest possible date. We are therefore tempted to conclude that assuming a fixed exercise date for these options has no severe effect on their values.

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[1]In this context, financial institutions primarily represent banks and insurance companies.

[2]See Committee on Banking Supervision (1988)

The protection period of the call feature corresponds to the maturity of the assumed European option.

We follow the approach by Black and Cox (1976) and Leland (1994), including full information and efficient market assumptions. In line with Goldstein, Ju, and Leland (2001) we assume that the issuing company's assets produce a stream of cashflows that follows a geometric Brownian motion. For a given capital structure, including an infinite horizon debt contract, there exists a constant asset value level where it is optimal for the company to go bankrupt. After introducing a finitely lived option on the debt, this bankruptcy level is no longer independent of the finite maturity of the option. The bankruptcy level after expiration of the option equals the constant Black and Cox (1976)-level.

One could alternatively consider the situation where third parties trade options on publicly traded debt. Naturally, the existence of such options would neither directly influence the pricing of the bonds at issue or in the marketplace nor the issuing company's optimal choice of bankruptcy level. We, however, consider a corporate setting where the issuer's call option is an integrated part of the bond(debt) contract. That is, the option is written by the debtholders in favor of the equityholders. We refer to such a call option as an *embedded* option. The existence of this option will influence both the issue-at-par coupon of the debt and the issuer's bankruptcy considerations before the option's expiration date. Intuition suggests that the coupon is increased to compensate for the embedded option, whereas the optimal bankruptcy level is decreased due to the option value - both compared to the case with no option.

We apply pricing formulas for down-and-out barrier European options on infinite horizon continuous coupon paying debt. Down-and-out barrier options are relevant since the debt options may only be exercised at the future time T if the issuing company has not gone bankrupt earlier. The asset-level which defines optimal bankruptcy before the option expires is thus the barrier used in the barrier option formulas.

For analytical tractability we assume that the shape of the time dependent barrier is an increasing exponential function. This is a straightforward way to model a time dependent barrier and a natural first attempt, but still an arbitrary choice. To investigate the significance of time dependency, we test the effect of alternative bankruptcy barrier assumptions. Our examples show that the effect of time dependency on the coupon-rate is limited, but increasing in cashflow volatility and time to expiration of the option. The changes in the value of barrier options as time elapses are different from the similar effects on plain vanilla options due to the barrier, here interpreted as the bankruptcy level of the issuing firm.

1.1. The debt payoff including the embedded option at expiration time T.
In this paper we denote the market value of total company assets at time t by A_t and the market value of its' debt at time t by $D(A_t)$.
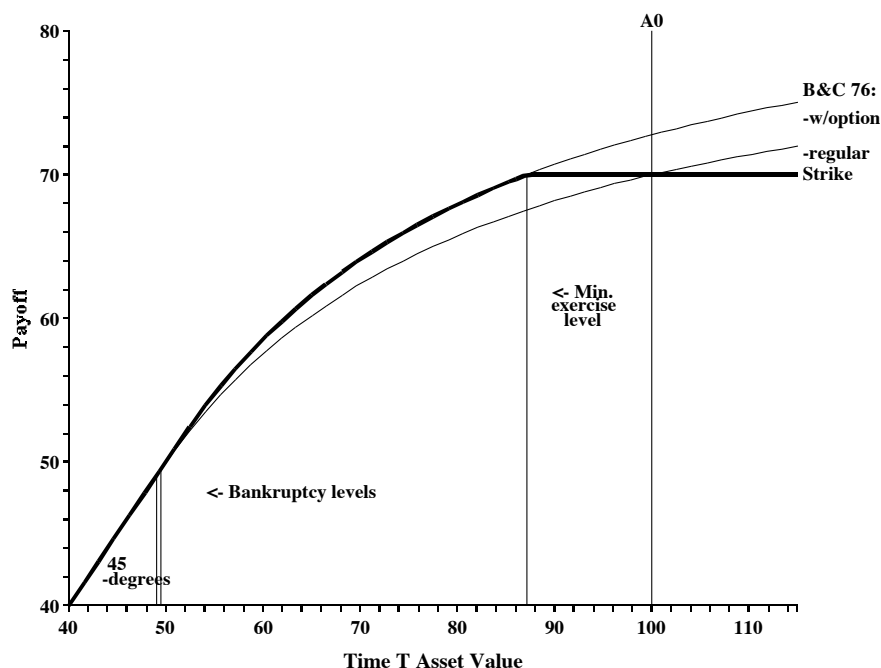

FIGURE 1. Payoff from perpetual debt with and without embedded option at time T as a function of asset level A_T. See Table 2 in Section 5 for parameter values.

The payoff to debtholders when the option expires is illustrated in Figure 1[3].

The payoff to debtholders is shown as a function of asset value A_T for debt with and without embedded option, assuming that the absolute priority rule is followed. The leftmost part of the graph shows that in bankruptcy, debtholders receive all assets, indicated by the 45-degree line. Further to the right, the thicker/upper line indicates the payoff to debt with embedded option whilst the thinner/lower line represents payoff to regular perpetual debt. The optimal bankruptcy asset levels (indicated in the figure) are different due to the different coupons. At

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[3]This and the next graphical presentation use the same base case parameters as in Table 2 in Section 5 of the paper: Time 0 EBIT (earnings before interest and tax) $\delta_0 = 3$, asset level $A_0 = 100$, par value of debt $D = 70$, expiration date of option $T = 10$ years, volatility of EBIT and assets $\sigma = 0.20$, constant riskfree interest rate $r = 5\%$ and drift of the EBIT(cashflow) process, $\mu = 2\%$. These parameters yield issue-at-par coupon rates of 5.718 % for perpetual debt without option and 5.998 % for the equivalent with embedded option, solved analytically.

time T the option does not impact optimal bankruptcy level anymore and it is only the higher coupon that causes a higher optimal long-term bankruptcy asset level.

The more interesting issue is for which levels of A_T the option is rationally exercised. *Ceteris paribus*, perpetual debt with a higher coupon will be more valuable than debt with a lower coupon. By not exercising the option at time T, the issuer is left with regular perpetual debt with a higher coupon than at time T issued identical debt. The explanation is that the coupons were fixed at time 0 and that a part of the historical coupon was a compensation to debtholders for the embedded, but at time T expired, option. The issuer is therefore willing to exercise the option at lower levels of A_T relative to the time 0 value of A_0 to avoid paying this relatively high coupon in the future. In the example in Figure 1, where the exercise level of the option is par value of the debt, (70), the indifference asset level is appr. 87, compared to the time 0 asset level[4] of 100. At this indifference level, when the company is in a worse state than at time 0, the issue-at-par optimal coupon for newly issued straight debt will exactly equal the original coupon for debt with option.

1.2. **Literature overview.** The related literature may broadly be separated into research on debt-based derivatives on one hand and on perpetual debt on the other hand.

Central to the classic literature on valuing bonds with embedded derivatives are papers like Ingersoll (1977) and Brennan and Schwartz (1977). Our paper differs from these primarily with regards to our explicit focus on the protection period of the embedded option, its impact on bankruptcy risk, the revised formulation of the underlying security in options on bonds and the use of barrier option methodology to reflect default risk before expiry of the security. Kish and Livingston (1992) test for determinants of calls included in corporate bond contracts. Their findings are that the interest rate level, agency costs and bond maturity significantly affect whether a bond comes with an embedded call option. Sarkar (2001) is the closest precedent to our paper in his focus on callable perpetual bonds modelled in the tradition of Leland (1994). The main difference is that the calls are assumed to be *American* and immediately exercisable, i.e., without a protection period, and a main part of the paper thus deals with the optimal timing of the exercise of the call. The paper does neither include analytical valuation of the options nor optimal coupon- or bankruptcy levels. Bank (2004) values call options on debt in a similar manner, but without calibrating

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[4]Our analysis provides the calibrated coupon level for debt with embedded option to ensure issue-at-par. The indifference level of A_T is found by using this coupon in the valuation expression for regular perpetual debt in expression (4) setting $D(A_T)$ equal to the exercise level (par) and solve for A_T.

coupons nor taking into account the fact that debt-values are not log-normally distributed. The paper also lacks a clear distinction between infinite securities and finite options.

Jarrow and Turnbull (1995) model various derivatives on fixed maturity debt securities, but do not include any analysis of the impact on endogenous bankruptcy decisions. Acharya and Carpenter (2002) develop valuation formulas for callable defaultable bonds with stochastic interest rates and asset values without including a protection period. Through decomposing the bonds into a riskfree bond less two options, they explore how the call option impacts optimal default in line with our results. They analyze fixed maturity bonds and the hedging aspects of callable bonds through the options' impact on bond duration. Toft and Prucyk (1997) develop modified equity option expressions based on Leland (1994) for leveraged equity and various capital structure and bankruptcy assumptions. The infinite horizon property of equity makes it comparable to our work although the specific issues related to embedded options on debt are not covered directly. Rubinstein (1983) is related to our approach with the use of a modified asset process, labelled a 'displaced diffusion process', to modify the standard Black-Scholes approach. Johnson and Stulz (1987) define the concept of 'vulnerable options' i.e. options where the counterparty may default on the contract. Hull and White (1995) categorize risky derivative contracts into classes by the counterparty default risk and the credit-risk of the underlying asset, respectively. Options embedded into debt contracts are in a class of vulnerable options on credit-risky underlying assets and the two risks may not be separated. By using barrier options one also include the risk of bankruptcy before the option matures.

In the perpetual debt pricing tradition, starting with Black and Cox (1976), our paper is related to the paper by Emanuel (1983) which develops a valuation of perpetual preferred stock, based on the option-methodology of Black-Scholes. Preferred stock can be viewed as perpetual debt for analytical purposes. Emanuel's analysis allows unpaid dividends to accumulate as arrearage due to the junior position of the instrument, which is relevant for financial institutions, but beyond the scope of our paper. He does not cover options on preferred stock as such. Sarkar and Hong (2004) extend Sarkar (2001) and analyze the impact from callability on the duration of perpetual bonds and find that a call reduces the optimal bankruptcy level and thus extends the duration of a bond. Their reduced optimal bankruptcy level matches our intuition and results.

1.3. **Contributions and outline of the paper.** Our main contributions are new valuation formulas for callable perpetual continuous coupon paying debt including a protection period. Our analysis also

quantifies the effects on optimal bankruptcy behavior during the protection period for an assumed exponential shape of the bankruptcy barrier. Numerical examples indicate that our closed form model produces correct coupon-rates compared to numerical solutions which have no restrictions on the shape of the protection period bankruptcy barrier.

We apply the valuation model from Mjøs (2007) based on barrier option formulas on perpetual debt contracts. Basically, this model expands the results of Black and Cox (1976) to integrate an issuer's deferred call option into the valuation.

Compared to market data the credit spread produced by our model is in a realistic order of magnitude.

The structure of the paper is as follows: In Section 2 we present the model and some basic results. Section 3 contains the complete expressions for perpetual debt with embedded options. Section 4 compares the base case analytical solutions with a numerically solved binomial tree, Section 5 tests the assumptions regarding the time dependency in the bankruptcy barrier, Section 6 presents numerical sensitivities, and Section 7 concludes the paper. Some proofs are included in an appendix.

2. The model and basic results

We consider the standard Black-Scholes-Merton economy and impose the usual perfect market assumptions:

- All assets are infinitely separable and continuously tradeable.
- No taxes, transaction cost, bankruptcy costs, agency costs or short-sale restrictions.
- There exists a known constant riskless rate of return r.

2.1. **The EBIT-based market value process.** We study a limited liability company with financial assets and a capital structure consisting of two claims, infinite horizon continuously coupon paying debt and common equity. In line with Goldstein, Ju, and Leland (2001), we assume that the assets generate an EBIT (earnings before interest and tax) cashflow denoted δ_t given by the stochastic differential equation

$$(1) \qquad\qquad d\delta_t = \mu\delta_t dt + \sigma\delta_t dW_t,$$

where μ and σ are constants representing the drift and volatility parameters respectively, and δ_0 is the fixed initial cashflow level. Here W_t is a standard Brownian motion under a fixed equivalent martingale measure. The total time t market value $\hat{A}_t$ of the assumed perpetual

EBIT stream from the assets equals

$$\hat{A}_t = E_t^Q \left[\int_t^\infty e^{-r(s-t)} \delta_s ds \right]$$

$$(2) \qquad = \frac{\delta_t}{r - \mu}$$

The market value of this EBIT stream is the solution to the stochastic differential equation process

$$d\hat{A}_t = (r\hat{A}_t - \delta_t)dt + \sigma\hat{A}_t dW_t$$

$$(3) \qquad = \mu\hat{A}_t dt + \sigma\hat{A}_t dW_t.$$

The quantity $\hat{A}$ is elsewhere in the literature referred to as *the unlevered value of the firm's assets*.

In this setting there is a level of $\hat{A}_t$ where it is optimal for the company to stop paying debt coupons and declare bankruptcy. In the classic case this level is independent of time, i.e., constant.

2.2. The standard Black and Cox (1976) results.

The time 0 market value of infinite horizon debt with continuous constant coupon payment is

$$(4) \qquad D(A) = \frac{cD}{r} - \left(\frac{cD}{r} - \bar{A}\right)\left(\frac{A}{\bar{A}}\right)^{-\beta},$$

where c is the constant coupon rate, D is the par value of the debt-claim and cD is the continuous coupon payment rate. The ratio $\left(\frac{A}{\bar{A}}\right)^{-\beta}$ can be interpreted as the *current market value* of one monetary unit paid upon bankruptcy, i.e., when the process A_t hits the bankruptcy level $\bar{A}$. Here

$$(5) \qquad \beta = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}.$$

Expression (4) for the market value of debt carries a nice intuition. Observe that $\frac{cD}{r}$ is the current market value of infinite horizon default-free debt. Upon bankruptcy the debtholder looses infinite coupon payments which at the time of bankruptcy have market value $\frac{cD}{r}$. On the other hand the debtholder receives the remaining assets with a value equal to $\bar{A}$. We can therefore interpret $\left(\frac{cD}{r} - \bar{A}\right)$ as the debtholder's *net loss* upon bankruptcy. The time 0 market value of this net loss, $\left(\frac{cD}{r} - \bar{A}\right)\left(\frac{A}{\bar{A}}\right)^{-\beta}$, therefore represents the reduction of the time 0 total market value of debt due to default risk. In our model this is the only source of risk in the debt.

The value of equity as the residual claim on the assets is in this setting determined by

$$(6) \qquad E(A) = A - D(A) = A - \frac{cD}{r} + \left(\frac{cD}{r} - \bar{A}\right)\left(\frac{A}{\bar{A}}\right)^{-\beta}$$

In the classic case of no embedded options, i.e., with a constant bankruptcy level, Black and Cox (1976) determine the *optimal* bankruptcy level for a given capital structure (E, D) from the perspective of the equityholders (found by differentiating expression (6) with respect to $\bar{A}$) as

$$(7) \qquad\qquad \bar{A} = \frac{\beta}{\beta + 1} \frac{cD}{r}.$$

2.3. **The modified process.** We price an embedded, finitely lived option on infinite horizon debt. Due to the finite horizon of this option, the optimal bankruptcy level of the issuer depends on remaining time to expiration. In order to capture this aspect we apply a time-dependent bankruptcy asset level B_t,

$$B_t = Be^{\gamma t},$$

for a given time 0 level B and a constant γ. The time of bankruptcy is given by the stopping time τ defined as

$$\tau = \inf\{t \geq 0, \hat{A}_t = B_t\}$$

where $\hat{A}_t$ is given in expression (2).

By modifying the asset process this stopping time can equivalently be expressed as

$$\tau = \inf\{t \geq 0, A_t = B\},$$

where A_t is

$$(8) \qquad\qquad dA_t = (\mu - \gamma)A_t dt + \sigma A_t dW_t,$$

Compared to equation (2), the modified process has a a negative drift adjustment of γ. Although γ determines the curvature of the bankruptcy level, it can formally be interpreted as a constant dividend yield on A_t. Again formally, this transformation allows us to analyze the simpler setting of a constant bankruptcy level B, although no economic fundamentals have been changed. The explanation for our choice of exponential bankruptcy level is thus only mathematical convenience, i.e., to facilitate this transformation. In Section 4, we numerically compare our analytical approximation with the optimal barrier derived numerically from a binomial tree. Some alternative specifications of time dependent bankruptcy barriers are analyzed in Section 5.

3. Value of perpetual debt including embedded option

We analyze a company with a simple capital structure, equity and one class of Black and Cox (1976)-debt including embedded option, and with net loss $J = \frac{cD}{r} - \bar{A}$ in case of bankruptcy.

In this section we assume that no further options are present after time T. We are then back to the classic Black and Cox (1976)-setting and the bankruptcy level from time T onwards is given by $\bar{A}$ in expression (7).

Let D_T^c denote the time T payoff of perpetual debt including an embedded option to repay debt at par value D, given no prior bankruptcy. Also denote the time 0 value of cashflows before time T, i.e., coupon and potential bankruptcy payments, by $L_0(A)$. The time 0 value of debt including the embedded option $D_0^c(A)$ equals the time 0 value of the time T cashflow D_T^c plus $L_0(A)$, i.e.,

$$(9) \qquad D_0^c(A) = V_0(D_T^c) + L_0(A),$$

where $V_0(\cdot)$ represents the time 0 market value operator.

3.1. The time T payoff of debt with embedded option.

We assume that $D > \bar{A}$, i.e., that the debt is risky in case of liquidation. The time T payoff of perpetual debt including an embedded option as a function of the market value of assets at time T, given by expression (3), is

$$D_T^c(\hat{A}_T) = \begin{cases} 0 & \text{for } \tau < T, \\ \hat{A}_T & \text{for } \tau > T \text{ and } \hat{A}_T < \bar{A}, \\ D(\hat{A}_T) & \text{for } \tau > T \text{ and } \bar{A} < \hat{A}_T < \dot{A}, \\ D & \text{for } \tau > T \text{ and } \hat{A}_T > \dot{A}, \end{cases}$$

where

$$\dot{A} = \theta \bar{A},$$

and

$$\theta = \left(\frac{J}{\frac{cD}{r} - K} \right)^{\frac{1}{\beta}},$$

and $D(\hat{A}_T)$ is given by expression (4) and τ is the time of bankruptcy as defined in Section 2. Here $\dot{A}$ represents the level of the market value process for which the embedded option is *at-the-money*, see Mjøs (2007).

If $B_T = \bar{A}$, i.e., the bankruptcy barrier is continuous at time T, case 2 above is redundant. There are strong arguments for a continuous bankruptcy barrier at time T. The payoff-profile in Figure 1 shows that the levels of A_T for which the option is in-the-money are well above the optimal bankruptcy level. The embedded option has therefore no impact on the optimal bankruptcy decision immediately before time T. Equivalently, the probabillity of non-negative option payoffs for distressed firms with asset values approaching the bankruptcy level is low.

In this paper we consider various approximations of the bankruptcy level before time T. Some of them violate the continuity property discussed above. We therefore have to include case 2 in the complete description of the time T payoff.

As indicated in the previous section, valuation takes place by using the modified asset value process in expression (8). For consistent valuation we express the time T payoff of debt in terms of the modified process A_T.

$$D_T^c(A_T) = \begin{cases} 0 & \text{for } \tau < T, \\ e^{\gamma T} A_T & \text{for } \tau > T \text{ and } A_T < e^{-\gamma T}\bar{A}, \\ D(e^{\gamma T} A_T) & \text{for } \tau > T \text{ and } e^{-\gamma T}\bar{A} < A_T < e^{-\gamma T}\dot{A}, \\ D & \text{for } \tau > T \text{ and } A_T > e^{-\gamma T}\dot{A}, \end{cases}$$

The time T payoff D_T^c is depicted in Figure 1. This expression can be rewritten as

(10)

$$D_T^c(A_T) = \begin{cases} 0 & \text{for } \tau < T, \\ e^{\gamma T} A_T & \text{for } \tau > T \text{ and } A_T < e^{-\gamma T}\bar{A}, \\ D(e^{\gamma T} A_T) - (D(e^{\gamma T} A_T) - D, 0)^+ & \text{for } \tau > T \text{ and } A_T > e^{-\gamma T}\bar{A}, \end{cases}$$

This shows how $D_T^c(A_T)$ equals $D(e^{\gamma T} A_T)$ minus the payoff from a *call*-option on the debt with exercise price *par*, in the case where $\tau > T$ and $A_T > e^{-\gamma T}\bar{A}$.

3.2. **Calculation of** $V_0(D_T^c)$. From expression (10) and standard financial pricing theory, see, e.g., Duffie (2001), the time 0 market value $V_0(D_T^c)$ can be written as

(11) $$V_0(D_T^c) = V_k(A) + C_2^{do}(A, 0) - C_1^{do}(A, D),$$

where

$$V_k(A) = E^Q[A_T e^{(\gamma - r)T} 1\{\tau > T\} 1\{A_T < e^{-\gamma T}\bar{A}\}],$$

$$C_2^{do}(A, 0) = E^Q[(D_T - 0)^+ e^{-rT} 1\{\tau > T\} 1\{A_T > e^{-\gamma T}\bar{A}\}],$$

$$C_1^{do}(A, D) = E^Q[e^{-rT}(D_T - D)^+ 1\{\tau > T\} 1\{A_T > e^{-\gamma T}\bar{A}\}].$$

Here $V_k(A)$ represents the part of the total value due to a possible discontinuity of the bankruptcy barrier at time T. Here $C_2^{do}(A, 0) - C_1^{do}(A, D)$ represents the time 0 market value of case 3 in expression (10). We recognize $C_2^{do}(A, 0)$ and $C_1^{do}(A, D)$ as the market values of barrier options with exercise prices 0 and D, respectively. Note, however, that these expressions contain an additional condition, $A_T > e^{-\gamma T}\bar{A}$, compared to the standard barrier condition, $\tau > T$.

Below we present the calculations of the three terms on the right hand side in separate propositions.

Proposition 1.

$$V_k(A) = Ae^{(\mu - r)T}\left(N(g_1) - N(g_2) + (\frac{A}{B})^{-\frac{2(\mu - \gamma)}{\sigma^2} - 1}[N(-g_3) - N(-g_4)]\right),$$

where g_1, g_2, g_3, and g_4 are given in Appendix A.

Proof. See Appendix A. □

Proposition 2.

$$C_2^{do}(A,0) = C_2(A,0) - (\frac{B}{A})^{(\frac{2(\mu-\gamma)}{\sigma^2}-1)}C_2(\frac{B^2}{A},0),$$

where

$$C_2(A,0) = \frac{cD}{r}e^{-rT}N(-f_2) - J(\frac{A}{\bar{A}})^{-\beta}N(-f_1).$$

where $J = \frac{cD}{r} - \bar{A}$, and f_1 and f_2 are given in Appendix A.

Proof. See Appendix A. □

Proposition 3.

$$C_1^{do}(A,D) = C_1^D(A,D) - (\frac{B}{A})^{(\frac{2(\mu-\gamma)}{\sigma^2}-1)}C_1^D(\frac{B^2}{A},K),$$

where

$$C_1^D(A,D) = (\frac{cD}{r} - D)e^{-rT}N(-d_2) - J(\frac{A}{\bar{A}})^{-\beta}N(-d_1),$$

where d_1 and d_2 are given in Appendix A.

Proof. See Appendix A. □

3.3. **Calculation of $L_0(A)$.** We now turn to the calculation of the time 0 value of cashflows before time T,

Proposition 4.

$$(12) \qquad L_0(A) = \frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A}{B})^{-\kappa} - C_3^{do}(A,0),$$

where

$$C_3^{do}(A,0) = C_3(A,0) - (\frac{B}{A})^{(\frac{2(\mu-\gamma)}{\sigma^2}-1)}C_3(\frac{B^2}{A},0),$$

$$C_3(A,0) = \frac{cD}{r}e^{-rT}N(-h_2) - \bar{J}(\frac{A}{B})^{-\kappa}N(-h_1).$$

and $\bar{J} = \frac{cD}{r} - B$, and κ, h_1 and h_2 are given in Appendix A.

Proof. See Appendix A. □

3.4. **The time 0 market value of perpetual debt with embedded option.**

Proposition 5. *The time 0 value of infinite horizon continuous coupon-paying debt claims including an embedded option to repay debt at par value D at time T is*

$$(13) \qquad D_0^c(A) = V_k(A) + C_2^{do}(A,0)$$

$$-C_1^{do}(A,D) + \frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A}{B})^{-\kappa} - C_3^{do}(A,0).$$

Our expression for $D_0^c(A)$ can be interpreted as follows: The first term represents the time 0 value of bankruptcy payoff if $\tau \geq T$, i.e., the firm has survived until time T, but $e^{\gamma T} A_T < \bar{A}$ and bankruptcy occurs at time T. The second term represents the time 0 market value of a call option on debt at time T with exercise price 0. The third term represents the short, embedded, call option on debt exercisable at time T with a strike price equal to par value D. This possibility to refinance in case of improved available terms at time T is exactly the purpose of the embedded option included in the time 0 debt contract. The last two terms represent the time 0 market value of all cashflows before time T, modelled as the difference between immediately starting perpetual debt and a forward starting perpetual debt expressed as a barrier call option with exercise price 0 at time T. This combined expression allows for calibrating both the "issue-at-par" coupon rate reflecting the embedded option and a time-dependent endogenously calibrated issuer bankruptcy level before the option expires.

4. BASE CASE PARAMETER CALIBRATIONS

In this section we calibrate coupon rates and calculate optimal bankruptcy levels for realistic parameter values. In particular, we test whether the closed form model produces correct coupon rates compared to numerical solutions. The difference between these coupon rates is a benchmark for the precision of our closed form approach.

In order to calibrate the protection period bankruptcy barrier parameters B and γ, we implement a binomial tree following the binomial lattice methodology from Broadie and Kaya (2007). This approach provides a coupon-rate, denoted by c_n, which is independent of any analytically assumed shape of the bankruptcy barrier.

The derived value of B, the initial bankruptcy level, is then used as a parameter value in the closed form solution to calculate the coupon-rate, here denoted by c_c. We do this by adjusting the coupon-rate c_c in equation (13) to achieve $D_0^c(A) = D$, i.e., that the time 0 market value of debt with embedded option is par.

In our numerical approach we apply the base case parameters in Table 2 and run 100,000 steps per year for 10 years. The chosen level of asset volatility and riskfree interest rate are common in similar illustrations, see e.g., Leland (1994). The base case time to expiration (protection period) of the option resembles the typical option maturities in publicly listed perpetual bonds issued by financial institutions.

Consistent with our assumed analytical shape of the bankruptcy barrier $B_t = Be^{\gamma t}$ we calculate $\gamma = \frac{1}{T} \ln(\frac{B_T}{B})$, where B is calculated by the binomial approach and B_T is equal to the long-term bankruptcy level, $\bar{A}$. Observe that by this formulation γ only depends on the time 0 and time T values of the bankruptcy barrier and not on intermediary values.

δ_0	3	Initial EBIT
μ	2 %	Drift of EBIT
D	70	Face value of debt
T	10	Expiration date of option
σ	20%	Volatility of EBIT
r	5 %	Riskfree interest rate
A_0	100	Total asset value at time 0

TABLE 2. Base case parameters, all rates are annualized.

We also test the assumed functional form $B_t = Be^{\gamma t}$ by using Ordinary Least Squares (OLS) to estimate γ. The estimation is based on the complete sequence of numerically calculated values of B_t and regress $\ln(B_t)$ on $\ln(B) + \gamma t$.

Figure 2 shows the development of B_t as a function of elapsed time to expiration from the binomial approach, the analytical approximation and the OLS-regression. The latter is shown by the lower line. We include two illustrations of the barriers to emphasize the limited impact of the time-dependencies compared to the long-term bankruptcy level. The appropriate choice of bankruptcy assumptions is further discussed in the next section.

Alternative solutions:	B	γ	c_c/c_n	$\bar{A}$
Analytical				
- regular(B&C'76)	Not applicable		5.718 %	49.04
- with option, B from model	4.91	0.250123	6.840 %	58.68
- with option, B from tree	50.15	0.0023026	5.998 %	51.437
Binomial				
- with option	50.15	0.0023026	5.969 %	51.18

TABLE 3. Calibrated values of the coupon-rates and corresponding bankruptcy levels using the base case parameters.

Table 3 compares the calibrated coupon-rates and bankruptcy barriers for alternative approaches. Our model yields a low starting-level B and high γ, compared to the binomial solution. We apply the starting level of B from the binomial tree as an input-variable in our calculations. Given this approach, our model generates results that are close to the binomial solution.

The results support our intuition that an embedded option increases the closed form coupon c_c (from 5.718 % to 5.998 %) even in a model with only bankruptcy risk. This increase also changes the long-term

Optimal bankruptcy before option expires - Truncated y-axis

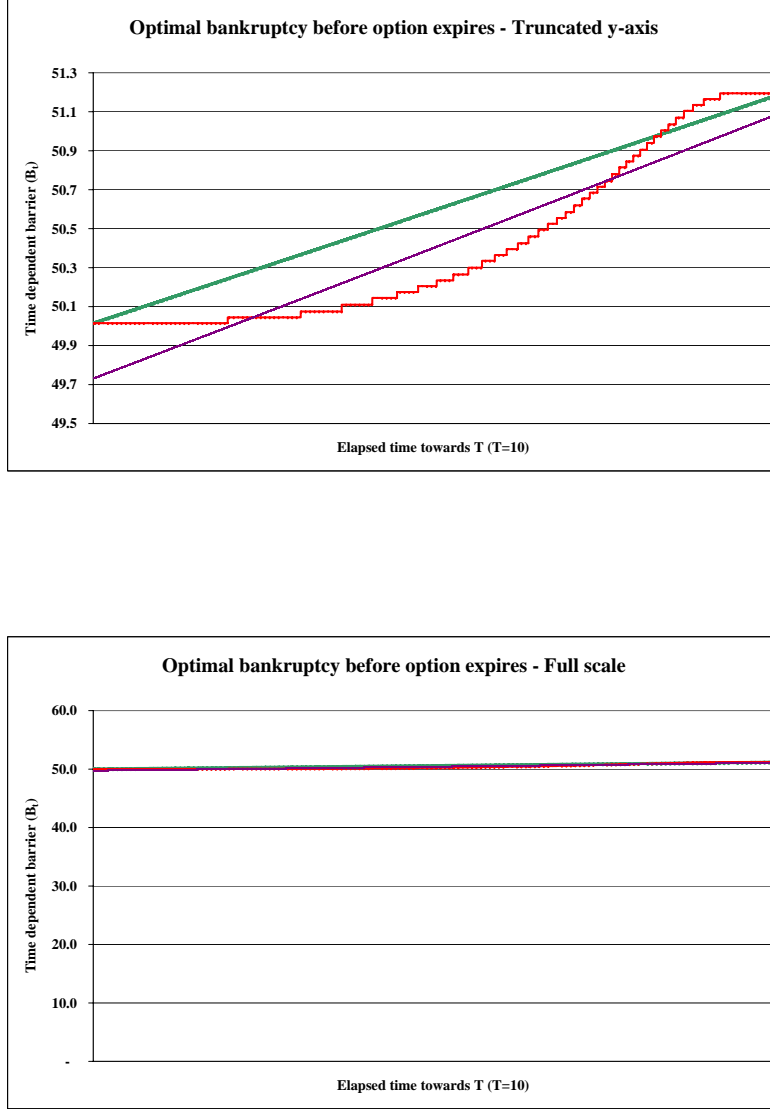**Optimal bankruptcy before option expires - Full scale**

FIGURE 2. Two plots with different scale of the y-axis to illustrate the magnitude of the time dependence. The numerically calculated optimal (red line), and the analytically solved bankruptcy asset level B_t (green line) as a function of elapsed time until expiration of the call option embedded in perpetual debt. The lower (purple) line represents an OLS-regression of the numerically calculated B_t-values. See Table 2 for parameter values.

bankruptcy-levels $\bar{A}$. As an overall assessment, we find that the coupon-rates c_c and c_n are reasonably close (the difference is less than 5 basispoints). The OLS-approach yields $\gamma = 0.00268$, $B = 49.73$, and
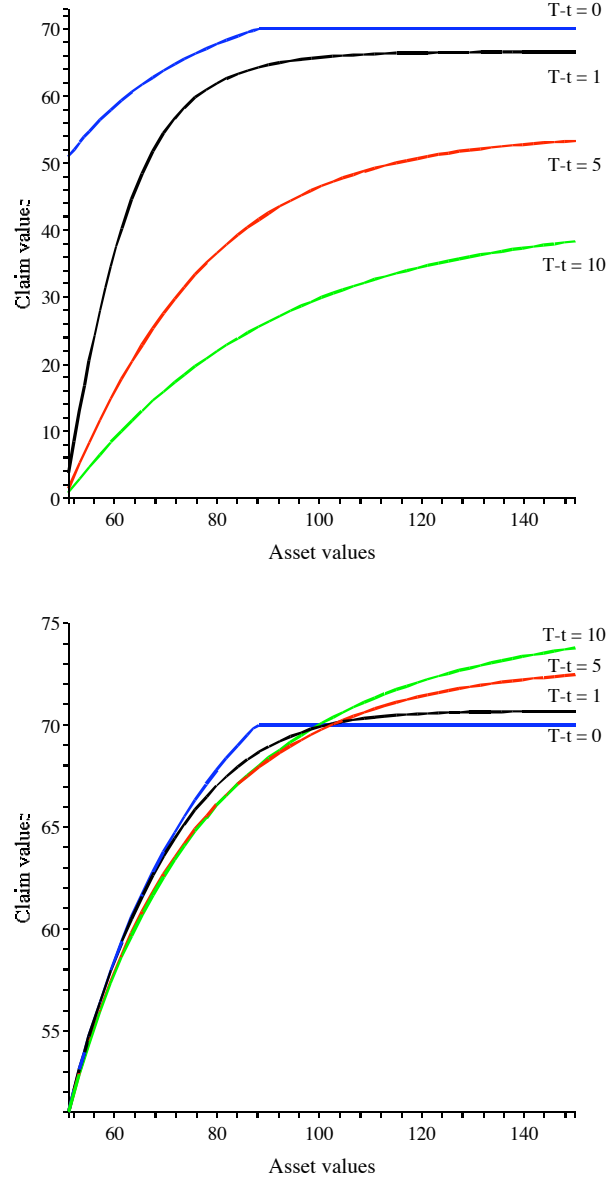
FIGURE 3. The top graph shows the market value of the time T barrier options, i.e., excluding the value of any cashflows before time T, for various remaining times to maturity. The lower graph shows the total market value of perpetual debt, i.e., including the value of cashflows before maturity from expression (13). See Table 2 for parameter values. In addition, we apply the parameter values $B = 50.15$, $\gamma = 0.0023026$, and $c = 5.998\%$ from Table 3.

$R^2 = 87.5\%$. The high value of R^2 supports the assumed linearity of $\ln(B_t)$. The estimated $\gamma's$ from the two approaches are close (The

ratio is 0.86.), and as expected, the OLS-approach underestimates the starting point B.

Observe in Figure 2 that both the numerically calculated and the modelled bankruptcy levels are below the constant long-run level $\bar{A}$. Additional analysis shows that for large values of T, the effect of the option disappears and B approaches $\bar{A}$.

The graphs in Figure 3 illustrate the sensitivites of the market values of hybrid capital, both excluding and including, respectively, intermediate cashflows before time T, for both the asset value level and remaining maturity of the option. The lower graph depicts the valuation formula (13) of Proposition 5. The upper graph illustrates the same formula excluding the latter terms representing the market value of intermediate payments $L_0(A)$ given in the valuation formula (12).

5. Relevance of the time-dependent bankruptcy barrier

An important assumption in our analysis is the exponential shape of the time-dependent bankruptcy barrier before the expiration of the embedded option. In this section, we compare the numerically calculated coupon-rate c_n to the closed-form coupon-rate c_c for variations in maturity and volatility to illustrate the effects of alternative assumptions regarding time-dependency. We compare our base case bankruptcy barrier to two alternatives profiles, a constant barrier and a constant barrier with two levels, as illustrated in Figure 4. The numerical results produce the correct bankruptcy barrier before time T whilst our analytical model necessarily represents an approximation.

We initially in Table 4 show the sensitivity of the time-dependency growth parameter γ for changes in volatility and option maturity. The values of γ are calculated from our binomial solutions as a benchmark for the choice of analytical assumptions. We find that γ is positively related to the value of our barrier-options, increasing in volatility σ and decreasing in option maturity T. As γ increases with volatility, time dependency becomes increasingly important.

γ - sensitivities	$Volatility(\sigma)$		
$Maturity(\text{T})$	0.10	0.20	0.30
5 years	0.000134	0.004716	0.011182
10 years	0.000219	0.002303	0.003786
20 years	0.000080	0.000652	0.000912

TABLE 4. Numerical values of the time-dependency parameter γ using alternative parameters for maturity(T) and annual EBIT-volatility (σ). 10,000 steps per year.

Bankruptcy level

Case A: Time dependent
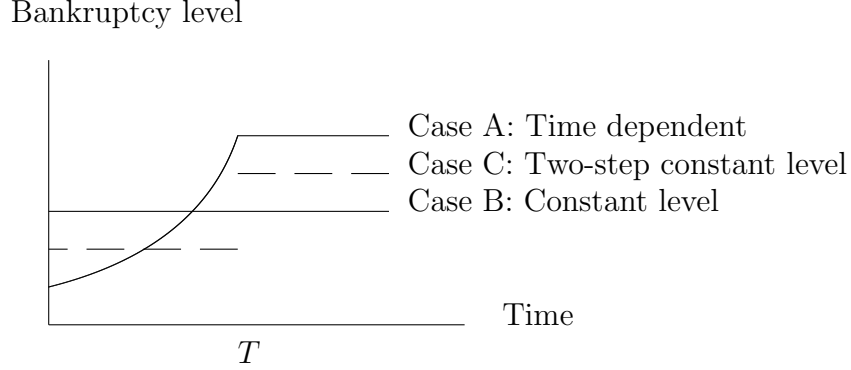Case C: Two-step constant level
Case B: Constant level

Time

T

FIGURE 4. Alternative bankruptcy barrier assumptions with different degrees of time dependency for $t < T$.

In Table 5 we show the coupon-rate differences, $c_c - c_n$. The differences are reported in basispoints for maturities 5, 10 and 20 years, respectively, for different values of σ and T and by alternative bankruptcy barrier assumptions. This test illustrates the relevance and precision of our assumed time-dependent bankruptcy barrier by comparing the following alternatives:

- *Case A:* The base case time dependent approach with assumed exponential barrier, $B \neq \bar{A}$ and $\gamma \neq 0$. B is found by solving a binomial tree, as explained in Section 5.
- *Case B:* A constant barrier for all t, $B = \bar{A}$ and $\gamma = 0$. This alternative disregards any impact the finite option may have on the optimal bankruptcy level.
- *Case C:* A two-step constant barrier with different levels before and after time T, $B \neq \bar{A}$ and $\gamma = 0$. This approach recognizes that the bankruptcy level may be lower before time T because of the option, but disregards any additional time dependencies.

The table shows that the coupon rate differences are relatively insensitive to the choice of barrier. When analyzing the longest maturities, 10 and 20 years, both alternatives with constant barriers (Cases B and C) produce comparably good results. Assuming a 5 year horizon, Case B with one, constant barrier performs best. The overall result is that in applications where one may test different combinations of σ and T, Case B with one, constant barrier is the preferable choice. This alternative also has the additional benefit of not requiring sny choice of B as input parameter, as well as computational simplicity. Our base case (Case A), performs equally well except for the shortest maturity and highest volatility. For long maturity (20 years) all barrier alternatives produce reasonably correct coupon rates.

Δ Coupon-rates, $(c_c - c_n)$, bp

Volatility (σ)	0.10	0.20	0.30
Maturity: 5 years			
Case A	-0.1	2.8	21.3
Case B	-0.1	-0.2	-0.4
Case C	N.A.	27.9	27.0
Maturity: 10 years			
Case A	-0.1	2.9	8.9
Case B	-0.1	-0.2	-0.9
Case C	N.A.	-0.2	-0.9
Maturity: 20 years			
Case A	0.1	1.0	2.7
Case B	0.0	-0.2	-0.3
Case C	0.0	-0.1	0.0

TABLE 5. Differences in basispoints between the closed form calibrated coupon rates c_c and the numerical solution c_n using alternative parameters for annual EBIT-volatility (σ) and alternative approaches for bankruptcy barriers denoted Cases A, B and C and described in the text. c_n is calculated using 10,000 steps per year.

Our conclusion may seem somewhat counterintuitive, but is that the effects of time-dependency are negligible for realistic parameter values. However, the fact that we assume 70 % debt financing to magnify the effects strengthen the conclusion.

In our discussions of sensitivities in Section 6 we apply the assumption of one constant bankruptcy barrier, Case B.

6. SENSITIVITIES AND MARKET REFERENCE

Table 6 shows the sensitivity of the analytically calculated couponrate c_c for alternative combinations of EBIT-volatility and time until the option expires. Here c_c is strongly increasing in volatility, in accordance with the classical result for option values. Contrary to the standard effect of maturity on plain vanilla option values, the barrier option specifications in our setting decreases value by longer maturities reflecting the risk of bankruptcy.

Table 7 shows the sensitivity of the long-term optimal bankruptcy level $\bar{A}$ following from the coupon-rates in Table 6. The main difference in the sensitivities is that $\bar{A}$ decreases both for increases in volatility and in option maturity.

Coupon-rates (%)	Volatility(σ)		
Maturity(T)	0.10	0.20	0.30
5 years	5.103	6.056	7.730
10 years	5.095	5.967	7.457
20 years	5.080	5.849	7.185

TABLE 6. Analytically calculated values of the coupon rate c_n for callable perpetual debt with embedded option expiring after T years using alternative parameters for maturity(T) and annual EBIT-volatility (σ). Assumed constant bankruptcy level.

Bankruptcy level $\bar{A}$	Volatility(σ)		
Maturity(T)	0.10	0.20	0.30
5 years	59.53	51.92	48.51
10 years	59.44	51.17	46.80
20 years	59.27	50.16	45.09

TABLE 7. Analytical values of the longterm bankruptcy asset level $\bar{A}$ for callable perpetual debt with embedded option after T years using c_n and alternative parameters for maturity(T) and annual EBIT-volatility (σ). Assumed constant bankruptcy level.

As a market reference, Figure 5 shows the yield spreads of Iboxx-indices[5] of UK Tier 1 perpetual debt securities including embedded option compared to UK banks' senior debt, both relative to UK government bonds(Gilts) reported weekly for the period December 2004 - November 2007. The perpetual debt yield-spread produced by our stylized model is of the same magnitude as the observed yield-spreads in the latter part of the period, except for the most recent months. The higher risk of these securities is exemplified both by the standard deviation of this annualized spread being 32 basispoints compared to 15 basispoints for senior debt, and the relatively larger credit spreads towards the end of the period. The latter relates to the credit-crisis beginning in Summer 2007. Both reflect that Tier 1 securities are more risk-exposed than senior bonds, as expected.

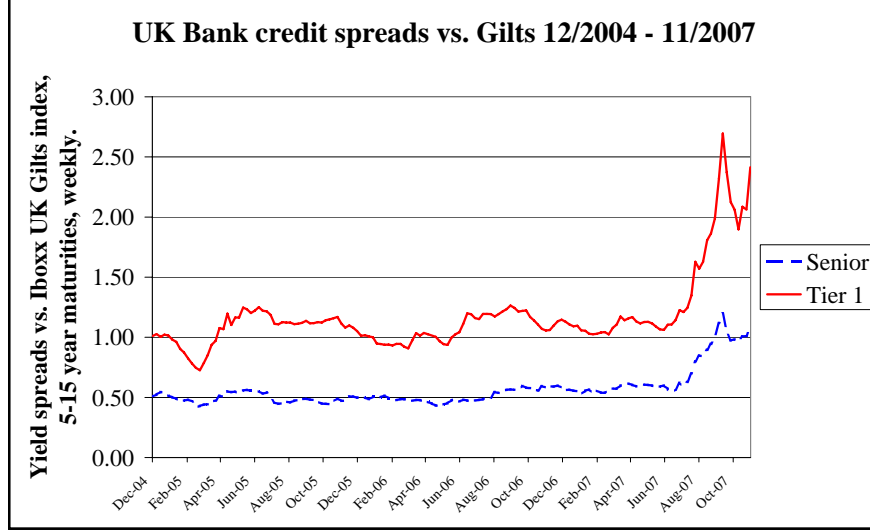[5]Index yields are sourced from Iboxx via Datastream.

FIGURE 5. The graph shows the redemption yield spread of
the Iboxx- indices for UK banks' senior debt and UK banks'
Tier I capital after deducting the yield on the UK Gilts index
for maturities 5 - 15 years. Weekly observations for the pe-
riod December 2004 until November 2007. Source: Iboxx(via
Datastream).

7. Concluding remarks and further research

We show how a European embedded option in perpetual debt im-
pacts both the value of debt and the issuer's rational economic be-
havior with regards to bankruptcy. Specifically, the embedded option
impacts the bankruptcy decision, level of debt coupons, and the opti-
mal exercise of the option. We show that for realistic parameter values
and assumed constant bankruptcy level our model produces correct
coupon-rates for alternative volatilities and option maturities. Per-
haps surprisingly, the model, with its stylized assumptions and only
incorporating bankruptcy risk, produces coupon spreads that appear
to be in a realistic order of magnitude compared to observed market
spreads.

The equityholders pay for the embedded option through a higher
fixed coupon on the perpetual debt, compared to regular perpetual
debt. The equityholders choice of optimal bankruptcy-level is im-
pacted by the debt with embedded option in two ways; an increased

coupon and the existence of a potentially valuable option. The increased coupon raises the optimal long-term bankruptcy-level, whilst the embedded option lowers it.

The market values of perpetual debt with and without option are different after expiration in the situation when the option has not been exercised. A higher coupon in the first case reflects the historical cost of the expired option and is a major motivation for the exercise of such options. This higher coupon rate causes exercises also in significantly worse future states compared to the situation at time of issue. It is common in the marketplace to contractually agree that coupons are even 'stepped-up' post-expiry of the option to further incentivice exercise.

The model can be extended along a number of dimensions such as introducing frictions (taxes, bankruptcy costs), different liquidation priorities (hybrid/preferred stock), coupon rate step-up, contractually omitted coupon payments, and American type options.

Appendix A. Proofs

Proofs of the valuation formulas from Section 3.

A.1. Proof of Propostition 1.

$$V_k(A) = E^Q[e^{(\gamma-r)T}A_T 1\{\tau > T\}1\{A_T < e^{-\gamma T}\bar{A}\}]$$
$$= Ae^{(\mu-r)T}\bar{Q}(1\{\tau > T\}1\{A_T < e^{-\gamma T}\bar{A}\})$$
$$= Ae^{(\mu-r)T}\left(N(g_1) - N(g_2) + (\frac{A}{B})^{-\frac{2(\mu-\gamma)}{\sigma^2}-1}[N(-g_3) - N(-g_4)]\right),$$

where

$$g_1 = \frac{\ln(\frac{A}{B}) + (\mu - \gamma + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$g_2 = \frac{\ln(\frac{A}{\max(\bar{A},B)}) + (\mu - \gamma + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$g_3 = \frac{\ln(\frac{A}{B}) + \ln(\frac{\max(\bar{A},B)}{B}) - (\mu - \gamma + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$g_4 = \frac{\ln(\frac{A}{B}) - (\mu - \gamma + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

For the second equality we use the change of numeraire technique using A_T as numeraire. Formally, the Radon-Nikodym derivative is $\frac{\partial \bar{Q}}{\partial Q} = \exp(-\frac{1}{2}\sigma^2 T + \sigma W_T)$ and the dynamics of A_T under $\bar{Q}$ is $dA_t = (\mu + \sigma^2)A_t dt + \sigma A_t dW_t$. Finally, $\bar{Q}(1\{\tau > T\}1\{A_T < e^{-\gamma T}\bar{A}\}) = N(g_1) - N(g_2) + (\frac{A}{B})^{-\frac{2(\mu-\gamma)}{\sigma^2}-1}[N(-g_3) - N(-g_4)]$.

A.2. Proof of Propostition 2.

In this case the exercise price $K = 0$. This implies the parameter value $\theta < 1$ in Mjøs (2007), and thus Proposition 5 in Chapter 3 applies. In this paper $\lambda = \gamma$, $M = \bar{A}$, $\kappa = \beta$, and our notation $C_2(A,0) = C_0(A,0)_\theta$ of Mjøs (2007). Furthermore,

$$f_1 = \frac{\ln(\frac{\bar{A}}{A}) - (\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta)T}{\sigma\sqrt{T}},$$

and

$$f_2 = f_1 - \sigma\beta\sqrt{T}$$

A.3. Proof of Propostition 3.

In this case the exercise price $K = D$. This implies the parameter value $\theta > 1$ in Mjøs (2007), and thus Proposition 4 in Chapter 3 applies. In this paper $\lambda = \gamma$, $M = \bar{A}$, $\kappa = \beta$, and our notation $C_1(A,D) = C_0^D(A,D)$ of Mjøs (2007). Furthermore,

$$d_1 = \frac{\ln(\frac{\bar{A}}{A}) - \frac{1}{\beta}(\ln(\frac{cD}{r} - K) - \ln J) - (\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\beta\sqrt{T}.$$

A.4. **Proof of Propostion 4.**

$$L_0(A)$$

(14)
$$= E^Q[\int_0^{\tau \wedge T} cDe^{-rs}ds] + E^Q[Be^{-r\tau}1_{\{\tau \leq T\}}],$$

$$= \frac{cD}{r} - \frac{cD}{r}e^{-rT}Q(\tau > T) - (\frac{cD}{r} - B)E^Q[e^{-r\tau}1\{\tau \leq T\}],$$

$$= \frac{cD}{r} - \frac{cD}{r}e^{-rT}Q(\tau > T) - (\frac{cD}{r} - B)E^Q[e^{-r\tau}(1 - 1\{\tau > T\})],$$

$$= \frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A}{B})^{-\kappa} - \frac{cD}{r}e^{-rT}Q(\tau > T) + (\frac{cD}{r} - B)E^Q[e^{-r\tau}1\{\tau > T\})],$$

$$= \frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A}{B})^{-\kappa} - \frac{cD}{r}e^{-rT}Q(\tau > T) + (\frac{cD}{r} - B)E^Q[e^{-rT}(\frac{A_T}{B})^{-\kappa}1\{\tau > T\})],$$

$$= \frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A}{B})^{-\kappa} - E^Q[e^{-rT}(\frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A_T}{B})^{-\kappa})1\{\tau > T\})],$$

$$= \frac{cD}{r} - (\frac{cD}{r} - B)(\frac{A}{B})^{-\kappa} - C_2^{do}(A, 0).$$

The first equality shows that $L_0(A)$ represents the time 0 market value of coupons until time T or the time of bankruptcy, τ, whichever comes first, plus the market value of any bankruptcy payoff before time T. For the second equality we solve the integral on the right hand side. For the fourth equality we use that $E^Q[e^{-r\tau}] = (\frac{A}{B})^{-\kappa}$. For the fifth equality we condition as follows:
$E^Q[e^{-r\tau}1\{\tau > T\}] = E^Q[e^{-rT}1\{\tau > T\}E^Q[e^{-r(s-T)}|\mathcal{F}_T]]$ and use (a conditional version of) the result in the previous line for the inner expectation.

We recognize the expectation in the second last line as the market value of a down-and-out barrier call option as analyzed in Mjøs (2007). In this case the exercise price $K = 0$. This implies the parameter value $\theta < 1$ in Mjøs (2007), and thus Proposition 5 in Chapter 3 applies. In this case $\lambda = 0$, $M = B$, and our notation $C_3(A, 0) = C_0(A, 0)_\theta$ of Mjøs (2007). Furthermore,

$$h_1 = \frac{\ln(\frac{B}{A}) - (\mu - \gamma - \frac{1}{2}\sigma^2 - \sigma^2\kappa)T}{\sigma\sqrt{T}},$$

$$h_2 = h_1 - \sigma\kappa\sqrt{T},$$

and

(15)
$$\kappa = \frac{\mu - \gamma - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \gamma - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}.$$

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