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Optimal Scale in Different Environments – The Case of Norwegian Electricity Distribution Companies

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Abstract

We study returns to scale in Norwegian electricity distribution companies. The scale issue of this sector has become an important political question, and it was for instance discussed by the Reiten commission (OED, 2014) in a study about the future structure and organization of the Norwegian electricity network industry. We use panel data from the Norwegian Water Resources and Energy Directorate (NVE) for the period from 2004 to 2010. The Data Envelopment Analysis (DEA) method and the Stochastic Nonparametric Envelopment of Data (StoNED) approach are applied to examine the scale issue. We show that a majority of the companies are smaller than the optimal size, in line with Kumbhakar et al. (2014). The performance of Norwegian distribution companies are influenced by a number of environmental factors, and some of these factors are negatively correlated with company size. However, our results show that controlling for environmental factors when estimating returns to scale does not have a big effect on the estimated optimal sizes.

Contents

1. Introduction

Norway was one of the pioneering countries in implementing market-oriented electricity sector reforms. The Norwegian electricity sector has been undergoing reorganization and restructuring after the Energy Act in 1991. However, its decentralized structure and ownership have remained largely unchanged. Recently, the structure of the industry has been discussed, e.g., by the Reiten-committee in a report prepared for the Norwegian ministry of petroleum and energy (OED). The report characterized smaller companies as being over-represented among the inefficient distribution companies (OED, 2014), and it suggested, among other things, increased co-operation and coordination among companies.

Returns to scale (RTS) addresses the input and output decisions of the organization. Nicholson (1985) defined return to scale as follows:

> "In intuitive terms, if a proportionate increase in inputs increases outputs by the same proportion, the production function exhibits constant returns to scale. If output increases less than proportionately, the function exhibits diminishing returns to scale. And if output increases more than proportionately, there are increasing returns to scale (p.247)".

In the standard empirical economics of efficient production, returns to scale is commonly quantified as scale elasticity, i.e., the proportionate increase in outputs resulting from the proportionate increase in inputs. The scale elasticity is often estimated using econometric approaches like stochastic frontier analysis (SFA) (Gary et al., 1999; Lawson et al., 2004; Kumbhakar and Tsionas, 2008). In this paper we examine the scale characteristics of the electricity distribution companies in Norway by means of the nonparametric data envelopment analysis (DEA) approach and the stochastic non-parametric envelopment of data (StoNED) method.

Unlike the parametric SFA approach, DEA does not have any assumptions on the underlying production or cost functional form as well as on the distribution of the inefficiency. Banker (1980) firstly proposed the standard model with single output for studying RTS in DEA. The RTS concept was extended from the single output case to the multiple output case by Banker et al (1984) and Banker and Thrall (1992). However, DEA does not distinguish inefficiency from noise in the data.

The stochastic non-parametric envelopment of data (StoNED), combining the virtues of SFA and DEA, was proposed by Johnson and Kuosmanen (2011). This approach has been applied to the Finnish electricity distribution regulation by Kuosmanen (2012). The main advantage of StoNED over SFA is the independence of the ad hoc parametric assumptions about the functional form of the production or cost function. In contrast to the fixed functional forms in SFA, one can impose more general monotonicity and concavity constraints in StoNED, without sacrificing the flexibility of the regression function. The main relative advantage of StoNED over DEA is the better robustness to outliers, data errors, and other stochastic noise in the data. Our study aims at quantifying returns to scale for electricity distribution companies using the StoNED approach.

Electricity distribution companies, even in the same country, do not operate under identical or even similar environmental and climatic conditions. It is well know that analyses of efficiency and productivity should control for factors beyond the companies' control, see e.g. Coelli and Battese (2008). Specifically, if the environmental factors are related to size, like in the Norwegian distribution sector, an analysis of economies of scale that does not control for these factors will probably be biased. See e.g. the discussion of the Canadian hospital sector by Asmild et al. (2013), where it is shown that the optimal hospital size depends on location. In this paper we investigate the environmental impact on the measured economies of scale for Norwegian electricity distribution companies. Norway is a suitable case for such a study because of its large number of distribution companies and detailed data of an extensive range of local geographic factors and weather conditions. We specify three different approaches based on the DEA model and the StoNED model to study whether environmental factors have any impact on estimated returns to scale.

The reminder of this paper is structured as follows: After the literature review in Section 2, Section 3 describes the data sample used in the estimation of various models. Section 4 reviews the theoretical foundation of DEA and StoNED and describes the methodology used for our analysis. The results are presented in Section 5. Section 6 contains concluding comments.

2. Review of previous studies

The scale issue in electricity distribution sectors for different countries has been studied by several authors. Many studies have found evidence of scale economies: Filippini (1996) for Switzerland; Kumbhakar and Hjalmarsson (1998) for Sweden; Yatchew (2000) for Canada and Kwoka (2005) for the US.

There are several studies that investigate the scale issue in Norwegian electricity distribution companies. Salvanes and Tjøtta (1994) studied the scale issue of 100 Norwegian electricity distribution industries in 1998. They found that no economies of scale were present in the industry, even for small companies. The total factor productivity development of Norwegian electricity distribution utilities of 157 firms in 1983 and 170 in 1989, respectively, was examined by Førsund and Kittelsen (1998). They concluded that the small firms experienced poor performance. Recently, there were conflicting results. Growitsch et al. (2009) used the method of SFA to estimate cost efficiency and scale economics for 499 electricity distribution companies from eight European countries: Finland, Ireland, Italy, Netherlands, Norway, Spain, Sweden and United Kingdom. The analysis of the relationship between firm size, technical efficiency and quality of service among these companies shows evidence of significant economies of scale in electricity distribution networks, even for the larger firms. Kumbhakar et al. (2014) used input distance functions to investigate scale economies, technical change and efficiency for 128 Norwegian electricity distribution companies from 1998 to 2010. They found evidence of scale economics for small companies.

The environmental influence on the performance in Norwegian electricity distribution companies has been studied by several authors. Growitsch et al. (2012) studied the effect of almost 100 geographic and weather variables on Norwegian electricity distribution companies for the 2001-2004 period using the input distance function and stochastic frontier method, and the results proved that the effect on companies' average efficiency was great. Miguéis et al. (2012) examined the productivity change for Norwegian electricity distribution companies between 2004 and 2007. The relationship between efficiency and environmental factors including size was studied, which indicated that size had no significant effect on efficiency levels. However, our study is the first to investigate the environmental impact on economies of scale in the Norwegian electricity distribution sector.

3. Data

The data used in this study comprise economic and technical information on 123 Norwegian electricity distribution companies from 2004 to 2010. The data were collected by Norway's regulatory agency (NVE). The variables in our data correspond to the variables used by the regulator in the benchmarking model that was implemented from 2007, i.e., it has a single input, five outputs and three environmental factors. The single input specified is total cost, which includes the four cost groups described in Table 1. The data for all years have been adjusted to the price level of a base year (2010). We use an industry-specific price index for adjusting operations and maintenance costs and the consumer price index for the quality costs. Thermal losses are valued at the average system price at Nord Pool for the base year, and the capital costs are calculated using the nominal rate of return set by the regulator for the base year.

Table 1 Elements of the single input cost variable

Cost group	Unit of measurement
Capital costs	NOK
Operations and maintenance costs	NOK
Quality cost (value of lost load, VOLL)	NOK
Cost of thermal power losses	NOK

Table 2 lists the five output variables. Energy delivered and customers are direct outputs from the production activity of the distribution companies. We distinguish between regular customers and cottage customers, since the latter customer type usually consume less energy than regular customers. Two of the variables (high voltage lines and network stations) are in fact input variables, however, they represent structural conditions that may influence the required network size and thereby the cost level of the companies.

Table 2 Output variables

Variable	Unit of measurement
Energy delivered	MWh
Customers (except cottage customers)	No. of customers
Cottage customers	No. of customers
High voltage lines	Kilometers
Network stations (transformers)	No. of stations

The environmental variables are listed in Table 3. They describe environmental conditions that may affect the cost of the companies, and are the only variables that are not based on data reported by the companies. The values of the environmental variables are size-independent index measurements and need to be scaled in the DEA model in order to avoid the bias problems described by 0. We use the length of the overhead high voltage network to scale the index variables for use in the DEA model, while unscaled variables are used in the StoNED model.

Table 4 shows descriptive statistics of the (unscaled) variables used in our analysis. The data set used here is the same as in Cheng et al. (2014). In this paper, however, we simplify by averaging the annual data to obtain a data set that is representative for the entire period 2004-2010.

Table 3 Environmental variables

Variable	Unit of measurement
Forest	Proportion (0–100) of area with high-growth forest
Snow	Average precipitation as snow (mm)
Coast	Average wind speed (m/s) / Average distance to coast (meters)

Table 4 Descriptive statistics of the variables used in our analysis

Variable	Mean	SD	Median	Min	Max
Total cost	899.70	1887.77	340.23	27.43	16418.60
Energy delivered	53331.80	156417.70	13927.00	913.50	1531464.30
Customers (except cottage customers)	1814.20	5341.68	469.50	26.80	52120.40
Cottage customers	208.31	336.34	104.14	10.31	2594.51
High voltage lines	73.82	126.98	31.17	3.09	830.29
Network stations (transformers)	92.32	179.66	33.60	2.97	1349.30
Forest	70.88	147.83	23.94	0.00	967.42
Snow	163341.00	262382.70	84623.00	1662.00	1542310.00
Coast	4.60	8.73	0.90	0.02	49.49

Table 5 Correlations

Env. var. / **Output**	Forest	Snow	Coast
Total cost	0.015	-0.210	-0.081
Energy	0.018	-0.204	-0.072
Customers	0.024	-0.193	-0.145
Cottage customers	0.013	-0.202	-0.076
High voltage lines	-0.021	-0.175	-0.115
Network stations	0.012	-0.205	-0.120

Table 5 shows the correlations between the outputs and environmental variables. Forest is positively correlated with the cost and the outputs, except for high voltage lines, while the corresponding correlation coefficient values for snow and coast are negative. A possible explanation for the observed negative correlation is that the environmental variables are related to size, i.e. smaller companies are located in areas that are more exposed to snow and costal climate. A priori, we expect this phenomenon to influence estimation results regarding both efficiency scores and returns to scale. Specifically, if the environmental effects are not controlled for, as in Kumbhakar (2014), we would expect an overestimation of returns to scale. In this paper we include the environmental factors and discuss whether the results are affected. This issue is important and relevant for the current discussion about the industry structure (OED, 2014).

4. Methodology

4.1 The DEA models

The DEA method is used to establish a best practice group among a set of observed units and to identify the units that are inefficient when compared to the best practice group (Charnes et al., 1978). The DEA models can be input-oriented or output-oriented. We consider the input-oriented model to be appropriate for the electricity distribution sector, since the objective of an electricity distribution company is to produce an exogenously given level of desirable outputs at minimum cost. In addition, DEA models can be specified as constant returns to scale (CRS) or variable returns to scale (VRS). Suppose we have n company observations $(\boldsymbol{x}_i, \boldsymbol{y}_i),\ i = 1,\cdots,n,$ where company i uses the vector of inputs $\boldsymbol{x}_i = (x_{1i},\cdots,x_{mi})$ to produce the vector of outputs $\boldsymbol{y}_i = (y_{1i},\cdots,y_{ri})$. In the regulation, a CRS model is used. However, since we are interested in the returns to scale characteristics of individual companies, we use a VRS model. The following set of equations and inequalities defines the DEA model that we utilize in our analyses (Banker et al., 1984):

$$
\begin{aligned}
&\theta_{vrs}^{j} = \max\left[\textstyle\sum_{l=1}^{r} u_l y_{lj} + w_j\right] && \\
&\text{s.t.} && \\
&\textstyle\sum_{k=1}^{m} v_k x_{kj} = 1 && \\
&\textstyle\sum_{l=1}^{r} u_l y_{li} - \sum_{k=1}^{m} v_k x_{ki} + w_j \leq 0 && i = 1,\cdots\cdots,n \qquad (1)\\
&u_l \geq 0 && l = 1,\cdots\cdots,r \\
&v_k \geq 0 && k = 1,\cdots\cdots,m
\end{aligned}
$$

In model (1), company j is the one under investigation, and v_k, u_l, and w_j are the shadow prices on the output and input constraints, and the VRS constraint, respectively, of the DEA model in envelopment form, to which (1) is the dual. The value of w_j identifies the returns to scale for company j. When $w_j > 0$, we have increasing returns to scale (IRS); $w_j < 0$ means that we have decreasing returns to scale (DRS); and $w_j = 0$ implies constant returns to scale (CRS).

There are different approaches in the DEA literature to investigate the impact of environmental factors on performance, see for instance Coelli et al. (2005) and Miguéis et al. (2012). In order to estimate the environmental impact on the scale economics in our analysis based on the DEA model, we specify three different DEA models as follows:

DEA_without EF: The VRS DEA model without environmental factors. The model considers only a single input and five outputs.

DEA_with EF: The VRS DEA model with environmental factors. The environmental factors are treated as outputs or cost drivers. One advantage of this approach is that we can obtain information about the shape of the frontier, e.g., local returns to scale for the companies.

Reverse DEA: The reverse two-stage VRS DEA model. Ruggiero (2004) and Simar and Wilson (2004) address the biased estimates resulting from correlation between inputs and environmental factors, and Barnum and Gleason (2008b) propose the reverse two-stage DEA model in order to mend the problem. The first step of this model is to regress the input (total cost) on the outputs and the environmental factors:

$$\log(x_i) = \alpha_i + \boldsymbol{\rho}_i \log(\boldsymbol{y}_i) + \boldsymbol{\delta_i} \boldsymbol{z}_i + \varepsilon_i. \qquad (2)$$

In this equation, x_i is the single input, $\boldsymbol{y_i}$ is the output vector, and $\boldsymbol{z}_i$ is the vector of environmental factors, of company i. The vector $\boldsymbol{\delta_i}$ contains the coefficients representing the environmental impact on the total cost of company i. Also, $\boldsymbol{\rho}_i$ is the vector of output coefficients, α_i is the intercept, and ε_i is the statistical error term, for company i. We then adjust the total cost by removing the estimated environmental impact as follows:

$$x_i^{adjust} = x_i \cdot \exp(-\boldsymbol{\delta_i} \boldsymbol{z}_i) \qquad (3)$$

In the second step, we use model (1) with the adjusted total cost as input to investigate the performance of the companies.

4.2 Measurement of scale elasticity in the DEA model

Førsund et al. (2007) specifies how to calculate the scale elasticities for inefficient and efficient companies, respectively. The inefficient companies are projected onto the efficient frontier. The projection can be input-oriented or output-oriented, and in our analysis we consider the former variant, in line with the discussion of model choice above. The scale elasticity for an inefficient company with unique shadow prices is

$$\varepsilon_j = \frac{\theta_{vrs}^j}{\theta_{vrs}^j - w_j}, j = 1, \dots, n. \tag{4}$$

For the efficient companies spanning the frontier and thus being corner points of the DEA technology set, Banker et al. (1984) and Banker and Thrall (1992) showed that the shadow prices may not be unique. We calculate the upper and lower bounds on the shadow prices in the way proposed by Banker and Thrall (1992). The upper bound w_j^{max} is found by maximizing the value of w_j, given that the objective function value in (1) is equal to 1, i.e., by solving the following optimization problem:

$$\begin{aligned}
&\max w_j \\
&\text{s.t.} \\
&\textstyle\sum_{l=1}^{r} u_l y_{lj} + w_j = 1 \\
&\textstyle\sum_{k=1}^{m} v_k x_{kj} = 1 \\
&\textstyle\sum_{l=1}^{r} u_l y_{li} - \sum_{k=1}^{m} v_k x_{ki} + w_j \leq 0 \qquad && i = 1, \cdots\cdots, n \\
&u_l \geq 0 && l = 1, \cdots\cdots, r \\
&v_k \geq 0 && k = 1, \cdots\cdots, m
\end{aligned} \tag{5}$$

To find the lower bound w_j^{min} of the shadow price, the sign in the objective function (5) is simply changed from positive to negative, i.e., $\{w\}$ is replaced with $\{-w_j\}$. From (5) we know that $w_j \in \{-\infty, 1]$, i.e., $w_j^{max} \leq 1$ and $w_j^{min} \geq -\infty$. The maximum and minimum scale elasticities, respectively, for the corner points is then calculated as (Førsund et al., 2007):

$$\varepsilon_j^{max} = \frac{1}{1 - w_j^{max}}, \ j = 1, \dots, n \tag{6}$$

$$\varepsilon_j^{min} = \frac{1}{1-w_j^{min}}, \ j = 1, \dots, n. \tag{7}$$

The maximal value of w_j corresponds to infinite scale elasticity, i.e., we are on a vertical frontier segment, and the minimal value implies zero scale elasticity, i.e., we are on a horizontal frontier segment. When computing the scale elasticity for efficient companies, we use the average of ε_j^{max} and ε_j^{min}, except when ε_j^{max} is infinite and we use ε_j^{min}.

4.3 The StoNED models

Johnson and Kuosmanen (2011) recently introduced the StoNED method in order to integrate a stochastic SFA-style noise term into the nonparametric DEA-style cost frontier, and to take the contextual variables, such as environmental variables, better into account. StoNED avoids the main disadvantage of SFA---its parametric nature---by using convex nonparametric least squares (CNLS) to estimate the cost frontier function. CNLS does not require an assumption about the functional form of the frontier function, but determines a frontier from the family of continuous, monotonically increasing, concave functions which best fits the data (Kuosmanen 2008). The StoNED method has two stages:

Stage 1: Estimate the shape of the cost frontier by the convex nonparametric least squares (CNLS).

Stage 2: Estimate additional distributional assumption about u_i and v_i and find the cost frontier function and efficiency scores.

We assume, as in Kuosmanen (2012), the cost frontier function

$$x_i = C(\boldsymbol{y}_i) \cdot \exp(\varepsilon_i) \qquad \text{where } \varepsilon_i = v_i + u_i, \ \ u_i \geq 0, \tag{8}$$

where x_i is the total cost of company i, C is the cost frontier function, $\boldsymbol{y}_i$ is the vector of the outputs of company i, ε_i is the residual of company i, and u_i and v_i represent inefficiency and a stochastic noise terms, respectively. In order to obtain the CNLS estimator in Stage 1, we solve the quadratic programming (QP) model (Kuosmanen, 2012; Kuosmanen and Kortelainen, 2012) given by

$$\min_{\gamma,\beta,\varepsilon} \sum_{i=1}^{n} \varepsilon_i^2$$

s.t.

$$\ln x_i = \ln \gamma_i + \varepsilon_i \qquad\qquad i = 1, \cdots, n \tag{9}$$

$$\gamma_i = \alpha_i + \boldsymbol{y}_i\boldsymbol{\beta}_i' \geq \alpha_h + \boldsymbol{y}_i\boldsymbol{\beta}_h' \qquad h = 1,\cdots,n$$

$$\boldsymbol{\beta}_i \geq 0 \qquad i = 1,\cdots,n,$$

where γ_i is the CNLS estimator of the expected total cost of producing outputs $\boldsymbol{y}_i$, $\boldsymbol{\beta}_i$ is the vector of marginal output costs of company i, and α_i is the intercept of company i. No restriction on the sign of α_i indicates that VRS is assumed. CRS can be imposed by assuming $\alpha_i = 0$, and IRS or DRS correspond to $\alpha_i \geq 0$ or $\alpha_i \leq 0$, respectively. The first constraint of model (9) can be interpreted as the regression equation, where the log transformation follows from the exponential formulation in (8). Non-concavity is ensured by the second constraint, and the third constraint guarantees monotonicity. Since we want to identify local returns to scale of each company, we make the VRS assumption in our StoNED models, i.e., we do not restrict the sign of α_i.

For stage 2 of the StoNED procedure, there are two approaches to estimate the variance parameters based on the optimal solution $\hat{\varepsilon}_i$ of model (9): the method of moments (MoM) (Aigner et al., 1977) and the pseudo-likelihood estimation approach (PSL) (Fan and Weersink, 1996). We only consider the former method, since the computation is simpler than for the latter one. As in Kuosmanen (2012), we assume that the stochastic noise term v_i follows a normal distribution $N(0,\sigma_v^2)$. The inefficiency term u_i follows a half-normal distribution with finite variance, σ_u^2, which implies that the expected value of inefficiency is $E(u_i) = \mu = \sigma_u\sqrt{2/\pi}$ (Aigner et al., 1977). Then, based on the vector of estimated errors $\hat{\boldsymbol{\epsilon}}$, the parameters of the two distributions can be obtained by

$$\hat{\sigma}_u = \sqrt[3]{\frac{\widehat{M}_3}{\left(\sqrt{\frac{2}{\pi}}\right)\left[\frac{4}{\pi}-1\right]}}, \text{and} \tag{10}$$

$$\hat{\sigma}_v = \sqrt[2]{\widehat{M}_2 - \left[\frac{\pi-2}{\pi}\right]\sigma_u^2}, \tag{11}$$

where $\widehat{M}_2 = \sum_{i=1}^{n}(\hat{\varepsilon}_i - \bar{\varepsilon})^2/n$ and $\widehat{M}_3 = \sum_{i=1}^{n}(\hat{\varepsilon}_i - \bar{\varepsilon})^3/n$ are estimates of the second and third central moments of the composite errors distribution, respectively.

Next, we estimate the cost frontier function for company i as

$$\hat{C}(\boldsymbol{y}_i) = \gamma_i \cdot \exp\left(-\hat{\sigma}_u\sqrt{\frac{2}{\pi}}\right) = (\alpha_i + \boldsymbol{\beta}_i\boldsymbol{y}_i) \cdot \exp\left(-\hat{\sigma}_u\sqrt{\frac{2}{\pi}}\right), \tag{12}$$

and the cost efficiency score for company i is the ratio of the minimum cost to the observed cost:

$$CE_i = \frac{\hat{C}(\boldsymbol{y}_i)}{x_i} \quad (13)$$

As for DEA, we specify three approaches with respect to how environmental factors are incorporated in the StoNED model:

StoNED_without EF: The VRS StoNED model without environmental factors.

StoNED_with EF: The VRS StoNED model with environmental factors. For this model, the regression constraints in Model (9) should be changed into

$$\ln x_i = ln\, \gamma_i + \boldsymbol{\delta z}_i + \varepsilon_i \qquad i = 1, \cdots, n, \quad (14)$$

where the coefficient vector $\boldsymbol{\delta}$ characterizes the environmental impact of company i. Note that, while in the corresponding DEA model described in Section 4.1, we have included the environmental variables as outputs, implying that their (dual) weights will be company-specific. In the StoNED model, however, the coefficient vector $\boldsymbol{\delta}$ applies to all the companies in the data set, so the two approaches are fundamentally different. Also, the The cost frontier function for this approach is

$$\hat{C}(\boldsymbol{y}_i, \boldsymbol{z}_i) = \gamma_i \cdot \exp\left(-\hat{\sigma}_u\sqrt{\frac{2}{\pi}} + \boldsymbol{\delta z}_i\right) = (\alpha_i + \boldsymbol{\beta}_i \boldsymbol{y}_i) \cdot exp\left(-\hat{\sigma}_u\sqrt{\frac{2}{\pi}} + \boldsymbol{\delta z}_i\right). \quad (15)$$

Reverse StoNED: The reverse VRS StoNED model. The first step in this approach is also to regress total cost on the environmental factors, which is the same as equation (2). We then use x_i^{adjust} in the two stages of the StoNED approach.

4.4 Measurement of scale elasticity in the StoNED model

Frisch (1965) introduced, for any production function, returns to scale or scale elasticity as a measure of the increase in output relative to a proportional increase in all inputs, evaluated as the marginal change at a point in input-output space. In our analysis, the cost frontier function is used. We then have that returns to scale can be measured as the increase in cost relative to a proportional increase in all outputs, as in Frisch (1965). Expanding outputs proportionally by factor φ we choose the minimal expansion of inputs $\omega = \omega(\varphi, \boldsymbol{x}_i, \boldsymbol{y}_i)$ allowed by the transformation function (Førsund et al., 2007)

$$F(\omega(\varphi, \boldsymbol{x_i}, \boldsymbol{y_i})\boldsymbol{x}_i, \varphi \boldsymbol{y_i}) = 0. \tag{16}$$

Scale elasticity is defined as the ratio between the relative change in outputs and inputs, respectively, i.e.

$$\varepsilon_i = \frac{\partial \varphi}{\partial \omega} \cdot \frac{\omega}{\varphi}. \tag{17}$$

Based on expression (17) for the general case, we can develop the scale elasticity for the StoNED model with a single input factor (total cost). Given that the proportional expansion of outputs is φ, the expanded cost level, based on (15), will be

$$\hat{C}(\varphi \boldsymbol{y}_i, \boldsymbol{z}_i) = (\alpha_i + \boldsymbol{\beta}_i \varphi \boldsymbol{y}_i) \cdot \exp\left(-\hat{\sigma}_u \sqrt{\frac{2}{\pi}} + \boldsymbol{\delta z}_i\right). \tag{18}$$

The marginal cost effect of expanding the output level is

$$\frac{\partial \hat{C}}{\partial \varphi} = \boldsymbol{\beta_i y}_i \cdot \exp\left(-\hat{\sigma}_u \sqrt{\frac{2}{\pi}} + \boldsymbol{\delta z}_i\right), \tag{19}$$

and the marginal change in the input expansion factor is therefore

$$\frac{\partial \omega}{\partial \varphi} = \frac{\boldsymbol{\beta_i y}_i \cdot \exp\left(-\hat{\sigma}_u \sqrt{\frac{2}{\pi}} + \boldsymbol{\delta z}_i\right)}{(\alpha_i + \boldsymbol{\beta}_i \varphi \boldsymbol{y}_i) \cdot \exp\left(-\hat{\sigma}_u \sqrt{\frac{2}{\pi}} + \boldsymbol{\delta z}_i\right)} = \frac{\boldsymbol{\beta_i y}_i}{\alpha_i + \boldsymbol{\beta}_i \varphi \boldsymbol{y}_i}. \tag{20}$$

Inserting the inverse of (20) into (17) and evaluating, without loss of generality, at $\omega = \varphi = 1$, gives

$$\varepsilon_i = \frac{\alpha_i + \boldsymbol{\beta_i y}_i}{\boldsymbol{\beta_i y}_i}. \tag{21}$$

Relating the elasticity measure in (21) to the discussion of returns to scale in Kuosmanen and Kortelainen (2012), we have the following cases:

- Constant returns to scale: $\alpha_i = 0 \Leftrightarrow \epsilon_\mathrm{i} = 1$
- Increasing returns to scale: $\alpha_i > 0 \Leftrightarrow \epsilon_\mathrm{i} > 1$
- Decreasing returns to scale: $\alpha_i < 0 \Leftrightarrow \epsilon_\mathrm{i} < 1$

5. Empirical results

The six models described in Section 4 have been applied to a data set with 123 companies, where the data variables for each company is obtained by taking averages over the period 2004-2010. We first discuss the efficiency scores, before moving on to considering returns to scale.

5.1 Efficiency scores

Figure 1 shows the efficiency scores obtained under the six different models that we are considering, and a summary is given in Table 6. We see that the inclusion of environmental variables will, for most companies, lead to an increase in the efficiency scores, and we also see that the StoNED efficiency scores are consistently higher than the corresponding DEA scores. These findings confirm the results in Cheng et al. (2014, and they can be important in a regulation framework since the current yardstick incentive regulation estimates the revenue caps based on the efficiency scores (Bjørndal et al., 2010). In the Norwegian context, however, the calibration methodology applied by the regulator reduces the importance of the efficiency score levels, although the combination of calibration and differences in level may lead to some redistribution effects.

Table 6 Efficiency scores.

Statistic	DEA or StoNED	Inclusion of env. factors		
		Without EF	With EF	Reverse
Mean	DEA	0.8252	0.8991	0.8312
	StoNED	0.9796	0.9806	0.9782
Median	DEA	0.8161	0.9046	0.8302
	StoNED	0.9702	0.9725	0.9686
Min	DEA	0.5722	0.6593	0.6005
	StoNED	0.6711	0.7735	0.7696
Max	DEA	1	1	1
	StoNED	1.455	1.303	1.3605

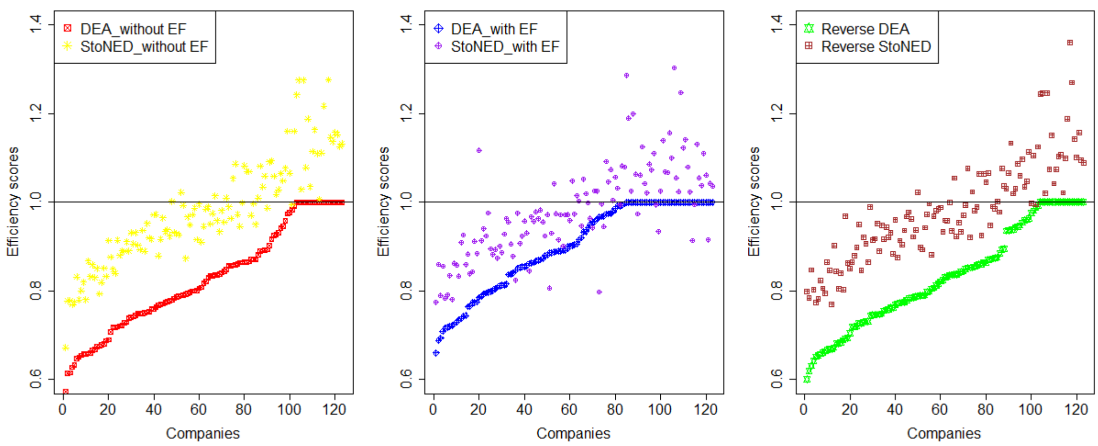

Fig.1 Efficiency scores under the DEA and StoNED models

5.2 Returns to scale

Figure 2 shows the number of companies with IRS, CRS and DRS for the respective models. The companies are predominantly IRS under all models, i.e., they appear to be smaller than the most productive size. However, this tendency is considerably stronger under StoNED models than under DEA. We also see that the number of IRS companies decreases when we include the environmental factors in either the DEA or the StoNED models, i.e., the optimal company sizes are smaller (Cheng et al., 2014). For the reverse DEA model we see only a slight reduction in the number of IRS companies, and for the reverse StoNED model we see a slight increase. One may conclude that the choice of estimation method to investigate environmental impact affects the scale issue.

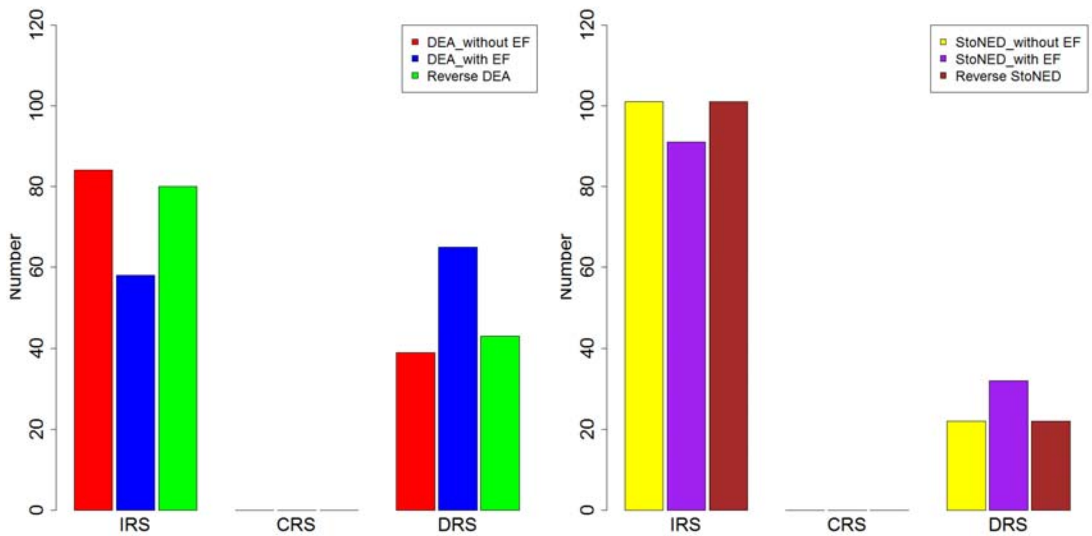

Fig.2 Distributions of returns to scale for the DEA models and StoNED models

Furthermore, in Figure 3, the scale elasticity estimates for the DEA models and the StoNED models are plotted against the number of customers for each company. In the DEA models, the scale elasticity estimates, irrespective of model alternative, mostly lie above the dotted line where scale elasticity is equal to one (CRS), i.e. the smaller companies are characterized by IRS. This is in line with the findings in Kumbhakar et al. (2014).

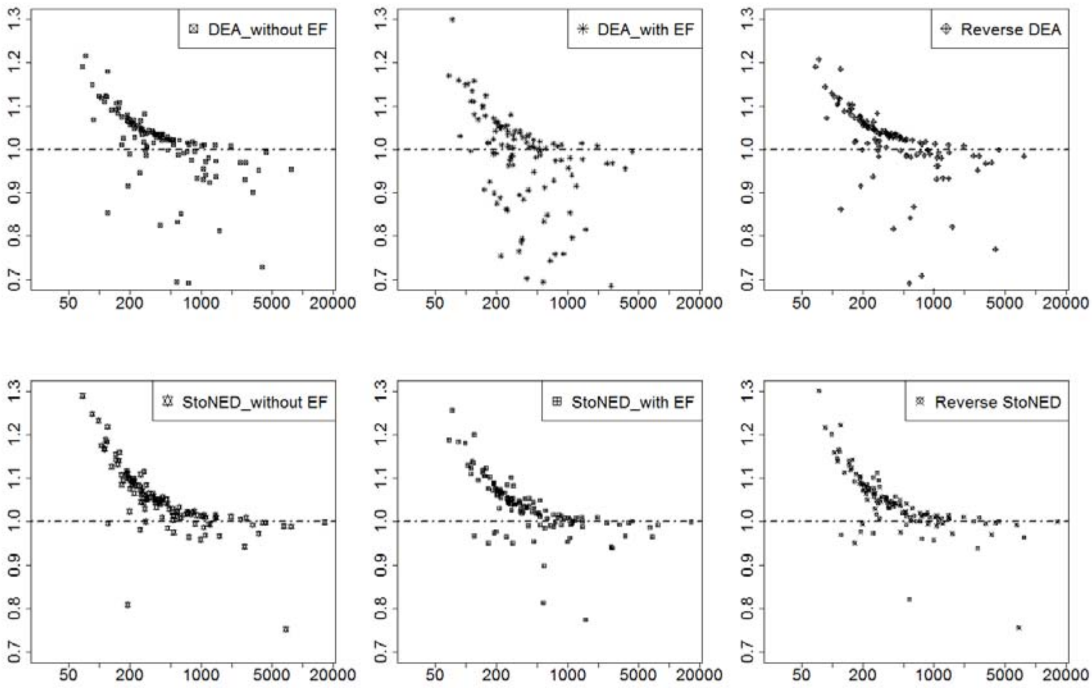

Fig.3 Scale elasticity (y-axes) versus number of customers (x-axes)

Given the relationship between size and environmental factors that we observed in Section 3, one might expect the inclusion of the environmental variables to affect the observed scale elasticities. Specifically, the negative correlation between size and two of the three environmental factors should result in lower estimates for the elasticities when the environmental factors are included in the analysis. Figure 4 compares the elasticity values obtained without and with controlling for environmental factors, shown on the x-axes and the y-axes, respectively. We see that inclusion of environmental factors as variables in the benchmarking models leads to lower estimated elasticity values, for most companies, under both DEA and StoNED, and the mean and median values in Table 7 confirms this. When total cost is adjusted for environmental effects in the reverse models, the picture is less clear. We see a slight reduction in the StoNED elasticities, although smaller than when the environmental variables are included in the benchmarking model. For the DEA elasticities, we observe almost no change.

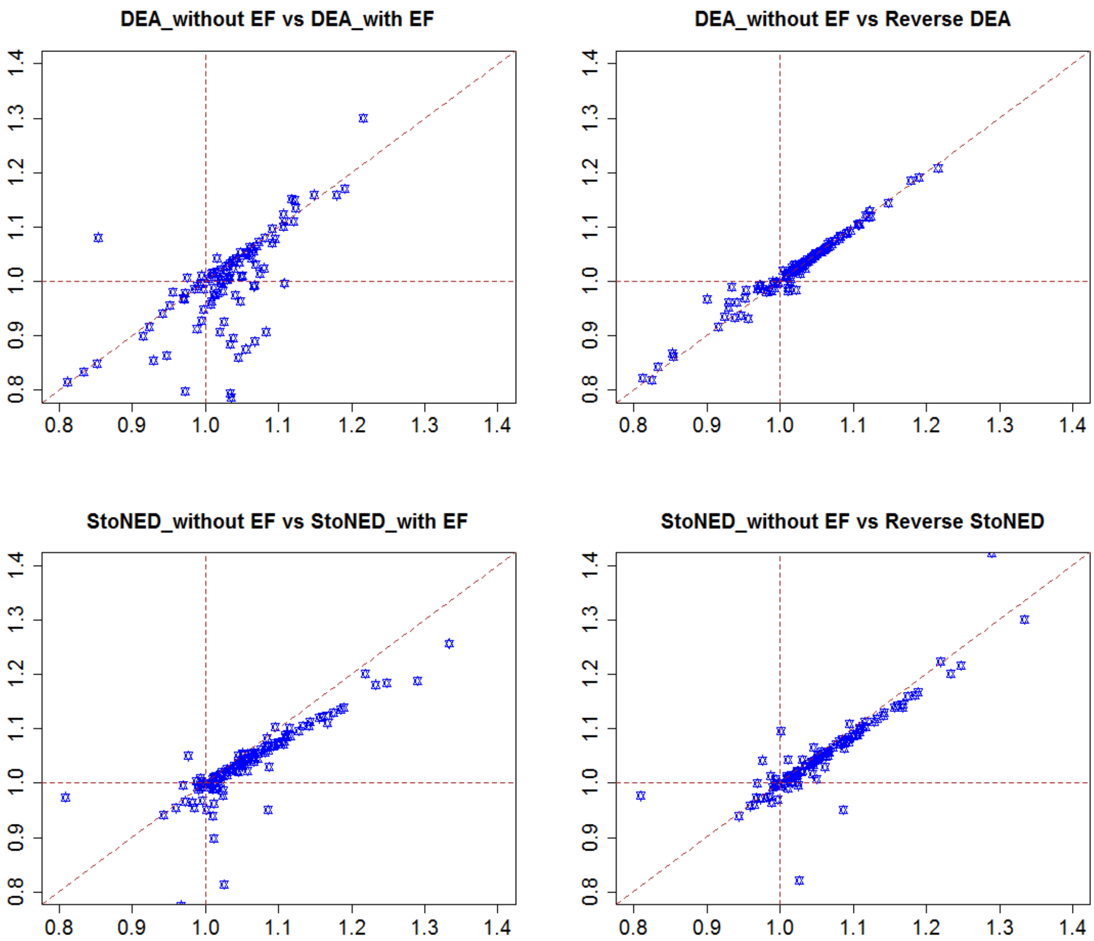

Fig.4 Scale elasticity without environmental factors (x-axes) versus scale elasticity with environmental factors (y-axes).

Table 7 Scale elasticities.

Statistic	DEA or StoNED	Inclusion of env. factors		
		Without EF	With EF	Reverse
Mean	DEA	0.9998	0.9798	1.002
	StoNED	1.0978	1.043	1.089
Median	DEA	1.0239	0.9959	1.0245
	StoNED	1.0458	1.0325	1.042
Min	DEA	0.4851	0.4794	0.4981
	StoNED	0.7525	0.6574	0.756
Max	DEA	1.2159	1.2995	1.2072
	StoNED	1.5762	1.9976	1.6893

6. Conclusion

In this paper, we examine returns to scale for Norwegian electricity distribution companies, based on average data for the period from 2004 to 2010. We compare results under the DEA and StoNED approach, respectively, and we also look at the effect of controlling for environmental factors. Our results show that a majority of the companies are below the optimal size. This is true for StoNED as well as for DEA, although the tendency is somewhat stronger under the former approach. Also, we see that controlling for environmental factors has the effect, except under the reverse DEA approach, of decreasing the optimal size. However, neither changing the estimation approach nor controlling for environmental factors changes the main conclusion, i.e., that the distribution companies are predominantly smaller than the optimal size. Hence, our research confirms the results in Kumbhakar et al. (2014).

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