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Abstract

We construct a Malmquist productivity index based on stochastic non-parametric envelopment of data (StoNED) method, and we study how the distributional assumptions in the second StoNED stage affect productivity change and its decompositions. Our discussion show that the distributional assumptions do not affect the estimates of overall productivity change and scale efficiency change, but that estimates of efficiency change and technical change are affected. Data on Norwegian electricity distribution companies is used to illustrate our discussion.

Key words: Productivity and competitiveness, StoNED, Malmquist productivity index, Wrong skewness issue

1. Introduction

During the past decades, many methods have been developed to study efficiency and productivity development. These methods are often classified as parametric and non-parametric methods. Parametric methods such as stochastic frontier analysis (SFA) estimate a cost or production function, whose functional form should be specified. In contrast, it is not necessary to estimate the cost and production function when using non-parametric methods. DEA is a non-parametric method that is capable of handling multiple inputs and multiple outputs. However, DEA, which does not consider the uncertainty in observations, only measures the inefficiency. Thus the estimated efficiency will not reflect the true performance of the units in question if there is any uncertainty in the dataset. In order to modify the measurement limitations in the DEA and SFA approach, Johnson and Kuosmanen (2011) proposed stochastic non-parametric envelopment of data (StoNED), combining the virtues of both DEA and SFA. This approach has been applied to the Finnish electricity distribution regulation (Kuosmanen and Kortelainen, 2012; Kuosmanen, 2012). Unlike SFA, StoNED has the advantage that the functional form of the production function or cost function does not need to be specified, except for some general assumptions about monotonicity, homogeneity and concavity. Compared to DEA, StoNED is also non-parametric in nature, and captures not only noise but also inefficiency.

In parametric stochastic frontier models, as well as in StoNED, the residual is specified as the sum of a two-sided noise component and a one-sided inefficiency component. A common assumption is that noise is normally distributed, while the inefficiency distribution is usually half-normal, exponential or truncated normal (Aigner et al., 1977; Meeusen and van den Broeck, 1977; Stevenson, 1980). In the widely used normal and half-normal model, the half-normal distribution on inefficiency implies that the residual distribution is skewed in one direction. However, in practice, the estimated residuals may display skewness in the wrong direction in finite samples (Waldman, 1982). This is often termed the “wrong skewness issue”. When the wrong skewness issue occurs, possible solutions are to increase the size of the sample or to respecify the model (Carree, 2002; Almanidis et al., 2011, Feng et al., 2012).

The main contribution of our paper is to apply the StoNED approach to estimate Malmquist productivity (Grifell-Tatje and Lovell., 1995 and 1999b; Pastor and Lovell., 2005), and to investigate the consequences of distributional assumptions, in the second stage of the StoNED procedure, on the productivity indices. We show that productivity change and scale efficiency change are not affected by the distributional assumptions, while efficiency change and technical change are affected. This means that estimates of productivity change and scale efficiency change can be based on the average-practice frontier for which the distributional assumptions play no role. Also, if the relative distance between the best-practice frontier and the average-practice frontier is constant over time, i.e., if the average inefficiency in the industry is constant, then efficiency change and technical change may also be based on the average-practice frontier. We illustrate our discussion with results based on a data for Norwegian electricity distribution companies in the period 2004-2013.

The remainder of the paper proceeds as follows: In Sections 2 and 3 we present the Malmquist and StoNED methodologies, respectively, and in Section 4 we explain how they can be combined in order to analyze productivity change. Section 5 discusses the impact of the distributional assumptions in StoNED on the Malmquist productivity index and its decompositions. An empirical illustration is presented in Section 6, and Section 7 concludes.

2. The Malmquist productivity index and its decomposition

The concept of the Malmquist productivity index originated from Caves et al. (1982a). In order to define it we need the specify the production technology as

$$P^t(\boldsymbol{y}^t) = \{\boldsymbol{x}^t : \boldsymbol{x}^t \text{ can produce } \boldsymbol{y}^t\}, \tag{1}$$

where $\boldsymbol{x}^t$ and $\boldsymbol{y}^t$ represent the input vector and output vector at each time period $t, t = 1, \cdots, T$, respectively. The set $P^t(\boldsymbol{y}^t)$ is assumed to be non-empty, closed, convex and bounded. It satisfies strong disposability of inputs and outputs, and also contains all input vectors that can produce output $\boldsymbol{y}^t$. A functional representation of the technology is constructed by Shephard's (1970) input distance function

$$D^t(\boldsymbol{y}^t, \boldsymbol{x}^t) = \sup\{\varphi : (\boldsymbol{x}^t/\varphi) \in P^t(\boldsymbol{y}^t), \varphi > 0\}. \tag{2}$$

The function $D^t(\boldsymbol{y}^t, \boldsymbol{x}^t)$ represents the maximum proportional contraction of inputs given outputs at each period t. The distance function satisfies $D^t(\boldsymbol{y}^t, \boldsymbol{x}^t) \geq 1$, with $D^t(\boldsymbol{y}^t, \boldsymbol{x}^t) = 1$ if and only if $\boldsymbol{y}^t \in \mathrm{Isoq}P^t(\boldsymbol{y}^t) <=> \{\boldsymbol{y}^t : \boldsymbol{y}^t \in P^t(\boldsymbol{y}^t), \lambda\boldsymbol{y}^t \notin P^t(\boldsymbol{y}^t), \lambda > 1\}$. The function $D^t(\boldsymbol{y}^t, \boldsymbol{x}^t)$ is defined in terms of period t dataset and technology, and adjacent-period input distances using period t or $t+1$ data and period $t+1$ or t technology are defined as

$$D^{t+1}(\boldsymbol{y}^t, \boldsymbol{x}^t) = \sup\{\varphi : (\boldsymbol{x}^t/\varphi) \in P^{t+1}(\boldsymbol{y}^t), \varphi > 0\} \tag{3}$$

and

$$D^t(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1}) = \sup\{\varphi: (\boldsymbol{x}^{t+1}/\varphi) \in P^t(\boldsymbol{y}^{t+1}), \varphi > 0\}, \tag{4}$$

respectively (Grifell-Tatje and Lovell, 1995).

Following Färe and Primont (1995), the input distance function $D^t(\boldsymbol{y}^t, \boldsymbol{x}^t)$ is reciprocal to Farrell's input oriented measure of efficiency, which is

$$E^t(\boldsymbol{y}^t, \boldsymbol{x}^t) = \min\{\theta: (\theta \boldsymbol{x}^t) \in P^t(\boldsymbol{y}^t), \theta > 0\}. \tag{5}$$

The efficiencies for the adjacent-period input distance functions can be obtained as

$$E^{t+1}(\boldsymbol{y}^t, \boldsymbol{x}^t) = \min\{\theta: (\theta \boldsymbol{x}^t) \in P^{t+1}(\boldsymbol{y}^t), \theta > 0\} \tag{6}$$

and

$$E^t(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1}) = \min\{\theta: (\theta \boldsymbol{x}^{t+1}) \in P^t(\boldsymbol{y}^{t+1}), \theta > 0\}. \tag{7}$$

The Malmquist productivity index between period t and $t+1$ can be expressed as

$$MPI(\boldsymbol{y}^t, \boldsymbol{x}^t, \boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1}) = \left[\frac{E_{crs}^t(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}{E_{crs}^t(\boldsymbol{y}^t, \boldsymbol{x}^t)} \frac{E_{crs}^{t+1}(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}{E_{crs}^{t+1}(\boldsymbol{y}^t, \boldsymbol{x}^t)}\right]^{\frac{1}{2}} = EC \cdot TC \cdot SEC, \tag{8}$$

where E_{crs}^t is the efficiency under constant returns to scale (CRS). Equation (8) also shows that the productivity index can be decomposed into efficiency change (EC), technical change (TC) and scale efficiency change (SEC) (Ray and Desli., 1997). We define E_{vrs}^t as efficiency under variable returns to scale (VRS), as well as

$$EC = \frac{E_{vrs}^{t+1}(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}{E_{vrs}^t(\boldsymbol{y}^t, \boldsymbol{x}^t)}, \tag{9}$$

$$TC = \left[\frac{E_{vrs}^t(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}{E_{vrs}^{t+1}(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})} \frac{E_{vrs}^t(\boldsymbol{y}^t, \boldsymbol{x}^t)}{E_{vrs}^{t+1}(\boldsymbol{y}^t, \boldsymbol{x}^t)}\right]^{\frac{1}{2}}, \text{ and} \tag{10}$$

$$SEC = \left[\frac{\frac{E_{crs}^t(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}{E_{vrs}^t(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}}{\frac{E_{crs}^t(\boldsymbol{y}^t, \boldsymbol{x}^t)}{E_{vrs}^t(\boldsymbol{y}^t, \boldsymbol{x}^t)}} \frac{\frac{E_{crs}^{t+1}(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}{E_{vrs}^{t+1}(\boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})}}{\frac{E_{crs}^{t+1}(\boldsymbol{y}^t, \boldsymbol{x}^t)}{E_{vrs}^{t+1}(\boldsymbol{y}^t, \boldsymbol{x}^t)}}\right]^{\frac{1}{2}}. \tag{11}$$

Productivity growth (decline) corresponds to $MPI(\boldsymbol{y}^t, \boldsymbol{x}^t, \boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1})$ greater (smaller) than one, and productivity is constant if $MPI(\boldsymbol{y}^t, \boldsymbol{x}^t, \boldsymbol{y}^{t+1}, \boldsymbol{x}^{t+1}) = 1$. Efficiency change (EC) greater (smaller) than unity indicates that the company has moved closer to the frontier from period $t+1$ to period t, and a value of unity means that the distance to the frontier is the same in the two periods. For technical change (TC) between periods t and $t+1$, a value of more (less) than unity means that the frontier technology in period $t+1$ is more (less) productive than the technology in period t. If the ratio of scale efficiency change (SEC) is larger (smaller) than unity, then the company has moved closer to (further away from) the optimal scale from period t to period $t+1$.

3. The StoNED method

The semi-nonparametric method termed stochastic nonparametric envelopment of data (StoNED) was proposed by Johnson and Kuosmanen (2011) to estimate efficiency, and it has been applied to the Finnish regulatory model by Kuosmanen (2012). We use the same cost frontier function and assumptions on the noise and inefficiency terms as Kuosmanen (2012).

According to Johnson and Kuosmanen (2011), the StoNED model can be a model of the production function or the cost function. When benchmarking for regulation, it is convenient to use a cost frontier function. With reference to the multiplicative model applied by Kuosmanen (2012), the following cost frontier function is used in this paper:

$$x_i^t = C(\boldsymbol{y}_i^t) \cdot exp(\boldsymbol{\delta}^t \boldsymbol{z}_i^t + \varepsilon_i^t) \qquad \text{where } \varepsilon_i^t = u_i^t + v_i^t,\ u_i^t \geq 0,\ t = 1, \cdots, T, \tag{12}$$

where x_i^t is the total cost and $\boldsymbol{y}_i^t$ the vector of outputs of firm i in period t, C is the cost frontier function, ε_i^t is the residual of firm i in period t, $\varepsilon_i^t = u_i^t + v_i^t$, and u_i^t and v_i^t are, respectively, the inefficiency term and the stochastic noise term in period t. The coefficient vector $\boldsymbol{\delta}^t$ represents the environmental impact and $\boldsymbol{z}_i^t$ is the vector of environmental variables for firm i in period t. The stochastic noise term v_i^t is assumed to follow a normal distribution $N(0, (\sigma_v^t)^2)$ while the inefficiency term u_i^t is assumed to follow a half-normal distribution with a finite variance $(\sigma_u^t)^2$. The expected value of inefficiency is denoted by $E(u_i^t) = \mu^t = \sigma_u^t\sqrt{2/\pi}$ (Aigner et al., 1977). Regarding the cost frontier function C, we do not impose a particular functional form, but it satisfies continuity, monotonicity, convexity and variable returns to scale (VRS), which is similar to the classical DEA model (Charnes et al., 1978).

The StoNED method consists of two stages:

Stage 1: Estimate the average-practice cost function by the convex nonparametric least squares (CNLS) method.

Stage 2: Estimate the variance parameters σ_u^2, σ_v^2, the expected values of inefficiency μ and the best-practice cost frontier function $\hat{C}^{best}$.

Stage 1 for period t can be expressed as the following optimization problem:

$$\min_{\gamma,\beta,\varepsilon} \sum_{i=1}^{N} (\varepsilon_i^t)^2$$

s.t.

$$\begin{aligned} &ln\, x_i^t = ln\, \gamma_i^t + \boldsymbol{\delta}^t \boldsymbol{z}_i^t + \varepsilon_i^t && i = 1, \cdots, n \\ &\gamma_i^t = \alpha_i^t + \boldsymbol{\beta}_i^{t\prime} \boldsymbol{y}_i^t \geq \alpha_h^t + \boldsymbol{\beta}_h^{t\prime} \boldsymbol{y}_i^t && h = 1, \cdots, n \\ &\boldsymbol{\beta}_i^t \geq 0 && i = 1, \cdots, n \end{aligned} \tag{13}$$

In (13), γ_i^t is the CNLS estimator of the average-practice total cost of producing $\boldsymbol{y}_i^t$ in period t, the intercept α_i^t of firm i in period t indicates its local returns to scale status ($\alpha_i^t > 0$ and $\alpha_i^t < 0$ represent DRS and IRS, respectively), and $\boldsymbol{\beta}_i^t$ is the marginal cost of outputs. The coefficient vector $\boldsymbol{\delta}^t$ represents the environmental impact and $\boldsymbol{z}_i^t$ is the vector of environmental variables for firm i in period t. The first constraint in (13) is the regression equation, and the second and third constraint ensures convexity and monotonicity, respectively. Model (13) has no sign restrictions on the intercept term α_i^t, which implies that we allow variable returns to scale (VRS). By imposing the constraint $\alpha_i^t = 0$ for all $i = 1, \cdots, n$, we can implement the assumption of constant returns to scale (CRS).

For stage 2 of the StoNED procedure, there are two commonly applied approaches to estimate the variance parameters based on the optimal solution $\hat{\varepsilon}_i$ of model (13): the method of moments (MoM) (Aigner et al., 1977) and the pseudo-likelihood estimation approach (PSL) (Fan et al., 1996). We will consider the former method. Under the maintained assumptions of half-normal inefficiency and normal noise, the estimators of $\hat{\sigma}_u^t$ and $\hat{\sigma}_v^t$ are obtained through the equations

$$\hat{\sigma}_u^t = \sqrt[3]{\frac{\widehat{M}_3^t}{\left(\sqrt{\frac{2}{\pi}}\right)\left[\frac{4}{\pi}-1\right]}}, \text{ and} \tag{13}$$

$$\hat{\sigma}_v^t = \sqrt[2]{\widehat{M}_2^t - \left[\frac{\pi-2}{\pi}\right](\hat{\sigma}_u^t)^2}, \tag{14}$$

where $\widehat{M}_2^t$ and $\widehat{M}_3^t$ are the second and third central moments of the composite errors from the solution of (13). They are given as

$$\widehat{M}_2^t = \sum_{i=1}^{n} (\hat{\varepsilon}_i^t - \bar{\varepsilon}^t)^2 / n, \text{ and} \tag{15}$$

$$\widehat{M}_3^t = \sum_{i=1}^{n} (\hat{\varepsilon}_i^t - \bar{\varepsilon}^t)^3 / n. \tag{16}$$

In Equation (14), $\widehat{M}_3^t$, which measures the skewness of the distribution, is related to the standard deviation of the inefficiency distribution. Given our distributional assumptions on u_i^t and v_i^t, we would expect $\widehat{M}_3^t$ to be positive. However, as we will discuss in Section 6, this is not always the case.

The best practice cost function for a given company is

$$\hat{C}_i^{t,best}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t) = \gamma_i^t(\boldsymbol{y}_i^t) \cdot \exp(\boldsymbol{\delta}^t \boldsymbol{z}_i^t) \cdot \exp\left(-\hat{\sigma}_u^t \sqrt{\frac{2}{\pi}}\right), \tag{17}$$

where $\gamma_i^t(\boldsymbol{y}_i^t) \cdot \exp(\boldsymbol{\delta}^t \boldsymbol{z}_i^t)$ is the average-practice cost frontier $\hat{C}_i^{t,avg}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t)$ (Johnson and Kuosmanen, 2011), i.e., $\hat{\sigma}_u^t = 0$. Based on Equation (18), we notice that the estimated standard deviation of inefficiency affects the best practice cost frontier, since the best-practice cost is obtained by multiplying the average-practice cost by the shift factor $\exp\left(-\hat{\sigma}_u^t \sqrt{2/\pi}\right)$.

The best-practice cost efficiency score of firm i in period t is defined as the ratio of the minimum cost to the observed cost, i.e.,

$$CE_i^{t,best}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t, x_i^t) = \frac{\hat{C}_i^{t,best}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t)}{x_i^t}, \tag{18}$$

and $CE_i^{t+1,best}(\boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^{t+1}, x_i^{t+1})$ can be obtained in a similar manner.

4. Productivity estimates based on StoNED

According to Kuosmanen et al. (2013), the estimated cost norm can also be calculated as

$$\gamma_i^t(\boldsymbol{y}_i^t) = \max_h(\alpha_h^t + (\boldsymbol{\beta}_h^t)' \boldsymbol{y}_i^t). \tag{19}$$

Adjacent-period estimated cost norms using period t or $t+1$ data and period $t+1$ or t technology are given by

$$\gamma_i^{t+1}(\boldsymbol{y}_i^t) = \max_h(\alpha_h^{t+1} + (\boldsymbol{\beta}_h^{t+1})' \boldsymbol{y}_i^t), \text{ and} \tag{20}$$

$$\gamma_i^t(\boldsymbol{y}_i^{t+1}) = \max_h((\alpha_h^t + (\boldsymbol{\beta}_h^t)' \boldsymbol{y}_i^{t+1}). \tag{21}$$

Based on Equations (21) and (22), the best-practice cost efficiency can be calculated as

$$CE_i^{t+1,best}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t, x_i^t) = \frac{\hat{C}_i^{t+1,best}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t)}{x_i^t} = \frac{\gamma_i^{t+1}(\boldsymbol{y}_i^t) \cdot \exp(\boldsymbol{\delta}^{t+1} \boldsymbol{z}_i^t) \exp(-\hat{\sigma}_u^{t+1} \sqrt{2/\pi})}{x_i^t}, \text{ and} \tag{22}$$

$$CE_i^{t,best}(\boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^{t+1}, x_i^{t+1}) = \frac{\hat{C}_i^{t,best}(\boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^{t+1})}{x_i^{t+1}} = \frac{\gamma_i^t(\boldsymbol{y}_i^{t+1}) \cdot \exp(\boldsymbol{\delta}^t \boldsymbol{z}_i^{t+1}) \cdot \exp(-\hat{\sigma}_u^t \sqrt{2/\pi})}{x_i^{t+1}}. \tag{23}$$

Based on Section 2, we define the Malmquist productivity index based on the best-practice StoNED frontier as

$$MPI_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = EC_i^{best} \cdot TC_i^{best} \cdot SEC_i^{best}. \tag{24}$$

The change in efficiency relative to the frontier between periods t and $t+1$, i.e.,

$$EC_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = \frac{CE_{i,vrs}^{t+1,best}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1})}{CE_{i,vrs}^{t,best}(y_i^t, z_i^t, x_i^t)}. \tag{25}$$

The technical frontier change between periods t and $t+1$ is

$$TC_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = \left[\frac{CE_{i,vrs}^{t,best}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1})}{CE_{i,vrs}^{t+1,best}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1})} \frac{CE_{i,vrs}^{t,best}(y_i^t, z_i^t, x_i^t)}{CE_{i,vrs}^{t+1,best}(y_i^t, z_i^t, x_i^t)}\right]^{\frac{1}{2}}, \tag{26}$$

and the scale efficiency change between periods t and $t+1$ is

$$SEC_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = \left[\frac{\frac{CE_{i,crs}^{t,best}(y^{t+1}, x^{t+1})}{CE_{i,vrs}^{t,best}(y^{t+1}, x^{t+1})}}{\frac{CE_{i,crs}^{t,best}(y^t, x^t)}{CE_{i,vrs}^{t,best}(y^t, x^t)}} \frac{\frac{CE_{i,crs}^{t+1,best}(y^{t+1}, x^{t+1})}{CE_{i,vrs}^{t+1,best}(y^{t+1}, x^{t+1})}}{\frac{CE_{i,crs}^{t+1,best}(y^t, x^t)}{CE_{i,vrs}^{t+1,best}(y^t, x^t)}}\right]^{\frac{1}{2}}. \tag{27}$$

5. Impact of distributional assumptions

In Section 3, the best-practice cost frontier is found by shifting the average-practice cost frontier with the estimated standard deviation of the inefficiency via Equation (18). Furthermore, in Equation (14), the estimate of the standard deviation depends on the skewness estimate. In order to examine the skewness' effect on productivity change, the productivity change based on the best-practice and the average-practice frontiers are compared in this section.

From equation (18) and (19) we can express the relationship between the efficiency scores of the best-practice and average-practice frontiers as

$$CE_{i,vrs}^{t,best}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t, x_i^t) = CE_{i,vrs}^{t,avg}(\boldsymbol{y}_i^t, \boldsymbol{z}_i^t, x_i^t) \cdot K_{vrs}^t\ , \tag{28}$$

where the scaling factor $K_{vrs}^t = \exp\left(-\hat{\sigma}_{u,vrs}^t \sqrt{2/\pi}\right)$ represents the estimated average efficiency under the assumption of half-normal inefficiency. This means that the distributional assumptions and estimates from the second StoNED stage affect the best-practice frontier through the scaling factor K_{vrs}^t.

Equations (26) and (29) imply that efficiency change is given as

$$EC_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = \frac{CE_{i,vrs}^{t+1,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{vrs}^{t+1}}{CE_{i,vrs}^{t,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{vrs}^t}$$

$$= EC_i^{avg}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) \cdot \frac{K_{vrs}^{t+1}}{K_{vrs}^t}, \qquad (29)$$

i.e., the ratio between efficiency change based on the best-practice frontier and the average-practice frontier, respectively, is given by the scaling factor $\frac{K_{vrs}^{t+1}}{K_{vrs}^t}$.

Technical change is, according to Equations (27) and (29), obtained by

$$TC_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = \left[\frac{CE_{i,vrs}^{t,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{vrs}^t}{CE_{i,vrs}^{t+1,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{vrs}^{t+1}} \cdot \frac{CE_{i,vrs}^{t,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{vrs}^t}{CE_{i,vrs}^{t+1,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{vrs}^{t+1}}\right]^{1/2}$$

$$= TC_i^{avg}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) \cdot \frac{K_{vrs}^t}{K_{vrs}^{t+1}}, \qquad (30)$$

i.e., the ratio between technical change estimate based on the best and average-practice frontiers, respectively, is equal to the inverse of the scaling factor in (30).

Let $K_{crs}^t = \exp\left(-\hat{\sigma}_{u,crs}^t \sqrt{2/\pi}\right)$. Then, Equations (28) and (29) imply that scale efficiency change is

$$SEC_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = \frac{\frac{CE_{i,crs}^{t,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{crs}^t}{CE_{i,vrs}^{t,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{vrs}^t}}{\frac{CE_{i,crs}^{t,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{crs}^t}{CE_{i,vrs}^{t,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{vrs}^t}} \cdot \frac{\frac{CE_{i,vrs}^{t+1,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{crs}^{t+1}}{CE_{i,vrs}^{t+1,avg}(y_i^{t+1}, z_i^{t+1}, x_i^{t+1}) \cdot K_{vrs}^{t+1}}}{\frac{CE_{i,vrs}^{t+1,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{crs}^{t+1}}{CE_{i,vrs}^{t+1,avg}(y_i^t, z_i^t, x_i^t) \cdot K_{vrs}^{t+1}}}$$

$$= SEC_i^{avg}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}), \qquad (31)$$

i.e., distributional assumptions and estimates in the second StoNED stage do not have any impact on scale efficiency change.

By combining Equations (25) and (30)-(32) we see that

$$MPI_i^{avg}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}) = MPI_i^{best}(\boldsymbol{y}_i^t, \boldsymbol{y}_i^{t+1}, \boldsymbol{z}_i^t, \boldsymbol{z}_i^{t+1}, x_i^t, x_i^{t+1}). \qquad (32)$$

Hence, the adjustments in the second StoNED do not have any impact on the total productivity change.

The StoNED approach is not the only benchmarking method where a best-practice frontier is obtained from an average-practice frontier. Other examples are Corrected Ordinary Least Squares (COLS) and Modified Ordinary Least Squares (MOLS) (Greene, 1993; Richmond, 1974). If such models are used to perform Malmquist productivity analysis, as in this paper, and if the adjustment from average-practice to best-practice takes the form of a multiplicative scaling of the cost or production values, i.e., an additive shift in terms of logged values, then (30)-(33) will be relevant. I.e., the overall productivity change and the scale efficiency change do not depend on the adjustment from average-practice to best-practice, but the efficiency change and the technical change will be affected.

6. Empirical illustration: electricity distribution in Norway

6.1 Data description

The data we will use to illustrate the StoNED Malmquist analysis is collected by the Norwegian Water Resources and Energy Directorate (NVE). It covers 123 Norwegian distribution companies for the period 2004-2013. The data set has one single input, three outputs, and five environmental variables, as described in Table 1 and Table 2.

Table 1

Inputs, outputs and environmental variables used in the model

Variable	Type	Sub-variable	Unit
Total cost	x	Operations and maintenance cost	1000NOK
		Value of lost load (quality cost)	1000NOK
		Thermal power losses	1000NOK
		Capital depreciation	1000NOK
		Return on capital	1000NOK
High voltage lines	y		Kilometers
Network stations (transformers)	y		No. of stations
Customers	y		No. of customers
Distance to road	z		Kilometers
HV underground	z		Share of HV network (0-1)
Forest	z		Share of HV lines affected (0-1)
Geol	z	Small scale hydro	Inst.cap. (MW)/cost norm[1]
		Average slope	Degrees (0-90)
		Deciduous forest	Share of HV lines affected (0-1)
Geo2	z	Wind/dist.to coast	$(m/s)^2/m$
		Islands	No. of islands /cost norm[1]
		HV sea cables	Share of HV network (0-1)

Total cost is the single input. The content of the total cost including five cost elements is listed in Table 1. Most of the companies also owns and operates part of the regional transmission network, and NVE reallocates part of this cost to the local distribution

[1] This variable is divided by the company's cost norm in order to ensure that the resulting variable is size independent. The cost norm is based on five-year average of inputs and outputs.

activity. The reallocated cost is not included in our study, so our results may therefore differ somewhat from the efficiency measurements published by NVE. The data for all years have been adjusted to the price level of a base year (2013). We use an industry-specific price index for adjusting operations and maintenance costs and the consumer price index for the VOLL (value of lost load) costs. Thermal losses are valued at the average system price at Nord Pool for the base year (300 NOK/MWh). Capital depreciation is based on reported (nominal) book values, and the return on capital is calculated using the nominal rate of return set by the regulator for the base year (7.12 %). In order to make the capital depreciation/return comparable across years, we have adjusted the capital values to the base year with an inflation rate of 2 % per year. This number corresponds, approximately, to the average inflation since the book values was established in the beginning of the 1990s, following the deregulation of the Norwegian power market.

The outputs are shown in the second part of Table 1 and include high voltage lines, network stations and customers. High voltage lines and network stations represent structural and environmental conditions which may affect required network size and thereby the cost level of the companies. The last part of Table 1 shows environmental variables. The environmental variables affect the performance of the companies, but they are out of the companies' control (Coelli et al. 1998).

Table 2

Descriptive statistics of variables

Variables	Mean	Min.	Median	Max.	Sd.dev
Total cost	108000.00	8884.00	39220.00	1771000.00	215719.80
High voltage lines	803.10	50.00	321.50	8744.00	1329.81
Network stations(transformers)	1012.00	52.00	367.00	13530.00	1888.21
Customers	22670.00	947.00	6428.00	570200.00	58710.64
Distance to road	226.00	70.37	142.90	1056.00	207.34
HV underground	0.34	0.06	0.31	0.86	0.18
Forest	0.12	0.00	0.12	0.39	0.10
Geol	0.02	-2.06	-0.43	4.72	1.49
Geo2	0.01	-0.64	-0.45	11.86	1.52

6.2 Results

As discussed in Section 3, the expected value of inefficiency is used to shift the estimates for average-practice frontier to obtain the best-practice frontier. The assumption of half-normally distributed inefficiency implies positive value for skewness, but in practice, this is not always observed. Table 3 lists the estimated skewness for each year in our data set. Under the CRS assumption, 4 out of 10 years exhibit negative skewness, and under the VRS assumption, it happens for 7 out of 10 years.

Table 3

Estimated skewness in the StoNED model

Model	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
CRS	-0.0005	0.0007	0.0012	0.0002	-0.0043	-0.0002	0.0008	-0.0006	0.0004	0.0001
VRS	-0.0014	0.0002	0.0002	-0.0007	-0.0041	-0.0019	0.0001	-0.0010	-0.0019	0.0001

Table 4 reports average productivity indices and their decompositions for the best-practice and the average-practice frontiers, respectively. The values for periods spanning more than one year are annualized by taking the geometric means over the included years. Table 4 confirms Equations (30)-(33), i.e., that the second stage StoNED adjustments do not have any impact on the productivity change and the scale efficiency change, but that the efficiency change and the technical change estimates, respectively, are affected.

For the periods 2007/08, 2008/09, and 2011/2012, the efficiency changes and the technical changes are the same for the best-practice and the average-practice frontiers, respectively. From Equations (26) and (27), we know that the efficiency change and the technical change only depend on the VRS efficiency scores. As shown in Table 3, the estimated skewness for the years 2007, 2008, 2009, 2011, and 2012 under are negative under VRS. In our application, we handle this problem as suggested by Kuosmanen (2012), i.e., by replacing the estimated skewness by a very small constant. If the standard deviation and mean of the inefficiency is the same in period t and period $t+1$, we have that

$$\mu_{vrs}^{t+1} = \mu_{vrs}^{t} => \frac{\exp(-\mu_{vrs}^{t+1})}{\exp(-\mu_{vrs}^{t})} = \frac{K_{vrs}^{t+1}}{K_{vrs}^{t}} = 1, \tag{33}$$

where $\mu_{vrs}^{t} = \sigma_{u,vrs}^{t}\sqrt{\frac{2}{\pi}}$ is the average inefficiency under VRS in period t. Hence, if the second stage StoNED adjustment is constant over time, e.g., if it is close to zero, we know from Equations (30) and (31) that EC and TC will be the same under best-practice and average-practice, respectively.

Table 5

Productivity change and its decompositions for the best- and average-practice frontiers

Period	Best-practice frontier				Average-practice frontier				Differences between the best and average practice frontier			
	MPI	EC	TC	SEC	MPI	EC	TC	SEC	MPI	EC	TC	SEC
2004/05	1.0659	0.9613	1.1095	0.9997	1.0659	1.0052	1.0520	0.9997	0.0000	-0.0440	0.0575	0.0000
2005/06	1.0334	0.9975	1.0348	1.0012	1.0334	1.0039	1.0822	1.0012	0.0000	-0.0064	-0.0474	0.0000
2006/07	1.0164	1.0579	0.9585	1.0016	1.0164	1.0061	1.0078	1.0016	0.0000	0.0517	-0.0493	0.0000
2007/08	0.9697	1.0069	0.9615	1.0012	0.9697	1.0069	0.9615	1.0012	0.0000	0.0000	0.0000	0.0000
2008/09	0.9765	1.0039	0.9725	1.0000	0.9765	1.0039	0.9725	1.0000	0.0000	0.0000	0.0000	0.0000
2009/10	1.0246	0.9702	1.0559	0.9998	1.0246	1.0056	1.0187	0.9998	0.0000	-0.0354	0.0372	0.0000
2010/11	0.9690	1.0437	0.9274	1.0010	0.9690	1.0069	0.9613	1.0010	0.0000	0.0368	-0.0339	0.0000
2011/12	1.0131	1.0049	1.0068	1.0008	1.0131	1.0049	1.0068	1.0008	0.0000	0.0000	0.0000	0.0000
2012/13	0.9802	0.9824	0.9898	1.0010	0.9802	1.0066	0.9757	1.0010	0.0000	-0.0242	0.0142	0.0000
2004/07	1.0383	1.0048	1.0324	1.0008	1.0383	1.0051	1.0469	1.0008	0.0000	-0.0003	-0.0144	0.0000
2007/10	0.9900	0.9935	0.9958	1.0003	0.9900	1.0055	0.9839	1.0003	0.0000	-0.0120	0.0118	0.0000
2010/13	0.9873	1.0100	0.9741	1.0009	0.9873	1.0061	0.9810	1.0009	0.0000	0.0039	-0.0070	0.0000
2004/13	1.0049	1.0027	1.0005	1.0007	1.0049	1.0056	1.0035	1.0007	0.0000	-0.0028	-0.0030	0.0000

As discussed above, the "wrong skewness issue" occurs in the StoNED model. We can resample the size of the dataset or respecify the model to solve the wrong skewness issue (Carree, 2002; Almanidis et al., 2011, Feng et al., 2012). In our application, the issue could not be solved by resampling the size of the data set. If we could assume that Equation (34) was true, i.e., that the relative distance between the best-practice frontier and the average-practice frontier was constant over time, we could use the average-practice results to study efficiency change and technical change as well. We see that average efficiency change estimates under the average-practice frontier are close to 1, which is consistent with (34), i.e., that the relative difference between the average and best performers, respectively, is constant. We do not claim that this observation reflects real tendencies in our data, it is merely a result of the assumption behind the average-practice frontier, as given by (34).

In any case, we can use the overall productivity change and scale efficiency change estimates, which do not depend on our distributional assumptions. We observe productivity growth for the period 2004/2007, which is consistent with Førsund and Kittelsen (1998) and Migueis et al. (2011), while productivity change is negative for 2007/2010 and 2010/2013. We observe very small scale efficiency changes, which is not surprising, since the industry structure in our data set is kept constant over time.

7. Conclusion

We have shown how the StoNED method can be combined with Malmquist analysis to investigate productivity change over time, with the usual decompositions of the overall productivity indices into efficiency change, technical change, and scale efficiency change. The distributional assumptions in the second StoNED stage influence some, but not all, of the results. Specifically, the overall productivity change and the scale efficiency change do not depend on the distributional assumptions, but the decomposition into efficiency change and technical change is affected. This implies that it does not matter whether we use the average-practice frontier or the best-practice frontier to analyze overall productivity change or scale efficiency change. Also, if the analyst can assume that the relative distance between the two frontiers are constant over time, then efficiency change and technical change can also be evaluated based on the average-practice frontier. Our results are due to the multiplicative form of the second stage adjustment in StoNED, and they will therefore also be valid when other benchmarking methods with a similar structure, such as COLS or MOLS, are used to perform Malmquist analysis. We have illustrated our discussion with data for Norwegian electricity distribution companies for the period 2004-2013.

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