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Riding the Low-Beta Wave

What drives the performance of Betting Against Beta?

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Abstract

This thesis seeks to explain the driving factors behind the Betting Against Beta (Frazzini and Pedersen, 2014) portfolio. We start by replicating the BAB factor, and then construct different portfolios in order to examine the factor's robustness, to which degree returns are driven by placements in extreme-beta stocks, and whether the factor is at all driven by industries. We find that using a different method of beta estimation dampens the portfolio's returns somewhat, but still generates positive and significant alphas. Equal-weighting and value-weighting the portfolio also leads to weaker returns, with the latter performing exceptionally poorly when looking at risk-adjusted returns, implying that returns are driven by heavy weightings in stocks of smaller market capitalisation. We also find that a portfolio that buys and sells the outermost beta-sorted deciles performs better than buying all stocks. This leads us to construct a version of the BAB portfolio that buys the outermost beta deciles and excludes micro-cap stocks, which again generates a higher alpha than the all-stocks portfolio, but produces lower risk-adjusted returns. Finally, by constructing three different kinds of industry portfolios, we corroborate findings by Asness et al. (2014) that BAB is not driven by industries.

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1 Introduction

The existence of a beta anomaly within asset pricing has become one of the most widely accepted anomalies within financial academics. It was first described by Jensen, Black and Scholes (1972), who found that the CAPM's predicted security market line was "too flat". Further studies by Fama and MacBeth (1973) and Haugen and Heinz (1975), among others, look closer into the cross-sectional relationship between risk and return, finding that stocks with low systematic risk outperform stocks with high systematic risk when measured in terms of risk-adjusted returns.

This beta anomaly was the basis for Frazzini and Pedersen's (2014) paper "Betting Against Beta", utilised by the authors in the implementation of the "BAB" portfolio. The BAB portfolio is constructed by buying low beta stocks, leveraged to a beta of one, and selling high beta stocks, deleveraged to a beta of one. The overall market-neutral portfolio achieves a significant alpha of 0.55% adjusted for the Fama and French (1993) three factors and the Carhart (1997) momentum factor for the period from 1926 to 2012.

Despite BAB's academic success, subsequent research has posed the question of whether the source of BAB's impressive returns actually is the low-beta anomaly, or rather the somewhat unorthodox methods utilised in the factor construction. This thesis probes into the elements actually driving the BAB factor, and hopes to complement existing literature by Asness et al. (2014) and Novy-Marx and Velikov (2022).

The approach in this thesis is best described as four-pronged. We identify four potential drivers of the impressive returns generated by BAB, and analyse the effect of these on a replicated BAB portfolio. The replication BAB is constructed using the methodology outlined by Frazzini and Pedersen (2014), and achieves an overall correlation of 84.70% with the daily maintained AQR BAB factors. We achieve similar means and standard deviations as the AQR BAB, but a slightly larger alpha. Nonetheless, we deem our replication adequate for analysis purposes.

We begin by examining the beta estimates. Frazzini and Pedersen use a non-standard technique to compute their ex-ante betas, which are estimated by calculating volatilities and correlations separately. The value-weighted mean of the Frazzini and Pedersen (FP) beta estimate is 1.05 with data from January 1980 to March 2022, which is in line with Novy-Marx and Velikov (2022). For comparison, we create another BAB portfolio, adjusted with a beta estimate calculated from simple rolling CAPM regressions. With a more conventional beta estimate, the BAB generates a lower alpha of 0.76% compared to 1.07% using FP betas when three-factor adjusted. The average monthly return is also lower, 0.81% compared to 1.00% using FP betas.

Secondly, we question Frazzini and Pedersen's (2014) choice of a rank-weighted portfolio. Novy-Marx and Velikov (2022) find that an equal-weighted portfolio that buys the top and bottom beta-sorted thirds of stocks performs near identically to the original rank-weighted BAB strategy. We opt to stay as close to BAB as possible by buying and selling all stocks, and find that equal-weighting the portfolio causes the Fama-French three-factor adjusted alpha to decrease from 1.07% to 0.73%. The nature of the rank-weighted portfolio also neglects the size of the stocks when assigning weights. Smaller stocks are associated with higher liquidity risk and are more expensive to trade than their larger counterparts (Achary and Pedersen, 2005). When opting for a value-weighting of the BAB portfolio, the Fama-French three-factor adjusted alpha drops to 0.41% - still a positive and significant result, but not nearly as impressive as the rank-weighted portfolio. When adjusting for robustness and investment pattern (Fama-French five-factor), the value-weighted portfolio alpha loses its significance.

Both the non-conventional beta calculation and the choice of a rank-weighted portfolio produce better risk-adjusted returns than using more conventional methods. However, the BAB methodology still produces positive, significant alphas even when using more established constructions of betas and portfolios. In order to get a grasp of the feasibility of implementing BAB as a real-world investment strategy, we look into how much of BAB's returns are driven by positions in both extreme-beta stocks as well as stocks with a relatively smaller market capitalisation.

For this, we construct decile portfolios. We sort the stocks into deciles by beta, and construct long/short portfolios using these. The first portfolio is long the bottom beta decile and short the top decile (*D1D10*). Performing factor-adjusted regressions, the *D1D10* portfolio achieves a Fama-French three-factor adjusted alpha of 1.94%, far outperforming the replication BAB (alpha of 1.07%). The portfolio also returns a higher Sharpe ratio of 1.03 compared to the replication BAB's 1.02. Value-weighting the same portfolio also outperforms the all-stocks BAB with a three-factor adjusted alpha of 1.40%, but has a lower Sharpe ratio of 0.40.

The findings from our *D1D10* portfolio point to BAB's returns being driven by positions in extreme-beta stocks. When constructing a portfolio that excludes the top and bottom deciles but buys all other stocks (*D2D9*), we find further support for this theory. Using both the *D1D10* and the *D2D9* portfolios as explanatory variables in a regression, we examine how much of the replication BAB's returns can be explained by the performance of the stocks in the different deciles. Adjusting for the *D2D9* portfolio causes the Fama-French five-factor adjusted alpha to fall from 0.74% to 0.30%. When we instead adjust for the *D1D10* portfolio, the replication BAB alpha falls into negative territory (-0.10%), implying that much of the BAB alpha can be attributed to positions in the highest and lowest beta stocks.

When segmenting the data by stock size, it becomes apparent that the BAB portfolio is biased towards stocks of a smaller market capitalisation. Recalling both Novy-Marx and Velikov's (2022), as well as our findings of lower returns when value-weighting, we construct the same *D1D10* portfolio, excluding the 40% smallest stocks. This portfolio returns a Fama-French three-factor adjusted alpha of 1.52%, implying that BAB is driven by positions in the outermost deciles even when excluding smaller-cap stocks. Using this *D1D10** portfolio as an explanatory regression variable causes the Fama-French five-factor adjusted all-stocks alpha to fall from 0.74% to 0.40%, meaning the replication BAB still generates a positive alpha. This implies that even though a lot of BAB's performance can be attributed to the outermost deciles, there is still a significant amount that is driven by smaller-cap stocks.

Finally, we investigate whether or not the BAB factor is driven by low-beta industry bets. Our findings when using 19 industry groupings, as opposed to Asness et al.'s (2014) 49, provide support for the aforementioned paper, but deviate somewhat. We form three sets of industry portfolios; an "industry BAB" that places pure industry bets by entering long and short positions in entire industries; an "industry neutral BAB" that buys (sells) stocks below (above) the median beta in each industry; and an "industry test BAB" which is designed to test whether one can achieve positive risk-adjusted returns from buying (selling) high (low) beta stocks in low (high) beta industries.

The industry BAB should produce positive and significant alphas as it solely bets on industries. However, we do not find significant alphas in our analysis, deviating from Asness et al. (2014), where their 49-industry BAB was significant. The industry-neutral BAB produces significant positive alphas, with the value-weighted version of the portfolio outperforming the equal-weighted variant. However, for us to conclude that BAB is driven by industries, these alphas would have to be negative. Our findings thereby point us in the opposite direction, corroborating the findings by Asness et al.. Finally, the industry test BAB produces significant, negative alphas, further backing the claim that BAB is not driven by industry bets.

The thesis is structured in the following manner. Section 2 presents previous literature that lays the theoretical foundation for the thesis. Section 3 details the sourcing and processing of data used in our empirical analysis, the findings of which are discussed in section 4. Section 5 summarises and concludes.

2 Literature Review

In this section we present previous literature that relates to our analysis, in order to provide the reader with the necessary theoretical background to interpret our results. We begin by presenting the theory and construction of the Betting Against Beta (BAB) portfolio. Thereafter, we discuss criticism of the original BAB article. Subsequently, we comment on previous studies that look into industry versus individual stock momentum. Finally, we present similar research performed that attempts to prove whether or not BAB's performance is industry-driven.

2.1 Betting Against Beta

Buying low-beta stocks and selling high-beta stocks has proven to deliver significant positive risk-adjusted returns. Jensen, Black and Scholes (1972) were the first to document the security market line being "too flat", implying that investors were not necessarily compensated with higher returns for taking on a higher degree of systematic risk. These findings were further corroborated by research from Fama and MacBeth (1973) and Haugen and Heinz (1975). Thus, the beta anomaly contradicts one of the most prominent models in financial academics, and it has become one of the most widely accepted anomalies in financial literature.

Subsequent literature has supported the beta anomaly across asset classes and geographies. Frazzini and Pedersen (2014) constructed a portfolio that is long low-beta assets levered to a beta of one and short high-beta assets de-levered to a beta of one. The BAB portfolio produces a significant alpha of 0.55% for US equities, adjusted for the Fama and French (1993) three-factor and the Carhart (1997) momentum factor over the period between 1926 and March 2012. In the documented model, the authors find that leverage constrained investors hold riskier assets in order to increase their expected returns, rather than buying low-beta assets and applying leverage.

Frazzini and Pedersen (2014) construct the BAB portfolio by estimating the beta of each stock. Thereafter they use a rank-weighting procedure where each stock is assigned to either the low or high-beta leg of the portfolio. Weights are calculated relative to the cross-sectional deviation of the stock's beta rank from the average rank. Ultimately, they construct a market-neutral portfolio (overall beta of zero) by buying the low-beta leg of the portfolio scaled to a beta of one and selling the high-beta leg of the portfolio, also scaled to a beta of one. We reconstruct the original BAB portfolio to use as a control. Our reconstruction achieves a daily return correlation with the AQR-maintained BAB dataset (AQR, 2022) of 84.70%, as well as similar means and standard deviations.

2.2 Critique of BAB

Despite Betting Against Beta's (Frazzini and Pedersen, 2014) great academic success and its position among the most cited articles in the Journal of Financial Economics, the authors' non-standard methods have become subject to criticism. For the purpose of this analysis, the most relevant critique of BAB is documented by Novy-Marx and Velikov (2022). The authors quantify the effects of the non-standard portfolio construction procedures in Betting Against Beta.

Novy-Marx and Velikov (2022) find that one of the factors driving BAB's performance is the chosen method of rank-weighting. Frazzini and Pedersen (2014) do not take stock market capitalisation into consideration when structuring the BAB portfolio. Novy-Marx and Velikov illustrate that rank-weighting leads to an overweighting of illiquid stocks subsequently resulting in high transaction costs. This affects the feasibility of implementing BAB in practice. In their methodology they look at the weighted average sorting variable rank, and find that an equal-weighted portfolio performs almost identically to a rank-weighted portfolio when buying the lower and selling the upper beta-sorted thirds of all stocks. Additionally, they create a value-weighted version of the same portfolio. The value-weighted portfolio performs, consistent with our findings, poorer than the rank-weighted and equal-weighted portfolios.

The second point that Novy-Marx and Velikov shed light on is BAB's unconventional ex-ante beta estimation, that involves the individual computations of volatilities and correlations. Recreating BAB with six different beta estimates, they find that computing volatilities and correlations separately leads to biased beta estimates. This stems from the value-weighted mean of beta often exceeding 1 in the period from 1968 to 2012, when the value-weighted mean of beta should be strictly 1. Our findings support this, as our shrunken beta estimate calculated using Frazzini and Pedersen (2014) methodology has a weighted time-series mean of 1.05 between 1980 and 2022.

2.3 Industry Momentum

As documented by Jegadeesh and Titman (1993), buying past winning stocks and selling past losing stocks produces positive abnormal returns. This refers to the individual stock momentum anomaly. Posterior literature has shed light on the topic that performance and momentum are mostly influenced by winning industries and not individual stocks (Moskowitz and Grinblatt, 1999). Moskowitz and Grinblatt document a strong and prevalent momentum effect in industry components of stock returns which accounts for most of the stock momentum anomaly documented by Jegadeesh and Titman.

We draw inspiration from research by Moskowitz and Grinblatt (1999) in order to look into whether BAB's performance is influenced by individual stocks or industries. Moskowitz and Grinblatt reason that their results indicate that momentum strategies are not fully diversified, as the winners and losers tend to be in the same industry. In addition, they defend their choice of industries as an explanation for the momentum anomaly because of the correlation tendencies among firms within the same industry. These correlation tendencies arise because the firms are in the same regulatory environment, have similar sensitivities to macroeconomic shocks, and are exposed to similar supply and demand fluctuations. We incorporate this in the last objective of our thesis, where we test whether BAB's performance is predominantly driven by the selection of stocks within the same industry.

2.4 Industry Driven BAB

To the best of our knowledge, previous literature considering whether the BAB portfolio is driven by industries or individual stocks is documented by Asness, Frazzini and Pedersen (2014). With data from 1926 to 2012, the authors aggregate 49 value-weighted industry portfolios. Thereafter they construct two BAB portfolios, one that is industry-neutral and one that solely bets on industries. Consequently, they find that both types of low-risk investing work, but that the industry-neutral BAB outperforms the industry BAB. By using a different industry split and different tests as seen in Moskowitz and Grinblatt (1999), we find that BAB is not industry-driven, strengthening the findings in Asness et al. (2014).

A finding of prominent interest in Asness et al. (2014) is that the BAB factor tends to be long safe and stable, and short cyclical and risky industries. An explanation for why the industry-neutral BAB delivers a higher Sharpe ratio than the industry BAB is that the industry-neutral requires more leverage. Hence, it is associated with more tail risk which explains why one can often achieve higher risk-adjusted returns. The authors explain that few investors can tolerate leverage beyond a certain point, thus raising the required rate of return for leverage-averse investors. This is consistent with the leverage aversion findings in the original Frazzini and Pedersen (2014) paper.

The Industry BAB portfolio earns an alpha of 0.31% from single-factor model regression, and the equal-weighted and value-weighted industry-neutral BAB earns an alpha of 0.66% and 0.62%, respectively. This deviates from our findings, where only the industry-neutral portfolios realise significant alphas once controlled for Fama and French's three factors. We reason that this is due to creating 19 value-weighted industry portfolios, instead of 49, in addition to the differences in time frame.

This thesis supplements the literature described above by deepening the understanding of the drivers behind the BAB factor by using different portfolio structures such as beta-sorted decile portfolios, as well as broader, more diversified industry groupings. Additionally, we find a decile-sorted portfolio structure that excludes nano and micro-cap stocks generates both a significant, positive alpha, as well as positive risk-adjusted returns. We believe this both complements preexisting literature, as well as potentially contributes to a more feasible implementation of BAB in a real-world scenario. The formation of, and findings from the aforementioned portfolio structures will be elaborated upon in the subsequent sections.

3 Data and Methodology

The sample used in our analysis contains the daily data of all common stocks listed on the NYSE, AMEX and NASDAQ from January 1980 to March 2022, collected from the Centre for Research in Security Prices (CRSP). The sample includes data on each stock's holding period return, price, shares outstanding and, if applicable, delisting return.

We ensure that all stocks in the sample are common shares listed on the three aforementioned exchanges by selecting stocks with share codes 10 or 11 and exchange codes 1, 2 or 3. To ensure non-missing returns, we remove any observations where holding period returns are equal to the following codes; -66, -77, -88 or -99, and where delisting returns are equal to the codes; -55, -66, -77 or -88. To accommodate for delisting returns, we add these to the holding period returns where applicable. In instances where a stock ceases trading without a corresponding delisting return, we opt not to add anything. Prices are transformed to absolute values in order to calculate market capitalisation, as closing prices aggregated from bid/ask averages are indicated by a negative sign.

In addition, we collect daily and monthly value-weighted stock market returns from CRSP which serve as a proxy for the market return. The 30-day and the daily aggregated T-bill rate from the Kenneth French Data Library are used as the risk-free rate. Lastly, we collect data on the Fama-French three-factor, Carhart momentum factor and Fama-French five-factor models from the Wharton WRDS database.

3.1 Beta Estimation

The purpose of this thesis is to look into which elements drive the impressive returns generated by the BAB portfolio. The first potential driver we investigate in the analysis is Frazzini and Pedersen's (2014) unusual beta construction. To look into how much of the BAB returns can be attributed to Frazzini and Pedersen's beta (hereafter referred to as FP betas), we wish to compare these to betas calculated in a different manner. We therefore employ two different methods for beta estimation. The first technique is the method described by Frazzini and Pedersen (2014) and involves individual computation of standard deviations and stock correlations with the market proxy. The second technique estimates betas from simple rolling CAPM regressions.

3.1.1 FP Beta Estimation

We follow the methodology outlined in Frazzini and Pedersen (2014) for estimating ex-ante betas. We use one day log returns for standard deviations and three-day overlapping log returns $r_{i,t}^{3d} = \sum_{k=0}^{2} ln(1 + r_{t+k}^{i})$ for correlations, to control for any non-synchronous trading. We use a one-year (252 trading days) window and require at least 120 observations within the window to calculate standard deviations. For correlations, we use a five-year (1260 trading days) window and require at least 750 observations, as correlations move slower than volatilities (Frazzini and Pedersen, 2014). Observations that do not fall within the rolling window requirements are removed, as beta calculations cannot be applied to these.

Having computed standard deviations and correlations separately, we calculate the daily beta for each stock: $\hat{\beta}_i^{TS} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m}$. To reduce the influence of outliers, we follow Vasicek (1973), Elton et al. (2014) and Frazzini and Pedersen (2014) and shrink the time series estimate of beta toward the cross-sectional mean.

$$\hat{\beta}_i = w_i\hat{\beta}_i^{TS} + (1 - w_i)\hat{\beta}^{XS} \tag{3.1}$$

Following Frazzini and Pedersen (2014) we set $w_i = 0.6$ and $\hat{\beta}^{XS} = 1$ for all periods across all stocks. The shrinkage factor does not affect the ranking of the betas as described in section 3.2. However, the shrinkage factor does affect the amount of leverage needed to scale the betas for each portfolio to one.

Due to limited processing power, we split the data frame by year into smaller sections. As a result, we end up with slightly different rolling windows for correlations and standard deviations than if we were to process the entire dataset at once. This could be the source of some slight discrepancies when comparing to the original FP betas. Still, with an overall return correlation of 84.70% in addition to similar means and extreme values compared to the original data, we conclude that our replication is accurate enough for further testing purposes.

3.1.2 CAPM Beta Estimation

We estimate CAPM betas as the slope coefficients from rolling CAPM regressions, where monthly excess stock returns are regressed on monthly excess market returns. We use a window of 60 months and require at least 36 months of observations to perform rolling CAPM regressions.

$$(r_i - r_f) = \alpha_i + \beta(r_m - r_f) + \epsilon_i \tag{3.2}$$

We follow the same procedure as in section 3.1.1 and shrink the CAPM betas towards the cross-sectional mean using the same shrinkage factor as shown in Equation 3.1. We construct monthly portfolios and calculate a monthly BAB factor when using CAPM betas, as using betas calculated on a monthly basis to scale daily portfolios leads to biased results.

Table 3.1: Beta Summary

Table 3.1 reports summary statistics for the two methods of beta calculation. FP betas (β^{FP}) are calculated using daily data from 1980 to 2022, while CAPM betas (β^{CAPM}) are calculated using monthly data from 1926 to 2022. Weighted averages are calculated as the daily value-weighted mean of FP betas and the monthly value-weighted mean of CAPM betas. The same holds true for the calculation of weighted standard deviations. Averages and standard deviations are calculated both for non-shrunken beta estimates and the shrunken beta estimates (denoted by β^{SR}).

	β^{FP}	β^{CAPM}
Number of observations	29,286,352	2,368,759
Weighted average	1.08	1.01
Weighted average (β^{SR})	1.05	1.00
Standard deviation	0.45	0.45
Standard deviation (β^{SR})	0.27	0.27

3.2 Long-Short Portfolio Construction

To test the robustness of the original BAB portfolio presented by Frazzini and Pedersen (2014), we construct different portfolio variations using different beta calculations and weightings. First, we replicate BAB as described in Betting Against Beta, to use as a control. We also calculate BAB factors using CAPM-estimated betas. Thereafter, we construct value and equal-weighted portfolios, to compare with Frazzini and Pedersen's choice of a rank-weighted portfolio.

3.2.1 Replicating Betting Against Beta

We follow the methodology described in Frazzini and Pedersen (2014). We rank all stocks in ascending order by their calculated ex-ante beta at the end of each month, before assigning them to either the long or short side of the portfolio.

The long side, with low-beta stocks, consists of every stock below the median beta at the end of each month. The short side, with high-beta stocks, includes every stock above the median beta at the end of each month. We then calculate the weights of each stock in its corresponding portfolio, based on its cross-sectional deviation from the mean rank. More formally, let z denote the $n \times 1$ vector of beta ranks and let $\overline{z} = \frac{1_n^{'} z}{n}$ be the average rank of all the stocks where n is the total number of stocks and $1_n^{'}$ is a vector of ones (Equation 3.3).

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{bmatrix} \qquad\qquad 1_n = \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix} \tag{3.3}$$

The formulas for the high and low portfolios are as following:

$$w_H = \frac{1}{k}(z - \overline{z})^{+} \qquad\qquad w_L = -\frac{1}{k}(z - \overline{z})^{-} \tag{3.4}$$

Where k is a normalizing constant calculated each month to ensure that the weights sum up to one:

$$k = \frac{1_n^{'}|z - \overline{z}|}{2} \tag{3.5}$$

The portfolios are rebalanced every month, thus we use the weights calculated at the end of the previous month to compute the weighted return for every stock in each portfolio in the current month.

The BAB factor is calculated by finding the daily sum of returns from the long side of the portfolio, r_{t+1}^{L} leveraged to a beta of 1, and the sum of returns from the short side, r_{t+1}^{H}, deleveraged to a beta of 1. This scales the overall portfolio to a beta of zero, and follows the methodology from Frazzini and Pedersen (2014):

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^{L} - r^f) - \frac{1}{\beta_t^H}(r_{t+1}^{H} - r^f) \tag{3.6}$$

Where r^f represents the daily risk-free rate, β_t^L and β_t^H are the scaling factors for the long (L) and short (H) sides of the portfolio, calculated by finding the weighted sum of daily beta estimates.

$$r_{t+1}^{L} = r_{t+1}^{'} w_L, \; r_{t+1}^{H} = r_{t+1}^{'} w_H, \; \beta_t^L = \beta_t^{'} w_L, \; \beta_t^H = \beta_t^{'} w_H \tag{3.7}$$

Table 3.2: Replication BAB, Summary Statistics, 1980 - 2022

Table 3.2 reports a summary of the replication BAB portfolio, using data in the period 1980 to 2022. The data is split into quintiles by market capitalisation, to illuminate how much of the portfolio is driven by stocks of different sizes. Average returns and average betas are calculated as a weighted average of daily returns and betas at the end of each month.

Size	Avg. MCAP	% of long	% of short	Avg. ret	Avg. beta
Lowest MCAP	27,339,740	30.80%	16.58%	0.0019	0.91
2	123,576,320	25.02%	18.05%	0.0014	0.97
3	414,623,250	16.90%	21.80%	0.0009	1.09
4	1,390,465,310	14.13%	21.91%	0.0008	1.10
Highest MCAP	18,184,770,950	13.15%	21.66%	0.0006	1.10

3.2.2 Equal-Weighted BAB Portfolio

To look into the effects of Frazzini and Pedersen's (2014) choice of a rank-weighted portfolio, we construct BAB portfolios using different weightings. We construct equal-weighted portfolios where all stocks are weighted as a proportion of the total monthly number of stocks in each leg of the portfolio at time t. As previously, weights are calculated at end-of-month and applied to the following month:

$$w^L_{t+1} = \frac{1}{n^L_t} \qquad w^H_{t+1} = \frac{1}{n^H_t} \tag{3.8}$$

where n^L_t and n^H_t denote the number of stocks in each leg of the portfolio.

These weightings deviate from the methodology utilised by Novy-Marx and Velikov (2022), where the authors construct equal-weighted portfolios by buying the top and bottom thirds of stocks sorted by beta estimates. We choose our approach where we buy and sell all stocks in order to closer follow the original BAB methodology. Additionally, we look closer into the effects of splitting up the stocks by deciles in section 4.4.1 of the analysis.

3.2.3 Value-Weighted BAB Portfolio

Likewise, we construct a value-weighted portfolio by following the same long/short split as previously. Instead of following the Frazzini and Pedersen (2014) rank-weighting, we find the total monthly market capitalisation in each leg of the portfolio, and weight stocks on their market capitalisation relative to the total market capitalisation at the end of each month. We again apply the weights to the following month:

$$w^L_{t+1} = \frac{MC^L_{i,t}}{\sum_{i=1}^{n} MC^L_{i,t}} \qquad w^H_{t+1} = \frac{MC^H_{i,t}}{\sum_{i=1}^{n} MC^H_{i,t}} \tag{3.9}$$

3.3 Decile Portfolios

When looking at the data, we observe that many of the stocks in both the long and short sides of the portfolio are stocks with a smaller market capitalisation[1], illustrated in Table

[1]The Financial Industry Regulatory Authority (FINRA) divides stocks by market capitalisation in the following way; micro-cap: market value below \$250 million; small-cap: between \$250 million and \$2 billion; mid-cap: between \$2 billion and \$10 billion; large-cap: above \$10 billion (The Financial Industry Regulatory Authority, 2022). In addition, we use the Investopedia (2022) definition of nano-cap stocks as having a market value below \$50 million.

3.2. Given that smaller stocks often tend to be less liquid than their larger counterparts, this leads to both increased risk and potential availability issues for the BAB portfolio. After sorting the data into quintiles by market capitalisation, we observe that 30.80% of the long position and 16.58% of the short position are placed in the 20% smallest stocks. To look into how the BAB factor is affected by both company size as well as positions in stocks with extreme beta values, we construct three different portfolios where stocks are sorted into deciles by their beta estimate.

3.3.1 D1D10 Portfolio

The nature of Frazzini and Pedersen's (2014) rank weighting procedure implies that the more extreme a stock's beta is, the greater it is weighted in the long-short portfolio. Part of the purpose of this analysis is to look into how much of BAB's performance is driven by positions in stocks with extreme beta values. We therefore construct a portfolio that consists of a long position in the lowest 10% of stocks (*D1*) and a short position in the top 10% of stocks (*D10*), sorted by beta. This portfolio will be used for comparison with the original BAB portfolio that assigns weights to all stocks above or below the median beta.

To find each stock's weight in either the long or short position of the portfolio, the betas are first sorted into deciles. We select the bottom 1st and upper 10th deciles to create the *D1D10* BAB portfolio, excluding all other stocks. When using a strict decile split, the Frazzini and Pedersen (2014) rank-weighting produces negative weights on certain observations. This is because the rank-weighting is determined by each stock rank's cross-sectional deviation from the mean rank, and the decile split is not necessarily consistent with the mean rank. However, Novy-Marx and Velikov (2022) find that an equal-weighted portfolio performs almost identically to a rank-weighted portfolio when buying the upper and lower thirds. The equal-weighted BAB portfolio constructed in section 3.2.2 has a correlation of 96.10% with our replication BAB portfolio, and the equal-weighted *D1D10* portfolio constructed in this section has a correlation of 93.90% with our replication BAB portfolio (Table 4.3). On this basis, we therefore proceed with equal-weighted portfolios, to ensure positive weights when buying the bottom decile and selling the top decile. Additionally, we construct a value-weighted decile portfolio for the sake of comparison, using the same deciles but instead weighting by market capitalisation.

Table 3.3: D1D10 Portfolio, Summary Statistics, 1980 - 2022

Table 3.3 reports a summary of the D1D10 portfolio, using data in the period 1980 to 2022. All stocks are sorted in ascending order by beta, and split into deciles. In the *D1D10* portfolio, stocks within the lowest beta decile comprise the long leg of the portfolio, whilst stocks within the top beta decile comprise the short leg of the portfolio. The stocks in the portfolio are equal-weighted. The data in this table is split into quintiles by market capitalisation to illuminate how much of the portfolio is driven by stocks of different sizes. Average returns and average betas are calculated as a weighted average of daily returns and betas at the end of each month.

Size	Avg. MCAP	% of long	% of short	Avg. ret	Avg. beta
Lowest MCAP	29,744,760	44.65%	20.57%	0.0024	0.93
2	116,336,790	29.32%	19.17%	0.0016	1.01
3	375,787,360	12.80%	22.54%	0.0010	1.32
4	1,257,880,600	7.64%	20.35%	0.0008	1.40
Highest MCAP	15,975,598,090	5.96%	17.37%	0.0006	1.32

3.3.2 D2D9 Portfolio

To quantify how much of BAB's performance is driven by extreme beta values, we construct an equal-weighted BAB portfolio consisting of the remaining stocks once we remove the upper and lower decile from the stock "universe". We will use this portfolio as a measure of comparison to the *D1D10* portfolio, in order to compare the alphas generated by the outermost beta values.

Table 3.4: D2D9 Portfolio, Summary Statistics, 1980 - 2022

Table 3.4 reports a summary of the D2D9 portfolio, using data in the period 1980 to 2022. All stocks are sorted in ascending order by beta, and split into deciles. In the *D2D9* portfolio, stocks in beta deciles D2 to D5 comprise the long leg of the portfolio, whilst stocks in beta deciles D6 to D9 comprise the short leg of the portfolio. The data in this table is split into quintiles by market capitalisation to illuminate how much of the portfolio is driven by stocks of different sizes. Average returns and average betas are calculated as a weighted average of daily returns and betas at the end of each month.

Size	Avg. MCAP	% of long	% of short	Avg. ret	Avg. beta
Lowest MCAP	26,199,950	19.83%	13.89%	0.0014	0.95
2	125,914,530	20.86%	17.03%	0.0012	0.98
3	422,911,280	20.13%	21.04%	0.0009	1.01
4	1,411,579,350	19.90%	23.17%	0.0007	1.01
Highest MCAP	18,488,763,020	19.30%	24.86%	0.0006	1.02

3.3.3 D1D10* Portfolio - Excluding Smallest Stocks

In addition to examining how much of the BAB factor is driven by the most extreme beta values, we also want to look into how much of the BAB factor's returns can be attributed to positions in nano and micro-cap stocks. When segmenting BAB into the five market cap. quintiles in Table 3.2, average returns are higher in the lower quintiles. Additionally, we see a bias towards stocks with low market capitalisation in the long position. We wish to observe what happens to the *D1D10* portfolio's performance when we exclude stocks that are relatively smaller. Therefore, we recreate the *D1D10* portfolio, but in this instance we only buy and sell stocks in the top 60% of market capitalisation.

Table 3.5: D1D10* - Excluding Smallest, Summary Statistics, 1980 - 2022

Table 3.5 reports a summary of the D1D10 portfolio, using data in the period 1980 to 2022 and excluding all stocks in the lowest 40% of market capitalisation. All stocks are sorted in ascending order by beta, and split into deciles. In the *D1D10** portfolio, stocks within the lowest beta decile comprise the long leg of the portfolio, whilst stocks within the top beta decile comprise the short leg of the portfolio. The stocks in the portfolio are equal-weighted. The data in this table is split into quintiles by market capitalisation to illuminate how much of the portfolio is driven by stocks of different sizes. As we only include the 60% largest stocks in this sample, the table only includes data for quintiles 3 through 5. Average returns and average betas are calculated as a weighted average of daily returns and betas at the end of each month.

Size	Avg. MCAP	% long	% short	Avg. returns	Avg. beta
3	375,787,360	46.97%	39.00%	0.0022	1.32
4	1,257,880,600	22.79%	33.82%	0.0016	1.40
Highest MCAP	15,975,598,090	31.39%	27.17%	0.0011	1.32

3.4 Industry Bets

3.4.1 Industry Portfolio Aggregation

To investigate whether BAB is driven by industries or individual stocks, we divide all stocks into 19 industry groups, following similar SIC code groupings done by Moskowitz and Grinblatt (1999), Boudoukh et al. (1994) and Jorion (1991). We originally followed the exact groupings done in Moskowitz and Grinblatt (1999), with 20 industry groups by SIC code. However, an own group for railroad stocks did not make sense to use in our data sample, averaging only 7 stocks over the period 1980 - 2022. We instead include railroads in the category for transport, in order to ensure large enough sample groups. Our industry groupings are shown in Table 3.6.

We aggregate daily industry betas and returns through a value-weighting of each individual stock's β^{FP} and returns by total market capitalisation in each industry. The average number of stocks in each industry group is 163. The least amount of stocks in an industry is 18. We argue that the number of stocks in each industry implies negligible firm-specific risk through ample diversification.

3.4.2 Industry Portfolio Construction

To investigate whether the BAB factor is driven by industries, we construct three different types of industry portfolios. For all portfolios, we calculate a BAB factor as in Equation 3.6 for comparison to the original FP BAB factor.

Table 3.6: Description and Summary Statistics of Industries

Table 3.6 contains monthly summary statistics for the 19 industry portfolios formed, in the period between 1980 to 2022. Groupings are done monthly by SIC code retrieved from the CRSP database, allowing for variation over time in the classification of industries. Excess returns are calculated as the time-series weighted monthly average of returns in each industry portfolio, in excess of the risk-free return. Abnormal returns are calculated as the difference between actual and expected returns, where expected returns are calculated as $r_E = r_F + \beta_i(r_M - r_F)$. Average overall no. of stocks, returns and abnormal returns are reported as well, in addition to an F-statistic ensuring that the mean of returns significantly differs across all industries.

	Industry	SIC code	Avg. no. stocks	Avg. % of mcap	Avg. excess ret	Avg. abnormal returns
1	Mining	10-14	124	3.06%	0.0066	-0.0021
2	Food	20	71	3.67%	0.0103	0.0043
3	Apparel	22-23	39	0.32%	0.0057	-0.0022
4	Paper	26	29	0.74%	0.0086	0.0007
5	Chemicals	28	219	11.05%	0.0123	0.0037
6	Petroleum	29	18	3.28%	0.0109	0.0034
7	Construction	32	19	0.22%	0.0087	0.0007
8	Prim. Metals	33	42	0.63%	0.0078	-0.0006
9	Fab. Metals	34	55	0.64%	0.0098	0.0022
10	Machinery	35	184	7.36%	0.0101	0.0015
11	Electrical Eq.	36	239	6.21%	0.0115	0.0031
12	Transport Eq.	37	64	2.78%	0.0098	0.0011
13	Manufacturing	38-39	202	3.88%	0.0113	0.0038
14	Transport	41-47	65	1.91%	0.0095	0.0003
15	Utilities	49	134	4.26%	0.0081	0.0026
16	Dept. Stores	53	23	2.52%	0.0074	-0.0001
17	Retail	50-52,54-59	290	6.30%	0.0104	0.0027
18	Financial	60-69	574	16.97%	0.0095	0.0027
19	Other	other	704	24.20%	0.0112	0.0029
	Average		163	5.26%	0.0095	0.0016
	F-stat (all the same)				0.999	0.793
	(p-value)				*(0.261)*	*(0.458)*

3.4.3 Industry BAB

The first portfolio formed places pure industry bets. The 19 industries are ranked monthly by their respective aggregated industry beta in ascending order, and an industry k is calculated for each month. This is used to calculate industry weights as in section 3.2.1. However, as opposed to in section 3.2.1, where weights were given to each individual stock, weights are instead assigned to each industry as a whole. Stocks within each industry are value-weighted, as betas and returns within each industry were aggregated by value-weighting, as in Asness et al. (2014).

Industries with an aggregated industry beta below the end-of-month median beta are assigned to the long side of the portfolio, while industries with an aggregated beta above the median are assigned to the short side of the portfolio. Lastly, we create three versions of the Industry BAB portfolio; rank-weighted, equal-weighted, and value-weighted.

3.4.4 Industry-Neutral BAB

For comparison to the industry BAB that places pure industry bets, we form an industry-neutral BAB portfolio. The industry-neutral BAB is constructed similarly to the industry-neutral portfolios in Moskowitz and Grinblatt (1999) and Asness et al. (2014). Within each industry, we go long stocks below the median industry beta, and short stocks above the median industry beta. We then construct a BAB factor aggregated from each industry's BAB. We create two versions of the industry-neutral BAB portfolio; one value-weighted and one equal-weighted.

This differs slightly from the methodology used to create an industry-neutral portfolio in Moskowitz and Grinblatt (1999), where the authors hold a long position in the top 30% of winning stocks and short the bottom 30% of losing stocks in each industry. We draw inspiration from the portfolio they create, and apply the methodology of going long low-beta stocks and short high-beta stocks within each industry portfolio. However, as our purpose is to look into how much of BAB is potentially driven by industries, we choose to buy and sell all stocks in order to stay as true to the original BAB construction as possible.

3.4.5 Industry Test BAB

The final industry portfolio is based on the construction of one of the portfolios in Moskowitz and Grinblatt (1999). The authors create a portfolio that takes a long position in losing stocks from winning industries, and shorts winning stocks from losing industries. We apply the same methodology used to examine the driving effects of industries on momentum to look into the driving effects of industries on the BAB factor.

First, all industries are ranked by their aggregated industry beta, and the three industries with the highest and lowest industry betas are selected. Thereafter, we choose the 30% of stocks with the highest beta in the low-beta industries, and the 30% of stocks with the lowest beta in the high-beta industries. As with all our other portfolios, the industry test portfolio is rebalanced on a monthly basis. Weights are calculated at the end of each prior month to determine positions in the current month.

As in the industry-neutral portfolio, we create both equal-weighted and value-weighted portfolios. We call it the "industry test" BAB because this portfolio assesses if one can achieve abnormal returns when buying (selling) high (low) beta stocks in low (high) beta industries.

4 Empirical Analysis

This section presents the results from the different BAB portfolios constructed. Our main objective is to identify the drivers of BAB's performance. We start by comparing our replication to the daily BAB factors updated by AQR. Further, we analyse how a different beta estimation affects BAB's performance. Thereafter we look at the effects of applying different weightings. In addition, we investigate how extreme-beta values affect BAB, by sorting the data into beta-sorted deciles and analysing decile-based portfolios. We also examine what happens within these decile portfolios once stocks with a lower market capitalisation are excluded from the mix. The final element we analyse is whether, and if so, how much the BAB factor is driven by industries.

Table 4.1: Summary of Individual Stock Portfolios

Table 4.1 reports the monthly summary statistics of the individual stock long-short portfolios from January 1985 to March 2022. Note that returns and alphas are reported in percentages, whilst standard deviations are in decimals. We extract alphas (α) after performing three-factor model regression with robust standard errors, thus the t-statistics are adjusted for heteroscedasticity. We use data from 1980, but due to a five-year window for beta calculations, the portfolio's returns are not generated until 1985. *AQR* is the AQR-maintained BAB factor, *Replication* is our rank-weighted (RW) replication BAB. All-Stocks refers to the three additional portfolios we construct using different beta estimates (β^{FP} and β^{CAPM}) and weightings. With β^{CAPM} we only construct a RW portfolio, whilst with β^{FP} we construct equal-weighted (EW) and value-weighted (VW) portfolios, in addition to the RW Replication portfolio. *D1D10* denotes the upper and bottom decile portfolio which has both equal and value-weighted iterations. *D2D9* denotes the decile portfolio that excludes the upper and bottom decile. Lastly, *D1D10** denotes the upper and bottom decile portfolio that excludes nano and micro-cap stocks.

Portfolio	AQR	Replication	All-Stocks			*D1D10*		*D2D29*	*D1D10**
Weigthing	RW	RW	RW	EW	VW	EW	VW	EW	EW
Beta Estimate	β^{FP}	β^{FP}	β^{CAPM}	β^{FP}	β^{FP}	β^{FP}	β^{FP}	β^{FP}	β^{FP}
Starting Year	1985	1985	1985	1985	1985	1985	1985	1985	1985
Monthly Return	0.78	1.00	0.81	0.60	0.28	1.67	0.97	0.40	1.18
Standard Deviation	0.037	0.034	0.032	0.027	0.031	0.056	0.084	0.021	0.070
Min. Return	-16.00	-11.90	-12.67	-10.54	-14.11	-15.77	-39.14	-10.29	-23.53
Max. Return	15.31	19.14	16.36	14.72	15.19	31.54	49.97	10.65	40.65
FF3 α	0.80	1.07	0.76	0.73	0.41	1.94	1.40	0.51	1.52
t-stat α	4.26	6.11	5.24	5.37	2.92	7.24	3.66	4.77	4.34

4.1 Replication versus AQR BAB

In addition to the dataset used in the original "Betting Against Beta" article (Frazzini and Pedersen, 2014), AQR has published an updated and extended version of the original paper data, available on AQR's website (AQR, 2022). As the original dataset ends in March 2012, we use the AQR-maintained dataset in the analysis in order to compare with data up till March 2022. When looking at the returns of our replication BAB we convert the daily data (as well as the daily equity factors from AQR's website) to monthly using formula 4.1 where n is the amount of daily observations in each month t, and r_i is the daily BAB return.

$$r_t = \left(\prod_{i=1}^{n} (r_i + 1) \right) - 1 \tag{4.1}$$

As mentioned in section 3.1.1, we split the data to compare our replication BAB with the maintained AQR BAB. The first BAB horizon is from 1926 to 1970, where the daily factors have a correlation of 79.39%. For the second horizon, we use data from 1964 to 1985 and achieve a correlation of 90.73%. The final split is from 1980 to March 2022, where the correlation between the AQR and replication BAB portfolios is 88.27%. Once we merge the years, we have an overall correlation of 84.70% with the daily maintained AQR BAB. With monthly returns, we achieve a correlation with the AQR sample from 1985 to 2022 of approximately 90%, as reported in Table 4.3.

To fully compare if we have managed to replicate BAB, we perform regressions with monthly data from 1985 to 2022 and compare our results with the AQR BAB. We use the technique of robust standard errors to obtain unbiased standard errors of OLS coefficients under heteroscedasticity. We perform the following regressions:

$$r_{BAB} = \alpha_i + \beta MKT_t + \epsilon_i \tag{4.2}$$

$$r_{BAB} = \alpha_i + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_i \tag{4.3}$$

$$r_{BAB} = \alpha_i + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \epsilon_i \tag{4.4}$$

$$r_{BAB} = \alpha_i + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t + \beta_5 CMA_t + \epsilon_i \tag{4.5}$$

Where *MKT* represents the excess market return in month *t*, and *SMB*, *HML*, *UMD*, *RMW* and *CMA* represent the return of the size, value, momentum, robustness and investment factors in month *t*.

As shown in Table 4.2, our replication BAB achieves an alpha of 1.14% with a t-statistic of 6.28 when we perform single-factor model regressions. The AQR BAB has a lower alpha of 0.89% with a t-statistic of 4.56. Further, we perform multi-factor model regressions. Once controlling for Fama and French's three factors and Carhart's momentum factor, our replication BAB achieves an alpha of 0.87%, significant with a t-statistic of 4.93. The AQR BAB achieves a slightly lower alpha of 0.59% (t-statistic 3.08). Alphas with their respective t-statistics, beta ex-post and Sharpe ratios are documented in Table 4.2. The Sharpe ratios of the replication and AQR BAB from the period 1985 to 2022 are 1.02 and 0.73, respectively. Although our replication BAB outperforms the AQR BAB, we conclude that we have replicated BAB to an adequate degree due to our high correlation, similar mean and standard deviation, and positive and significant alphas.

4.2 BAB's Performance with CAPM Beta

As described in section 3.1, we construct two separate beta estimates for comparison purposes; Frazzini and Pedersen's (2014) ex-ante beta and a more conventional beta estimate extracted as the slope from CAPM regressions. Table 3.1 documents that the value-weighted average of the FP beta estimate exceeds one, both for the original and shrunken beta estimates. The value-weighted mean is supposed to be equal to the market beta, which is strictly one. However, when computing various beta estimates, Novy-Marx and Velikov (2022) also find that the value-weighted FP beta often exceeds one, implying a biased beta estimate. Our value-weighted CAPM beta is 1.0005 after applying the shrinkage factor.

When performing factor model regression with the BAB portfolio constructed using CAPM betas, we realise lower alphas than our replication BAB, documented in Table 4.2. Once controlled for Fama and French's three factors and Carhart's momentum factor, the portfolio achieves an alpha of 0.56% with a t-statistic of 4.05. Controlling for Fama and French's five factors, the portfolio achieves an alpha of 0.53%, compared to replication BAB which earns an alpha of 0.74% (Table 4.2). We observe that the CAPM BAB realises positive and significant alphas, but the alphas are of a smaller magnitude than the replication BAB. We therefore reason that the Frazzini and Pedersen beta estimate is one of the elements that drives BAB's performance.

We note that a biased beta estimate especially affects the leverage needed to scale the portfolios to a beta of one. Asness et al. (2014) argue that a strategy that needs more leverage is associated with more tail risk. Hence, a biased beta estimate results in applying more leverage than needed, thereby raising the required rate of return. This assumption is supported when we observe that the BAB portfolio with betas estimated from rolling CAPM regressions has a lower kurtosis than our replication BAB (Table 4.2), although it is negatively skewed.

4.3 Equal and Value-Weighting BAB

Shifting the focus to Frazzini and Pedersen's choice of a rank-weighted portfolio, we examine BAB's performance when constructing equal and value-weighted portfolios. Novy-Marx and Velikov (2022) found that an equal-weighted portfolio performs almost identically to a rank-weighted portfolio when buying the upper and lower beta-sorted thirds. This section considers the strategy of buying all stocks, in order to stay as close to the original BAB construction as possible. Hence, we achieve lower means than the replication BAB when constructing equal and value-weighted stock portfolios, documented in Table 4.1.

When controlling for Fama and French's three factors, the equal-weighted portfolio achieves a positive and significant alpha of 0.73% , whilst the value-weighted BAB has a significant alpha of 0.41% (Table 4.2). Once adjusted for Fama and French's five factors, the equal-weighted BAB has a significant alpha of 0.45%, whilst the value-weighted BAB alpha falls to 0.05% and is rendered statistically insignificant (t-statistic 0.38). The equal-weighted portfolio, on the other hand, produces positive and significant alphas throughout all factor model regressions (Table 4.2). Comparing the Sharpe ratios of the different portfolios, the value-weighted BAB falls short of the mark, with an annualised Sharpe ratio of 0.31. By contrast, the market's Sharpe ratio is 0.60[2]. indicating the value-weighted version of BAB is a poor choice in terms of risk-adjusted return.

Table 4.2: Performance Measures: All-Stocks Portfolios

In Table 4.2 we extract the alphas and t-statistics from monthly returns regressed on monthly factors. *AQR* is the AQR-maintained BAB-factor, *Replication* is our replication BAB, which is constructed following methodology from Frazzini and Pedersen (2014). CAPM is the BAB portfolio constructed using betas from CAPM regression. EW and VW denote the portfolios constructed using β^{FP}, but equal or value-weighting instead of Frazzini and Pedersen's rank-weighting. We report the ex-post beta, annualised Sharpe ratios, skewness and kurtosis for each portfolio, where the Sharpe ratios consider returns from 1985 to 2022. This explains the deviation from the Sharpe ratios reported in Frazzini and Pedersen (2014) where they report a Sharpe ratio of 0.78 from 1926 to 2012. For information purposes, the monthly average risk free rate is 0.26% from 1985 to March 2022.

BAB Portfolios	AQR	Replication	CAPM	EW	VW
CAPM	0.89%	1.14%	0.84%	0.78%	0.47%
	(4.56)	(6.28)	(5.20)	(5.69)	(3.20)
FF3	0.80%	1.07%	0.76%	0.73%	0.41%
	(4.26)	(6.11)	(5.24)	(5.37)	(2.92)
FF3 + MOM	0.59%	0.87%	0.56%	0.58%	0.26%
	(3.08)	(4.93)	(4.05)	(4.14)	(1.82)
FF5	0.45%	0.74%	0.53%	0.45%	0.05%
	(2.40)	(4.38)	(3.59)	(3.43)	(0.38)
Beta ex-post	−0.15	−0.18	−0.04	−0.24	−0.26
Sharpe Ratio	0.73	1.02	0.88	0.77	0.31
Skewness	-0.44	0.41	-0.19	0.41	0.00
Kurtosis	6.57	7.29	6.42	8.20	7.13

The equal-weighted portfolio has a monthly mean of 0.60%, twice as large as the value-weighted portfolio with a mean return of 0.28% (Table 4.1). This illustrates how a rank-weighted portfolio that assigns larger weights to extreme beta values performs better

[2]We collect the monthly excess market return from Kenneth French Data Library from January 1985 to March 2022. The Sharpe ratio is computed by dividing the average excess market return with its standard deviation. Sharpe ratios (SR) are annualised by $SR\sqrt{12}$.

than both the equal and value-weighted portfolios. Looking at the correlation between the different portfolios in Table 4.3, the equal and value-weighted BAB have a correlation with the replication BAB of 96.10% and 66.00%, respectively. When adjusting the three-factor regressed replication BAB for the highly correlated equal-weighted portfolio, the alpha falls from 1.07% to 0.10%. When adjusting the replication BAB for the value-weighted portfolio, however, we achieve an alpha of 0.80%, supporting the notion that the choice of rank-weighting drives a large amount of BAB's returns.

4.4 Decile Portfolios

4.4.1 D1D10 Portfolio

Considering our findings from section 4.3, we see that a value-weighted portfolio yields far less impressive returns than both a rank-weighted and an equal-weighted portfolio. This indicates greater weightings are being assigned to stocks of more extreme beta values and driving BAB's performance. The *D1D10* decile portfolio was constructed in order to closer examine exactly how much of the BAB portfolio performance can be attributed to these positions in the most extreme-beta stocks. As documented in Table 4.5, the *D1D10* portfolio achieves a Fama-French three-factor adjusted alpha of 1.94% (t-statistic 7.24) when performing factor-adjusted regressions. The portfolio thereby outperforms the replication BAB with an alpha of 1.07% (t-statistic 6.11) (Table 4.2). Additionally, the *D1D10* portfolio appears to generate higher risk-adjusted returns than the all-stocks replication BAB portfolio, with a Sharpe ratio of 1.03 (Table 4.5) compared to the replication BAB with a Sharpe ratio of 1.02 (Table 4.2). This bears implications that the replication BAB portfolio's performance is being pulled in a positive direction by positions in the outermost beta deciles.

The value-weighted *D1D10* portfolio also outperforms the all-stocks replication BAB, achieving a three-factor adjusted alpha of 1.40% (t-statistic 3.66) (Table 4.5). However, with a Sharpe ratio of 0.40, the risk-adjusted return achieved is less than both the equal-weighted *D1D10* portfolio with a Sharpe ratio of 1.03 (Table 4.5), as well as the equal-weighted all-stocks portfolio with a Sharpe ratio of 0.77 (Table 4.2).

With this in mind, we recall Table 3.3 in section 3.3.1. In this table, the data is split by market capitalisation, sorting the stocks into size quintiles. Where the long leg of the portfolio is heavily dominated by small-cap stocks, the short leg is more evenly distributed between stocks of all sizes. This points to stocks of higher market capitalisation having relatively higher betas, and stocks with lower market capitalisation having relatively lower betas on average. As the Sharpe ratio is calculated as the excess returns of a risky asset divided by its standard deviation, stocks with a higher standard deviation, ceteris paribus, increase the numerator in $\hat{\beta}_i = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m}$ and increase the beta estimate. This implies that the more (relatively) high-beta stocks a portfolio contains, the lower its risk-adjusted return, all else held equal. As stocks with higher market capitalisation - and higher FP betas on average as seen in Table 3.3 - are assigned greater weights in the value-weighted portfolio, the Sharpe ratio takes a hit.

Looking at the data, we see that several of the lowest-beta stocks also are those with the lowest market capitalisation, with several of the smallest and most illiquid stocks producing negative betas. As these are assigned greater importance in both the equal and rank-weighted versions of the portfolios, this seems to be an important factor driving the BAB returns. However, as documented by Novy-Marx and Velikov (2022), this leads to issues in terms of both transaction costs and availability, as the smallest stocks are generally more illiquid and expensive to trade (Achary and Pedersen, 2005). We look into the effects of removing the smallest stocks from the stock "universe" in section 4.4.3.

4.4.2 Excluding the Upper and Bottom Decile

Our findings of BAB being driven by extreme-beta stocks are corroborated when we reconstruct the same portfolio as in section 4.4.1, but instead exclude the two outermost deciles. Constructing a portfolio that buys beta deciles D2 through D5 and sells beta deciles D6 through D9 results in a Fama-French three-factor adjusted alpha of 0.51% (Table 4.5). The alpha is still positive and significant, but considerably lower than alphas generated from both the replication and the *D1D10* BAB portfolios. In other words, the stocks in the upper and lower beta deciles appear to contribute positively to the BAB factor, whereas the stocks in the remaining deciles dampen the abnormal returns achieved through the BAB strategy.

Table 4.3: Correlations: Monthly Individual Stock Portfolio Returns

Table 4.3 presents the monthly return correlation between the individual stocks portfolios. The sample runs from January 1985 to March 2022. AQR is the maintained BAB factor, Replication is the rank-weighted portfolio with β^{FP}. CAPM is the rank-weighted portfolio estimated with β^{CAPM}. EW and VW BAB are the equal and value-weighted BAB portfolios. EW *D1D10* and VW *D1D10* is the equal and value-weighted upper and bottom decile portfolio. *D1D10** is the equal-weighted upper and bottom decile portfolio that excludes micro-cap stocks. *D2D9* denotes the portfolio that excludes the upper and bottom decile.

	AQR	Replication	CAPM	EW BAB	VW BAB	EW $D1D10$	VW $D1D10$	*D1D10**	*D2D9*
AQR	1								
Replication	0.899	1							
CAPM	0.685	0.650	1						
EW BAB	0.856	0.961	0.597	1					
VW BAB	0.599	0.660	0.471	0.723	1				
EW $D1D10$	0.827	0.939	0.612	0.911	0.623	1			
VW $D1D10$	0.516	0.578	0.462	0.644	0.779	0.589	1		
*D1D10**	0.661	0.746	0.515	0.794	0.719	0.735	0.779	1	
D2D9	0.824	0.903	0.542	0.970	0.702	0.802	0.609	0.741	1

Table 4.4: Deciles as Explanatory Variables

Table 4.4 shows the results from regression performed using the replication BAB as the dependent variable. Fama and French´s three factors (2), Carhart's momentum factor (3), Fama and French´s five factors (4), as well as the three decile portfolios; *D1D10* (5), *D2D9* (6), and *D1D10** (7) are used as explanatory variables. Standard deviations are reported in parentheses underneath the coefficients. ***Significance at 1% level.

	REPLICATION 1985 - 2022						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Alpha	0.011^{***}	0.011^{***}	0.009^{***}	0.007^{***}	-0.001^{**}	0.003^{***}	0.004^{***}
	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
MKT	-0.183^{***}	-0.146^{**}	-0.075	-0.057	0.118^{***}	0.065^{***}	0.041
	(0.066)	(0.064)	(0.061)	(0.061)	(0.017)	(0.021)	(0.042)
SMB		-0.038	-0.034	0.137^{**}	0.033	0.079^{**}	0.189^{***}
		(0.066)	(0.070)	(0.068)	(0.022)	(0.039)	(0.049)
HML		0.273^{***}	0.381^{***}	0.036	0.001	0.115^{**}	0.105
		(0.082)	(0.089)	(0.092)	(0.028)	(0.052)	(0.081)
UMD			0.269^{***}				
			(0.057)				
RMW				0.564^{***}	0.140^{***}	0.059	0.210^{***}
				(0.095)	(0.027)	(0.045)	(0.067)
CMA				0.303^{**}	0.010	-0.107^{*}	0.014
				(0.137)	(0.041)	(0.061)	(0.097)
EW $D1D10$					0.588^{***}		
					(0.014)		
$D2D9$						1.528^{***}	
						(0.063)	
$D1D10*$							0.349^{***}
							(0.031)

By using the *D1D10* and *D2D9* portfolios as explanatory variables in a regression, we examine exactly how much of the replication BAB portfolio can be explained by stock performance in these deciles, documented in Table 4.4. When adding the *D2D9* portfolio as an explanatory variable, the replication BAB alpha falls to 0.30%, but is still positive and significant, implying that the replication portfolio that includes all stocks still generates returns after having been corrected for deciles 2 through 9. When adding equal-weighted *D1D10* as an explanatory variable, the replication BAB alpha falls to -0.10%, with a significant loading of 0.59 on the EW *D1D10* portfolio. In other words, when correcting for the *D1D10* deciles, the replication BAB no longer generates positive returns, implying that the positive alphas can be attributed to positions in the top and bottom deciles.

4.4.3 D1D10*: Excluding Bottom Market Cap. Quintiles

When segmenting the *D1D10* decile portfolio by company size, it becomes apparent that a large part of both the long and short legs of the portfolio are comprised of stocks with a low market capitalisation, as seen in Table 3.3. This is in line with what we observe in Table 3.2, where we segment our replication BAB by market capitalisation. This is further corroborated by our findings in both section 4.3 and section 4.4.1, where we observe significantly lower returns from value-weighted portfolios, due to a reduced weighting of small-cap stocks. On average, 44.65% of the long leg of the *D1D10* portfolio and 20.57% of the short leg of the *D1D10* portfolio between 1980 and 2022 is positioned in the smallest 20% of stocks (Table 3.3). This is in line with findings from Novy-Marx and Velikov (2022), where the authors find that both the long and short sides of the portfolio invest "aggressively" into stocks of nano and micro-cap sizes. As Novy-Marx and Velikov critique, this has implications when attempting to implement the BAB portfolio strategy in a real-world situation, as smaller stock sizes tend to go hand-in-hand with liquidity limitations (Achary and Pedersen, 2005).

The *D1D10* and *D2D9* decile portfolio constructions indicate that much of the BAB factor's results are driven by positions in stocks in the outermost beta deciles. However, from these findings we are unable to draw any conclusions regarding exactly how much of the BAB factor is driven by positions in micro-cap stocks. In order to look into this, we reconstruct the *D1D10* portfolio, but exclude the bottom two market capitalisation quintiles as described in section 3.3.3.

Once again, we observe that the long position is biased towards stocks with a smaller market capitalisation, with 50.52% of the position in the third market cap quintile (Table 3.5). The short side is more evenly distributed between company sizes, but also has a slight bias towards the stocks in the third market capitalisation quintile.

From Table 4.5 we see that the *D1D10** portfolio also produces a significant Fama-French three-factor adjusted alpha of 1.52% (t-statistic 4.34), again outperforming the replication BAB factor (alpha 1.07%, t-statistic 6.11) (Table 4.2). The returns generated are slightly less than when including stocks of all sizes, but three times larger than the returns from the *D2D9* portfolio. This indicates BAB returns are driven by placements in smaller, more illiquid stocks to a certain degree. However, as returns are still greater than when buying and selling all stocks, from our results it appears probable that BAB returns are mostly driven by placements in the outermost beta deciles.

Regressing the *D1D10** portfolio as an explanatory variable on the replication BAB portfolio backs up these findings, illustrated in Table 4.4. The alpha falls from 0.74% (Fama-French five-factor) to 0.40%, implying that although a large part of the replication BAB alpha is explained by the *D1D10** portfolio, the replication BAB still generates a positive, significant alpha. Especially when taking into consideration that the replication BAB alpha falls into negative territory when adjusted for the original *D1D10* portfolio, this becomes apparent. Furthermore, with a Sharpe ratio of 0.58 (Table 4.5), the *D1D10** portfolio generates lower risk-adjusted returns than both the *D1D10* and the replication BAB portfolio. When excluding smaller stocks, the portfolio is less volatile, as documented in Table 4.1, which decreases the denominator in the Sharpe ratio formula:

$$SR = \frac{\overline{r}_P - \overline{r}_f}{\sigma_{r_P r_F}} \tag{4.6}$$

However, we attribute the lower risk-adjusted return to the overall reduction in the average monthly return for the *D1D10** portfolio.

Table 4.5: Performance Measures: Decile Portfolios

Table 4.5 contains an overview of alphas and their corresponding t-statistics extracted from regressions performed on the decile portfolios. The decile portfolios are used as dependent variables, with monthly returns regressed on monthly factors. EW and VW *D1D10* denote the equal and value-weighted upper and bottom decile portfolios. *D2D9* is the portfolio that excludes the upper and bottom deciles. *D1D10** represents the upper and bottom decile portfolio that excludes micro and nano-cap stocks. The excess market return, Fama-French three and five-factor models, as well as the Carhart momentum factor (MOM) are used as explanatory variables. In addition, we report ex-post betas and annualised Sharpe ratios.

Decile BAB Portfolios	EW $D1D10$	VW $D1D10$	*D2D9*	*D1D10**
CAPM	2.04%	1.52%	0.54%	1.62%
	(7.33)	(3.83)	(5.14)	(4.67)
FF3	1.94%	1.40%	0.51%	1.52%
	(7.24)	(3.66)	(4.77)	(4.34)
FF3 + MOM	1.66%	1.04%	0.39%	1.15%
	(6.22)	(2.70)	(3.49)	(3.39)
FF5	1.48%	0.59%	0.29%	0.84%
	(5.54)	(1.71)	(2.80)	(2.63)
Beta ex-post	−0.48	−0.73	−0.17	−0.58
Sharpe Ratio	1.03	0.40	0.66	0.58

4.5 Is BAB Driven by Industries?

The final element this thesis investigates is whether BAB is driven purely by individual stocks, or if the strategy unwittingly takes industry bets. We hope to complement research by Asness et al. (2014), by using different portfolio constructions and industry groupings. Asness et al. construct 49 different industry portfolios, which arguably are relatively less diversified and more exposed to firm-specific risk. We instead follow groupings done by Moskowitz and Grinblatt (1999), Boudoukh et al. (1994), and Jorion (1991), splitting the data into 19 value-weighted industry portfolios. Additionally, we focus on a more recent time frame, from 1980 to 2022, as opposed to 1926 to 2012.

4.5.1 Industry BAB

The first portfolio constructed from our aggregated industry portfolios takes pure industry bets by buying (selling) industries below (above) the monthly median industry beta. Three versions of the pure industry BAB are created using different weighting strategies; a rank-weighted, an equal-weighted and a value-weighted iteration. Both the rank-weighted and the value-weighted portfolios have a mean BAB return of approximately 0.08%. The value-weighted portfolio has a slightly lower standard deviation of 0.027, compared to 0.035 for the rank-weighted portfolio (Table 4.6). The equal-weighted portfolio has a mean of 0.06% and a standard deviation of 0.027. Thus, the value-weighted industry BAB achieves the highest risk-adjusted return with a Sharpe ratio of 0.10 (Table 4.8), but is still weak when compared to the individual stock portfolios. Out of the three industry BAB portfolios, the rank-weighted portfolio has the highest correlation with the replication BAB of 54.40% (Table 4.9).

Table 4.6: Summary of Industry BAB Portfolios

Table 4.6 reports the monthly summary statistics of the seven industry BAB portfolios from January 1985 to March 2022. Note that the monthly returns and alphas are reported in percentages, whilst standard deviations are in decimals. We extract alphas (α) after performing three-factor model regression with robust standard errors, thus the t-statistics are adjusted for heteroscedasticity. The Industry BAB portfolio goes long and short high-beta industries, and has rank (RW), equal (EW) and value-weighted (VW) iterations. The Industry-Neutral BAB goes long (short) industries below (above) each industry's monthly median beta and is equal and value-weighted. The final portfolio, the Industry Test, goes long high-beta stocks in low-beta industries and vice versa, and is equal and value-weighted.

Portfolio	Industry BAB			Industry-Neutral		Industry Test	
Weigthing	RW	EW	VW	EW	VW	EW	VW
Starting Year	1985	1985	1985	1985	1985	1985	1985
Monthly Return	0.081	0.056	0.076	-0.028	-0.026	-0.987	-0.377
Standard Deviation	0.035	0.027	0.027	0.039	0.047	0.056	0.079
Min. Return	-16.04	-11.82	-16.47	-16.91	-19.19	-58.30	-74.20
Max. Return	17.46	11.40	10.57	18.57	21.26	14.01	68.35
FF3 α	0.22	0.16	0.13	0.34	0.41	-1.05	-0.33
t-stat α	1.41	1.34	1.09	2.09	2.72	-4.14	-0.89

We perform the same regressions with robust standard errors for our industry portfolios as documented in Equations 4.2 through 4.5. The industry BAB portfolios produce alphas, but none of them are significant. When controlling for Fama and French's five factors, they realise negative, but still insignificant alphas (Table 4.8). As a result, we cannot draw the same conclusion as Asness et al. where they find that betting on low-beta industries proves profitable. If BAB is driven by industries, we would expect the industry BAB to produce significant alphas. Nonetheless, due to the portfolio's relatively high degree of correlation with the replication BAB, we opt to perform more tests to investigate how much of BAB's performance potentially can be attributed to industries.

As mentioned, if BAB is driven by low-beta industries, a portfolio that buys low and sells high-beta industries should, in theory, achieve positive and significant alphas. Additionally, once controlling for industries, we would expect the replication BAB to realise a lower alpha and become less profitable. When adding the industry BAB (Equation 4.7) as an explanatory variable in a regression with our replication BAB as the dependent variable, we observe that the alpha decreases with 7 bps from 0.87% to 0.80% (Table 4.7). This indicates that once we control for industries, the BAB portfolio becomes less profitable. Nevertheless, the statistical significance of the BAB intercept is still strong and we cannot use this to conclude that industry bets are among the elements driving BAB's performance.

$$r_{BAB} = \alpha_i + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \beta_5 RWIndustry_t + \epsilon_i \quad (4.7)$$

4.5.2 Industry-Neutral BAB

Asness et al. (2014) find that an industry-neutral BAB outperforms the industry BAB, producing positive and significant alphas. Our industry-neutral BAB does, to some extent, complement this finding.

Out of the seven different industry portfolios constructed, the industry-neutral BAB is the only portfolio that produces positive and significant alphas once controlled for Fama and French's three factors (Table 4.8). Using the excess market return as the only explanatory variable, the equal-weighted industry-neutral BAB realises an alpha of 0.36% (t-statistic 2.15) and the value-weighted industry-neutral BAB produces an alpha of 0.43% (t-statistic 2.82). When performing three-factor model regression, the equal and value-weighted portfolios realise alphas of 0.34 % (t-statistic 2.09) and 0.41 % (t-statistic 2.72), respectively (Table 4.8). We note that if BAB were industry-driven, the industry-neutral BAB should exhibit significant negative alphas.

We again perform regressions where the industry-neutral BAB is used as an explanatory variable and the replication BAB is used as the dependent variable (Equation 4.8). We choose the equal-weighted industry-neutral BAB as an explanatory variable because it is the industry portfolio with the highest degree of correlation with the replication BAB, with a correlation of 69.30%, documented in Table 4.9.

$$r_{BAB} = \alpha_i + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \beta_5 IndustryNeutral_t + \epsilon_i \tag{4.8}$$

Using the equal-weighted industry-neutral portfolio as a control, we observe that the four factor-adjusted alpha again decreases by 7 bps, from 0.87% to 0.80% (Table 4.7). This implies that once controlled for a BAB portfolio within each industry, the strategy becomes less profitable. We note that the industry-neutral BAB has a larger loading on the replication BAB than the industry BAB. Where the industry BAB has a loading of 0.45, the industry-neutral has a loading of 1.03.

We again perform a regression with the replication BAB as the dependent variable, and add the rank-weighted industry BAB and the equal-weighted industry-neutral BAB as explanatory variables (Equation 4.9).

$$r_{BAB} = \alpha_i + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 Industry_t + \beta_5 IndustryNeutral_t + \epsilon_i \tag{4.9}$$

Our results corroborate the findings of Asness et al. (2014) where the authors find that

BAB has a larger loading on the industry-neutral BAB than the industry BAB. From our results, the loading on the industry-neutral BAB is 0.90, whilst the industry BAB has a coefficient of 0.17 (Table 4.7). The alpha declines from 1.07% to 0.7% when controlling for Fama and French's three factors and the industry portfolios. Given the larger loading on the industry-neutral BAB, the relationship between the replication BAB and industries bears characteristics of being more industry-neutral than industry-driven.

The industry-neutral BAB can be decomposed into each of the 19 industries' individual BAB returns. The portfolio achieves a negative monthly average, and therefore a negative annualised risk-adjusted return. Regardless of the overall negative average return, we draw inspiration from Asness et al. (2014), and look at each industry's individual industry-neutral BAB portfolio. We begin by looking at the daily equal-weighted BAB returns in each industry-neutral BAB portfolio. We observe that none of the individual BAB factors in each of the 19 industries signify positive daily returns on average. The best performing category is Other, with a daily average BAB return of -0.001%, followed by Financial (-0.005%) and Retail (-0.034%). The three worst performing industries are Construction (-0.49%), Apparel (-0.42%) and Fabricated Metals (-0.37%).

The value-weighted industry-neutral portfolio, on the other hand, achieves an overall better performance than the equal-weighted portfolio. Despite a negative mean BAB factor overall, we observe that eight of the industries achieve positive daily average returns. The three industries with the strongest BAB performance are Petroleum, Department Stores and Transport, with daily BAB returns of 3.33%, 3.13% and 1.99% on average. The three industries with the lowest BAB returns are, in descending order, Apparel (-12.61%), Construction (-8.50%) and Fabricated Metals (-3.66%). We find the results from the industry-neutral BAB to be contradictory to the findings in Asness et al. (2014), where they find that all industry-neutral BAB factors in each of the 49 industry portfolios have a positive Sharpe ratio. We attribute the difference in findings to the different ways the portfolios are constructed, given that we have larger groupings and fewer industry "buckets".

4.5.3 Industry Test BAB

The final portfolio structure formed to look into the effect of industries on BAB was the test portfolio, based on framework from Moskowitz and Grinblatt (1999). As the portfolio buys (sells) high (low) beta stocks in low (high) beta industries, we reason it should produce positive and significant alphas if we are to draw the conclusion that BAB is driven by low-beta industries. In our case, however, it produces significant negative alphas (Table 4.8), pointing towards BAB not being industry driven, thereby supporting the reasoning of Asness et al. (2014).

Table 4.7: Industry Portfolios as Explanatory Variables

Table 4.7 shows the results from regression performed with Replication BAB as the dependent variable and using Fama and French´s three factors (1), Carhart's momentum factor (2), as well as the rank-weighted industry portfolio (3), equal-weighted industry-neutral portfolio (4) as explanatory variables on their own. Column (5) shows the regression results when using both as explanatory variables. Standard deviations are reported in parentheses underneath the coefficients.
***Significance at 1% level.

	REPLICATION 1985 - 2022				
	(1)	(2)	(3)	(4)	(5)
Alpha	0.011***	0.009***	0.008***	0.008***	0.007***
	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
MKT	−0.145**	−0.075	−0.013	0.598***	0.552***
	(0.064)	(0.061)	(0.057)	(0.042)	(0.038)
SMB	−0.039	−0.035	0.106*	0.316***	0.325***
	(0.066)	(0.070)	(0.058)	(0.045)	(0.043)
HML	0.272***	0.381***	0.268***	0.235***	0.236***
	(0.082)	(0.089)	(0.075)	(0.037)	(0.034)
UMD		0.269***	0.188***	−0.071**	
		(0.057)	(0.052)	(0.029)	
RW Industry			0.449***		0.165***
			(0.052)		(0.031)
EW Neutral				1.026***	0.898***
				(0.050)	(0.047)

As with the other portfolios, both an equal and value-weighted version of the test portfolio were formed, using data spanning the period between 1980 and March 2022. The equal and value-weighted portfolios achieve a mean of -0.99 % and -0.38 % respectively (Table 4.6). Standard deviation for the equal-weighted test portfolio is 0.056 and the value-weighted portfolio has a standard deviation of 0.079. Both the equal-weighted and value-weighted test portfolios have a negative risk-adjusted return.

Table 4.8: Performance Measures: Industry Portfolios

Table 4.8 contains an overview of alphas and their corresponding t-statistics extracted from regressions performed on the various industry portfolios, using the Fama-French three and five-factor models, as well as the Carhart momentum factor as explanatory variables. The portfolios denoted as Industry take pure industry bets, portfolios denoted as Neutral are industry-neutral, and the portfolios of type Test buy high-beta stocks in low-beta industries and vice versa. RW, EW and VW denote the weighting each portfolio is structured with; rank, equal or value-weighting. Ex-post betas and annualised Sharpe ratios are also reported for each of the seven industry portfolios.

Portfolio	RW Industry	EW Industry	VW Industry	EW Neutral	VW Neutral	EW Test	VW Test
CAPM	0.28%	0.20%	0.19%	0.36%	0.43%	-0.97%	-0.28%
	(1.70)	(1.60)	(1.49)	(2.15)	(2.82)	(-3.69)	(-0.78)
FF3	0.22%	0.16%	0.13%	0.34%	0.41%	-1.05%	-0.33%
	(1.41)	(1.34)	(1.09)	(2.09)	(2.72)	(-4.14)	(-0.89)
FF3 + MOM	0.08%	0.08%	0.05%	0.08%	0.27%	-1.01%	-0.25%
	(0.51)	(0.64)	(0.40)	(0.53)	(1.74)	(-3.96)	(-0.64)
FF5	−0.09%	-0.05%	-0.13%	-0.01%	-0.01%	-1.07%	-0.07%
	(-0.58)	(-0.45)	(-1.08)	(-0.04)	(-0.04)	(-4.22)	(-0.18)
Beta ex-post	−0.25	−0.18	−0.14	−0.79	−0.59	-0.02	-0.12
Sharpe ratio	0.08	0.07	0.10	-0.02	-0.02	-0.61	-0.17

All in all, our findings using industry groupings and methodology from Moskowitz and Grinblatt (1999) provide support to the claims made by Asness et al. (2014). We do not find statistical evidence that BAB is driven by industries, even when applying different techniques and larger, more diversified industry groupings. However, our findings when using 19, rather than 49 industry groupings differ somewhat from Asness et al., as few of our industry portfolios achieve positive risk-adjusted returns.

Table 4.9: Correlations: Monthly Industry Portfolio Returns

Table 4.9 presents the monthly return correlation between replication BAB and the industry portfolios. The sample runs from January 1985 to March 2022. RW Industry, EW Industry and VW Industry denote the rank, equal and value-weighted industry BAB portfolios. EW Neutral and VW Neutral are the equal and value-weighted iterations of the industry-neutral BAB portfolio. EW Test and VW Test are the equal and value-weighted industry test BAB portfolios.

	Replication	RW Industry	EW Industry	VW Industry	EW Neutral	VW Neutral	EW Test	VW Test
Replication	1							
RW Industry	0.544	1						
EW Industry	0.473	0.957	1					
VW Industry	0.527	0.831	0.828	1				
EW Neutral	0.693	0.534	0.492	0.470	1			
VW Neutral	0.550	0.685	0.661	0.635	0.811	1		
EW Test	−0.022	0.301	0.249	0.296	0.022	0.072	1	
VW Test	0.021	0.108	0.037	0.140	0.016	−0.003	0.437	1

5 Conclusion

In this thesis, we apply various methods to examine the robustness of the techniques in the original Betting Against Beta portfolio proposed by Frazzini and Pedersen (2014). We start by replicating the portfolio, which is designed to capture the beta anomaly by buying low-beta stocks and selling high-beta stocks, applying leverage to create a market-neutral portfolio.

Having replicated the portfolio, we recreate BAB with a different beta estimate and create equal and value-weighted portfolios to discover whether BAB is driven by the effects of the low-beta anomaly, or if it is mainly driven by Frazzini and Pedersen's non-standard methodology. In line with what Novy-Marx and Velikov (2022) describe, we find that both a different beta estimate, as well as the use of equal-weighting, dampen the BAB factor but still produce positive and significant alphas. A value-weighted portfolio realises an annualised Sharpe ratio below that of the market, as well as an insignificant four and five-factor adjusted alpha.

BAB's poor performance when value-weighting points to the choice of rank-weighting as a driver of returns. This is due to stocks with more extreme beta values being assigned greater weights when rank and equal-weighting. Constructing a portfolio that only buys (sells) the lower (upper) beta-sorted stock deciles, we see the full effects of this, with a higher realised alpha and Sharpe ratio than the all-stocks portfolios. Examining a portfolio consisting of all stocks but excluding the outermost deciles corroborates this, dampening the alpha generated by BAB. The shift away from extreme-beta stocks when value-weighting is indicative of the small size of these stocks. The replication BAB portfolio is biased towards small-cap stocks. Constructing the same portfolio that buys and sells the outermost deciles, but excludes the 40% smallest stocks produces a higher alpha than when buying and selling all stocks. However, the risk-adjusted returns are lower.

Finally, we look into whether the returns generated by BAB actually are the result of exploiting the low-beta anomaly, or if it unwittingly takes industry bets. We construct three different sets of industry portfolios; one that takes pure industry bets, one industry-neutral and one that buys (sells) high (low) beta stocks in low (high) beta industries. Our findings cannot conclude that BAB is driven by the selection of stocks in the same industries, but rather point in the opposite direction - that BAB is not driven by industries.

Our contribution with this thesis is to deepen the understanding of the Betting Against Beta strategy in terms of the drivers behind it. Additionally, we believe our findings that constructing a portfolio excluding nano and micro-cap stocks in the top and bottom beta deciles produces both a significant alpha and positive risk-adjusted returns can contribute to a more feasible implementation of BAB in practice.

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