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The Magic Formula

An empirical study of Joel Greenblatt's magic formula, backtested on the Oslo Stock Exchange

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Abstract

This study analyzes the performance of Joel Greenblatt's *magic formula* on the Oslo Stock Exchange from May 2003 to May 2022. The investment strategy involves ranking stocks based on their earnings yield and return on capital. We employ the four-factor model of Fama and French (1993) and Carhart (1997) to measure the strategy's alpha. Our results indicate that the *magic formula* generates risk-adjusted excess, with a monthly alpha of 0.5% over the sample period, statistically significant at the $p < 0.05$ level. Additionally, it outperforms the OSEAX by 8.02 percentage points in compound annual growth rate and has a Sharpe ratio of 44 decimal points higher. However, when implementing transaction costs, the alpha is only significant at the $p < 0.1$ level, suggesting that risk-adjusted excess returns are not achievable in real-world conditions.

Keywords – Value investing, *magic formula*, backtesting, Oslo Stock Exchange

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1 Introduction

"Choosing individual stocks without any idea of what you are looking for is like running through a dynamite factory with a burning match. You may live, but you are still an idiot." - Joel Greenblatt (2005)

Value investing, first introduced by Graham and Dodd in the 1920s, is a well-established investment strategy focusing on purchasing stocks priced below their intrinsic value. This approach challenges the efficient-market hypothesis, which posits that such bargains do not exist in markets where participants have complete and symmetric information. Value investing has been subject to intense scrutiny, and various strategies have emerged from its origins. The topic has attracted significant attention from market professionals and academics, including Joel Greenblatt, who developed a *magic formula* for outperforming the market over time. Greenblatt backtested the *magic formula* on the US Market from 1988 to 2004.

In this study, we backtest the *magic formula* on the Oslo Stock Exchange from May 2003 to May 2022. We examine the strategy's performance in a smaller market and a more recent time period than Greenblatt did. We compute simple returns and adjust for risk using the Sharpe ratio and the four-factor model of Fama and French (1993) and Carhart (1997), with the Oslo Børs All Share Index (OSEAX) as the benchmark. This approach allows us to evaluate the strategy's ability to generate significant risk-adjusted excess returns, as measured by the alpha. Additionally, we conduct several robustness tests on the strategy, including implementing transaction costs, value-weighting the portfolio, selecting varying numbers of stocks in the portfolios, uneven metrics weighting, and applying alternative proxy metrics.

The initial results are promising, and after adjusting for risk, we find that the strategy generates risk-adjusted excess returns over the sample period. However, when we implement transaction costs, the excess returns are not significant at a $p < 0.05$ level, indicating that the strategy may not be profitable in real-world conditions. In addition, our findings indicate that the strategy marginally improves by substituting EBIT for operating cash flow in the ranking process and selecting 15 stocks in the portfolio.

Joel Greenblatt is a renowned finance academic, hedge fund manager, an alumnus of the Wharton School of the University of Pennsylvania, and an adjunct professor at Columbia University Graduate School of Business. Greenblatt argues that systematically combining value and profitability is a profound way to beat the market. He quantifies this value-profitability paradigm in his book *The Little Book That Beats the Market* (2005). Greenblatt claims to have discovered a *magic formula* for investing in the stock market and asserts that the strategy outperforms the market, most professional traders, and index funds in the US over time. He bases the formula on fundamental investing principles and explains it logically.

The fundamental principles of the *magic formula* revolve around companies' earnings yield and return on capital. Greenblatt presents his logic clearly and compellingly. For instance, in the case of earnings yield: if a company earned 1 USD last year and the price of a share in that company is 10 USD, the earnings yield is 10%. Now, consider a bond yielding 5% as an alternative risk-free investment. The investment in the stock yielded more. However, it is uncertain whether earnings will increase or decrease next year. Since we cannot predict the future of a business, we focus on the things we know. If the company's earnings were 2 USD last year and the stock price is still at 10 USD, the earnings yield is 20%. Investing in the business becomes more attractive than before. For the sake of logic, let earnings be 5 USD with the stock price remaining at 10 USD, resulting in an earnings yield of 50%. Now, all else being equal, the investment is more desirable as the company earns relative more to the price. Based on this logical argument, Greenblatt concludes that in a perfect scenario, a rational individual will always prefer the highest earnings yield when investing (Greenblatt, 2005).

The second metric of the *magic formula* is the return on capital. Greenblatt explains that a high return on capital is a clear indication of a company's profitability. For instance, a company that earned 50,000 USD last year on assets worth 500,000 USD has a return on capital of 10%, while another company that earned 250,000 USD on assets worth 500,000 USD has a return on capital of 50%. So, all else being equal, an investor would prefer the second company due to its higher return on capital. Greenblatt argues that a business model with high earnings relative to the cost of making the earnings is a clear indication of how profitable the company is (Greenblatt, 2005).

Greenblatt proposes a method for ranking stocks based on their earnings yield and return on capital. He argues that companies with high earnings yield represent good bargains, and those with a high return on capital represent good businesses. As such, Greenblatt suggests that investors should prioritize stocks that are good businesses at bargain prices. In Greenblatt's backtest, he has a dataset of 3,500 stocks available for trading in the US. To rank stocks, Greenblatt first assigns each stock a rank based on its earnings yield, with the highest-ranked stock receiving a rank of 1 and the lowest-ranked stock receiving a rank of 3,500. He then repeats this process for return on capital, with the highest-ranked stock receiving a rank of 1 and the lowest-ranked stock receiving a rank of 3,500.

The final rank for each stock is the sum of its earnings yield and return on capital ranks. For instance, a company with the second-highest earnings yield and the 500th-highest return on capital will have a combined rank of 502. Another company with the 200th-highest earnings yield and the third-highest return on capital will have a combined rank of 203. The company assigned a rank of 203 has the better combined rank in this case. Greenblatt form a portfolio of the 20–30 best-ranked stocks each year. He claims that this strategy yielded an average annual return of 30.8% from 1988 to 2004, outperforming the S&P500 by 18.4 percentage points. The strategy beat the S&P500 14 out of the 17 years in the sample period (Greenblatt, 2005).

We structure this thesis into five main sections, including the introduction. Section 2 reviews relevant literature on the *magic formula*. Section 3 describes the data collection, the adjustment processes, and the empirical methodology employed in this study. Section 4 presents and analyzes the empirical results, including initial returns, risk-adjusted performance, and robustness tests. Finally, in section 5, we draw a conclusion.

2 Literature review

Since the publication of Joel Greenblatt's *The Little Book That Beats the Market* in 2005, several academics have questioned the validity of his findings. In an article published in Barron's, Alpert (2006) critiques Greenblatt's strategy and identifies several shortcomings. Despite extensive research on the formula by academics, the consensus indicates that it provides an excess return compared to the market. However, achieving an annual return of almost 30% seems unlikely, leading some to suggest that transaction costs and other frictions may have impaired the performance.

The main point of contention is that the results may be too data-specific. The article presents several examples of replications of the formula using different databases, which all show excess returns, but not to the same extent as reported in Greenblatt's book. Robert Haugen, a renowned finance researcher, believes that Greenblatt's approach effectively identifies profitable companies at bargain prices. Haugen states that the strategy naturally favors companies that sell software, services, or brand-name products, as they typically have high profits relative to tangible assets (Alpert, 2006).

Moreover, Alpert (2006) notes that it is unclear whether the formula used by Greenblatt takes advantage of a persistent feature in the economy and employs frequent data mining techniques to do so. Additionally, Greenblatt's method of filtering companies is not well understood, as he states that he deducts excess cash when calculating enterprise value in return on capital, but the exact nature of this deduction remains unclear.

Davydov et al. (2016) conducted a backtest of the *magic formula* on the Finnish Stock Market from 1993 to 2013. They found that the *magic formula* generated an average annual return of 19.26%, outperforming the market's 13.63%. However, other value investing strategies outperformed the market as well, including an E/P strategy yielding 20.50%, an EBIT/EV strategy yielding 20.57%, a B/P strategy yielding 16.74%, a CF/P strategy yielding 19.04%, and a cash flow-augmented *magic formula* (MF-CF) strategy yielding 20.17%.

The study also tested the risk associated with the returns using the three-factor model of Fama and French (1993) and the Sharpe ratio. The results showed that the Sharpe ratio was higher than the market's for every value investing strategy, with EBIT/EV being the best and the *magic formula* ranking fourth. The *magic formula*, E/P, EBIT/EV, and MF-CF all showed significant alphas of approximately 7% annually. The researchers found that an EBIT/EV strategy had the best risk-adjusted performance in the Finnish stock market from 1993 to 2013. They found that the *magic formula* was not among the best value investing strategies in the Finnish stock market during this period.

Blackburn and Cakici (2017) investigate the *magic formula's* efficacy in generating risk-adjusted excess returns and explain the cross-section of returns across several stock markets in four global regions: North America, Europe, Japan, and Asia. They backtest the strategy in the period from January 1991 to December 2016. They form a portfolio that is long the quintile of profitable value stocks and is short the quintile of unprofitable growth stocks. Their results from the four-factor model of Fama and French (1993) and Carhart (1997) contradict Greenblatt's, as the strategy yields significant risk-adjusted excess returns only in Europe. In North America, Japan, and Asia, the returns are insignificant and, occasionally, negative.

They also test an altered version of the *magic formula*, replacing EBIT with gross profit, motivated by Novy-Marx (2013). This altered *magic formula* yields significant risk-adjusted excess returns across all global regions. Correspondingly, Blackburn and Cakici (2017) conclude that the profitability hypothesis of Novy-Marx (2013) indicates a stronger predictor of the cross-section of returns. Furthermore, they test if small and illiquid stocks drive the returns. They find the risk-adjusted returns to be positive and significant for both small- and large-cap stocks, even though the spreads are wider for small-cap stocks. Lastly, they use independent double sorts and Fama-MacBeth regression to test whether size, book-to-market, and momentum explain the relationship between the strategy and the cross-section of returns. By double sorting on the *magic formula* and market capitalization, as well as on book-to-market, they find significant return differentials for almost all sizes and most book-to-market quintiles. The Fama-MacBeth regression also generates significant results. They conclude that the altered *magic formula*, using gross profit instead of EBIT, can indeed predict the cross-section of returns globally.

3 Data and Methodology

We conduct a backtest of the *magic formula* from May 2003 to May 2022 for equities listed on the Oslo Stock Exchange. We risk-adjust the performance of the strategy using the four-factor model (i.e., *MKT*, *SMB*, *HML*, *PR1YR*) of Fama and French (1993) and Carhart (1997) to measure for a significant alpha. Transaction costs are subsequently implemented to detect if the alpha is still significant. Finally, we conduct several robustness tests to ensure the validity of the results.

3.1 Data collection

We collect most of the data from the Compustat database from Wharton Research and Data Services, a reliable database that provides financial- and market data for more than 80,000 active and inactive publicly traded companies (Wharton Research Data Services, 2022).

We collect data from 2002 to 2022, where the first year of trading is 2003. We use this sample period because the Compustat database has insufficient data for previous years. A sample period of 19 years allows us to sufficiently evaluate the strategy's performance over a long time horizon and capture a wide range of market conditions.

We use the Global Company Key (GVKEY), a unique six-digit number earmarked to each company in the Compustat database, as an identifier for the companies included in the backtesting process. We filter the database for all companies listed on the Oslo Stock Exchange, except for utilities, financial stocks, and companies with incomplete or untimely information. We use Compustat's North American Industry Classification System (NAICS) to exclude utilities and financial stocks. We exclude financial firms as they have business models where high leverage is common, whereas, for nonfinancial firms, high leverage could indicate distress (Fama and French, 1992). Utilities also tend to be highly levered as they usually require expensive infrastructure and operate in excessively regulated markets. These sectors have business models and characteristics different from the other sectors and may invalidate the ranking system (Greenblatt, 2005).

The Compustat database includes some companies traded over-the-counter (OTC). We exclude those companies because our research focuses on stocks listed on the Oslo Stock Exchange. Furthermore, these companies may have insufficient liquidity and turnover to provide reliable stock returns.

The market capitalization cut-off suggested by Greenblatt is in the range of \$50M – \$100M. However, Greenblatt tested the strategy in the US market, which contains a larger number of stocks and a higher fraction of stocks with large market capitalization than the Norwegian market. Even if we set the cut-off at the lower end of Greenblatt's suggested range, the number of companies in the dataset reduces significantly. Therefore, we set the cut-off at 200 MNOK to ensure a sufficient number of stocks in the dataset.

We collect the necessary accounting data for each selected company to properly rank and form portfolios. The data includes EBIT, long-term interest-bearing debt, cash, net property, plant, and equipment, and net working capital for each reported fiscal year. In addition, we collect daily data on close prices, an adjustment factor for stock splits, a total return factor for dividends, and common shares outstanding for each company. We also collect additional annual accounting data and daily data from the Compustat database to conduct robustness tests. We collect daily bid-ask data for individual stocks from the Refinitiv Eikon Datastream, one of the world's leading providers of financial markets data (Refinitiv, 2022). We use this data to calculate the indirect transaction cost of the bid-ask spread.

We collect time-series data for daily OSEAX returns, the monthly Fama and French (1993) three-factor framework, and the momentum factor from Carhart (1997), from the website of Bernt Andre Ødegaard, a professor at the University of Stavanger. Ødegaard calculates *SMB*, *HML*, and *PR1YR* using the same methodology as Fama and French (1993) and Carhart (1997) on Norwegian data. He sources the data from the Oslo Stock Exchange Data Service until November 2020 and the remaining data from Yahoo finance. Risk-free rates are monthly forward-looking, estimated by Norges Bank using government securities and NIBOR (Ødegaard, 2022).

3.2 Variables

3.2.1 Adjusted stock prices

To ensure accurate returns calculations, we implement an adjustment in compliance with the Compustat manual (Wharton Research Data Services, n.d.). This adjustment is necessary because companies could have distributed dividends or undertaken a stock split during the sample period. These actions affect a company's stock price and distort the calculated returns. To prevent this distortion, we divide the stock price by the adjustment factor for stock splits (AJEXDI) and multiply it by the total return factor that corrects for cash equivalent distributions, reinvestment of dividends, and the compounding effect of dividends paid on reinvested dividends (TRFD). This adjustment ensures that our returns calculations are not affected by company stock price changes due to dividends or stock splits.

$$Adjusted\,Close\;Price = \frac{Close\;Price}{AJEXDI} * TRFD \tag{3.1}$$

3.2.2 The *magic formula* metrics

Return on Capital

Return on capital is one of the two metrics in the *magic formula* Greenblatt uses to rank companies. Using this metric, he intends to evaluate companies' core business profitability. Greenblatt uses EBIT instead of earnings to avoid distortions caused by different taxation regiments and debt levels. He uses tangible capital, instead of total assets, to identify the essential assets required to conduct a company's core business. He explicitly excludes goodwill because it usually arises from acquiring companies and is a historical cost that does not need constant replacement. As such, tangible assets consist of net fixed assets and net working capital. We use net property, plant, and equipment as net fixed assets because these are the bare minimum of tangible assets companies need to conduct their business. Greenblatt uses net working capital because it is necessary to fund a company's receivables and inventory (Greenblatt, 2005).

We observe some anomalies while analyzing the companies' return on capital. For instance, some companies have a return on capital at unusually high levels ranging from 1,000% to 150,000%, which may be an error in the data. Alternatively, this anomaly could result from a company experiencing financial distress, which may have led to a write-down of some of its capital assets and a reduction in long-term debt. We winsorize return on capital at the 95th percentile to prevent these outliers from biasing the results. After winsorizing, the highest return on capital is approximately 100%.

$$Tangible\,Capital = Net\,Fixed\,Assets + Net\,Working\,Capital \tag{3.2}$$

$$ROC = \frac{EBIT}{Tangible\,Capital} \tag{3.3}$$

Earnings Yield

The second metric in the *magic formula* is earnings yield. Greenblatt measures earnings yield as the ratio of EBIT-to-enterprise value. His intention in using earnings yield is to determine how much a company earns relative to the price paid for the company. While price-to-earnings is a more commonly used valuation ratio, Greenblatt highlights its limitations. He uses enterprise value instead of the market value of equity, as it also considers the level of debt financing used to generate operating income (Greenblatt, 2005). We measure the pre-tax earnings yield on the total purchase price of a company as the ratio of EBIT-to-enterprise value, allowing us to fairly compare companies with different capital structures and tax rates.

We observe some anomalies while analyzing the companies' earnings yields. For instance, some companies have earnings yields above 100%, meaning that EBIT is higher than enterprise value. Such unusually high levels of earnings yield may be errors in the data. Alternatively, companies with high EBIT the previous year but now experiencing a decreasing stock price because of financial distress may have unusually high earnings yield. Therefore we winsorize earnings yield at the 95th percentile to prevent these outliers from biasing the results.

$$Market\,Capitalization = Close\,Price * Common\,Shares\,Outstanding \tag{3.4}$$

$$Enterprise\,Value = Market\,Capitalization + Long\text{-}term\,Debt - Cash \tag{3.5}$$

$$Earnings\,Yield = \frac{EBIT}{Enterprise\,Value} \tag{3.6}$$

3.3 Backtesting

In this study, we employ backtesting as an empirical methodology. Backtesting aims to simulate the historical performance of an investment strategy. While it can be a helpful tool, Schumann (2018) highlights several pitfalls and criticisms of its viability. For instance, some finance professionals argue that running multiple backtests can lead to strategies that appear profitable on a given data set but are actually profitable by chance. In addition, the problem of intentionally overfitting, where simple models with few parameters perform well in-sample, can mislead investors into supposedly successful investment strategies. Look-ahead bias can also be an issue, as some data and information may have been publicly available later than accounted for in the backtest. Additionally, backtesting must include poorly performing companies that go bankrupt to avoid survivorship bias. In the backtest of the *magic formula*, we employ a methodology that mitigates these pitfalls to obtain the most reliable results possible.

3.4 Portfolio formation

To rank the companies in the *magic formula* in year *t*, we use accounting data from the annual reports for year *t-1* and market equity data from the day of the ranking. This approach allows us to use the most current and accurate data on past performance and asset values, which is essential in estimating future performance according to Greenblatt (2005). Norwegian law requires companies listed on the Oslo Stock Exchange to publish their annual report within four months of the end of the reporting period (cf. Verdipapirhandelloven

§5-5). Therefore, we conduct all rankings, and stock purchases in the backtest on the first trading day of May in year t to avoid look-ahead bias. Market capitalization is calculated as the close price times the number of shares outstanding on the first trading day of May in year t. To calculate the enterprise value, we add long-term interest-bearing debt and subtract cash from the market capitalization.

We rank stocks in year t by assigning a rank based on the companies' earnings yield, where we assign a rank of 1 to the company with the highest earnings yield. Then, we assign a rank for return on capital, using the same approach. Finally, we combine the assigned ranks for earnings yield and return on capital for each stock to get a combined ranking. For instance, a company with the highest earnings yield and the 18th-highest return on capital in the dataset will have a combined rank of 19. Greenblatt recommends buying shares in the 20–30 companies with the best combined ranking. However, he used a much larger dataset than ours, with 3,500 stocks available in the US stock market. After data cleansing, our annual dataset ranges from 59 to 166 companies. To avoid survivorship bias, we include delisted companies. Suppose a company in the portfolio delists during the holding period. In that case, we calculate the return until the delisting and hold the cash from that position until the next portfolio rebalancing. Given that our dataset contains significantly fewer stocks than Greenblatt's, we choose to have 20 stocks in the portfolio, at the lower end of Greenblatt's recommendation. If a company remains among the top 20 ranked companies in May of year $t+1$, we rebalance that particular position to maintain equal weighting of the portfolio, following Greenblatt's approach.

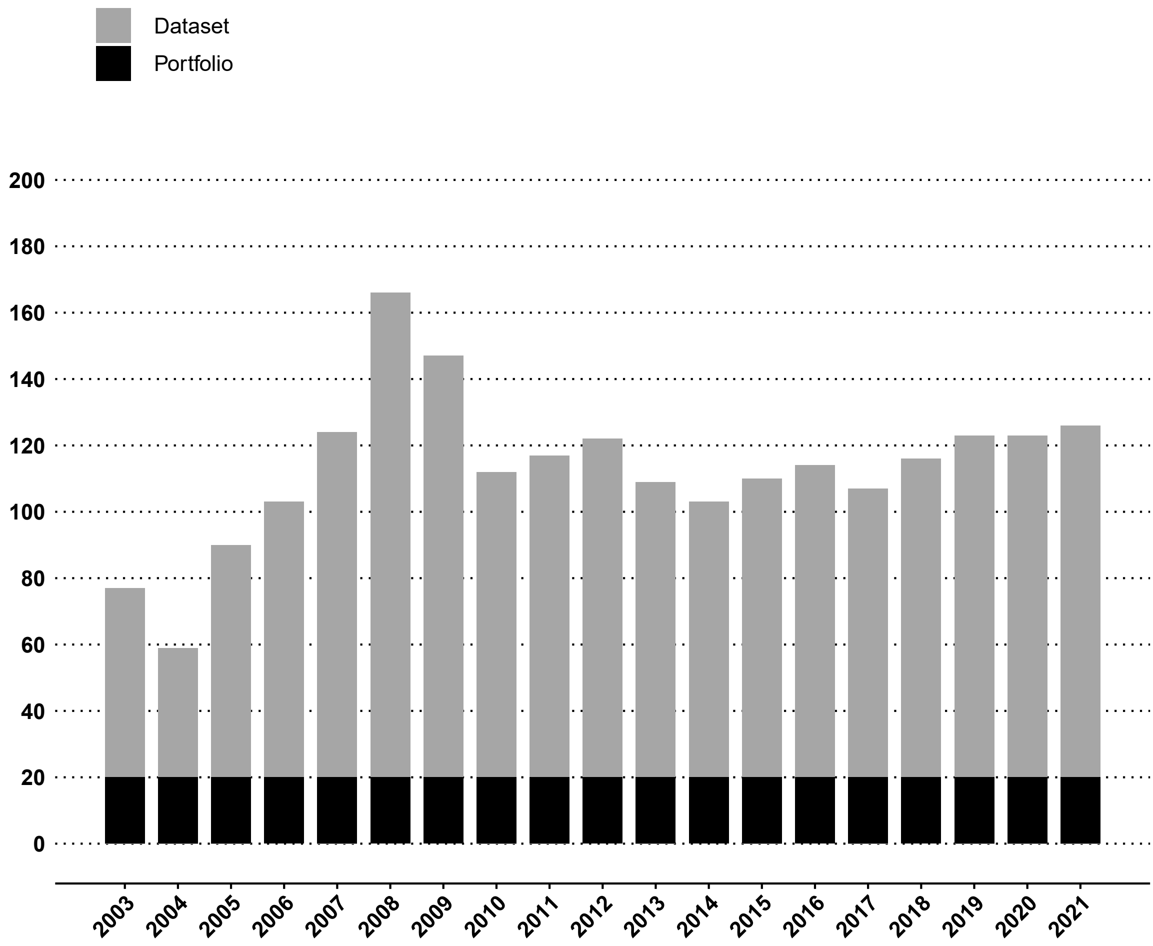

Figure 3.1: Annual number of stocks in the dataset

The figure presents the number of stocks in the dataset from May 2003 to May 2021. We exclude utilities, financial stocks, companies with incomplete or untimely information, and companies with a market capitalization of less than 200 MNOK. We use these stocks to rank and form portfolios. We exclude 2022 as we liquidate the portfolio that year.

Greenblatt suggests buying the 5–7 best-ranked stocks every 2–3 months until the portfolio contains 20–30 stocks and holds each position for one year. In this approach, the portfolio contains fewer stocks in the first year until it reaches its intended composition of 20–30 stocks. In the subsequent years, the portfolio will continuously consist of 20–30 stocks. This approach is also known as dollar-cost averaging, where investors can spread out their purchases over time and potentially reduce the risk of purchasing stocks at an unfavorable time.

In our approach, we buy 20 stocks on the first trading day of May of the first year and hold them for one year. After one year, we rank the stocks in the dataset and form a new portfolio of the 20 best-ranked stocks. The reason for this approach is that the first year will be the only year where the number of stocks in the portfolio differs, as Greenblatt uses the first year to build the portfolio of 20–30 stocks. Thus, deviating from his approach will not significantly affect the results over a long time horizon. Additionally, the impact of dollar-cost averaging may be less significant over a long period because market volatility tends to decrease as the investment horizon lengthens. Moreover, Leggio and Lien (2003) found that dollar-cost averaging is inferior to other investing strategies using risk-adjusted performance measures. As a result, buying all the stocks at once can be a simpler and more efficient way to invest in the market.

Greenblatt suggests implementing a taxation strategy when investing with the *magic formula*. He recommends selling losers just before the end of the fiscal year to maximize tax deductions and selling winners one week after the end of the fiscal year. Despite this suggestion, Greenblatt does not consider capital gains tax in his backtest and also states that taxes become irrelevant when using a tax-free trading account. Therefore, our analysis does not consider capital gains tax because a tax-free trading account is available in Norway for stocks traded on the Oslo Stock Exchange. A taxable event only occurs when withdrawing gains from the trading account. Although this account restricts trading for stocks listed on Euronext Growth and companies registered outside the EEA, they are too few to make a significant difference. Therefore, capital gains tax strategies are not relevant in this study.

3.5 Calculating returns

To compute daily simple returns, we first scale the adjusted close prices for each stock i in the portfolio to have a starting value of 1, to account for equal weighting in the portfolio. Then, we sum the scaled stock prices to obtain the scaled values of the portfolio. Next, we compute daily simple returns for each holding period by dividing the scaled portfolio value at day $t{+}1$ by the scaled portfolio value at day t and subtracting 1.

$$Scaled\ Adj.\ Close\ Price_{it} = \frac{Adj.\ Close\ Price_{it}}{Adj.\ Close\ Price_{i1}} \tag{3.7}$$

$$Scaled\,Portfolio\,Value_t = \sum_{i=1}^{n=20} Scaled\,Adj.\,Close\,Price_{it} \tag{3.8}$$

$$r_{pt} = \frac{Scaled\,Portfolio\,Value_{t+1}}{Scaled\,Portfolio\,Palue_t} - 1 \tag{3.9}$$

Scaling the adjusted prices is essential to ensure equal weighting, as we want the portfolio value to compound over the holding period. When we form the portfolio in May of year *t+1*, we repeat the scaling process to maintain equal weighting at the start of the holding period. After computing the simple returns for each day in all holding periods, we combine the return vectors to create a time series for the entire sample period. We then transform this time series into a continuously compounded portfolio with a starting value of 1. The portfolio value will compound over the sample period by the daily simple returns.

$$Portfolio\,Value_t = Portfolio\,Value_{t-1} * (1 + r_{pt}) \tag{3.10}$$

We calculate the sample period return by subtracting the starting value from the final value divided by the starting value. We calculate the compound annual growth rate (CAGR) as follows:

$$CAGR = \left(\frac{Ending\,Value}{Starting\,Value}\right)^{\frac{1}{T}} - 1 \tag{3.11}$$

3.6 Benchmark

We use the Oslo Børs All-share Index (OSEAX) as the benchmark in our backtest. This index is value-weighted based on market capitalization and includes all stocks admitted to the primary listing on the Oslo Stock Exchange. Thus, it excludes stocks traded on Euronext Growth. The index is revised semi-annually, with changes made after the market close of the last trading day of January and July (Euronext, 2022). We do not use the more well-known OSEBX index because it contains fewer stocks and discriminates on size and turnover. Therefore, we consider the OSEAX to be the most relevant benchmark and proxy for the Norwegian stock market.

3.7 Risk-adjusted performance

The rational investor demands compensation for risk, so the higher the risk, the higher the expected return. To measure the risk-adjusted performance of the *magic formula*, we calculate the Sharpe ratio and run a regression on the Fama and French (1993) three-factor model, expanded by the momentum factor from Carhart (1997). The Fama-French three-factor model aims to explain our strategy's monthly excess returns by considering monthly returns of portfolios of the three factors *MKT*, *SMB*, and *HML* (Fama and French, 1993). *MKT* captures the market returns less the risk-free rate, i.e., the market risk premium. *HML* captures the returns of a portfolio of high book-to-market stocks, less the returns of a portfolio of low book-to-market stocks. *SMB* captures the returns of a portfolio of small-cap stocks, less the returns of a portfolio of large-cap stocks. Carhart (1997) expands the model by adding the *PR1YR* factor based on the effect of momentum. Jegadeesh and Titman (1993) identified the momentum effect to explain persistence in the returns of mutual funds in the US. The factor captures the returns of buying stocks with strong returns and selling stocks with low returns in the last 3 to 12 months. We aim to determine the alpha, i.e., the intercept of the regression, of the *magic formula* in the Norwegian stock market.

$$r_p - r_f = \alpha_p + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \beta_4 PR1YR + \varepsilon_t \tag{3.12}$$

Sharpe (1966) proposed using a reward-to-variability ratio, later known as the Sharpe ratio, to measure the performance of mutual funds. This ratio is a well-known measure of the excess return per unit of risk taken. We calculate the Sharpe ratio using monthly observations.

$$S_p = \frac{r_p - r_f}{\sigma_p} \tag{3.13}$$

$$\textit{Annualized Sharpe Ratio} = \frac{\mu_p}{\sigma_p} \tag{3.14}$$

where:

$$\mu_p = \left(\left(1 + \frac{1}{n}\sum_{i=1}^{n} r_i - r_f\right)^{12}\right) - 1 \tag{3.15}$$

and

$$\sigma_p = \sqrt{12} * \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - r_f)^2} \tag{3.16}$$

3.8 Robustness

Many outperforming trading strategies exist in the literature, but they often fail to generate excess returns for investors after publication (Malkiel, 2003). To determine the *magic formula's* stability and reliability, we apply several methods to scrutinize its performance. These methods include implementing transaction costs, value-weighting the portfolio, selecting varying numbers of stocks in the portfolios, uneven metrics weighting, and applying alternative proxy metrics. In practice, transaction costs can make it difficult for investors to achieve backtested returns. Greenblatt (2005) did not account for transaction costs in his backtest, which may cause the strategy's performance to be less remarkable than claimed. A commonly used academic measure to adjust for this bias is considering the spread of the stock on trading days (Będowska-Sójka and Echaust, 2020). The quoted price of the trades used in the backtest is historical close prices, which may not be achievable in actual trading. As a result, the seemingly high profits from the backtested trading strategy may deteriorate in implementation due to transaction costs offsetting gains measured by close prices. To address this, we use the bid-ask spread, combined with the portfolio turnover and the standard commission fee, to estimate transaction costs. Implementing transaction costs allows us to evaluate the strategy's feasibility more accurately.

The ask price is the lowest price a seller is willing to accept. Similarly, the bid price is the highest price a buyer is willing to pay. This mismatch between the close price and the actual bid and ask prices creates an upward bias for our returns, as the bid price is typically lower than the quoted price, and the ask price is typically higher.

First, we calculate the relative spread for each stock bought in year t by dividing the difference between the ask and close price by the close price. Accordingly, for each stock sold, we divide the difference between the bid and close price by the close price. Then, we calculate the average relative close-ask spread of all stocks bought in year t and the average relative bid-close spread of all stocks sold in year t. Lastly, we sum the relative spreads and multiply it by the portfolio turnover to get the weighted relative bid-ask spread for the stocks we trade in the portfolio in year t. The turnover is the fraction of the portfolio that gets replaced.

For instance, we need to sell stocks A and B because they have been in the portfolio for one year and are not ranked top 20 in the current year. Stock A has a price of 20 and a bid price of 19.5, while stock B has a price of 30 and a bid price of 29. The relative spread related to the sale of stocks A and B will then be 2.5% and 3.33%, respectively. Next, we consider the relative spread of all the stocks sold that year and calculate the average. In this example, considering only stocks A and B, the average relative spread related to the sale will be 2.92%.

Now, consider that stocks C and D will replace stocks A and B in the portfolio. Stock C has a price of 10 and an ask price of 10.3, while stock D has a price of 15 and an ask price of 15.2. Their relative spread will then be 3% and 1.33%, respectively. In this case, the average relative spread related to buying is 2.17%. We then sum the average spreads of both selling and buying to get the total relative spread related to the transactions. Finally, we multiply this percentage by the turnover of the portfolio, which is the ratio of stocks replaced relative to the total number of stocks in the portfolio. In this case, the turnover is 10%. As a result, our measure of the transaction cost related to the bid-ask spread of the stocks bought and sold is 0.51%. This measure will provide a more accurate estimate of the bid-ask spread than simply using the total bid-ask spread.

In addition, we calculate the transaction cost of the stocks that remain in the portfolio. Consider stocks E and F, are still ranked top 20 after being held in the portfolio for one year. The stock price of stock E has increased, and stock F has decreased during the holding period. As such, they need rebalancing. The bid price of stock E is 19.5 and the ask price is 20, resulting in a bid-ask spread of 0.5 and a relative bid-ask spread of

2.5%. The bid price of stock F is 10.5 and the ask price is 11, resulting in a bid-ask spread of 0.5 and a relative bid-ask spread of 4.55%. We calculate the relative bid-ask spread divided by two of all the stocks that remain in the portfolio that year and take the average of those. In this case, the relative spread is 1.76%. We multiply this amount by the fraction of the portfolio not replaced, i.e., one minus the turnover ratio. We multiply this percentage by the total compounded annual return (CAGR), which we see as the best proxy for the average fraction of the stocks' value that needs rebalancing. In our example, if we consider the average relative bid-ask spread of 1.76%, a turnover of 80%, and a CAGR of 20%, the transaction cost related to the bid-ask spread of the remaining stocks will be 0.07%. The total transaction cost related to the bid-ask spread will then be as follows:

$$Bid\text{-}ask\ Spread_p = Turnover_p * (Bid\text{-}ask\ Spread_{Buy} + Bid\text{-}ask\ Spread_{Sell}) + \frac{Bid\text{-}ask\ Spread_{Keep}}{2} * CAGR * (1 - Turnover_p) \tag{3.17}$$

We do not consider bid-ask spreads for stocks that delist during the holding periods because there is often no transaction cost related to a delisting. The turnover for 2003 and 2022 is 100% as we buy and liquidate the entire portfolios, respectively. A limitation in integrating transaction costs using bid-ask spreads is that it could understate the costs of larger transactions, meaning that the quoted bid- and ask price is only available for a given number of shares.

While using indexes as benchmarks for examining trading strategies, we must account for the standard commission fees to brokerage firms because index investing is considerably cheaper. Following *Nordnet*, a commonly used brokerage firm in Norway, we set the price of OSEAX index investing as a flat rate of 0.19% p.a. of total funds invested and 0.049% p.a. commission per trade for individual stocks (Nordnet, 2022). We multiply the commission rate with the turnover and then multiply by two as we both sell and buy an equal number of stocks on the day of rebalancing. We adjust the returns on the rebalancing days using the relative spreads and commission fees.

$$Transaction\ Costs_p = Bid\text{-}ask\ Spread_p + (Commission\ Fee_p * Turnover_p) \tag{3.18}$$

3.9 Econometric approach

Standard errors

In our analysis, we use OLS regression to estimate the coefficients of the four-factor model. In OLS regression analysis, assumptions of the model must be satisfied to get valid statistical tests of significance. Heteroscedasticity or autocorrelation may invalidate the OLS standard errors and, thus, the test statistics in the model (Wooldridge, 2015). In such cases, the OLS estimator is still unbiased but inefficient. A solution to address the issues of heteroscedasticity and autocorrelation is to use standard errors that are robust to these effects. This adjustment can provide a more accurate assessment of the statistical significance of the regression coefficients. To detect the presence of heteroscedasticity and autocorrelation, we use the Breusch-Pagan test (Breusch and Pagan, 1979) and the Durbin-Watson test (Durbin and Watson, 1951), respectively.

As presented in Table A5.1, some of the models indicate the presence of autocorrelation and heteroscedasticity. To assess the statistical significance of the coefficients more accurately, we use heteroscedasticity- and autocorrelation-consistent standard errors, as introduced by Newey and West (1987).

T-test

In our analysis, we conduct a two-sided paired t-test to the annual Sharpe ratios of the *magic formula* and the benchmark to evaluate the statistical significance of their differences. We use this test to compare the means of two related groups, which in this case is the Sharpe ratio of the portfolio and the benchmark. If the p-value of the t-test is less than the predetermined significance level, we can reject the null hypothesis that the means of the two groups are equal.

4 Empirical Results

4.1 Initial results

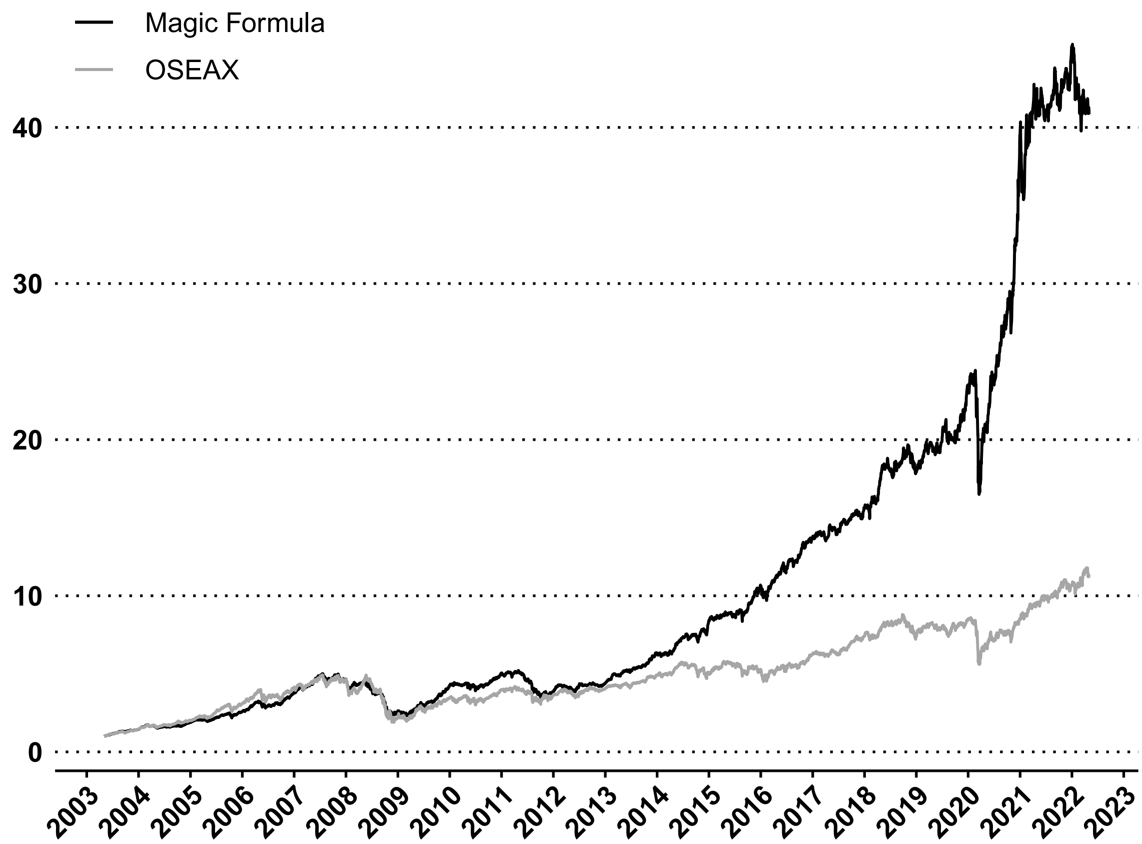

Figure 4.1: The *magic formula's* cumulative simple returns

The figure compares the simple cumulative returns of the *magic formula* and the benchmark from May 2003 to May 2022. The y-axis displays the compounded value of an investment of NOK *1* in May 2003.

The initial results in Figure 4.1 indicate that the *magic formula* outperforms the benchmark by a significant margin. Over the sample period, *magic formula* yields a 41x return on the initial investment, while the benchmark yields an 11x return, resulting in compound annual growth rates of 21.56% and 13.54%, respectively.

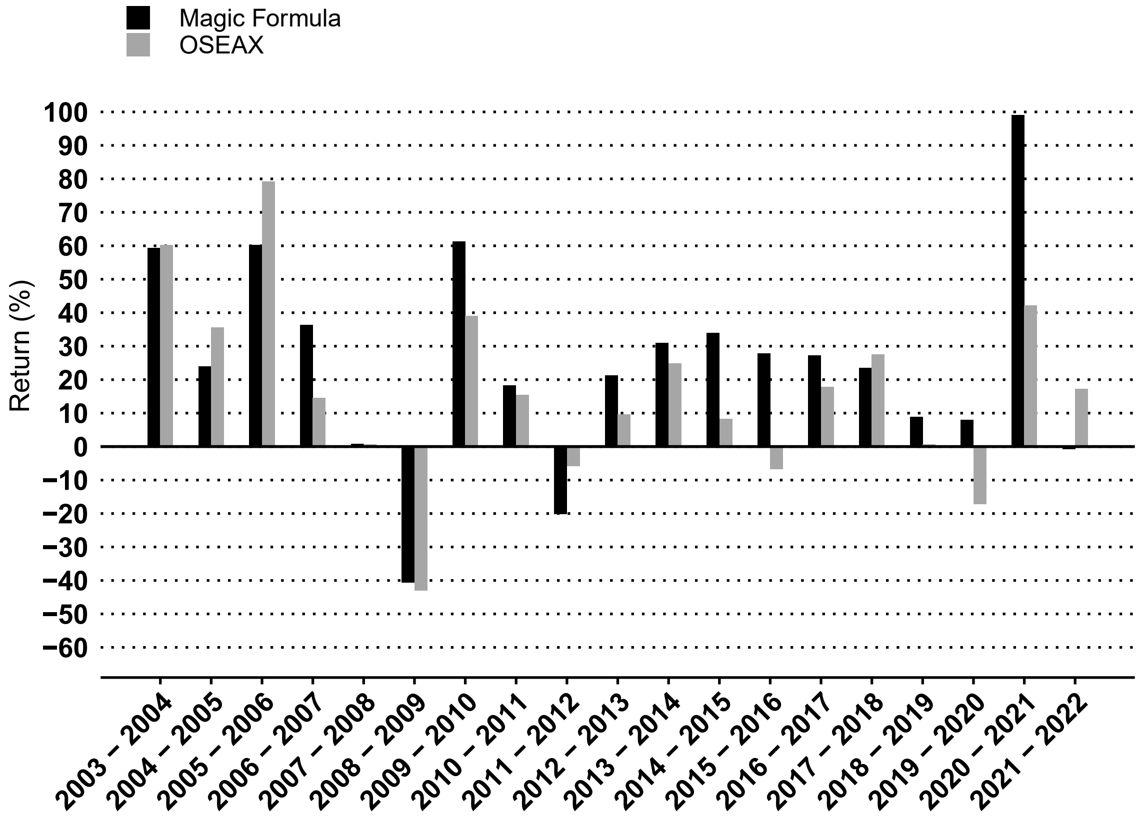

Figure 4.2: The *magic formula's* annual returns

The figure presents the *magic formula's* and the benchmark's annual returns for each holding period from May 2003 to May 2022.

The *magic formula* outperforms the benchmark 13 years of the 19-year sample period. The *magic formula's* highest holding period return is 99.02% compared to the benchmark's 79.2%. Prior to the recession in 2008 and 2009, the portfolio and the benchmark show similar returns. During the recession, both experience significant losses, as presented in the logarithmic returns in Figure 4.3. In the subsequent recovery years, the *magic formula* performs better than the benchmark. We see a similar trend after the COVID-19-induced crash in February and March 2020.

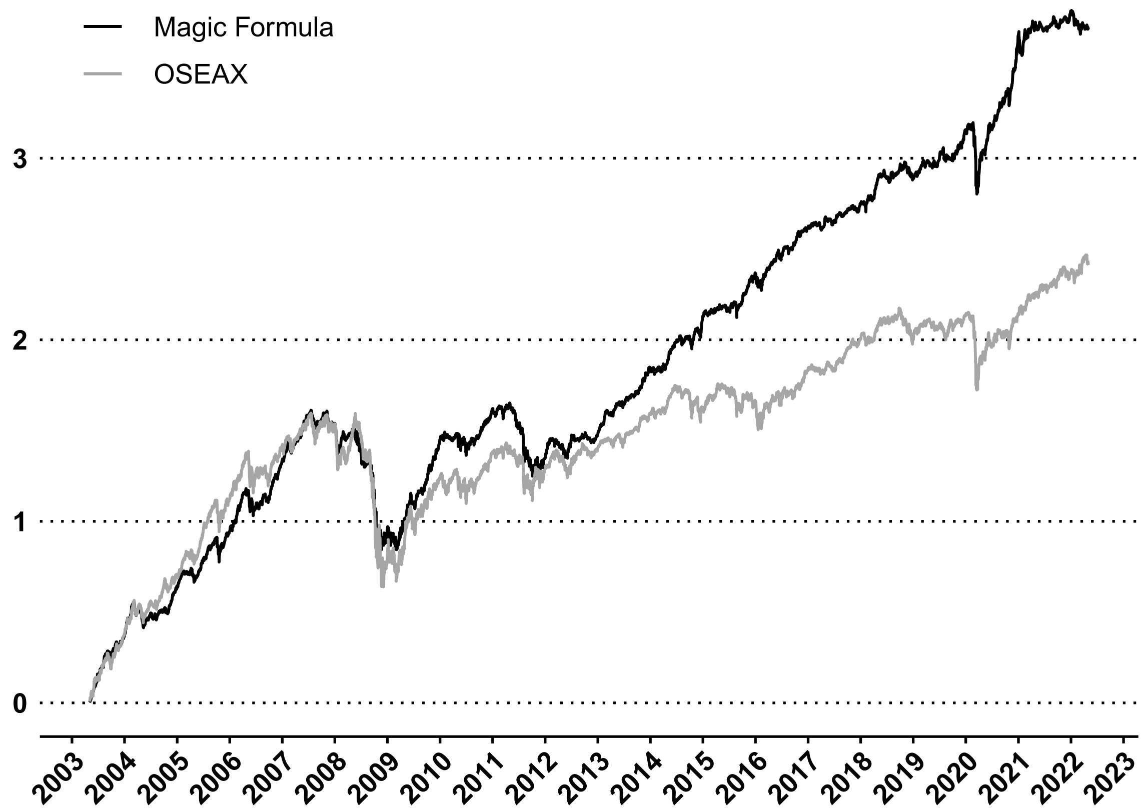

Figure 4.3: The *magic formula's* logarithmic returns
The figure compares the *magic formula* and the benchmark's logarithmic returns from May 2003 to May 2022.

The initial results suggest that the *magic formula* outperforms the market in Norway. However, this does not necessarily imply that it is a superior investment strategy. The impressive performance may be due to the strategy taking on additional risk.

4.2 Portfolio analysis

We analyze the portfolios further to get an overview of what characterizes the stocks selected by the *magic formula*. We analyze the stocks' market capitalization and the portfolios' sector exposure compared to the Oslo Stock Exchange's statistics throughout the sample period.

When assessing the market capitalization of the stocks selected by the *magic formula*, we focus on the median value, as a few large companies account for a significant fraction of the Oslo Stock Exchange's value. Ødegaard (2021) shows that the four largest companies accounted for between 35.8% and 55.4% of the exchange's value from 2003 to 2020. These large companies, such as Equinor ASA, tend to push the average market capitalization to a high level. Our analysis shows that the *magic formula* has significantly higher average market capitalizations than the median, with an average median market capitalization of 2.74 BNOK in the sample period. The median varies over time, with the lowest value in 2009 of 595 MNOK and the highest in 2015 of 5.96 BNOK, indicating that the strategy selects stocks of varying sizes throughout the sample period. The comparison of the average market capitalization between the *magic formula* and the Oslo Stock Exchange shows that, on average, the portfolios selected by the *magic formula* are composed of higher market capitalization stocks than those on the exchange, even when examining the median. We analyze this further in Section 4.3.

Table 4.1: The *magic formula's* market capitalization statistics

The table presents the average, median, maximum, and minimum market capitalization of the *magic formula* at the start of each holding period from May 2003 to May 2022. In addition, it presents the difference in the median and average market capitalization between the *magic formula* and the Oslo Stock Exchange. We exclude companies on the Oslo Stock Exchange with a market capitalization of less than 200 MNOK.

Numbers in MNOK

	Average	Median	Max	Min	Difference in median from the OSE	Difference in average from the OSE
2003	16,244	1,842	158,745	215	603	10,176
2004	4,390	1,423	49,775	254	535	-941
2005	12,058	1,615	107,429	230	710	5,978
2006	8,522	1,244	92,768	286	324	1,852
2007	46,694	4,055	454,339	451	2,789	36,943
2008	13,625	1,894	188,191	252	438	4,252
2009	7,359	595	108,466	208	-725	-2,249
2010	28,084	3,474	419,307	283	2,795	21,287
2011	27,879	3,921	457,890	293	2,588	18,759
2012	16,263	3,057	150,536	305	1,786	6,569
2013	16,838	3,283	169,182	351	1,889	7,305
2014	14,566	2,066	202,169	231	830	4,351
2015	48,619	5,957	575,232	234	4,288	36,964
2016	17,415	1,770	255,698	205	346	5,542
2017	14,848	1,692	206,901	311	19	3,857
2018	16,564	3,173	208,402	605	1,235	4,593
2019	22,701	4,938	266,959	202	2,767	8,560
2020	25,594	2,436	250,314	306	254	10,898
2021	21,855	2,321	226,827	216	-165	5,454
2022	48,420	4,133	554,915	205	2,894	42,352

To examine the sectoral composition of the portfolios selected by the *magic formula*, we group the companies according to their primary business activity using the Global Industry Classification Standard (GICS). The resulting sectors include materials, industrials, consumer discretionary, energy, IT, real estate, communication services, consumer staples, and health care, with the average portfolio composition ranging between 2.75%–24.75%. The lowest weighted sector is health care, while the largest is industrials. In comparison, the sectoral composition of the Oslo Stock Exchange has a range of 1.06%–25.74%, with the lowest being communication services and the largest being energy, as presented in Table A3.2.

Table 4.2: The *magic formula's* sector exposure

The table presents the sector exposure for the *magic formula* at the start of each holding period from May 2003 and May 2022. We group the companies in the sectors from the Global Industry Classification Standard (GICS).

	Materials	Industrials	Consumer Discretionary	Energy	IT	Real Estate	Communication Services	Consumer Staples	Health Care
2003	10%	15%	35%	20%	10%	5%	5%	-	-
2004	5%	10%	25%	10%	20%	5%	10%	10%	5%
2005	15%	20%	20%	5%	15%	5%	15%	-	5%
2006	5%	35%	15%	5%	25%	-	10%	5%	-
2007	5%	35%	15%	15%	5%	5%	10%	10%	-
2008	-	40%	15%	5%	30%	-	5%	5%	-
2009	5%	25%	10%	15%	30%	-	5%	10%	-
2010	-	25%	10%	15%	10%	-	15%	20%	5%
2011	-	30%	10%	10%	10%	-	-	40%	-
2012	5%	40%	5%	5%	10%	5%	10%	15%	5%
2013	5%	20%	5%	15%	20%	5%	5%	20%	5%
2014	5%	25%	5%	10%	20%	5%	10%	20%	-
2015	15%	15%	10%	10%	20%	5%	5%	15%	5%
2016	-	20%	10%	5%	25%	5%	10%	20%	5%
2017	-	20%	15%	10%	25%	-	10%	15%	5%
2018	-	25%	15%	5%	15%	5%	10%	25%	-
2019	10%	20%	10%	10%	10%	5%	5%	30%	-
2020	5%	10%	10%	15%	20%	-	15%	20%	5%
2021	-	25%	15%	-	25%	-	10%	15%	10%
2022	-	40%	15%	15%	10%	-	10%	10%	-
Average	4.5%	24.75%	13.5%	10%	17.75%	2.75%	8.75%	15.25%	2.75%

The industrial, consumer discretionary, and IT sectors are the only industries consistently represented in the portfolios selected by the *magic formula*. Among these, the industrials sector has the most considerable average annual exposure. The significant exposure to IT companies in the portfolio, which averages 17.75%, compared to 13.6% for the Oslo Stock Exchange (Appendix A3.2), supports the notion that the *magic formula* naturally selects software companies because of their low tangible assets and high earnings, as stated by Haugen (Alpert, 2006).

4.3 Risk-adjusted performance

The initial returns may be the result of taking on excessive risk. However, the *magic formula's* daily average standard deviations are lower than the benchmark, as shown in Figure 4.4. Moreover, the *magic formula's* average daily standard deviation is lower in 17 of 19 holding periods, only higher in 2020–2021. By this measure, the findings suggest that the magic formula has lower volatility than the benchmark.

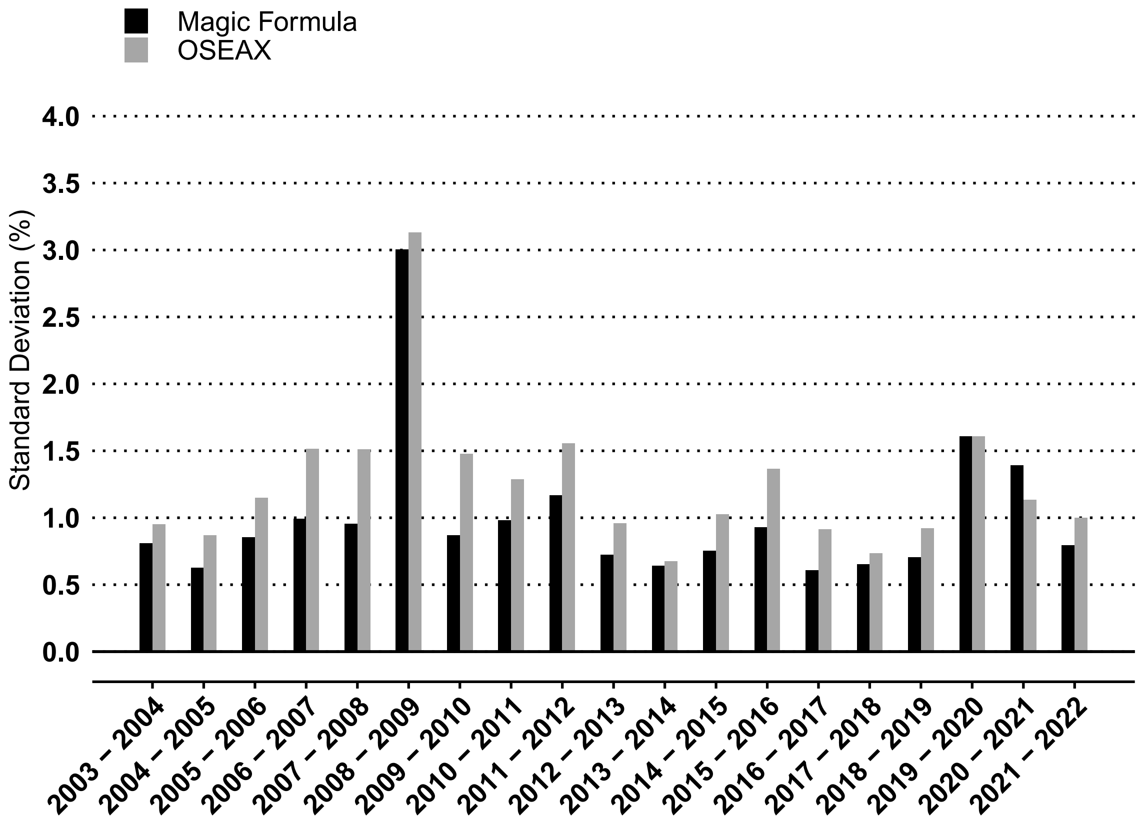

Figure 4.4: Average daily volatility

The figure presents the annual average daily standard deviation from May 2003 to May 2022.

When risk-adjusting the returns using the Sharpe ratio, the *magic formula* outperforms the market by 44 decimal points. The benchmark has an annualized Sharpe ratio of 0.72 compared to the *magic formula's* 1.16. Analyzing the annual Sharpe ratios, the strategy outmatches the market overall, beating it 14 times out of 19. Moreover, the *magic formula* has more than twice the Sharpe ratio of the benchmark in eight holding periods. The two-sided paired t-test calculates a p-value of 0.007, indicating that the difference in Sharpe ratios is significantly different from zero at the $p < 0.05$ level.

Table 4.3: The Sharpe ratio

The table presents the *magic formula* and the benchmark's annual and annualized Sharpe ratio based on monthly observations from May 2003 to May 2022. We calculate the annual Sharpe ratio for each holding period based on monthly data for each holding period, and the annualized Sharpe ratio based on monthly data over the entire sample period.

Holding Period	Magic Formula	OSEAX
2003 - 2004	4.39	3.81
2004 - 2005	2.22	2.44
2005 - 2006	4.26	4.21
2006 - 2007	2.08	0.46
2007 - 2008	-0.31	-0.21
2008 - 2009	-0.96	-0.97
2009 - 2010	4.25	1.59
2010 - 2011	1.00	0.64
2011 - 2012	-1.21	-0.34
2012 - 2013	1.66	0.51
2013 - 2014	2.85	2.19
2014 - 2015	2.67	0.42
2015 - 2016	1.79	-0.36
2016 - 2017	2.68	1.16
2017 - 2018	2.15	2.30
2018 - 2019	0.70	-0.02
2019 - 2020	0.25	-0.73
2020 - 2021	4.39	2.32
2021 - 2022	-0.11	1.04
Annualized Sharpe ratio	**1.16**	**0.72**

Furthermore, we examine if the four factors of Fama and French (1993) and Carhart (1997) explain the *magic formula's* excess returns. The results from the OLS regression show a positive monthly alpha of 0.5% that is statistically significant at the $p < 0.05$ level. Annualizing this alpha results in an annual excess return of 6%, indicating that the strategy yields significant risk-adjusted excess returns.

Table 4.4: OLS regression of the *magic formula*

The table presents the results of OLS regression analyses of the *magic formula's* monthly excess returns on CAPM, the Fama and French (1993) three-factor model, and the Carhart (1997) four-factor model for the period from May 2003 to May 2022. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:		
	Excess returns of the *magic formula*		
	CAPM	FF3	C4
MKT	0.769***	0.781***	0.791***
	(0.051)	(0.052)	(0.051)
SMB		0.201***	0.219***
		(0.064)	(0.064)
HML		−0.077**	−0.075*
		(0.037)	(0.040)
PR1YR			−0.010
			(0.046)
Alpha	0.008***	0.005**	0.005**
	(0.002)	(0.002)	(0.002)
Obs.	228	228	225
R^2	0.599	0.636	0.650
Adj. R^2	0.597	0.631	0.644
Res. SE	0.034 (df = 226)	0.032 (df = 224)	0.032 (df = 220)
F Stat.	337.108*** (df = 1; 226)	130.448*** (df = 3; 224)	102.159*** (df = 4; 220)
Note:			*p<0.1; **p<0.05; ***p<0.01

The CAPM suggests a beta of the *magic formula* of 0.769, indicating a lower level of systematic risk than the benchmark. The beta remains consistent and statistically significant when regressing on the extended models. The statistically significant coefficients of *SMB* suggest that the strategy favors small-cap stocks. However, Chen and Bassett (2014) states that a positive *SMB* coefficient does not necessarily indicate a preference for small-cap stocks, and many large-cap portfolios have positive significant *SMB* coefficients. The idea that the *magic formula* is a value investing strategy makes the negative *HML* coefficient surprising. However, the coefficient is only statistically significant at the $p < 0.1$ level. Fama and French (2021) suggest that the premium of the traditional value investing factor has decreased in recent times, which could explain the negative *HML* coefficient.

The *PR1YR* coefficient is not statistically significant, indicating that the momentum factor does not significantly affect the excess returns of the *magic formula*. In conclusion, the *magic formula* generates excess returns when applying the four-factor model of Fama and French (1993) and Carhart (1997).

4.4 Robustness

4.4.1 Transaction costs

We implement transaction costs to test the *magic formula's* robustness. We measure the transaction costs using bid-ask spread and commission fees. The results from the OLS regression on the four-factor model indicate a correlation with the returns of small-capitalization stocks. Ødegaard (2008) finds that the largest spreads for stocks listed on the Oslo Stock Exchange are in small-cap stocks. Therefore, we test if the *magic formula's* returns deteriorate when incorporating transaction costs. We find that the total relative bid-ask spread varies between 0.17% and 4.14%, and the turnover of the portfolios varies from 35% to 70%, excluding the first and last year, as we buy 20 stocks in the first year and liquidate the portfolio in the last. The logarithmic returns deteriorate after implementing transaction costs. The total compounded return decreases from 41x to 31x. However, the strategy still provides higher returns than the benchmark over the sample period.

Table 4.5: The bid-ask spread and the turnover

The table presents the portfolios' annual average relative bid-ask spread and turnover from May 2003 to May 2022. The buy-spread is the average relative spread between the ask and close price for stocks we buy in year *t*. The sell-spread is the average relative spread between the close and bid price for stocks we sell in year *t*. The total spread shows the relative bid-ask spread for all traded stocks in year *t*. Finally, the turnover is the ratio of stocks replaced in year *t*.

	Buy spread	**Sell spread**	**Total spread**	**Turnover**
2003	4.14%	-	4.14%	-
2004	1%	2.44%	3.44%	40%
2005	1.08%	0.89%	1.97%	50%
2006	0.62%	0.67%	1.29%	45%
2007	0.09%	0.50%	0.59%	50%
2008	0.95%	0.30%	1.25%	55%
2009	0.88%	1.37%	2.25%	45%
2010	0.09%	3.76%	3.85%	70%
2011	0.57%	1.71%	2.28%	50%
2012	0.07%	0.40%	0.47%	45%
2013	-0.39%	1.19%	0.80%	45%
2014	0.27%	1.98%	2.26%	35%
2015	-0.15%	0.32%	0.17%	40%
2016	1.79%	0.39%	2.18%	45%
2017	0.15%	0.46%	0.61%	45%
2018	0.08%	1.11%	1.19%	60%
2019	0.26%	1.46%	1.72%	40%
2020	0.88%	0.11%	0.99%	50%
2021	0.30%	0.35%	0.65%	45%
2022	-	0.88%	0.88%	-

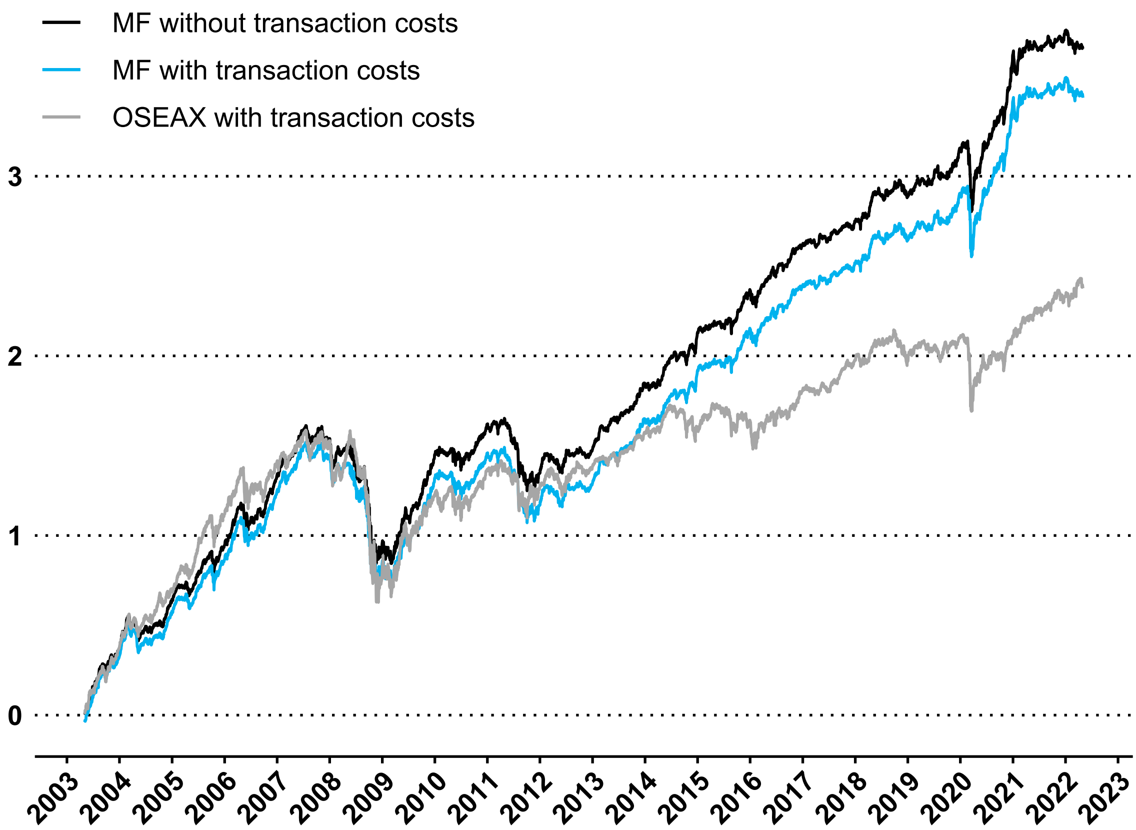

Figure 4.5: Logarithmic returns after implementing transaction costs

The figure presents the *magic formula's* logarithmic returns with- and without transaction costs from May 2003 to May 2022. In addition, the graph includes the benchmark's logarithmic returns with transaction costs.

To analyze the impact of transaction costs on the excess returns, we regress them on the four-factor model to determine if the strategy still provides a significant alpha. The coefficients are similar to those obtained in the previous analysis. However, the alpha is only significant at the $p < 0.1$ level, suggesting that the applied risk factors explain a significant portion of the excess returns. The coefficients of *SMB* and *MKT* remain significant at the $p < 0.01$ level, indicating that large bid-ask spreads of small-cap stocks in the portfolio deteriorate the excess returns. In conclusion, transaction costs considerably affect the *magic formula's* performance, highlighting the latest alpha.

Table 4.6: OLS regression after implementing transaction costs

The table presents the results of OLS regression analysis of the *magic formula's* monthly excess returns after implementing transaction costs on the four-factor model of Fama and French (1993) and Carhart (1997) from May 2003 to May 2022. We calculate the transaction costs for the *magic formula* using the bid-ask spread and the commission fees. For the benchmark, we include an annual fee for index investing. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:
	Magic formula with transaction costs
MKT	0.792***
	(0.052)
SMB	0.226***
	(0.066)
HML	−0.071*
	(0.040)
PR1YR	−0.0004
	(0.046)
Alpha	0.004*
	(0.002)
Obs.	225
R^2	0.647
Adj. R^2	0.640
Res. SE	0.032 (df = 220)
F Stat.	100.638*** (df = 4; 220)
Note:	*p<0.1; **p<0.05; ***p<0.01

4.4.2 Altered *magic formula*

Value-weighted

As the results of the OLS regression indicate, the *magic formula's* returns correlate significantly with the returns of small-cap stocks. We analyze this further by constructing a value-weighted portfolio to examine whether the excess returns are affected. The portfolio contains the same stocks as the original strategy but weighted in correspondence to their market capitalization on the rebalancing day.

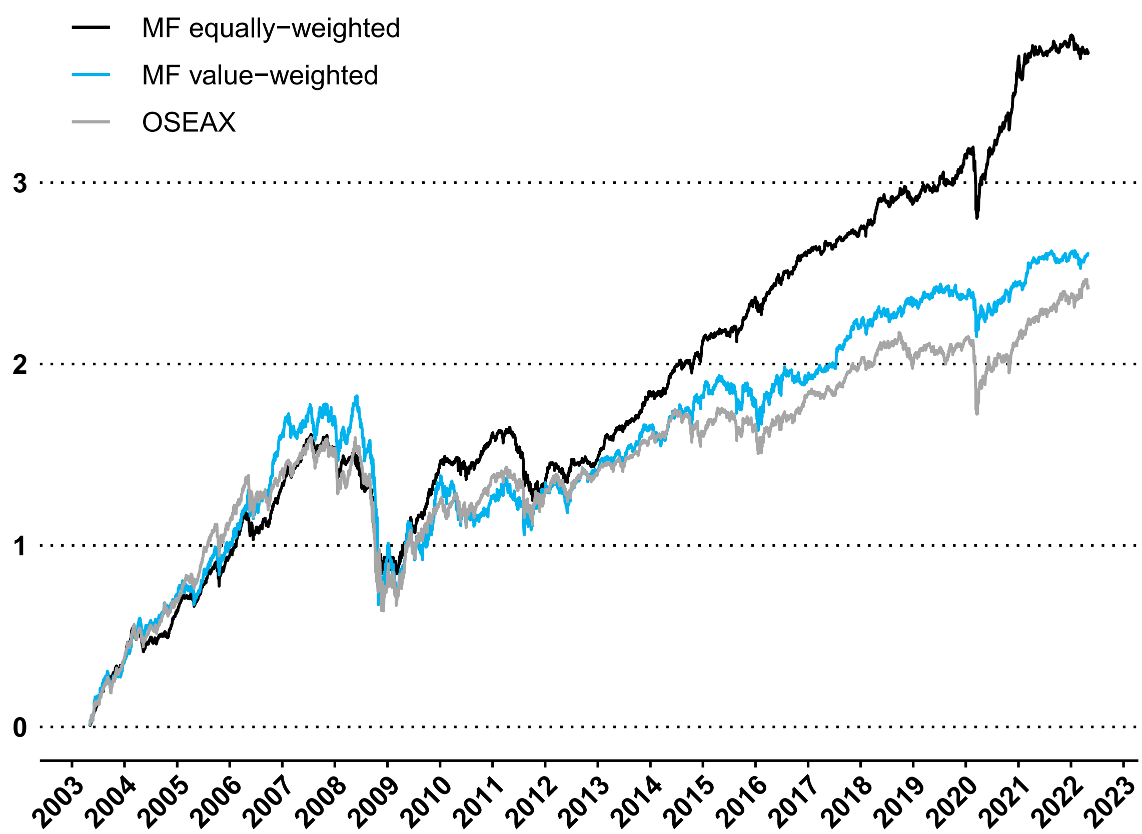

Figure 4.6: Value-weighted logarithmic returns

The figure presents the *magic formula's* equal- and value-weighted logarithmic returns from May 2003 to May 2022. We weight the stocks in correspondence to their market capitalization on the rebalancing day. We include the benchmark's logarithmic returns for comparison.

The results of the OLS regression indicate that the value-weighted strategy does not produce a statistically significant alpha. The beta of 0.922 indicates that the excess returns of the value-weighted strategy follow the market, and could explain the insignificant alpha. In addition, the equal-weighted portfolio outperforms the value-weighted in terms of logarithmic returns, indicating that the returns of the small-cap stocks drive the *magic formula's* returns. These results are similar to those of Malladi and Fabozzi (2017), who studied equal-weighted and value-weighted portfolios using simulation and real-world data from 1926 to 2014. They found that an equal-weighted portfolio outperforms a value-weighted in that period, with 85% of the excess returns attributed to the rebalancing effect and 15% to the size effect.

Table 4.7: OLS regression of the value-weighted *magic formula*

The table presents the results of OLS regression analyses of the *magic formula's* equal- and value-weighted monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997) for the period from May 2003 to May 2022. The stocks are weighted based on their market capitalization on the rebalancing day. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:	
	Equally-weighted	Value-weighted
MKT	0.791***	0.922***
	(0.051)	(0.084)
SMB	0.219***	−0.061
	(0.064)	(0.059)
HML	−0.075*	−0.095**
	(0.040)	(0.039)
PR1YR	−0.010	0.013
	(0.046)	(0.060)
Alpha	0.005**	0.003
	(0.002)	(0.003)
Obs.	225	225
R^2	0.650	0.595
Adj. R^2	0.644	0.588
Res. SE (df = 220)	0.032	0.041
F Stat. (df = 4; 220)	102.159***	80.794***
Note:		*p<0.1; **p<0.05; ***p<0.01

Unevenly metrics weight

In the original ranking approach, we assign equal weights to earnings yield and return on capital. However, these metrics may have different explanatory power and should be weighted differently. To test this, we conduct a backtest where we weight the metrics differently in 10% increments.

First, we weight the metrics in favor of earnings yield, ranging from 50%-100% in 10% increments. We find that weighting earnings yield at 80% and return on capital at 20%

generates the highest monthly alpha of 0.58%, significant at the $p < 0.05$ level. This alpha is 8 basis points higher than the original alpha. The 60/40 and 80/20 earnings yield-weighted portfolios are the only combinations that generate significant excess returns at the $p < 0.05$ level, in addition to the original weighting. The regression coefficient of the *MKT* factor generally decreases, while the coefficients of *SMB* and *HML* increase slightly. The coefficient of the *PR1YR* factor, on the other hand, tends to fluctuate. However, only the coefficients of *MKT* and *SMB* are statistically significant.

Table 4.8: OLS regression of the *magic formula* weighted in favor of earnings yield

The table presents the results of OLS regression analyses of the *magic formula's* monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997) using different earnings yield weights in the ranking procedure, from May 2003 to May 2022. The weighting of earnings yield is increased from 50% to 100% in increments of 10%. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:					
	50/50	60/40	70/30	80/20	90/10	100/0
MKT	0.7908***	0.7700***	0.7632***	0.7329***	0.7136***	0.7152***
	(0.0511)	(0.0508)	(0.0518)	(0.0490)	(0.0486)	(0.0476)
SMB	0.2189***	0.2080***	0.2233***	0.2255***	0.2301***	0.2274***
	(0.0645)	(0.0567)	(0.0638)	(0.0685)	(0.0672)	(0.0665)
HML	−0.0749*	−0.0732*	−0.0589	−0.0416	−0.0381	−0.0230
	(0.0404)	(0.0430)	(0.0444)	(0.0477)	(0.0447)	(0.0452)
PR1YR	−0.0100	0.0118	0.0270	−0.0243	−0.0285	−0.0471
	(0.0459)	(0.0481)	(0.0517)	(0.0495)	(0.0524)	(0.0537)
Alpha	0.0050**	0.0049**	0.0043*	0.0058**	0.0041*	0.0035
	(0.0020)	(0.0021)	(0.0022)	(0.0023)	(0.0025)	(0.0025)
Obs.	225	225	225	225	225	225
R^2	0.6500	0.6454	0.6290	0.6155	0.5941	0.6065
Adj. R^2	0.6437	0.6389	0.6223	0.6085	0.5868	0.5994
Res. SE (df = 220)	0.0318	0.0311	0.0320	0.0321	0.0328	0.0322
F Stat. (df = 4; 220)	102.1589***	100.0969***	93.2569***	88.0557***	80.5152***	84.7876***

Note: *p<0.1; **p<0.05; ***p<0.01

When weighting in favor of return on capital, the results from the OLS regression show a slight decrease in performance. We find that weighting the metric at 60/40 and 70/30 produces a significant alpha, but heavier weightings do not result in excess returns. Interestingly, the coefficient of the *HML* factor decreases as the weight assigned to return on capital increases, suggesting that the return on capital metric favors low book-to-market companies. A significant negative coefficient of *HML* indicates that the strategy typically selects growth stocks. Novy-Marx (2013) finds that companies with a high return on capital, as measured by gross-profits-to-assets, have characteristics and covariances similar to low book-to-market companies. He states that while they may appear to be growth companies, they are actually high-quality growth companies, outperforming the market despite having a low book-to-market ratio. Although his measure of return on capital is slightly different from Greenblatt's, the same could be true in this case. However, too much weight on return on capital may not be effective in identifying these high-quality stocks, as a weighting of more than 70% generates an insignificant alpha.

Table 4.9: OLS regression of the *magic formula* weighted in favor of return on capital

The table presents the results of OLS regression analyses of the *magic formula's* monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997) using different return on capital weights in the ranking procedure, from May 2003 to May 2022. The weighting of return on capital is increased from 50% to 100% in increments of 10%. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:					
	50/50	60/40	70/30	80/20	90/10	100/0
MKT	0.7908***	0.7468***	0.7646***	0.7706***	0.8123***	0.8207***
	(0.0511)	(0.0515)	(0.0480)	(0.0516)	(0.0542)	(0.0470)
SMB	0.2189***	0.2180***	0.2557***	0.3284***	0.3513***	0.3265***
	(0.0645)	(0.0620)	(0.0565)	(0.0776)	(0.0814)	(0.0707)
HML	−0.0749*	−0.0964**	−0.1112***	−0.1437***	−0.1452**	−0.1238**
	(0.0404)	(0.0398)	(0.0386)	(0.0556)	(0.0602)	(0.0570)
PR1YR	−0.0100	−0.0092	0.0257	0.0714	0.0927*	0.0807
	(0.0459)	(0.0453)	(0.0411)	(0.0542)	(0.0562)	(0.0556)
Alpha	0.0050**	0.0051***	0.0040**	0.0014	0.0004	−0.0003
	(0.0020)	(0.0019)	(0.0019)	(0.0025)	(0.0025)	(0.0023)
Obs.	225	225	225	225	225	225
R^2	0.6500	0.6363	0.6644	0.5591	0.5636	0.5858
Adj. R^2	0.6437	0.6297	0.6583	0.5511	0.5556	0.5782
Res. SE (df = 220)	0.0318	0.0310	0.0299	0.0384	0.0401	0.0384
F Stat. (df = 4; 220)	102.1589***	96.2235***	108.8797***	69.7508***	71.0172***	77.7716***

Note: *p<0.1; **p<0.05; ***p<0.01

To further investigate the relationship between return on capital and the *HML* factor, we analyze the sector exposure of portfolios increasingly weighted by return on capital. Increasing the weight of return on capital leads to increased exposure to the energy and IT sectors. One characteristic of the energy sector is its sensitivity to global economic and political conditions, and the demand for energy products is closely tied to the overall state of the economy. As these companies are typically cyclical, a higher selection of energy stocks due to their high return on capital during cyclical periods could explain the increasing exposure to this sector.

A characterization of the IT sector is its high profit margins and low capital expenditure, which can lead to a high return on capital. It is also known for its rapid innovation and disruption, leading to a high potential for growth. Thus, it is not surprising that a strategy that leans towards selecting growth stocks would experience a decrease in the coefficient of *HML*. Additionally, we observe a reduction in the exposure to consumer staples, a sector including companies that produce and sell essential goods and services that consumers purchase regularly. These companies usually have steady cash flows but offer lower growth potential than other sectors due to limited opportunities for new product development or market expansion. Given that the strategy seems to lean towards growth stocks when increasing the weight of return on capital, decreasing exposure to this sector makes sense.

Table 4.10: Sector exposure when weighted in favor of return on capital

The table presents the average sector exposure of the magic formula with a gradually increasing weight in favor of return on capital from May 2003 to May 2022. The displayed percentages represent the average sector exposure for each sector over the sample period. The weighting of return on capital is increased from 50% to 100% in increments of 10%. The companies in the portfolios are grouped into sectors according to the Global Industry Classification Standard (GICS).

Weight	Materials	Industrials	Consumer Discretionary	Energy	IT	Real Estate	Communication Services	Consumer Staples	Health Care
50/50	4.5%	24.75%	13.5%	10%	17.75%	2.75%	8.75%	15.25%	2.75%
60/40	3.5%	25.75%	13%	11.25%	19.25%	1%	9.25%	13.75%	3.25%
70/30	3%	24.75%	12.75%	13.5%	20%	0.5%	9.25%	11.75%	4.5%
80/20	2.5%	24.5%	13%	15%	21.5%	-	8.75%	10%	4.75%
90/10	1.75%	24.25%	12.75%	16.75%	22%	-	8.25%	8.25%	6%
100/0	2.25%	24%	11.5%	16.25%	23%	-	8%	8.25%	6.75%

The results of the analysis of unevenly weighted metrics indicate that, when considered individually, the strategy does not perform well. As a result, we suggest combining the metrics to generate excess returns. However, the results from this robustness test are ambiguous.

Number of stocks

According to Bessembinder (2018), most individual common stocks do not outperform Treasury bills over the long run. He argues that the overall market performance largely attributes to a relatively small number of stocks. Based on this information, we hypothesize that the top-ranked stocks in the *magic formula* contribute significantly to the overall returns of the strategy. We form portfolios of 5, 10, and 15 of the highest-ranking stocks to test this hypothesis. Additionally, we form portfolios of the 25 and 30 highest-ranking stocks to test if diversification can mitigate some of the unsystematic risk.

We find that the portfolios composed of 15 stocks generate the highest significant monthly alpha, marginally higher than the original composition. Selecting fewer than 15 or more than 25 stocks does not generate a significant alpha at the $p < 0.05$ level. Thus, the original portfolio composition seems suitable. The results also support the findings of Ødegaard (2021) that most of the diversification effect on the Oslo Stock Exchange is achieved after picking 15 stocks, as the standard deviation curve flattens out at that point. Moreover, the results support our claim that the lower end of Greenblatt's suggestion of 20–30 stocks is preferable in a smaller market. Arguably, the range should be 15–25 stocks, based on the results from the OLS regression. Furthermore, the *HML* coefficient decreases the fewer stocks the portfolio contains and is statistically significant at the $p < 0.01$ level for 5–15 stocks. This finding indicates that the top-ranked companies in the *magic formula* have low book-to-market values.

Table 4.11: OLS regression of the *magic formula* with different number of stocks

The table presents the results of OLS regression analyses of the *magic formula's* monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997) using different numbers of stocks in the portfolios, from May 2003 to May 2022. We include portfolios of 5, 10, 15, 20, 25, and 30 stocks. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:					
	The *magic formula*					
	5 stocks	10 stocks	15 stocks	20 stocks	25 stocks	30 stocks
MKT	0.7617***	0.7590***	0.7257***	0.7908***	0.7889***	0.7835***
	(0.0629)	(0.0649)	(0.0503)	(0.0511)	(0.0494)	(0.0456)
SMB	0.2379***	0.3185***	0.2600***	0.2189***	0.2125***	0.2197***
	(0.0632)	(0.0813)	(0.0610)	(0.0645)	(0.0548)	(0.0482)
HML	-0.2090***	-0.1863***	-0.1361***	-0.0749*	-0.0528	-0.0389
	(0.0618)	(0.0542)	(0.0410)	(0.0404)	(0.0389)	(0.0379)
PR1YR	0.0185	-0.0013	0.0058	-0.0100	0.0069	0.0157
	(0.0740)	(0.0727)	(0.0513)	(0.0459)	(0.0416)	(0.0398)
Alpha	0.0029	0.0035	0.0052**	0.0050**	0.0043**	0.0032*
	(0.0035)	(0.0028)	(0.0022)	(0.0020)	(0.0018)	(0.0018)
Obs.	225	225	225	225	225	225
R^2	0.4376	0.5088	0.6030	0.6500	0.6848	0.7199
Adj. R^2	0.4274	0.4999	0.5958	0.6437	0.6791	0.7148
Res. SE (df = 220)	0.0479	0.0423	0.0327	0.0318	0.0292	0.0267
F Stat. (df = 4; 220)	42.8002***	56.9699***	83.5495***	102.1589***	119.5168***	141.3254***

Note: *p<0.1; **p<0.05; ***p<0.01

Intangible Assets

Greenblatt (2005) uses tangible assets in his calculation of return on capital. However, he published his book in 2005, when intangible assets were more uncommon. Since then, some companies have noticeably more intangible assets, especially IT companies, that they use to generate operating revenue. For this reason, an upward bias can be present when calculating return on capital based on tangible assets. As such, including the total assets when calculating return on capital may be a method to preserve actual capital efficiency. Thus, we replace tangible assets with total assets when forming portfolios to analyze if it has a significant effect.

Table 4.12: OLS regression of the *magic formula* based on total assets

The table presents the results of OLS regression analyses of the *magic formula* based on total assets' monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997), from May 2003 to May 2022. We replace tangible capital with total assets in the calculation of return on capital in the ranking procedure. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:	
	Magic Formula	Total Assets
MKT	0.791***	0.738***
	(0.051)	(0.050)
SMB	0.219***	0.194***
	(0.064)	(0.059)
HML	−0.075*	−0.047
	(0.040)	(0.044)
PR1YR	−0.010	−0.031
	(0.046)	(0.047)
Alpha	0.005**	0.003
	(0.002)	(0.002)
Obs.	225	225
R^2	0.650	0.633
Adj. R^2	0.644	0.626
Res. SE (df = 220)	0.032	0.031
F Stat. (df = 4; 220)	102.159***	94.857***
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 4.12 shows that the *magic formula* using total assets instead of tangible assets has an insignificant alpha. These results indicate that using tangible assets is a better measure for predicting excess returns than when including intangible assets. These findings align with Greenblatt's hypothesis that tangible capital represents the actual amount of capital needed to conduct the company's business.

Similar measures

Joel Greenblatt presents his two metrics which he argues are the best to determine a company's profitability and value. However, there are numerous methods to measure those. For instance, Asness et al. (2019) measure profitability using gross profit-to-assets, return on assets, return on equity, gross margin, cash flow over assets, and the fraction of earnings composed of cash. To measure a company's value, well-known measures, such as book-to-market, cash flow-to-price, and debt-to-equity, are applicable. We use the same method as previously when ranking and forming portfolios in the robustness tests. We apply some of the measures mentioned above and explain the reasoning below. As there is missing data in the early years, especially for the cost of goods sold, we only backtest to 2009 in this robustness test.

Gross Profit

Novy-Marx (2013) argues that gross profit-to-assets has as much ability to predict stock returns as conventional value measures. He posits that this gross profitability premium comes from it being a better proxy for true economic profitability. Furthermore, the metric is situated higher in the income statement and is free of pollution from accounting items possibly unrelated to expenses for generating operating revenue. Motivated by Novy-Marx (2013), Blackburn and Cakici (2017) substitute EBIT for gross profit in their backtest of the *magic formula* and find that the strategy improves. Therefore, we test if the gross profit can replace EBIT in the *magic formula*. First, as the proxy for return on capital, we use gross profit-to-total assets. Then, as the proxy for earnings yield, we use gross profit-to-enterprise value.

Price-to-Earnings and Return on Equity

Greenblatt (2005) argues that using EBIT instead of net income allows for comparing companies' operating earnings with different tax rates and debt levels. However, the most taxed companies could get overvalued using EBIT instead of net income on the *magic formula*. For instance, the Oslo Stock Exchange contains many oil companies. These companies are subject to an aggressive tax policy from the Norwegian government. As the tax policy is irrelevant in the selecting process of the *magic formula*, they would get selected, even though the aggressive taxation could reduce most of the potential profits to equity holders. Therefore, we test whether price-to-earnings and return on equity can be better measures. The only change in the ranking process is that the companies' price-to-earning ranks from lowest to highest to get the equivalent of the earnings yield.

Operating Cash Flow

Several academics have studied metrics based on cash flow as a measure of operating profitability in examining the cross-section of expected returns. Sloan (1996) argues that accruals are less persistent than cash flows. He states that an accrual anomaly arises, where the accrual element of earnings exhibit less persistence than the cash flow element of earnings. Desai et al. (2004) find that using the ratio of operating cash flow-to-price as a measure of value has explanatory power for the accrual anomaly. Foerster et al. (2017) state that measures of operating profitability based on cash flow are superior to more commonly used income-statement metrics, such as earnings yield and return on assets. Ball et al. (2016) also studied measures of operating profitability based on cash flow, which is free of accounting accruals adjustment. They found that profitability based on cash flow is the more robust indicator of future returns.

We substitute the metrics in the *magic formula* for the cash flow-based equivalent. Specifically, the metrics in this robustness test are operating cash flow-to-enterprise value and operating cash flow over tangible assets.

Results

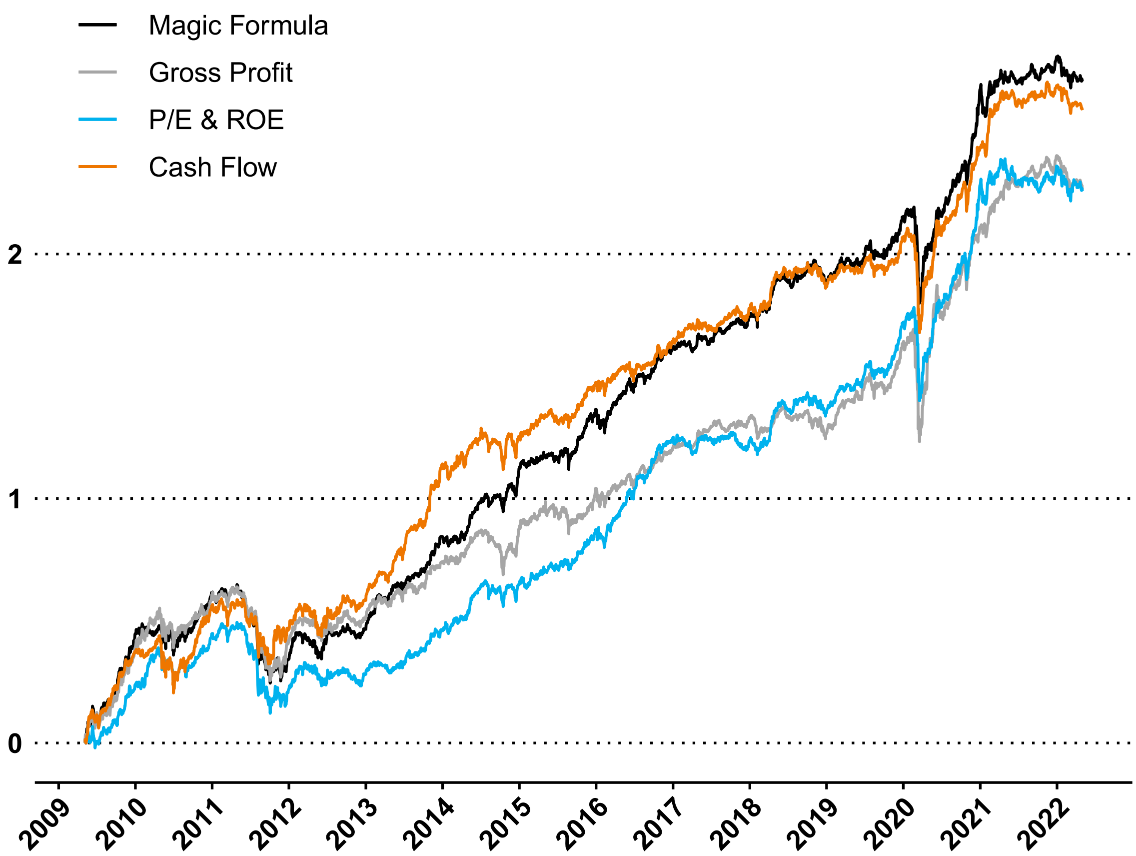

Figure 4.7: Logarithmic returns for the altered *magic formulas*

The figure presents the altered *magic formula's* logarithmic returns from May 2009 to May 2022. We alter the *magic formula* by replacing earnings yield and return on capital with proxy variables in the ranking procedure. We use gross profit-to-assets and gross profit-to-enterprise value, price-to-earnings and return on equity, and operating cash flow-to-enterprise value and operating cash flow-to-tangible assets.

When we backtest from May 2009 to May 2022, we find that the *magic formula* still provides a significant alpha at the $p < 0.05$ level. The results from the strategies based on P/E and ROE, and gross profit, are inferior. For the strategy based on P/E and ROE, there is an indication that using EBIT instead of net income is an improved estimation of the operational efficiency, especially when companies have widely different taxations. We do not find any hold for the hypothesis of Novy-Marx (2013) either. However, the cash flow-based strategy's performance shows potential. The strategy provides a monthly alpha of 0.7%, statistically significant at the $p < 0.01$ level. The logarithmic returns show that the cash flow-based strategy performs marginally worse than the *magic formula*, but when adjusting for risk, it outperforms. These results are in line with Desai et al. (2004), Sloan (1996), Foerster et al. (2017), Ball et al. (2016), and Davydov et al. (2016) which suggest that using operating cash flow provides a stronger indication of future returns.

Table 4.13: OLS regression of the altered *magic formulas*

The table presents the results of OLS regression analyses of the altered *magic formula's* monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997), from May 2009 to May 2022. We use price-to-earnings and return on equity, operating cash flow-to-enterprise value and operating cash flow-to-tangible assets, and gross profit-to-assets and gross profit-to-enterprise value, as proxies for earnings yield and return on capital. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:			
	Magic Formula	P/E and ROE	Cash Flow	Gross Profit
MKT	0.800***	0.738***	0.825***	0.798***
	(0.099)	(0.108)	(0.081)	(0.102)
SMB	0.255***	0.244***	0.196***	0.340***
	(0.083)	(0.073)	(0.046)	(0.109)
HML	−0.090*	−0.078*	−0.027	−0.138
	(0.052)	(0.047)	(0.046)	(0.092)
PR1YR	−0.008	0.035	−0.011	−0.028
	(0.064)	(0.059)	(0.051)	(0.060)
Alpha	0.006**	0.003	0.007***	0.002
	(0.003)	(0.003)	(0.002)	(0.003)
Obs.	153	153	153	153
R^2	0.556	0.488	0.621	0.555
Adj. R^2	0.544	0.474	0.611	0.543
Res. SE (df = 148)	0.033	0.034	0.029	0.035
F Stat. (df = 4; 148)	46.274***	35.287***	60.581***	46.210***
Note:				*p<0.1; **p<0.05; ***p<0.01

To further explore this relationship, we extend the analysis to include data for the cash flow equivalent strategy from 2003. The results from the OLS regression show that the performance of the *magic formula* and the cash flow equivalent strategy is similar, providing a significant alpha at the $p < 0.05$ and $p < 0.01$ levels, respectively. However, the cash flow strategy generates a 2 basis points higher monthly alpha. These findings indicate that using operating cash flow instead of EBIT can improve the *magic formula*.

Table 4.14: OLS regression of the cash flow equivalent *magic formula*

The table presents the results of OLS regression analyses of the *magic formula's* cash-flow equivalent strategy's monthly excess returns on the four-factor model of Fama and French (1993) and Carhart (1997) from May 2003 to May 2022. We use operating cash flow-to-enterprise value and operating cash flow-to-tangible assets in the ranking procedure. Newey and West (1987) heteroscedasticity and autocorrelation consistent standard errors are in parentheses.

	Dependent variable:	
	Magic Formula	Cash Flow Equivalent
MKT	0.7908***	0.7883***
	(0.0511)	(0.0421)
SMB	0.2189***	0.2127***
	(0.0645)	(0.0455)
HML	−0.0749*	−0.0527
	(0.0404)	(0.0383)
PR1YR	−0.0100	−0.0015
	(0.0459)	(0.0416)
Alpha	0.0050**	0.0052***
	(0.0020)	(0.0017)
Obs.	225	225
R^2	0.6500	0.7051
Adj. R^2	0.6437	0.6997
Res. SE (df = 220)	0.0318	0.0279
F Stat. (df = 4; 220)	102.1589***	131.5098***
Note:		*p<0.1; **p<0.05; ***p<0.01

5 Conclusion

In this study, we backtest Joel Greenblatt's *magic formula* on the Oslo Stock Exchange from May 2003 to May 2022. We aim to identify risk-adjusted excess returns in a smaller market and a more recent period than Greenblatt's test. Our approach stands out from other research by carefully examining the impact of transaction costs. In addition, we apply several robustness tests to scrutinize the *magic formula's* performance.

The results of this study indicate that the *magic formula* is an effective investment strategy for the Oslo Stock Exchange over the sample period. We uncover four main findings. (1) We show that the *magic formula* yields a 41x return on the initial investment over the sample period, equalling a compound annual growth rate (CAGR) of 21.56%, compared to the benchmark's 11x and 13.54%. (2) We find that the risk-adjusted performance is strong, with an annualized Sharpe ratio of 1.16, and a significant annual alpha of 6% from the four-factor model of Fama and French (1993) and Carhart (1997). (3) We find that risk-adjusted excess returns may not be achievable in real-world conditions due to the impact of transaction costs, as the alpha is only significant at the $p < 0.1$ level. (4) The robustness tests indicate that the strategy could be improved by using operating cash flow instead of EBIT and forming portfolios of 15–25 stocks. There is also an indication that earnings yield and return on capital should be weighted unevenly, but this test does not yield unanimous answers.

Further research can be analyzing the individual stock's turnover and the market's liquidity, as these factors could impact the strategy's ability to generate excess returns in real-world conditions.

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Appendix

A1 Stocks in the portfolios

Table A1.1: Portfolios 2003 - 2007

The table presents the 20 stocks in the portfolios of the *magic formula* each holding period from May 2003 to May 2007.

2003 - 2004	2004 - 2005	2005 - 2006	2006 - 2007
EQUINOR ASA	TANDBERG DATA ASA	NORSK HYDRO ASA	NEKKAR ASA
NORSK HYDRO ASA	GRESVIG ASA	YARA INTERNATIONAL ASA	VEIDEKKE A/S
VEIDEKKE A/S	EKORNES ASA	SCHIBSTED ASA	TELENOR ASA
EKORNES ASA	EXPERT ASA	GRESVIG ASA	AKASTOR ASA
KOMPLETT ASA	KOMPLETT ASA	NEKKAR ASA	LEROY SEAFOOD GROUP ASA
VOICE ASA	TELENOR ASA	AF GRUPPEN ASA	HAG ASA
P4 RADIO HELE NORGE ASA	SCHIBSTED ASA	VEIDEKKE A/S	SCANA ASA
AF GRUPPEN ASA	ELKEM GROUP A/S	EKORNES ASA	SUPEROFFICE AS
KONGSBERG GRUPPEN ASA	NORMAN ASA	POLARIS MEDIA ASA	WILSON ASA
FARSTAD SHIPPING ASA	OLAV THON EIENDOMSSELSKAP	TELENOR ASA	Q-FREE ASA
TGS ASA	SMEDVIG A/S	EXPERT ASA	STAVANGER AFTENBLAD ASA
NORMAN ASA	PROFDOC ASA	PROFDOC ASA	NRC GROUP ASA
SOLSTAD OFFSHORE ASA	VEIDEKKE A/S	NORMAN ASA	VISMA ASA
EXPERT ASA	VISMA ASA	AKASTOR ASA	EVRY ASA
OLAV THON EIENDOMSSELSKAP	GYLDENDAL ASA	ELKEM GROUP A/S	SCHIBSTED ASA
GYLDENDAL ASA	RIEBER & SON AS	BELSHIPS ASA	KOMPLETT ASA
TANDBERG AS	SOLSTAD OFFSHORE ASA	OLAV THON EIENDOMSSELSKAP	AF GRUPPEN ASA
ELKEM GROUP A/S	TANDBERG AS	ANDVORD TYBRING-GJEDDE ASA	YARA INTERNATIONAL ASA
SCHIBSTED ASA	KONGSBERG GRUPPEN ASA	TANDBERG TELEVISION ASA	EKORNES ASA
CHOICE HOTELS SCANDINAVIA	LEROY SEAFOOD GROUP ASA	EVRY ASA	ITERA ASA

Table A1.2: Portfolios 2007 - 2011

The table presents the 20 stocks in the portfolios of the *magic formula* each holding period from May 2007 to May 2011.

2007 - 2008	2008 - 2009	2009 - 2010	2010 - 2011
EQUINOR ASA	BWG HOMES ASA	NEXTGENTEL HOLDING ASA	NEXTGENTEL HOLDING ASA
NORSK HYDRO ASA	AF GRUPPEN ASA	BWG HOMES ASA	AF GRUPPEN ASA
TELENOR ASA	EVRY ASA	AF GRUPPEN ASA	BOUVET ASA
BWG HOMES ASA	SALMAR ASA	BOUVET ASA	EQUINOR ASA
ODIM ASA	BOUVET ASA	KITRON ASA	SIMRAD OPTRONICS ASA
CERMAQ GROUP AS	DATA RESPONSE ASA	EKORNES ASA	TELENOR ASA
EXPERT ASA	AKASTOR ASA	ITERA ASA	LEROY SEAFOOD GROUP ASA
AF GRUPPEN ASA	VEIDEKKE A/S	KONGSBERG GRUPPEN ASA	SCHIBSTED ASA
LEROY SEAFOOD GROUP ASA	INTELECOM GROUP ASA	EVRY ASA	SALMAR ASA
SCANA ASA	WILSON ASA	NORWAY PELAGIC AS	DOLPHIN DRILLING ASA
EKORNES ASA	ITERA ASA	CRAYON GROUP HOLDING ASA	KONGSBERG GRUPPEN ASA
VEIDEKKE A/S	KONGSBERG GRUPPEN ASA	VEIDEKKE A/S	VEIDEKKE A/S
POLARIS MEDIA ASA	KITRON ASA	BJORGE GRUPPEN ASA	EKORNES ASA
ITERA ASA	EXPERT ASA	DATA RESPONSE ASA	KOMPLETT ASA
WILSON ASA	COMROD COMMUNICATIONS ASA	SIMTRONICS ASA	SCANA ASA
OLAV THON EIENDOMSSELSKAP	EKORNES ASA	SYNNOVE FINDEN ASA	TGS ASA
NEKKAR ASA	TELENOR ASA	GRENLAND GROUP ASA	MEDISTIM ASA
ALTINEX ASA	SCANA ASA	FARSTAD SHIPPING ASA	MOWI ASA
AKASTOR ASA	GOODTECH ASA	YARA INTERNATIONAL ASA	RIEBER & SON AS
AKER ASA	NORWEGIAN AIR SHUTTLE ASA	COMROD COMMUNICATIONS ASA	Q-FREE ASA

Table A1.3: Portfolios 2011 - 2015

The table presents the 20 stocks in the portfolios of the *magic formula* each holding period from May 2011 to May 2015.

2011 - 2012	2012 - 2013	2013 - 2014	2014 - 2015
EQUINOR ASA	BOUVET ASA	BWG HOMES ASA	EVRY ASA
CERMAQ GROUP AS	EVRY ASA	TELENOR ASA	SALMAR ASA
LEROY SEAFOOD GROUP ASA	BWG HOMES ASA	ELTEK ASA	LEROY SEAFOOD GROUP ASA
SALMAR ASA	HAVFISK ASA	BOUVET ASA	AGR GROUP ASA
KONGSBERG GRUPPEN ASA	KONGSBERG GRUPPEN ASA	TOMRA SYSTEMS A/S	KVAERNER ASA
MOWI ASA	TOMRA SYSTEMS A/S	EVRY ASA	BOUVET ASA
GOODTECH ASA	YARA INTERNATIONAL ASA	SALMAR ASA	NORWAY ROYAL SALMON AS
ELTEK ASA	ELTEK ASA	KONGSBERG GRUPPEN ASA	BWG HOMES ASA
AUSTEVOLL SEAFOOD ASA	DOLPHIN DRILLING ASA	DNO ASA	MOWI ASA
BOUVET ASA	AF GRUPPEN ASA	MEDISTIM ASA	NEXTGENTEL HOLDING ASA
BWG HOMES ASA	TELENOR ASA	YARA INTERNATIONAL ASA	TELENOR ASA
TOMRA SYSTEMS A/S	PRONOVA BIOPHARMA ASA	HAVFISK ASA	DATA RESPONSE ASA
AF GRUPPEN ASA	ENDUR ASA	GOODTECH ASA	GOODTECH ASA
CRAYON GROUP HOLDING ASA	SOLVTRANS ASA	KITRON ASA	STRONGPOINT ASA
RIEBER & SON AS	POLARIS MEDIA ASA	NORWAY ROYAL SALMON AS	BORGESTAD ASA
COMROD COMMUNICATIONS ASA	OLAV THON EIENDOMSSELSKAP	AUSTEVOLL SEAFOOD ASA	VEIDEKKE A/S
DOLPHIN DRILLING ASA	NORWEGIAN AIR SHUTTLE ASA	KVAERNER ASA	ELTEK ASA
HAVFISK ASA	NORWAY PELAGIC AS	AKASTOR ASA	TOMRA SYSTEMS A/S
NORWAY PELAGIC AS	GOODTECH ASA	DATA RESPONSE ASA	OLAV THON EIENDOMSSELSKAP
EKORNES ASA	CERMAQ GROUP AS	OLAV THON EIENDOMSSELSKAP	KONGSBERG GRUPPEN ASA

Table A1.4: Portfolios 2015 - 2019

The table presents the 20 stocks in the portfolios of the *magic formula* each holding period from May 2015 to May 2019.

2015 - 2016	2016 - 2017	2017 - 2018	2018 - 2019
KVAERNER ASA	STRONGPOINT ASA	NORWAY ROYAL SALMON AS	TELENOR ASA
EVRY ASA	BOUVET ASA	NEKKAR ASA	BOUVET ASA
SALMAR ASA	KITRON ASA	STRONGPOINT ASA	KID ASA
BWG HOMES ASA	MULTICONSULT ASA	AKER SOLUTIONS ASA	SALMAR ASA
AKVA GROUP ASA	TELENOR ASA	ITERA ASA	PHILLY SHIPYARD ASA
EQUINOR ASA	AKVA GROUP ASA	BOUVET ASA	ARCUS ASA
TELENOR ASA	NEXTGENTEL HOLDING ASA	TELENOR ASA	KVAERNER ASA
ELTEK ASA	AKER SOLUTIONS ASA	LEROY SEAFOOD GROUP ASA	AKVA GROUP ASA
DATA RESPONSE ASA	HAVFISK ASA	NEXTGENTEL HOLDING ASA	EUROPRIS ASA
STRONGPOINT ASA	AUSTEVOLL SEAFOOD ASA	SALMAR ASA	KITRON ASA
VEIDEKKE A/S	OLAV THON EIENDOMSSELSKAP	KVAERNER ASA	POLARIS MEDIA ASA
BOUVET ASA	ITERA ASA	KID ASA	GYLDENDAL ASA
MEDISTIM ASA	DATA RESPONSE ASA	KITRON ASA	NTS ASA
HURTIGRUTEN GROUP ASA	SALMAR ASA	WEIFA ASA	GRIEG SEAFOOD AS
OLAV THON EIENDOMSSELSKAP	LEROY SEAFOOD GROUP ASA	LINK MOBILITY GROUP ASA	VEIDEKKE A/S
MOWI ASA	RENONORDEN ASA	MULTICONSULT ASA	NORWAY ROYAL SALMON AS
YARA INTERNATIONAL ASA	TOMRA SYSTEMS A/S	EUROPRIS ASA	Q-FREE ASA
HAVFISK ASA	MEDISTIM ASA	AKER ASA	OLAV THON EIENDOMSSELSKAP
BORREGAARD ASA	GYLDENDAL ASA	SAGA PURE ASA	AUSTEVOLL SEAFOOD ASA
BORGESTAD ASA	EKORNES ASA	EKORNES ASA	TOMRA SYSTEMS A/S

Table A1.5: Portfolios 2019 - 2022

The table presents the 20 stocks in the portfolios of the *magic formula* each holding period from May 2019 to May 2022.

2019 - 2020	2020 - 2021	2021 - 2022
KID ASA	KID ASA	NEKKAR ASA
EUROPRIS ASA	SCHIBSTED ASA	KID ASA
SALMAR ASA	EUROPRIS ASA	MULTICONSULT ASA
AUSTEVOLL SEAFOOD ASA	ITERA ASA	EUROPRIS ASA
GRIEG SEAFOOD AS	SELF STORAGE GROUP ASA	KITRON ASA
VOW ASA	KITRON ASA	NATTOPHARMA ASA
TELENOR ASA	NORSK HYDRO ASA	VEIDEKKE A/S
BOUVET ASA	STRONGPOINT ASA	ARCUS ASA
LEROY SEAFOOD GROUP ASA	BOUVET ASA	BOUVET ASA
ELKEM ASA	SAGA PURE ASA	TELENOR ASA
NORWAY ROYAL SALMON AS	TELENOR ASA	DATA RESPONSE ASA
ARCUS ASA	ARCUS ASA	VISTIN PHARMA ASA
FJORD1 AS	AUSTEVOLL SEAFOOD ASA	POLARIS MEDIA ASA
KITRON ASA	SALMAR ASA	CRAYON GROUP HOLDING ASA
BORGESTAD ASA	POLARIS MEDIA ASA	SELF STORAGE GROUP ASA
AKER SOLUTIONS ASA	MAGNORA ASA	KONGSBERG GRUPPEN ASA
OLAV THON EIENDOMSSELSKAP	ORKLA ASA	XXL SPORT & VILLMARK AS
AKVA GROUP ASA	ELECTROMAGNETIC GEOSERVICE	SALMAR ASA
SELF STORAGE GROUP ASA	PGS ASA	NYKODE THERAPEUTICS AS
MAGNORA ASA	MEDISTIM ASA	STRONGPOINT ASA

A2 Statistics of the *magic formula*

Table A2.1: Summary statistics of the daily returns

The table presents summary statistics for the daily returns of the *magic formula* and the benchmark from May 2003 to May 2022.

Statistic	n	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
The *magic formula*	4,782	0.0008	0.0115	-0.1117	-0.0039	0.0061	0.0826
OSEAX	4,782	0.0006	0.0136	-0.0936	-0.0057	0.0076	0.0962

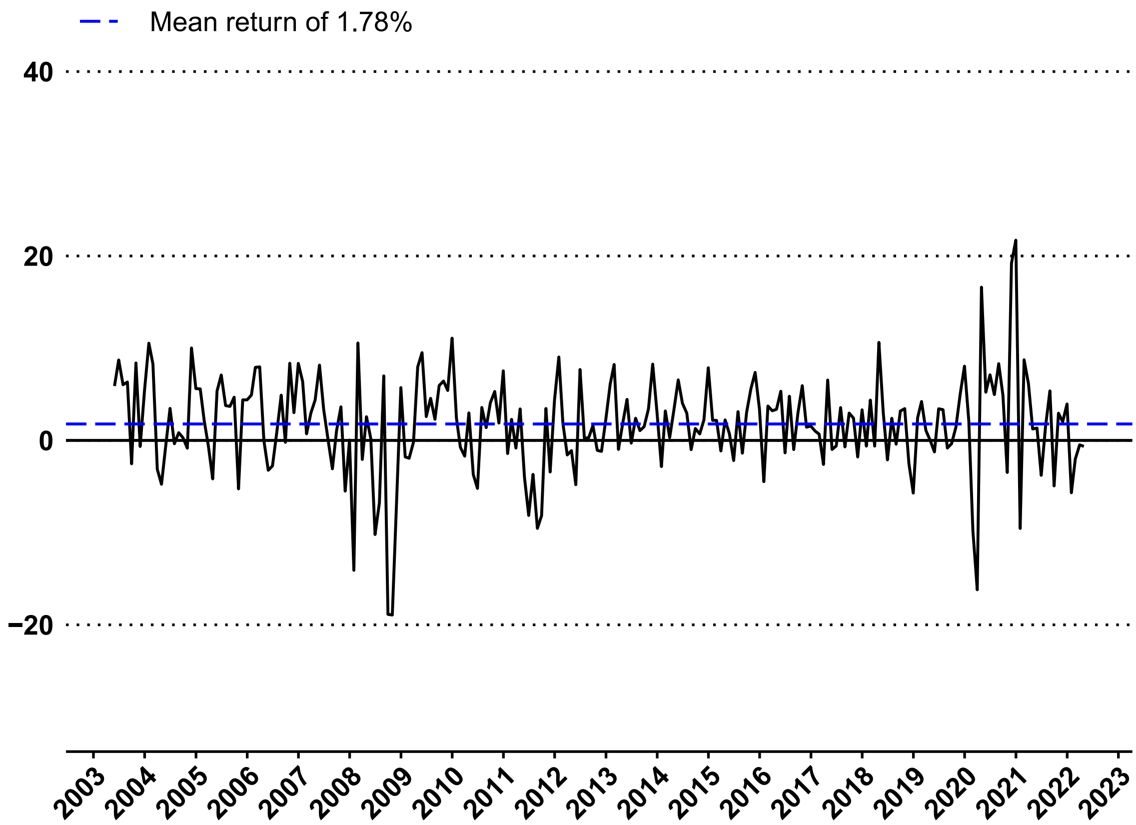

Figure A2.1: Monthly simple returns for the *magic formula*

The figure presents the simple returns for the *magic formula* from May 2003 to May 2022, expressed by monthly datapoints.

A3 Statistics of the Oslo Stock Exchange

Table A3.1: The Oslo Stock Exchange's market capitalization

The table presents the Oslo Stock Exchange's average, median, maximum, and minimum market capitalization for the start of each holding period from May 2002 to May 2022. We include statistics for all stocks and for a cut-off of market capitalization at 200 MNOK.

Numbers in MNOK

	All stocks				**Market cap. > 200 MNOK**			
	Average	**Median**	**Max**	**Min**	**Average**	**Median**	**Max**	**Min**
2002	4,218	527	158,745	13	6,068	1,239	158,745	201
2003	2,992	320	124,806	4	5,331	888	124,806	219
2004	4,538	558	188,304	3	6,080	905	188,304	208
2005	5,661	666	243,044	1	6,670	920	243,044	211
2006	8,525	942	454,339	3	9,751	1,266	454,339	209
2007	8,292	1,186	373,660	1	9,373	1,456	373,660	221
2008	7,757	813	607,756	0	9,608	1,320	607,756	208
2009	4,451	363	419,307	1	6,797	679	419,307	204
2010	6,354	543	457,890	0	9,120	1,333	457,890	205
2011	7,184	655	487,544	3	9,694	1,271	487,544	212
2012	6,665	598	487,225	2	9,533	1,394	487,225	218
2013	7,310	515	448,643	1	10,215	1,236	448,643	204
2014	9,091	861	575,232	0	11,655	1,669	575,232	209
2015	9,469	821	506,995	1	11,873	1,424	506,995	205
2016	8,502	789	445,773	1	10,991	1,673	445,773	209
2017	9,700	1,214	462,871	1	11,971	1,938	462,871	214
2018	11,770	1,307	686,095	0	14,141	2,171	686,095	200
2019	11,489	1,331	640,530	0	14,246	2,151	640,530	206
2020	9,462	972	477,443	0	12,334	1,860	477,443	204
2021	10,319	1,684	554,915	0	12,024	2,081	554,915	201
2022	12,143	1,378	1,035,293	0	14,137	1,744	1,035,293	201

Table A3.2: The Oslo Stock Exchange's sector composition

The table presents the Oslo Stock Exchange's sector composition from 2003 to 2020 (Ødegaard, 2021). The sectors are grouped by the Global Industry Classification Standard (GICS).

	Materials	Industrials	Consumer Discretionary	Energy	IT	Consumer Staples	Health Care	Financials	Utilities
2003	3.8%	20.1%	10.0%	17.2%	19.1%	3.8%	3.8%	20.1%	1.0%
2004	4.4%	20.2%	8.9%	16.7%	20.2%	4.4%	4.9%	18.7%	1.0%
2005	3.8%	18.2%	6.8%	22.0%	19.5%	5.5%	4.7%	18.2%	0.8%
2006	4.0%	17.9%	7.1%	24.2%	18.7%	5.6%	5.2%	15.9%	0.8%
2007	4.9%	17.4%	4.5%	29.2%	14.9%	6.2%	5.6%	16.0%	0.7%
2008	4.2%	18.3%	4.6%	27.8%	15.1%	7.0%	6.3%	15.1%	0.7%
2009	4.5%	18.9%	4.5%	26.9%	13.6%	7.2%	6.4%	16.3%	0.8%
2010	4.7%	18.4%	5.1%	26.6%	12.5%	7.4%	6.6%	17.2%	0.8%
2011	4.7%	17.8%	4.3%	28.5%	11.1%	7.1%	7.1%	17.8%	0.8%
2012	4.5%	18.5%	4.5%	27.6%	10.3%	7.4%	7.0%	18.5%	0.8%
2013	4.1%	17.4%	4.1%	29.3%	11.2%	7.4%	7.0%	16.9%	1.2%
2014	4.2%	19.0%	4.6%	29.5%	11.0%	5.5%	6.3%	16.9%	1.7%
2015	3.9%	18.9%	4.8%	28.1%	11.8%	4.4%	7.0%	18.0%	1.8%
2016	3.2%	19.2%	4.6%	27.9%	10.0%	5.0%	7.3%	19.6%	1.8%
2017	3.1%	19.9%	4.0%	26.5%	11.5%	4.4%	7.1%	20.4%	1.8%
2018	3.2%	20.0%	4.1%	25.0%	12.3%	5.0%	6.4%	20.9%	1.8%
2019	3.5%	19.0%	4.4%	25.2%	11.9%	4.9%	7.1%	20.4%	1.8%
2020	3.1%	19.3%	4.5%	25.1%	11.7%	5.4%	7.2%	20.6%	1.8%
Average	4.0%	18.8%	5.3%	25.7%	13.7%	5.8%	6.3%	18.2%	1.2%

A4 T-test Sharpe ratio

Table A4.1: T-test of the Sharpe ratio

The table presents the results from the two-sided paired t-test of the differences of the annual Sharpe ratios of the *magic formula* and the bechmark from May 2003 to May 2022. We use 95 percent confidence interval.

Mean of diff.	t-statistic	p-value	Parameter	Conf. low	Conf. high	Method	Alternative
0.754	t = 3.039	0.007	df = 18	0.233	1.275	Paired t-test	Two-sided

A5 Standard errors

A5.1 Durbin-Watson test

The Durbin-Watson statistic (DW) is a measure of the autocorrelation of the residuals of a regression model. The residuals are regressed on the lagged residuals. The test statistic is calculated as the ratio of the sum of the squared residuals to the sum of the squared lagged residuals and compared to a critical value from a standard normal distribution. The test statistic ranges from 0 to 4, with a value of 2 indicating no autocorrelation. Conversely, values closer to 0 or 4 indicate the presence of positive or negative autocorrelation, respectively.

$$DW = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2} \tag{.1}$$

where:

DW is the Durbin-Watson test statistic

e_t is the residual at time t

n is the number of observations

A5.2 Breusch-Pagan test

The Breusch-Pagan test measures the relationship between the regression model's residuals and the square of the independent variable. The test statistic (BP) is calculated as the ratio of the explained variance in the residuals to the unexplained variance in the residuals. It is then compared to a critical value from a chi-squared distribution with a certain number of degrees of freedom. In the absence of heteroscedasticity in the data, the test statistic will be close to zero. However, if there is heteroscedasticity in the data, the test statistic will be significantly larger than zero.

$$BP = n \cdot R_{lm}^2 \cdot \frac{k}{(n-k)} \tag{.2}$$

where:

BP is the Breusch-Pagan test statistic

n is the number of observations

R_{lm}^2 is the squared multiple correlation coefficient

k is the number of independent variables

$n - k$ is the degrees of freedom

A5.3 Test of autocorrelation and heteroscedasticity

Table A5.1: Durbin-Watson and Breusch-Pagan test

The table presents the results of the Durbin and Watson (1951) test to assess the presence of autocorrelation and the Breusch and Pagan (1979) test to evaluate the presence of heteroscedasticity in the regression models. The Durbin-Watson statistic (DW) measures the autocorrelation of the residuals of the regression models. It ranges from 0 to 4, with a value of 2 indicating no autocorrelation. Values closer to 0 or 4 indicate the presence of positive or negative autocorrelation, respectively. The BP statistic will be close to zero if there is no heteroscedasticity in the data. If there is heteroscedasticity in the data, the test statistic will be significantly larger than 0.

	Durbin-Watson test		**Breusch-Pagan test**	
Model	**p-value**	**DW statistic**	**p-value**	**BP statistic**
CAPM	0.060*	1.796	0.080*	3.056
FF3	0.367	1.958	0.046**	7.986
C4	0.426	1.979	0.106	7.639
Transaction costs	0.808	2.583	0.129	7.142
Value-weighted	0.746	2.504	0.173	6.371
60/40 favor of earnings yield	0.915	2.775	0.195	6.060
70/30 favor of earnings yield	0.894	2.730	0.159	6.600
80/20 favor of earnings yield	0.794	2.565	0.211	5.841
90/10 favor of earnings yield	0.588	2.335	0.286	5.011
100/0 favor of earnings yield	0.533	2.281	0.388	4.136
60/40 favor of ROC	0.910	2.765	0.354	4.409
70/30 favor of ROC	0.970	2.945	0.609	2.701
80/20 favor of ROC	0.979	2.987	0.246	5.429
90/10 favor of ROC	0.979	2.987	0.246	5.429
100/0 favor of ROC	0.984	3.022	0.756	1.890
5 stocks	0.059*	1.797	0.777	1.775
10 stocks	0.485	1.999	0.030**	10.702
15 stocks	0.513	2.008	0.013**	12.728
25 stocks	0.536	2.016	0.011**	12.998
30 stocks	0.767	2.100	0.016**	12.133
Total assets	0.979	2.988	0.493	3.405
P/E and ROE	0.135	1.825	0.060*	9.030
Cash Flow	0.446	1.980	0.233	5.579
Gross Profit	0.084*	1.781	0.000***	27.911
Cash flow from 2003	0.621	2.045	0.009***	13.593
Note:	*p<0.1; **p<0.05; ***p<0.01			

A5.4 Newey and West (1987) standard errors

Newey and West (1987) standard errors are calculated using a weighting matrix that discounts the contribution of observations that are more distant from each other in time. This method helps account for the potential for autocorrelation in the residuals, which can occur when the error terms in a regression model are not independent. In addition, Newey and West (1987) standard errors are also robust to heteroscedasticity, meaning that they can provide accurate inferences about the significance of the coefficients in the model even when the errors are not consistently distributed across all values of the independent variable(s).

$$\hat{\sigma}^2_{Newey-West} = \frac{1}{T}\sum_{t=1}^{T}\hat{\epsilon}_t^2 + \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{T}\hat{\epsilon}_t\hat{\epsilon}_j(1-L)^{|t-j|} \quad (.3)$$

where:

$\hat{\sigma}^2_{Newey-West}$ is the Newey-West estimator of the variance

$\hat{\epsilon}_t$ is the residual for time period t

T is the number of time periods

L is the lag length