Norwegian School of Economics
Bergen, Autumn 2022

Don't put all your eggs in one basket – spread them around!

Diversification using alternative assets and the benefits of hand-picking parameters for portfolio models

Yilin Huang & Stine Sjøhaug

Supervisor: Francisco Santos

Master thesis in Economics and Business Administration
Major: Financial Economics

NORWEGIAN SCHOOL OF ECONOMICS

Acknowledgment

This master's thesis was completed in the fall semester of 2022 as a part of our Master of Science in Financial Economics at the Norwegian School of Economics (NHH). The thesis constitutes 30 ECTS.

We want to thank our supervisor, Francisco Santos, for his feedback and positive outlook while writing the thesis. The process has been challenging but very educational, thanks to the guidance we received along the way. We also want to thank our classmates for their insightful discussions and help. Finally, we appreciate the support our friends and family have shown us during the past semester and for our entire degree.

Abstract

After the global financial crisis, alternative assets have become increasingly popular as an investment option due to their potential to generate higher returns and abilities to diversify portfolios. This thesis studies if constructed portfolios containing traditional and multiple alternative assets are better investments than holding any single assets. Further, the paper investigates the performance of a mean-variance framework versus a naïve 1/N constructed portfolio. In addition, the role of different parameters such as lookback window, rebalancing frequency, and weight constraints is analyzed to determine the optimal portfolio strategy for an alternative portfolio. At last, the paper highlights the benefits of holding a portfolio containing alternative assets compared to a traditional stock-bond portfolio.

Our results show that some single assets outperform a constructed naïve 1/N portfolio. The mean-variance portfolio framework tends to be a better investment object than holding single assets, with a few exceptions. Overall, our results state that constructed mean-variance alternative portfolios seem to distribute risk, resulting in a higher Sharpe ratio.
Regarding parameters, our results suggest that the optimal parameter for an alternative portfolio is a long-only strategy with a five-year lookback window and monthly rebalancing, considering Sharpe as the primary performance measure. Moreover, we look at the effect of differentiating between positive and negative volatility, where the optimal portfolio parameters are still to utilize a five-year lookback window with monthly rebalancing. However, the favored portfolio framework changes to a no-constrains weight strategy. At last, we provide evidence that investing in a portfolio containing alternative assets outperformed a traditional stock-bond portfolio.

Table of contents

List of Figures

List of tables

1. Introduction

Warren Buffet, CEO and Chairman of Berkshire Hathaway, said the following about diversification in June 2020: "I think it's important to diversify. That's been a tenet of mine for many years, and I think it still works. I think it's still important to spread your investments around and not put all your eggs in one basket." (Buffet, 2020). Paradoxically, he said the following about diversification 20 years earlier: "Diversification is protection against ignorance. It makes very little sense for those who know what they're doing." (Buffet, 1997). One of the most successful investors through time have expressed mixed signals about the topic of diversification. Investors can choose from endless financial products in today's markets, including various alternative assets. In recurring incidents, individuals have lost a large portion of their savings over traded volatile assets like cryptocurrencies or hyped stocks such as GameStop.
Therefore, a crucial topic is clarifying the caution an investor should take when investing in these assets and validating the truth behind the investments to guarantee that the choice is based on secure performance.

Alternative assets are investments that are not traditional Equity, Bonds, or cash. Examples of alternative assets are real estate, private equity, emerging markets, commodities, precious metals, cryptocurrencies, and hedge funds. Although alternative assets have existed for decades, investing in them has been limited compared to traditional assets. Following the global financial crisis in 2008, which taught investors a lesson about the importance of risk spreading, public markets saw dropping interest rates and subdued returns. As a result, an increasing proportion of institutional investors resorted to alternative assets in quest of higher returns (Liu et al., 2017). Understanding alternative investments' potential rewards, risks, and complexities is essential for informed decision-making and successful portfolio construction.

The growing popularity of investing in alternatives has led to increased investments in these assets due to their supposedly appealing qualities. Yet, a rise in incidents where investors have lost large sums exhibits otherwise. This thesis is motivated by a wish to unravel the conflicting messages on the advantages and disadvantages of investing in alternative assets. We want to examine if any of the alternative assets’ attributes remain valid when incorporated into a portfolio rather than just holding them individually.

Moreover, we conduct our study based on two main research questions:
Will a constructed portfolio including alternative assets be a better investment object than holding any single asset? Furthermore, what is the benefit of holding the portfolio with alternatives compared to a traditional equity-bond portfolio?

To carry out the analysis, we collect historical prices of traditional and alternative assets from 1970 to 2022. We include traditional assets of equities, government bonds, corporate bonds, and equity portfolios formed on Fama-French factors SMB, HML, RMW, and CMA. The alternative assets we incorporate are emerging markets, hedge funds, gold, real estate, private equity, commodities, and Bitcoin. In light of our first research question, we use the mentioned assets to construct thirteen portfolios with different lookback windows, rebalancing frequencies, and weight constraints. We utilize two different lookback windows, with a two-year and five-year rolling window technique. The portfolios are either rebalanced monthly or yearly, and they are either not imposed any weight constraints, capped with a range of 100% to -100%, or only investing long in assets. The portfolios are based on a 1/N method and the suggested framework mentioned in Markowitz's widely cited paper "Portfolio Selection" from 1952.

We examine the performance and risk profile differences between single assets and the constructed portfolios, with the Sharpe ratio being our main performance measure. Our results show that the 1/N portfolio achieved a Sharpe of 0.435, which is lower than six single assets, including Bitcoin, Hedge funds, Corporate bonds, CMA, RMW, and Government bonds. However, five of the Markowitz portfolios outperform all single assets, and the highest Sharpe ratio is 1.280, generated by the Long portfolio with a five-year lookback window and monthly rebalancing; Bitcoin generates the highest single asset Sharpe of 1.028. When comparing these results, the optimal constructed portfolio outperforms the highest single asset Sharpe.

We also examine portfolio parameters, where we find that all our Markowitz portfolios with the five-year lookback window perform better than the two-year lookback window in terms of the Sharpe ratio, and monthly rebalancing outperforms yearly. The effect of this can be seen as the No constraints portfolio framework with monthly rebalancing goes from having a Sharpe of 0.292 to 1.172 as the lookback window is expanded from two to five years. In regard to rebalancing frequency, we find that monthly outperforms yearly, illustrated when the No constraint portfolio with a five-year lookback window, goes from having a Sharpe of 0.983 to 1.172 as the rebalancing frequency increases.

Finally, we develop two portfolios only employing traditional assets to assess the second research question. The aim is to compare distinctions between the portfolios containing alternative assets and those not. Our analysis shows that when alternative assets are incorporated, the excess returns increase, the standard deviation decreases, and the Sharpe ratio increases significantly. As alternative assets are incorporated, the 5M - No restrictions portfolio's Sharpe increases from 0.400 to 1.075. This suggests that introducing alternative assets into the portfolio enhances overall performance and creates a diversification impact that aids in the reduction of portfolio risk.

We contribute to the existing literature by incorporating the effects of optimizing a portfolio with alternatives to a traditional portfolio while extending it to include seven alternative assets. The majority of previous studies have only looked at how an equity-bond portfolio is affected by adding a single alternative asset, like Amin & Kat (2003), which looked at the effect of including hedge funds. The study conducted by Platanakis et al. (2019) was found to include the largest number of five alternative assets but lacked to include Bitcoin and Gold. Additionally, the study only employs a single equity index and bonds in general to form the traditional portfolio. While we differentiate between various equities and bonds by adding the Fama-French factors as individual Equity assets and distinguishing between Corporate and Government-issued bonds. These publications also exclusively study how the addition of alternative assets affects the traditional portfolio while we also look at whether the portfolios outperform the single assets. Bessler and Wolff's study (2015) showed that different commodities can have varying effects when included in a portfolio, which is why we decided to distinguish between Gold and other Commodities. A few previous studies have examined the effect of changing parameters within the mean-variance framework when talking about in-sample calculations. For instance, Potrykus (2019) studied the optimal lookback window for Gold using daily data. However, no previous studies have looked at the optimal parameters for a mean-variance model when constructing portfolios with alternative assets.

The thesis comprises the following parts: Section 2 reviews the relevant literature on using alternative assets and optimizing portfolios. Continuing in section 3, we present the collected data representing the asset classes that form our dataset. Section 4 introduces the utilized risk analysis measures and how we construct the portfolio models. Furthermore, in section 5, we examine asset characteristics and analyze the findings on the portfolio performances in terms of parameters and the inclusion of alternatives. Lastly, sections 6 discuss and conclude our findings.

2. Literature Review

This chapter will highlight how prior research has addressed the problems regarding qualities for different alternative assets, optimal portfolio framework, and the choice of portfolio parameters.

In 1952 Harry Markowitz published the paper "portfolio selection," where he established the concept of Modern Portfolio Theory, which changed how investors think about portfolio construction and risk management. Markowitz claims that the ideal portfolio is optimized to produce an acceptable return while also considering lowering risk rather than solely choosing the highest projected return. He also introduced the concept of diversification, which states that investors should spread their investments over various assets. The model was revolutionary compared to the naïve portfolio model, which assumed that a portfolio's risk and return could not be improved by diversification. The naïve portfolio framework does not consider the correlation between assets or the volatility of different assets.

Several studies have been done on the different portfolio models, and the results are mixed. DeMiguel et al. (2009) evaluated the out-of-sample performance of a naïve 1/N portfolio and a sample-based mean-variance model and found that none of the mean-variance portfolios is consistently better than 1/N in terms of Sharpe ratio. Kirby and Ostdiek (2012) disagreed with DeMiguel et al.'s finding and suggested their results were due to their research design. In addition, Platanakis et al. (2021) also analyzed the two model frameworks, but they hypothesized that mean-variance strategies are superior to the naïve 1/N model in terms of asset allocation. On the other side, they state that the naïve 1/N model is superior in stock picking. The study confirmed their hypothesis and found that the superiority of mean-variance over the naïve 1/N increases stems from a lower cross-sectional idiosyncratic volatility. The inconsistency between the three studies implies that there is not an explicit portfolio strategy to favor.

The mean-variance optimization model has been criticized for its poor estimation of expected returns and volatility. The performance of a mean-variance portfolio depends on the asset's return, volatility, and correlation, but also the choice of self-selected parameters such as lookback window and rebalancing frequency. Potrykus (2019) discussed the optimal length of the lookback window for Gold within the Markowitz framework. The study used daily data and concluded that the optimal length for the lookback window was between 144 to 160 days. Additionally stating that other lookback windows could be appropriate, depending on the investment objective and the portfolio characteristics. Further, Jobson (1981) stated in his paper that the optimal lookback window with monthly data seems to be between four and seven years. However, all papers reviewing the lookback window have a complete dataset where all assets are available simultaneously. This is different for our dataset, as our paper use assets with different amounts of data, which is what set our research apart from previously conducted studies.

Traditional assets are commonly viewed as safer investments that give a consistent income and are also relatively easy to buy and sell. Conversely, alternative investments are typically viewed as riskier but potentially able to provide larger returns. As alternative assets are in general less established financial assets, they are also less liquid and may be more challenging to obtain, making them less transparent and less accessible as investment objects. However, the rise of ETFs has increased the availability for an average investor to invest in alternative assets, making the qualities of alternative assets more and more interesting for an average investor.

There are several alternative asset classes, and more studies have examined each asset class's qualities. For instance, Bessler and Wolff (2015) analyzed if commodities added value in multi-asset portfolios. Their results showed that the effect the asset had on performance differed depending on which commodities they looked at, whereas precious metals were one of the commodities that generated increased performance. Due to the findings of Bessler and Wolff (2015), we chose to look at Gold as a separate alternative asset and therefore used commodities that excluded precious metals as a measure of Commodities as an alternative.

In 1992 Divecha et al. studied Emerging markets from a quantitative perspective and found that even though emerging markets are risky individually, they had a low correlation with the developed markets. They concluded that a diversification-free lunch existed but indicated that this effect would reduce as the market became more developed. Whether the diversification effect has disappeared, reduced, or maintained or if the asset's riskiness has increased or decreased will probably change the usage of emerging markets as an alternative asset in portfolios. This change motivates us to include emerging markets as an alternative asset in our study 30 years later.

There are several other papers that examine the effect of adding one alternative asset to a portfolio containing Equity and Bonds. Platanakis and Urquhart (2020) examined the benefits of including Bitcoin, Amin & Kat (2003) analyzed the diversification effects of holding Hedge funds, Kuhle (1987) and Bond et al. (2007) studied the effect of adding Real estate. Examined the effect of adding Private equity. All papers concluded that adding one alternative to a traditional Equity-Bond portfolio was beneficial. Platanakis et al. (2019) took it further and looked at how adding five alternative assets to Equity and Bond portfolios would affect diversification. In contrast to several single asset studies, they incorporated transaction costs and examined optimized portfolios using several frameworks. They also looked at the out-of-sample performance. Their findings conclude that adding alternatives harms diversification due to transaction cost, non-normality, and estimation risk. This conclusion is the total opposite of what previous studies have concluded.

Our research differs in several ways from all the presented studies regarding the effect of adding alternative assets to a traditional portfolio. First, we have added seven alternative assets instead of Platanakis et al. (2019) five assets. Further, we have included the Fama-French factors and split the bonds into Government and Corporate bonds, which the previous papers have not included. Also, our paper analyzes different mean-variance parameters and which parameters are optimal for portfolios containing several alternative assets. In addition, we have looked at how different weight constraints might affect the performance of a portfolio containing several alternative assets added at different points in time. During our paper, we touch on the work of several of these papers.

3. Data

In this section, we present the process of gathering the pricing data used to construct the dataset. Primarily, we review the features of the indices that represent each asset class. Subsequently, we present relevant descriptive statistics.

Sample construction

The dataset we used to construct all portfolio models comprises 14 assets. The time series consists of monthly data from December 1969 to July 2022, where the asset data are available at different points in time. In the following section, we describe the procedure for collecting and calculating asset class data. Asset classes included in the portfolios are Equity, Government bonds, Corporate bonds, Emerging markets, Hedge funds, Commodities, Gold, Real estate, Private equity, Bitcoin, and the Fama-French portfolios formed on SMB, HML, RMW, and CMA.

We chose the major index equivalent for all assets in the portfolio rather than the ETF due to data limitations[1]. Therefore, we emphasize that when a portfolio takes short positions in the asset, one could short the ETF equivalent or either buy a put option or enter a futures contract to sell the long position.

Data sources

The Bloomberg Terminal was used to retrieve several of the following indices: MSCI World Total Return Index (Equity), FTSE World Government Bond Index (Government bonds), S&P 500 Investment Grade Corporate Bond Index (Corporate bonds), MSCI Emerging Markets Index (Emerging markets), HFRI Fund of Funds Composite Index (Hedge funds), NYSE Arca Gold GUDS Index (Gold), Bloomberg Commodity ex-Precious Metals Index (Commodities), MSCI World Real Estate Index (Real estate), LPX50 Listed Private Equity Index TR (Private equity) and Bitcoin/ U.S. Dollar perpetual inverse swap (XBTUSD) (Bloomberg L.P., 2022). All downloaded indices are quoted in USD. To represent the returns of the assets, we use the monthly value changes of the indices adjusted for dividends. We calculate simple returns to measure the performance of the asset.

[1] One example is that by choosing the ETF of, e.g., MSCI World Index = URTH, we got our data shortened from 1970 to 2012.

The four Fama-French factors plus the monthly risk-free rate are available on Kenneth French's website (French, 2022). The 3-month U.S. Treasury bill used to measure the risk-free rate is downloaded from the Federal Reserve Bank of St. Louis' database (U.S. Bureau of Economic Analysis, 2022).

Equity

To capture a more global diversification effect within listed equities, we choose the traditional MSCI World Index[2] rather than an index like the S&P 500. The index is a broad measure of equity performance in developed countries because it contains 1511 large and mid-cap firms from 23 nations in all sectors (MSCI Inc., 2022a). MSCI World Index launched on March 31st, 1986, with historical prices backtested to December 1969.

Government bonds

We use the FTSE World Government Bond Index[3] to represent government bonds. It is a broad measure of fixed income as it invests in over 20 developed countries' government bonds, is quoted in several currencies, and is weighted after market capitalization (FTSE Russell, 2022). All fixed-rate bonds included in the index have at least one year to maturity and an outstanding value of at least USD 25 million.

Corporate bonds

The S&P 500 Investment Grade Corporate Bond Index[4] is a proxy for corporate bond returns. It invests in corporate debt, with at least one month left to maturity, issued by companies included in the S&P 500 while considering the associated investment-grade rating. The index launched on the 8th of July 2015, including asset data backtested data until 1995.

Emerging markets

We use the MSCI Emerging Markets Index[5] to estimate the evolution of emerging markets. The index measures the performance of rapidly expanding economies by investing in a broad selection of large and mid-cap companies in 25 countries (MSCI Inc., 2022b). The data

[2] Equity, Bloomberg ticker: MXWO
[3] Government bonds, Bloomberg ticker: SBWGU
[4] Corporate bonds, Bloomberg ticker: SP5IGBIT
[5] Emerging markets, Bloomberg ticker: MXEF

includes backtested prices lasting from December 1987 until the inception, which was 1st January 2001.

Hedge funds

HFRI Fund of Funds Composite Index[6] was created to broadly capture the performance of hedge funds across all regions and strategies. The index is highly recognized as a global benchmark measuring the performance of hedge funds and was launched on the 31st of December, 1989.

Gold and Commodities - excluding precious metals

We download the price change of the NYSE Arca Gold GUDS Index[7], also known as the "HUI" index, which invests in companies operating in the gold mining business. It is the most watched pure gold investing index on the world market. The index was incepted on the 15th of March 1996, with a base value of 200.
Furthermore, the Bloomberg Commodity ex-Precious Metals Index[8] measures all other commodities, including investments in energy, agricultural products, and industrial metals. The silver mining business will be the only industry left out of the analysis.

Real Estate

To measure real estate value, we use the approximation MSCI World Real Estate Index[9], consisting of large and mid-cap stocks across 23 developed market countries. All invested securities are classified as operating in the real estate sector. Backtested data of the index runs from 1995 until the launch on December 31st, 1998.

Private equity

Investing directly in private equity-backed investments is challenging to accomplish for a private investor. However, the listed Private Equity firms reflect the sector's prospects, which motivates using LPX50 Listed Private Equity Index TR[10]. The index has existed since March

[6] Hedge funds, Bloomberg ticker: HFRIFOF
[7] Gold, Bloomberg ticker: HUI
[8] Commodities, Bloomberg ticker: BCOMXPMT
[9] Real Estate, Bloomberg ticker: MXWO0RE
[10] Private Equity, Bloomberg ticker: LPX50TR

2004 and invests in the 50 most highly capitalized and liquid exchange-traded private equity firms, spreading it across regions, strategies, and vintage years.

Bitcoin

Bitcoin was the first decentralized digital currency created on the 31st of October 2008 and is considered the most liquid and tradable cryptocurrency. Although the currency began to trade in an open source in 2009, we could only download pricing data from August 2010 using an index that traced the performance of the Bitcoin measured in USD[11].

Fama-French factors

We include four Fama-French factors as assets in calculating the optimal portfolio. The factors are based on the New York Stock Exchange, American Stock Exchange, and Nasdaq common stocks. The factor returns are the average of several portfolios where the strategy is to go long in the desirable traits and short in the undesirable traits (French, 2021). These factors are formed upon Zero-Investment Portfolios.

Monthly risk-free rate

The risk-free rate is calculated using the 3-month U.S. Treasury bill, which has also been cross-referenced with the monthly risk-free rate provided in the Fama-French five-factor dataset.

[11] Bitcoin, Bloomberg ticker: XBTUSD

Descriptive statistics

Table 1: Summary statistics of all asset class data

Table 1: Summary statistics of all asset class data. All statistics are stated in annual terms. Column "N" explain the total number of observations included for each asset. The assets in the table are ranked from those with the most data points to those with the fewest. Excess returns are the average monthly portfolio returns adjusted for the monthly nominal risk-free rate. The standard deviation is calculated using the sample standard deviation. Skewness and kurtosis are calculated using Pearson's standardized third and fourth central moment of distribution.

Summary statistics of the dataset								
Asset class	Abbreviation	Data available from	N	Excess return	Standard deviation	Sharpe ratio	Skewness	Kurtosis
SMB	SMB	December 1969	632	1.88 %	10.50 %	0.178	0.37	6.44
HML	HML	December 1969	632	3.79 %	10.70 %	0.35	0.1	5.16
RMW	RMW	December 1969	632	3.69 %	7.90 %	0.46	-0.32	14.54
CMA	CMA	December 1969	632	3.96 %	7.10 %	0.55	0.34	4.39
Equity	Equity	December 1969	633	3.04 %	14.80 %	0.201	-0.53	4.53
Government Bonds	Gov	January 1985	452	3.01 %	6.80 %	0.439	0.11	3.48
Emerging markets	EM	December 1987	417	6.54 %	22.10 %	0.288	-0.55	4.82
Hedge funds	Hedge	December 1989	392	3.72 %	5.60 %	0.661	-0.76	7.48
Corporate bonds	Corp	December 1994	333	3.47 %	5.30 %	0.646	-0.67	7.53
Gold	Gold	December 1994	333	6.55 %	39.40 %	0.161	0.54	4.41
Real-Estate	RE	December 1994	333	2.77 %	19.00 %	0.144	-0.58	6.18
Private Equity	PE	January 1999	284	9.06 %	25.50 %	0.342	-0.37	7.18
Commodities	Com	December 2001	249	2.38 %	17.60 %	0.134	-0.57	4.41
Bitcoin	Bitcoin	July 2010	146	545.51 %	196.30 %	1.028	4.43	29.48

4. Methodology

In this section, we discuss how we calculate several risk estimations for assets and portfolios and how we build the various portfolios. The first section outlines the calculations and metrics utilized for the research, while the second section details the portfolio frameworks employed for further study, as well as the parameters associated with them.

4.1 Risk-adjusted performance measures

We introduce several crucial financial risk measures to the analysis of the portfolios: The Sharpe ratio, skewness, kurtosis, Value at Risk (VaR), and Sortino ratio. Therefore, we give an overview of these metrics and explain how they were derived.

Sharpe ratio

The Sharpe Ratio is used as an estimate to look at the portfolio models' performance. William F. Sharpe developed the Sharpe ratio in 1966, which measures how much an investor receives in excess return for every unit of risk. Excess return is employed because investors need to be rewarded for the extra risk they take beyond the risk-free asset. The ratio is one tool that can be used to determine which investment choice that produces the highest returns while also considering risk. The formula to compute Sharpe Ratio is as stated:

$$\text{(1) } Sharpe\ Ratio = \frac{r_p - r_f}{\sigma_{r_p - r_f}}$$

$$r_p \text{ is the portfolio return, } r_f \text{ is the risk} - \text{free rate, and}$$

$$\sigma_{r_p - r_f} \text{ is the standard deviation of the portfolio's excess return.}$$

Skewness & Kurtosis[12]

We evaluate the impact of the third and fourth moments of the distribution to include additional elements of risk. Skewness is an indication of the dataset's level of symmetry or asymmetry. When a dataset is positively skewed, the majority of observations are greater than the mean. In the case of negatively skewed data, most observations are smaller than the mean. A return distribution with positive skewness will likely result in frequent small losses and a

[12] Skewness and Kurtosis calculated in R using the Performance Analytics package

few bigger rewards. On the other hand, a negatively skewed distribution denotes an investment with a high number of small wins and a few larger losses.

We use Pearson's second coefficient of skewness to measure the asset and portfolio returns asymmetry. A normal distribution has a skewness equal to 0. The formula for skewness is presented below:

$$(2)\ Skewness = \frac{\sum_i^n (r_i - \bar{r})^3}{(N-1) * \sigma^3}$$

where N is the number of returns, $\bar{r}$ *is the mean of the return distribution and* σ *is the sample standard deviation.*

Kurtosis signifies whether the dataset is light-tailed or heavy-tailed compared to a normal distribution. If the dataset has high kurtosis, it indicates a heavier presence of extremely high or extremely negative returns, which are associated with increased risk. However, a smaller kurtosis indicates a moderate risk because the likelihood of extreme returns is relatively low. A kurtosis equal to 3 follows the tails of a normal distribution. Kurtosis is calculated as follows:

$$(3)\ Kurtosis = \frac{\sum_i^n (r_i - \bar{r})^4}{(N-1) * \sigma^4}$$

where N is the number of returns, $\bar{r}$ *is the mean of the return distribution and* σ *is the sample standard deviation.*

Value at Risk[13]

Value at Risk (VaR) measures the magnitude of potential losses. It is defined as the maximum loss an investment will likely suffer over a given period with a certain confidence level. VaR can be calculated using various methods, where we use the historical simulation method. Our estimates are based on how much the portfolio might lose on a monthly basis with a 99% confidence level. The Value at Risk is calculated as the following:

$$(4)\ VaR^{14} = E(r_p - r_f) - (2.576 * \sigma_{r_p - r_f})$$

[13] VaR calculated in R using the Performance Analytics package

[14] Z-score of a 99% confidence interval

Sortino Ratio[15]

The Sortino ratio is a variant of the Sharpe ratio. The ratio compares an investment's return only to its downside risk. Instead of dividing the excess returns by the total standard deviation, the Sortino ratio is divided by the Lower Partial Standard Deviation (LPSD). The standard deviation of the downside is computed by only including the variability of returns that falls below zero.

$$(5)\ LPSD = \sqrt{\frac{\sum_{i=1}^{k}(x_i - \mu)^2}{k-1}}\ for\ x_i < 0$$

$$(6) Sortino\ Ratio = \frac{R_p - r_f}{LPSD}$$

$$R_p\ is\ the\ portfolio\ return, R_f\ is\ the\ risk - free\ rate, and$$
$$LPSD\ is\ the\ standard\ deviation\ of\ the\ downside$$

4.2 Portfolio models

In the following section, we will explain the frameworks for the different portfolio models and clarify how we have calculated the parameters within each portfolio. First, we present how we calculate the naïve 1/N portfolio, which invests equally in all assets available. Secondly, we show how our mean-variance portfolios with different degrees of constraint are computed.

1/N - Naïve portfolio model

No matter an asset's return or standard deviation, the equally weighted portfolio allocates the same amount in each asset. In our model, we rebalance the portfolio every time a new asset becomes available, which ensures that the weights between all the available assets are allocated uniformly. The following formula computes the portfolio weights:

$$(7)\ W_t^{eqw} = \frac{1}{N_t}$$

where W is the portfolio weights at time t and N is the number of assets available at time t.

[15] Sortino Ratio calculated in R using the Performance Analytics package

Further, the portfolio return is computed with the following formula:

$$(8) E\,[r]_t^{eqw} = W_t^{eqw} * E[r_a] + W_t^{eqw} * E[r_b] + \cdots + W_t^{eqw} * E[r_i]$$

$$\rightarrow E[r]_t^{eqw} = \sum W_t^{eqw} * E[r_i]$$

Where $E[r]_t^{eqw}$ is the portfolio's expected return at time t, W_t^{eqw} is the portfolio weights at time t, and $E[r_i]$ is the expected return of each asset.

Mean-Variance portfolio

The second portfolio framework used in our analysis is the Markowitz portfolio model. The classical framework of modern portfolio theory assumes that the investor only cares about the first two moments of the return distribution: mean and variance. The return-to-risk efficient portfolio is generated by examining different portfolio combinations based on anticipated mean returns and standard deviations of the assets.

The portfolio expected return is computed by multiplying a vector of asset expected return and a vector of portfolio weights, where the sum is set not to exceed 100%.

$$(9)\ E[R_p] = \mu_p = [w_1, w_2, \dots, w_n] \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_n \end{bmatrix} = \sum_{j-1}^{n} w_j * E(R_j)$$

To calculate the total combined risk, the weights vector is multiplied by the covariance matrix, which is then multiplied by the vector of the same weights transposed. In our portfolios, assets are incepted at different times, which means that the length of the vectors will change as one asset adds to the portfolio. However, the concept is the same; but we must expand the vectors by N + i.

$$(10)\ Var[R_p] = \sigma_p^2$$

$$= [w_1, w_2, \dots, w_n] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2n} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \cdots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix}$$

We use the Markowitz framework to calculate the assets' monthly weights. The input data are computed based on a rolling window approach for the lookback window. Depending on the length, we use the previous two- or five years of data to compute our portfolio's mean return and covariance matrix. The process is advanced one month at a time. Furthermore, the optimal weights are rebalanced monthly or yearly[16], depending on the portfolio's rebalancing frequency. Finally, we retrieve the optimal weights each month, multiply them by the realized return of each asset, and sum the resulting values to compute the portfolio return.

We have established three different weight constraints within the Markowitz framework. First, a portfolio model where we allow for all asset allocations. Secondly, a portfolio model with capped weights with a minimum of shorting 100% and a maximum of longing 100%. Finally, a long-only strategy that does not allow for short-selling.

Below is a summary of the different portfolio models within the mean-variance framework:

Table 2: Utilized mean-variance portfolio models

A more detailed explanation of the abbreviations is found in the Appendix.

No constraints				Capped: 100% to -100%				Long			
2M	2Y[16]	5M	5Y	2M	2Y	5M	5Y	2M	2Y	5M	5Y

[16] The weights are rebalanced on January 1st for the yearly rebalanced portfolios

Table 3: Performance for Single assets and Portfolios

Summary statistics of all asset classes and portfolios. All measures are stated in annual terms. Excess returns are the average monthly portfolio returns adjusted for the monthly nominal risk-free rate. The standard deviation is calculated using the sample standard deviation. Panel A displays performance for single assets, and we use return observation from the month the assets become available until July 2022. Panel B displays the performance for portfolios containing traditional and alternative assets. The 1/N portfolio contains observations from 1970 to July 2022, the portfolios with a two-year lookback window contain observations from 1972 to July 2022, and the portfolios with a five-year lookback window contain observations from 1975 to July 2022. Panel C displays performance for two portfolios only containing traditional assets, and two portfolios containing traditional and alternative assets (called "traditional portfolio" and "alternative portfolio"). The observations in panel C are from 1993 to July 2022, since the first alternative asset is available from 1993.

Panel A

	Annualize Excess Return	Annualized SD	Annualized Sharpe
Single Assets			
SMB	1.88 %	10.51 %	0.178
HML	3.79 %	10.65 %	0.350
RMW	3.69 %	7.87 %	0.460
CMA	3.96 %	7.07 %	0.550
Equity	3.04 %	14.92 %	0.201
Gov	3.01 %	6.76 %	0.439
EM	6.54 %	22.05 %	0.288
Hedge	3.72 %	5.53 %	0.661
Corp	3.47 %	5.29 %	0.646
Gold	6.55 %	39.41 %	0.161
RE	2.77 %	18.99 %	0.144
PE	9.06 %	25.49 %	0.342
Com	2.38 %	17.62 %	0.134
Bitcoin	545.51 %	196.32 %	1.028

Panel B

	Annualize Excess Return	Annualized SD	Annualized Sharpe
Portfolio Models incorporating alternative assets			
2M no constraints	8.56 %	28.04 %	0.294
2Y no constraints	3.23 %	36.28 %	0.088
5M no constraints	4.79 %	4.00 %	1.172
5Y no constraints	3.76 %	3.76 %	0.983
2M capped	6.09 %	6.69 %	0.886
2Y capped	4.74 %	5.83 %	0.795
5M capped	4.50 %	3.45 %	1.279
5Y capped	3.65 %	3.66 %	0.981
2M long	5.27 %	4.39 %	1.173
2Y long	4.14 %	4.05 %	1.004
5M long	4.64 %	3.55 %	1.280
5Y long	3.95 %	3.69 %	1.050
1/N	4.45 %	10.03 %	0.435

Panel C

	Annualize Excess Return	Annualized SD	Annualized Sharpe
Traditional vs Alternative portfolios			
Alternative - No constraints	5.02 %	4.57 %	1.075
Traditional - No constraints	2.72 %	6.72 %	0.400
Alternative - Long	4.77 %	3.99 %	1.171
Traditional - Long	3.36 %	3.48 %	0.950

5. Analysis

5.1 Single Asset

In the following section, we look at asset performance on a total level through the measures of excess return, standard deviation, skewness, kurtosis, and Sharpe ratio. Return conveys the asset's prospect of generating value, and the standard deviation states the asset's volatility. In addition, we look at the correlations between the different assets.

5.1.1 Asset performance

We look at the assets' skewness and kurtosis to analyze the third and fourth moments of the distribution and, therefore, gain a complete risk profile. Our results show that most assets are negatively skewed, indicating that they have recurring small positive returns and a few significant losses. Further, all assets have a kurtosis above 3, indicating a high presence of extreme values. Government bonds are closest to resembling a normal distribution, with a skewness of 0.11 and a kurtosis of 3.48. In addition, Table 3 Panel A shows that all assets generate a positive return and that there are large differences in the standard deviation.

Table 4: Skewness and Kurtosis for Single assets

Skewness and Kurtosis for all the single assets calculated of returns.

	SMB	HML	RMW	CMA	Equity	Gov	EM	Hedge	Corp	Gold	RE	PE	Com	Bitcoin
Skewness	0.37	0.10	-0.32	0.34	-0.53	0.11	-0.55	-0.76	-0.67	0.54	-0.58	-0.37	-0.57	4.43
Kurtosis	6.44	5.16	14.54	4.39	4.53	3.48	4.82	7.48	7.53	4.41	6.18	7.18	4.41	29.48

Bitcoin outperforms all other assets with an excess return of 545.51% and a Sharpe ratio of 1.028, as illustrated in Table 3 Panel A. However, it is also the most volatile, with the highest standard deviation of 196.32%, skewness of 4.43, and kurtosis of 29.48 (Table 4). Nonetheless, Government bonds are the most stable asset regarding all three moments of risk and might be favored by more risk-averse investors. It is also worth mentioning that since all the assets, except Bitcoin, are based on indices that often invest in several companies, the assets generate diversification within themselves. Therefore, by investing in several indices, one is additionally diversified across companies, sectors, and markets.

5.1.2 Asset correlation

To study how a portfolio could be potentially affected by adding several assets, we examine the correlation between them. The lower the correlation between the assets, the greater the potential advantage of diversification. By computing a correlation matrix, we can study the correlation between the assets and at which magnitude they experience co-movements. The aim is to see if some of the assets would be a good diversifier when joined with certain assets, based solely on the correlation coefficient.

Table 5: Correlation matrix for Single assets

*The correlation between assets is calculated as the assets are available. This means that the correlation between SMB, available since 1970, and Bitcoin, available from August 2010, are calculated from August 2010 to July 2022. Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***.*

Correlation Matrix														
	SMB	*HML*	*RMW*	*CMA*	*Equity*	*Gov*	*EM*	*Hedge*	*Corp*	*Gold*	*RE*	*PE*	*Com*	*Bitcoin*
SMB	1.000													
HML	-0.032	1.000												
RMW	-0,369***	0,126**	1.000											
CMA	-0.075	0,685***	0.026	1.000										
Equity	0,180***	-0,142***	-0,186***	-0,290***	1.000									
Gov	-0,146**	-0.055	0.080	-0.065	0,256***	1.000								
EM	0,271***	-0.088	-0,297***	-0,230***	0,729***	0,134**	1.000							
Hedge	0,336***	-0,132**	-0,361***	-0,251***	0,624***	0.045	0,673***	1.000						
Corp	-0.030	-0.064	0.012	-0,137*	0,291***	0,589***	0,239***	0,280***	1.000					
Gold	0,140*	-0.022	-0.040	-0.018	0,254***	0,382***	0,413***	0,273***	0,295***	1.000				
RE	0,188***	0.069	-0,158**	-0,111*	0,774***	0,286***	0,733***	0,545***	0,367***	0,320***	1.000			
PE	0,362***	-0.007	-0,432***	-0,240***	0,882***	0,183**	0,790***	0,773***	0,297***	0,210***	0,796***	1.000		
Com	0,198**	0,282***	-0.106	0.043	0,485***	0,154*	0,535***	0,546***	0.109	0,319***	0,424***	0,473***	1.000	
Bitcoin	0.032	-0.042	0.024	-0.066	-0,180*	0.111	0.085	0,200*	0.111	-0.037	0.125	0,201*	0.080	1.000

The strongest positive correlation was found between Equity and Private equity, with a correlation coefficient of 0.882 (Table 5), indicating that combining these two assets might not be the best in reducing risk. Further, Equity is highly correlated with several alternative assets, such as Emerging markets, Hedge funds, Real estate, and Commodities. In theory, combining Equity with any of these alternatives would not generate a high diversification effect. The high correlation might be due to the MSCI World Index, our proxy for Equity, already containing stocks in sectors similar to these alternative asset classes, such as energy, materials, financials, and Real estate.

Emerging markets also experience a high correlation with almost all other assets, which might be attributable to the proportion invested in each sector in MSCI Emerging Markets and MSCI World being quite comparable (MSCI Inc., 2022). This suggests that the free lunch previously attained by investing in both Emerging markets and Equity, found in 1992 by Divecha et al. seems considerably reduced. The diversification effect points to Emerging markets becoming more developed. Also, the reduced differences in correlation levels between Emerging markets and Equity suggest that changes in these assets become more tied to the same market factors. On the contrary, Emerging markets and Equity have a low correlation with the other traditional assets, Gold, and Bitcoin. Although, several assets, such as the Fama-French factors, Gold, Bitcoin, Government bonds, and Corporate bonds, have a low correlation with almost all the other assets.

Overall, our findings suggest that several assets are highly correlated, but some asset combinations have the potential to increase the diversification when combined.

5.2 Portfolio analysis

In this section, we analyze whether constructed optimal portfolios improve performance compared to holding a single asset. If so, which factors contribute to the improved outcomes. We begin by studying how the portfolios are differentiated in terms of asset allocation. Furthermore, we investigate how the different portfolio frameworks, weight constraints, and parameters impact the performance and risk to identify the best-performing portfolio. Finally, we will analyze other risk metrics to assess portfolio complete risk profiles and study how they behave in times of crisis.

5.2.1 Portfolio weights

Continuing, we look at how the portfolios have invested in different assets on a general level to ensure that the portfolios are diversified over multiple assets. We look at this by taking the average of the weights for all assets within each portfolio model since there are over 600 months of weight combinations. On the next page are figures of how all portfolio models allocate the assets:

Figure 1: Portfolio allocation in traditional assets

The figure shows all the portfolios' average allocation in traditional assets in the following order: SMB, HML, RMW, CMA, Equity, Government bonds, and Corporate bonds.

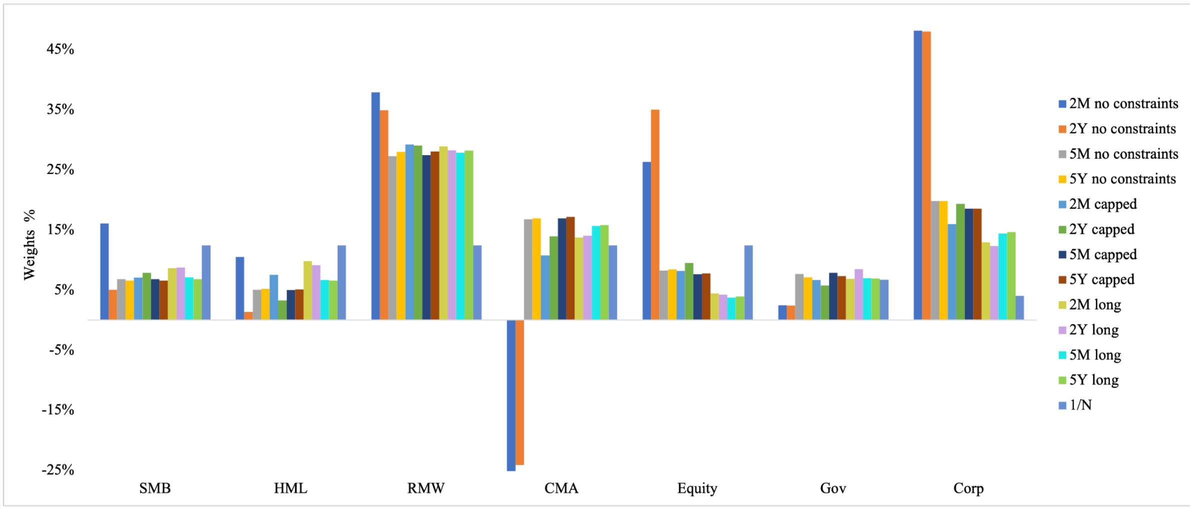
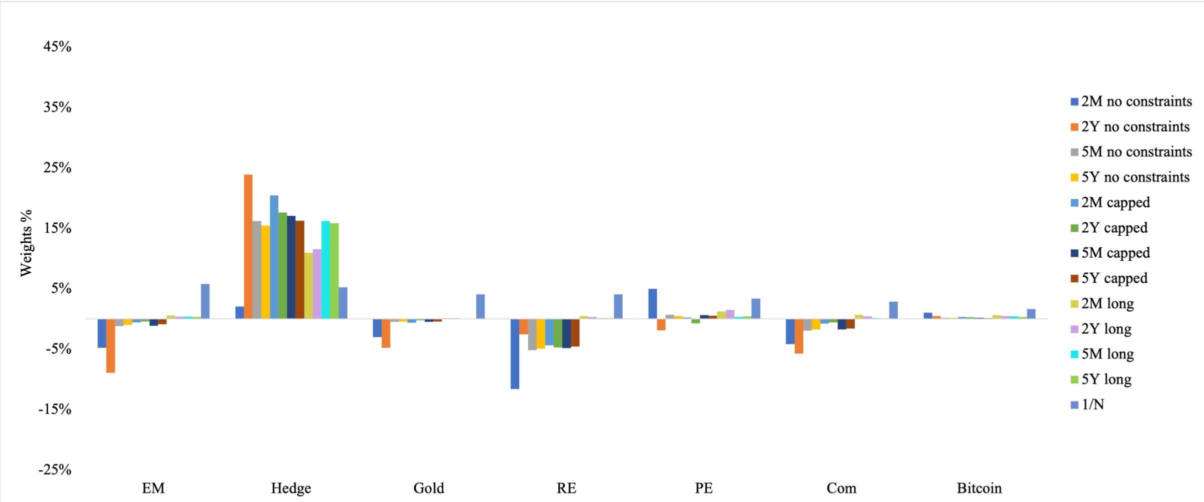


Figure 2: Portfolio allocation in alternative assets

The figure shows all the portfolios' average allocation in alternative assets in the following order: Emerging markets, Hedge funds, Gold, Real Estate, Private Equity, Commodities, and Bitcoin.



We immediately notice that ten out of the twelve mean-variance portfolios invest almost identically at an average level, where the differences are as minor as 2-3%. We can also see that all the portfolios are invested in both traditional and alternative assets. Corporate bonds and RMW are the highest allocated traditional assets, while Hedge funds are the most preferred alternative asset.

If we look at the exceptions, the 2M and 2Y – No constraints, and compare them to their equivalent 5-year-based portfolios, we see that the 5M/5Y invest more in CMA and

Government bonds. As an example, 2M/2Y heavily shorts CMA by roughly 25%, while 5M/5Y goes long by around 17%. Additionally, 2M/2Y invests 28% more in Corporate bonds. CMA is relatively close to a normal distribution, with skewness and kurtosis of respectively 0.34 and 4.39, whereas Corporate bonds have skewness and kurtosis of respectively -0.67 and 7.53. This might point to the 5-year window pulling more of the value in less risk-associated assets.

In general, all portfolios tend to invest more heavily in traditional assets; the only exception being Hedge funds. This allocation might be due to the low correlation Hedge funds have with almost all traditional assets except equity or due to the asset class generating one of the highest Sharpes of 0.661. Additionally, the No constraints and Capped portfolios tend to short a couple of percentages in alternative assets such as Real Estate, Emerging markets, Gold, and Commodities. The shorting of these alternative assets suggests that they are primarily being utilized as means to further invest more in other assets.

5.2.2 Single asset vs. Portfolio performance

When we compare the Sharpe ratios obtained from only holding a single asset to our portfolio performance in Table 3, Panel A and B, we can see that five portfolios outperform all single assets. Additionally, ten portfolios outperform most assets except Bitcoin. Diversifying into several assets seems to help distribute risk by lowering the standard deviation, resulting in a higher performance measured in the Sharpe ratio. For instance, 5M - Long achieves the highest portfolio Sharpe of 1.280, while the highest single asset Sharpe is generated by Bitcoin of 1.028. The high Sharpe is caused by the portfolio's low annual standard deviation of 3.55%, which is significantly less than Bitcoin's 196.32%, indicating the presence of the diversification effect in the portfolio.

Conversely, not all the constructed portfolios outperform single assets. The 2Y - No constraint, which attains a Sharpe ratio of 0.081, is outperformed by all single assets. In addition, the 2M - No constraint portfolio performs mediocrely, with a Sharpe of 0.292.
We also observe that the Sharpe of Bitcoin, Corporate bonds, Hedge funds, Government bonds, CMA, and RMW surpass the 1/N portfolio because the 1/N is failing in lowering the standard deviation. Among the single assets that beat the 1/N are two Fama-French factors that exist for the entire dataset. Ultimately, investors who choose to invest in CMA or RMW

for the entirety of the 1/N portfolio will receive a higher Sharpe, which depicts that diversifying for the sake of it will not always pay off.

Additionally, eight portfolios are outperformed by Bitcoin, indicating that holding Bitcoin over these eight portfolios would be preferred. Bitcoin has historically generated sufficient realized returns to cover the increased risk. However, we must stress that Bitcoin has a relatively short time frame and some extraordinary returns throughout its shorter existence. We expect the returns for Bitcoin to eventually stabilize as the market for cryptocurrencies matures and becomes more efficient.

In general, with only a few exceptions, we see that constructing portfolios rather than holding single assets is preferred, due to diversification reducing the portfolio's standard deviation. There are few instances that depict holding portfolios not always being the best option in terms of Sharpe, where the 1/N, 2M - No constraints, and 2Y - No constraints are surpassed by several single assets. The best performing asset being Bitcoin which beats eight portfolios. However, five of our portfolios still beat every single asset.

5.2.3 The effect of weight constraints, lookback window, and rebalancing

The portfolios' Sharpe differ greatly, indicating that the techniques we employ significantly influence the portfolio performance. Following, we examine how the choice of weight constraints, the lookback window, and the rebalancing frequency in the Markowitz framework affects performance.

It is also crucial to note that results based purely on historical returns should be interpreted cautiously since past performance cannot guarantee future returns. However, this is only partially true when we look at asset classes rather than individual equities. Asset classes are more determined by some elemental macro factors. To a certain extent, these factors tend to be repeated, and a retroactive approach will therefore be appropriate when assigning asset classes. We keep this in mind as we proceed with the analysis.

Looking at the Sharpe ratios in Table 3 Panel B, we can infer that portfolios with a Long-only strategy tend to attain the highest Sharpes. However, the four capped portfolios and the no constraints portfolios based on a 5-year lookback window are not performing significantly worse. The standard deviations for all Markowitz portfolios lie between 3.45%-6.69%, except the 2M/2Y - No constraints portfolios with a significantly higher standard deviation of respectively 28.04% and 35.85%. Pointing to the combination of a shorter window with no constraints is a non-optimal approach to optimization.

As portfolio constraints become stricter, the differences between the lookback windows become less pronounced. For example, when comparing the No constraints portfolios, the standard deviation of the 2M portfolio is 28.04% whilst the 5M portfolio is 4%. However, when the portfolio is capped, the standard deviation of the 2M portfolio is 6.69%, while the 5M portfolio has a standard deviation of 3.45%. This decrease in disparity is due to the stricter constraints limiting the portfolios from taking on extreme allocations, creating less volatile portfolios.

Furthermore, we see that across the three portfolio weights strategies, the five-year lookback window always performs better than the two-year lookback window, in terms of the Sharpe ratio. An example from Table 3 Panel B, is where the Capped portfolio with monthly rebalancing increases from a Sharpe of 0.983 to 1.279 as the lookback window is extended from two to five years. Therefore, we can deduce that more data might lead to higher portfolio performance. However, many believe that utilizing data from empirical series that are too far from the present may not be acceptable since they include stale information. By testing the portfolios[17] with a ten-year lookback window, we were able to achieve a Sharpe ratio of 1.033, which is lower than the Sharpe of 1.172 found when using a five-year lookback window. Despite this, the ten-year lookback window portfolio still outperformed the two-year lookback window portfolio, whose Sharpe ratio was 0.292. The results depict that a too long window will worsen the performance, but that it is still preferred over a shorter one. A "sweet spot" might be somewhere between five and ten years.

When looking at the portfolios in terms of realized returns, the portfolios with a two-year lookback window tend to outperform the respective five-year portfolios. Investing in the 2M – No constraints portfolio yields the highest return of 8.51%, whereas the highest yielding portfolio with a five-year lookback window lies at 4.79% (5M - No constraints). However, when we look at the standard deviation, portfolios with a five-year lookback window consistently attain a lower standard deviation. The decrease in standard deviation, might

[17] This is tested for the portfolios based on a no constraints strategy with monthly rebalancing.

indicate that a longer lookback window results in an increased diversification compared to a shorter lookback window. The results indicate that an extended lookback window is better at recognizing some of the repeating macro patterns, depicted through their capability to deliver returns at a lower risk level.

Allowing more frequent rebalancing ensures that an investor's portfolio remains aligned with the intended risk profile. Rebalancing also intends to prevent long-term exposure to undesirable risks, by capturing the value changes in the assets. From looking at the Sharpes in Table 3 Panel B, we see that the monthly rebalancing frequency always results in a higher Sharpe ratio and portfolio excess return. For example, when we go from a 5M - No constraints to a 5Y - No constraints the Sharpe increases by 0.189, the excess returns also increase by an additional 1.03%.

To summarize, the portfolios constructed of a No constraints weight strategy and a two-year lookback window resulted in the lowest Sharpes. As stricter constraints are imposed on a portfolio, we see a less prominent effect the lookback window has on the portfolio performance. In addition, a two-year lookback window tends to generate the highest return, whereas a five-year lookback window consistently achieves a lower standard deviation. Regarding Sharpe, a five-year lookback window is always preferred over a two-year lookback window. At last, monthly rebalancing is always the best frequency, no matter the lookback window and weight constraints. The highest-performing portfolio is the 5M - Long portfolio, which incorporates all the ideal parameters.

5.2.4 Additional risk measures

Among the constructed portfolios, the best-performing model is the 5M - Long portfolio model. However, we only look at the Sharpe ratio, realized return, and standard deviation. Our portfolios have been held for over 50 years, so one performance criterion might not accurately reflect the true long-term performance. Therefore, we examine other ways to interpret the risk related to the portfolios. There are various approaches for calculating downside risk, with the standard deviation being the most common measure. In the previous section, we covered the evaluation of the portfolios based on their standard deviation. However, it measures variability in data regardless of the direction of variation. This section will examine downside risk using other metrics: portfolio drawdown, Value at Risk (VaR), skewness, kurtosis, and the Sortino ratio.

Figure 3: Drawdown of 5M - Long vs. 1/N

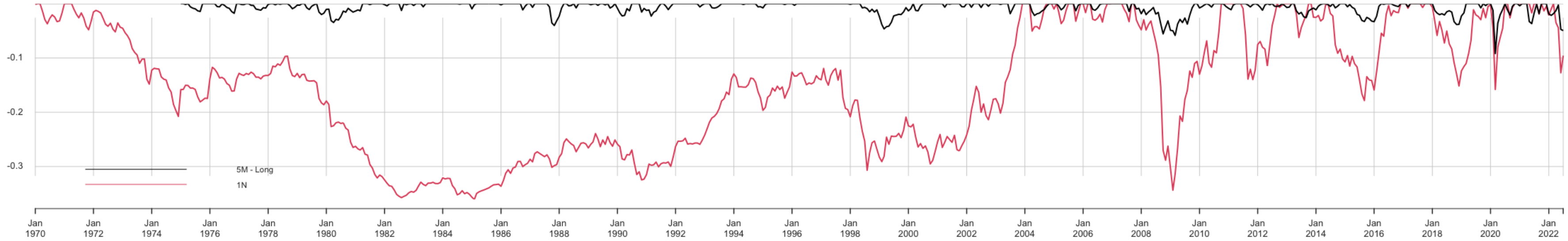


Figure 4: Drawdown of 5M - Long vs. the No constraints portfolios

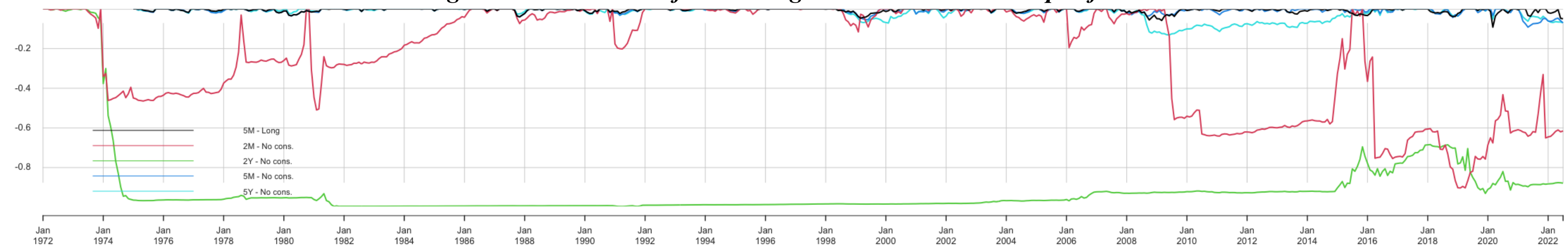


Figure 5: Drawdown of 5M - Long vs. the Capped portfolios

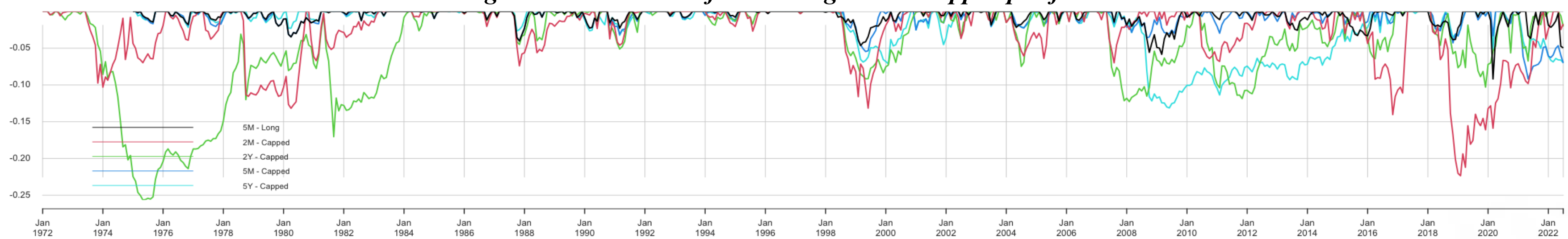
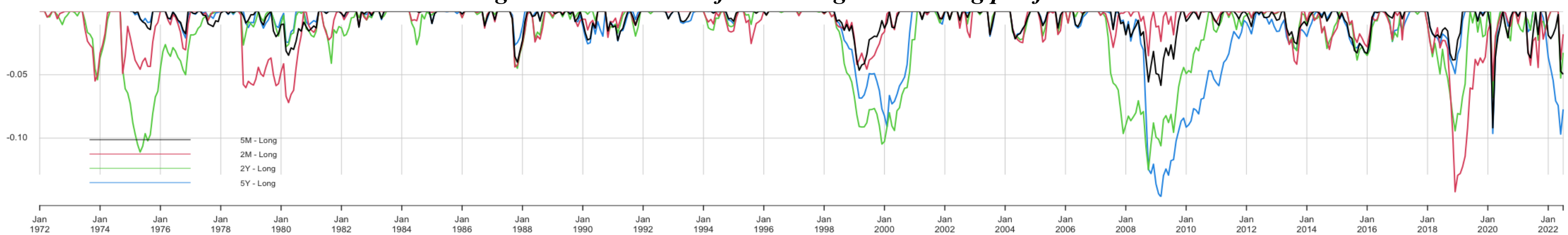


Figure 6: Drawdown of 5M - Long vs. the Long portfolios



Drawdown

The figures above (Figure 3-6) depict the portfolio drawdown for all models with the same weight constraints. The optimal portfolio, 5M - Long, is included as a baseline, illustrated as the black graph in all figures. Portfolio drawdown is the maximum percentage of a portfolio's value lost after an investment peak. The goal of measuring portfolio drawdown is to analyze the risk associated with an investment and assess an investment's ability to recover from losses.

We observe that, across all plots, portfolios with a five-year lookback window experience fewer drawdowns than those with a two-year lookback window. This effect suggests that a longer lookback window is more resilient during adverse market conditions. Furthermore, portfolios based on a Long only strategy have lower drawdown than the other weight strategies, indicating that stricter constraints lead to greater return stability in times of crisis.

According to our drawdown calculations, annually rebalanced portfolios took longer to recover from losses than monthly rebalanced portfolios. One of the most extreme cases is the 2Y - Capped portfolio, spending almost nine years recovering from the greatest loss of 25.65%. In comparison, amongst the portfolios based on a monthly rebalancing frequency, the 2M - Capped portfolio spent three years regaining losses at 22.36%.
Moreover, the 1/N portfolio, which only rebalances in the presence of new assets, requires almost 19 years to fully recover from a loss of -36.06%. This considerable disparity demonstrates that more frequent rebalancing leads to faster recoveries after experiencing a loss. Highlighting the consequences of not rebalancing regularly, as it leads to long-term exposure to undesirable losses.

When we study all figures containing the 5M - Long portfolio, we see that this is the most stable portfolio throughout its entire holding period, experiencing fewer drawdowns than the rest. The portfolio experiencing the highest drawdown is the 2Y - No constraints, falling as much as 99.58% from its peak in July 1973 and is never able to recover fully. The 2M - No constraints also experienced a significant loss of 90.29% in February 2019 and has still not recovered. These two portfolios that undergo the highest drawdowns also have the lowest Sharpe ratios, while the 5M - Long portfolio with the lowest drawdown had the highest

Sharpe. This implies that the greater the drawdowns in a portfolio are the lower the Sharpe ratio will be.

Value at Risk – VaR

Value at Risk assesses the magnitude of left tail risk in a portfolio over a given period with a specified degree of certainty. Our calculations are based on one month with a 99% confidence level and reveal a higher left tail risk for the 2M and 2Y - No constraints portfolios. For example, with a 99% certainty, the two portfolios will not experience a loss of more than 53.85% (2M) and 58.29% (2Y) in one month. In comparison, the maximum VaR amongst the other Markowitz portfolios lies at -7.55% (2M - Capped).

Table 6: Portfolio VaR

Value at Risk is calculated with a 99% confidence level for all portfolios.

	No constraints				Capped				Long				
	2M	2Y	5M	5Y	2M	2Y	5M	5Y	2M	2Y	5M	5Y	1/N
VaR	-53.85 %	-58.29 %	-4.85 %	-3.41 %	-7.55 %	-4.69 %	-2.53 %	-3.41 %	-4.54 %	-3.77 %	-5.06 %	-3.41 %	-11.54 %

Skewness & Kurtosis

We examine skewness and kurtosis to see whether any portfolios have a left- or right-shifted distribution, as well as if they have light or heavy tails. At first glance, by looking at Table 7, we notice that there is no apparent pattern in how portfolios within the same weight constraints, lookback window, or rebalancing method are distributed in terms of either skewness or kurtosis.

Skewness relates to the asymmetry in the probability distribution of returns. We uncover varied skewness for the thirteen portfolio returns, where five of them have a negative skewness, and eight are positively skewed. The eight positively skewed portfolios contain a few extremely high returns and frequent small losses. Implying that the calculated standard deviation overestimates the portfolio's true risk level, as the extreme positive returns dominate the negative (Bodie et al., 2021, p. 139). The opposite applies to the five positively skewed portfolios where extreme bad returns are more recurring.

In terms of weight constraints, portfolios with capped weights are overall closest to a skewness conforming to a normal distribution, indicating that if an investor prefers less skewed portfolio returns, the portfolio should be allowed some shorting but at a limited level.

Kurtosis is a measure of distribution that assesses the likelihood of extreme values by evaluating the fatness of the tails. A normal distribution has a kurtosis of 3, while our portfolio returns have a kurtosis and hence a leptokurtic distribution, thus implying a higher probability of extreme values. All the portfolios have a high kurtosis, with some extreme values, particularly the 5M - No constraints with a kurtosis of 32.27, the 1/N of 31.24, and the 2M - No constraints of 20.52.

Table 7: Portfolio skewness and Kurtosis

Skewness and kurtosis are calculated using Pearson's standardized third and fourth central moment of distribution. All terms are annualized.

	No constraints				Capped				Long				
	2M	2Y	5M	5Y	2M	2Y	5M	5Y	2M	2Y	5M	5Y	1/N
Skewness	-0.65	-0.05	2.66	0.28	0.20	0.07	0.08	0.14	0.27	-0.39	-0.55	-0.81	2.83
Kurtosis	20.52	16.84	32.27	8.75	11.56	6.20	5.79	8.79	10.83	7.13	14.55	16.45	31.24

Sortino ratio

The Sortino ratio does not penalize a portfolio for its positive movements since it only measures deviations caused by negative returns. All the portfolios experience a higher Sortino than their Sharpe ratio (Table 3 Panel B), indicating that all the portfolios are penalized for risk related to positive returns. The 5M - No constraints is the most penalized portfolio in terms of its positive volatility, as it shifts from being suboptimal in terms of Sharpe to optimal in terms of Sortino. This is reflected in its Sharpe ratio being 1.172, which is lower than the highest Sharpe achieved by the 5M - Long portfolio of 1.280, but its Sortino ratio of 2.492 surpassing that of the 5M - Long's at 2.236. While the weight strategy that generates the highest ratio changes from the Long strategy to a No constraints one, our earlier findings of the optimal lookback window and rebalancing method still hold.

Table 8: Sortino ratio for all portfolios

The table contains the Sortino Ratio for all portfolios.

	No constraints				Capped				Long				
	2M	2Y	5M	5Y	2M	2Y	5M	5Y	2M	2Y	5M	5Y	1/N
Sortino	0.421	0.122	2.492	1.757	1.469	1.333	2.376	1.757	2.086	1.689	2.236	1.757	0.826

In summary, none of the thirteen portfolios resemble a normal distribution in terms of skewness or kurtosis. The five positively skewed portfolios' standard deviation underestimates their true risk, and risk is overestimated for the remaining eight negatively skewed portfolios. The high positive kurtosis across all portfolio's points to a high presence of extreme returns. The five-year lookback window, Long weight strategy, and more frequent rebalancing are parameters making portfolios more robust for losses. In a worst-case scenario, the 2M/2Y - No constraints portfolios potentially experience a much higher maximum loss (VaR) than the other portfolios. When we employ the Sortino ratio to not penalize return movements in the positive direction, the 5M - No constraints achieve the greatest ratio in contrast to the optimal model assessed on the Sharpe; the 5M - Long.

5.2.5 Portfolio performance during crisis

Although losses are experienced in different magnitudes, it is still apparent from studying Figures 3-6 depicting drawdowns that all portfolios tend to experience losses in the same periods. Also, we see that some of the portfolios are never able to recover from early losses and are thus penalized for them for the entirety of their holding period. The best overall portfolio strategy might not necessarily be the one that performs best at every point in time, which is why it is interesting to see how the portfolios perform during different crisis periods. A portfolio's performance during a crisis is a critical factor in determining the success of the portfolio strategy. During a crisis, markets can experience significant drops in value, and the aim is to see which portfolio strategy is more capable of minimizing losses. The following section will examine the portfolio's performance during four crises with different qualities and duration.

We plot the dollar-investing and risk-reward for all crises to discuss the portfolio performances in these times. Dollar investing is computed by investing 1$ at the beginning of the period and holding it throughout the crisis. Two dollar-investing graphs are given for each crisis, categorized based on the portfolios with the same lookback windows. The 1/N is present in both graphs. The risk-reward plots depict the portfolios' Sharpe during the crisis (pink dots) compared against their Sharpe based on the total level (black dots).

Stagflation (1970-1981)

Between 1970 and 1981, the world economy experienced periods of high inflation and uneven economic growth, also known as stagflation. Stock prices fell as firms fought to remain profitable in the face of rising prices and stagnating salaries. Furthermore, investors were afraid to invest in equities due to economic uncertainties. As a result, stock values remained low for the decade, making it hard to maintain a portfolio that generated value.

Figures 7 and 8, on the next page, present the value generated for each portfolio. We see from Figure 7 that three portfolios are not able to generate value during the crisis; 2Y - No constraints, 2Y - Capped, and 1/N, which ended up at values of \$0.005, \$0.898, and \$0.639. The clear insufficient portfolio is the 2Y - No constraints, that experience a loss of 99% of its portfolio value in this period. This loss happens early in the crisis, and the portfolio is not able to regain its losses.

Figure 9 compares the crisis period with the overall performance in a risk-return diagram. We see from the figure that the 2Y - no constraints portfolio performs poorly due to the extreme negative returns, which we also saw from the dollar investing. Most of the portfolios perform poorer during the stagnation, except 5Y - No constraints, 5Y - Capped and 5Y - Long, who perform better due to decreased standard deviation compared to the total level.

Figure 7: Dollar investing 1970-1981 - Two-year lookback window portfolios

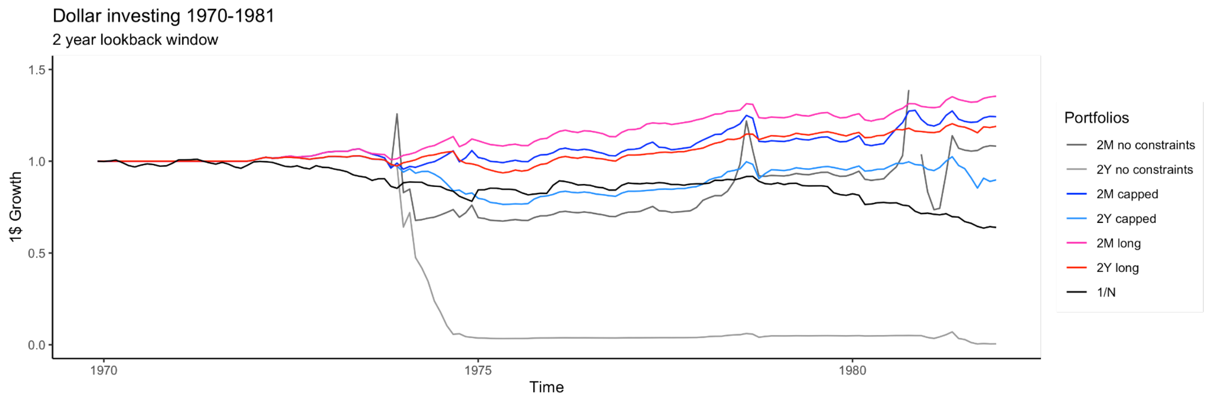


Figure 8: Dollar investing 1970-1981 - Five-year lookback window portfolios

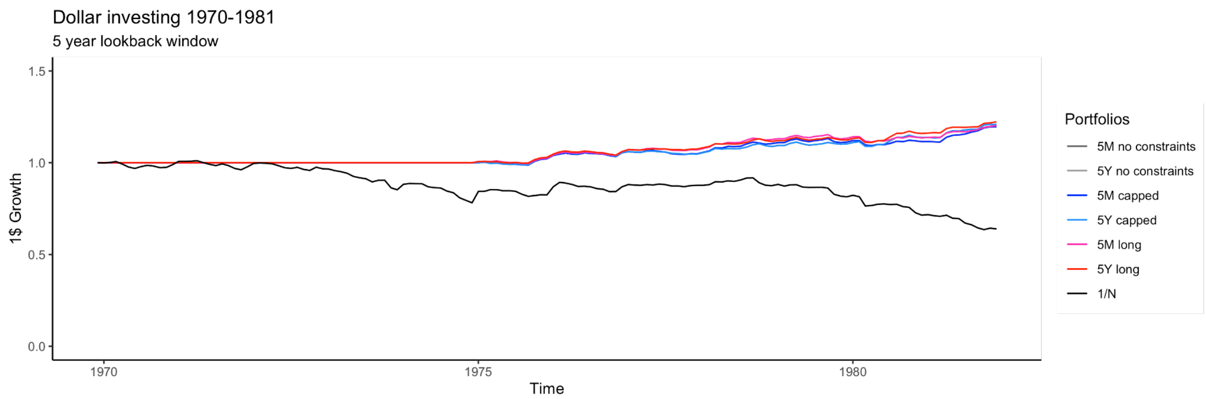
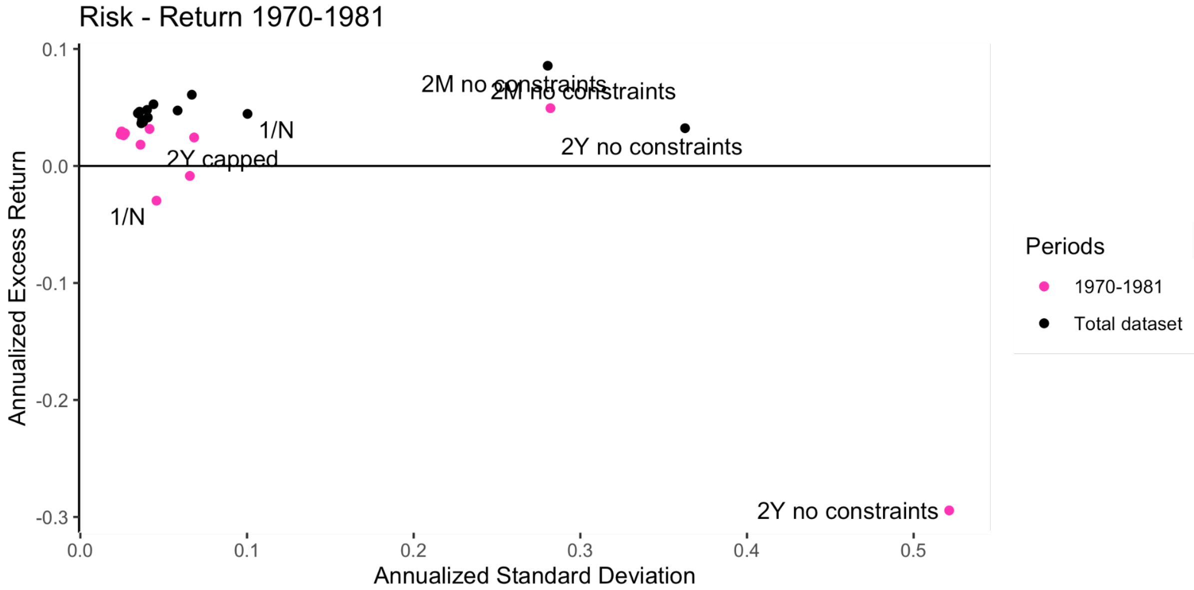


Figure 9: Risk-reward plot for all portfolios between 1970-1981



Dot-Com (2000-2002)

In the late 90s, the market experiences a high demand for tech stocks, generating a financial bubble. At the beginning of 2000, the bubble burst, which had a tremendous influence on the stock market as the highly speculated tech companies plunged in value. The burst resulted in a recession that lasted until 2003 and had long-term consequences for the stock market. The recession made it hard for investors to generate value for their investments during this period.

At the end of the crisis, all the assets managed to generate more value than they started with, illustrated in Figures 10 and 11. All Markowitz portfolios are able to end up with a dollar value of approximately \$1.20, whilst the 1/N had a lower value at \$1.04.
Moreover, the risk-return diagram plotted in Figure 12 shows that all the Markowitz portfolios generated a higher Sharpe ratio during the crisis compared to the total portfolio performance, whilst 1/N ended up with a lower Sharpe during the Dot-com. The portfolio experiencing the biggest difference is the 2Y - Capped, which had a Sharpe equal to 0.795 at the total level, and 1.790 during the Dot-com. A reason for the improved performance might be the Markowitz portfolios not holding heavily in stocks, and thus the tech sector. It also shows how investing in multiple assets protects the portfolios to reduce the overall risk in a volatile market.

Figure 10: Dollar investing 2000-2002 for 2-year window portfolios

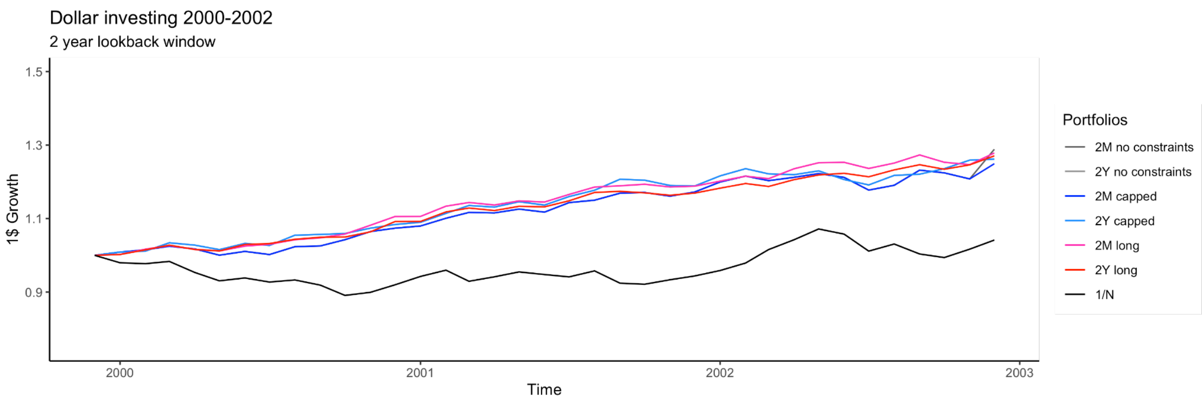


Figure 11: Dollar investing 2000-2002 for 5-year window portfolios

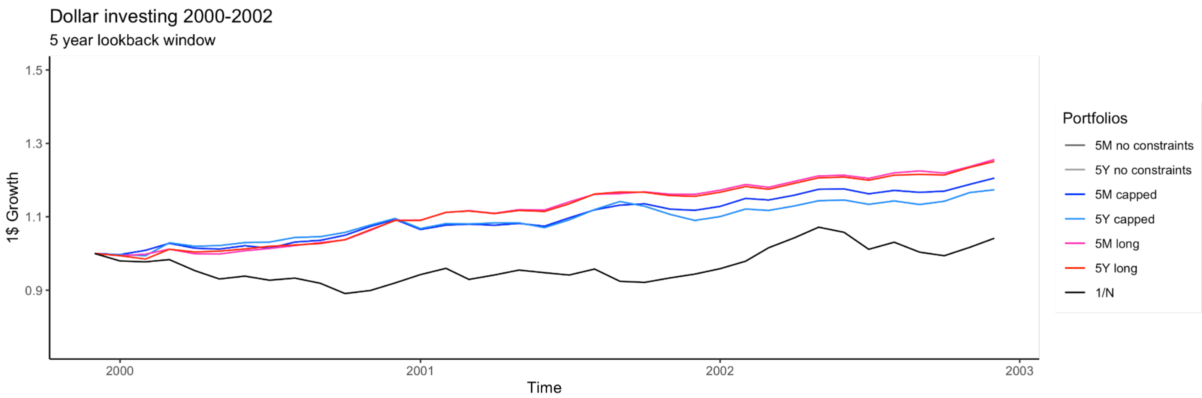
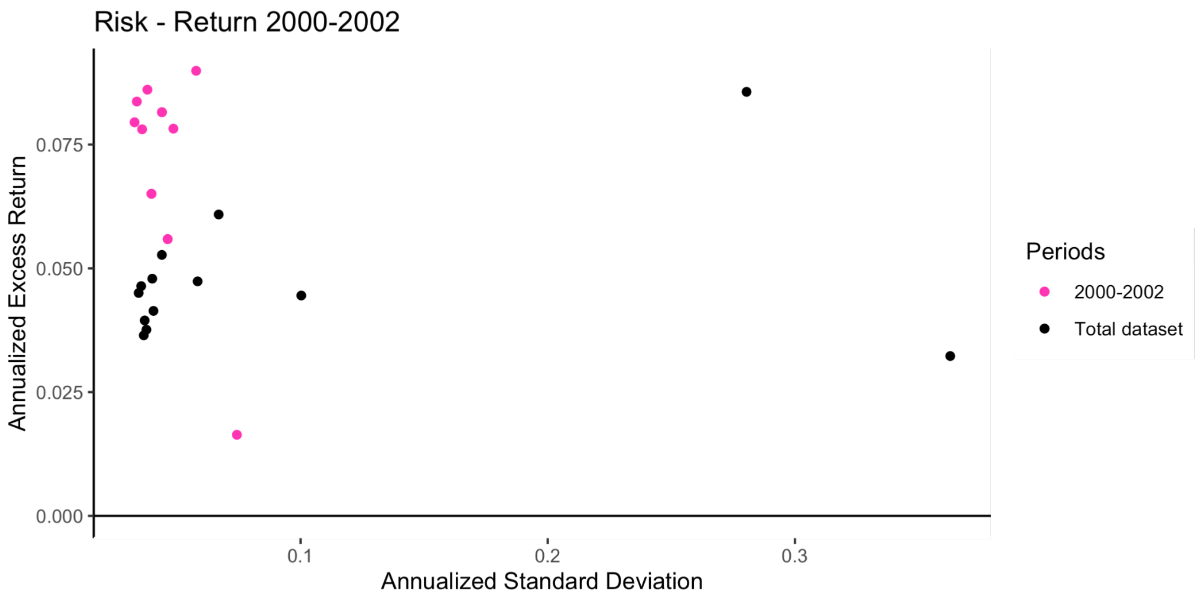


Figure 12: Risk-reward plot for all portfolios between 2000-2002



The Financial Crisis (2008-2009)

The Global Financial Crisis of 2008 is defined by many as the most significant crisis of the modern times. The financial markets and the global economy were severely impacted, and global stock markets crashed, resulting in enormous losses for investors. The financial crisis had many ripple effects that spread to other markets, which is why the world economy spent a long time recovering from the crisis.

During the crisis graphed in Figures 13 and 14, only six of the portfolios are able to generate dollar value, and five of these portfolios used a two-year lookback window. The highest dollar value is generated by the 2M - Capped at $1.106.
Further, we notice several deviations from the overall Sharpe performance earlier discussed in subsection 5.2.4 by looking at Figure 15. First, 2Y - No constraints perform better during the crisis period compared to the total level Sharpe. Additionally, five out of six five-year lookback window portfolios yield negative returns during this period. Previously, we found these portfolios to perform exceptionally well during periods of economic instability. However, in this crisis, they performed the worst regarding the Sharpe ratio. For instance, a 5Y - Capped goes from an overall Sharpe of 0.981 to -0.982.

Figure 13: Dollar investing 2008-2009 for 2-year window portfolios

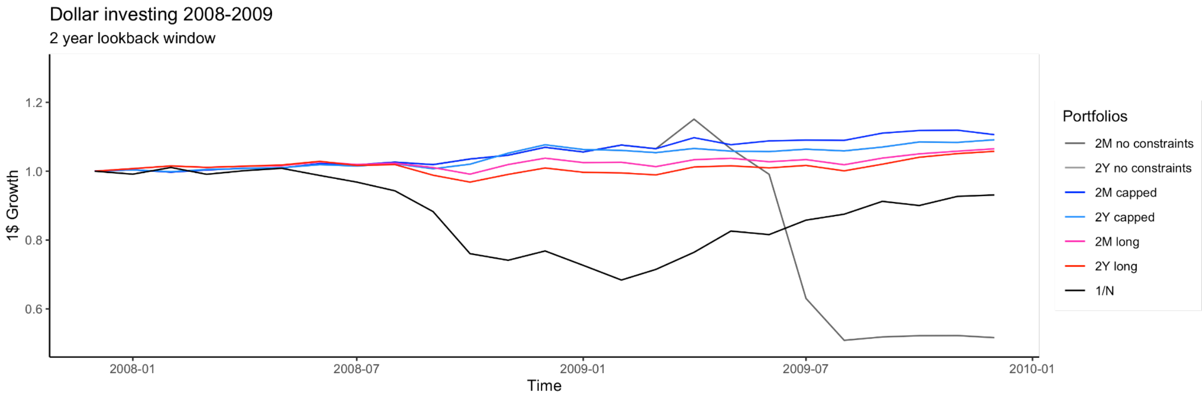


Figure 14: Dollar investing 2008-2009 for 5-year window portfolios

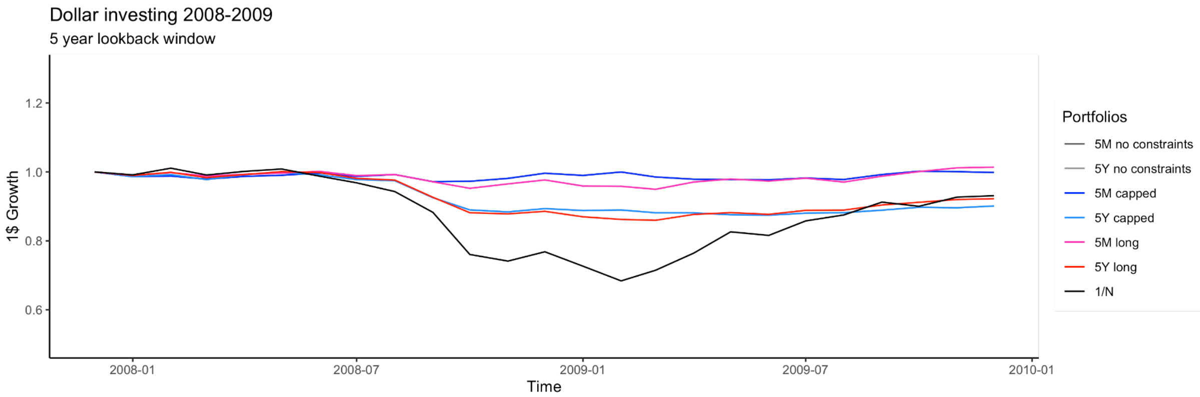
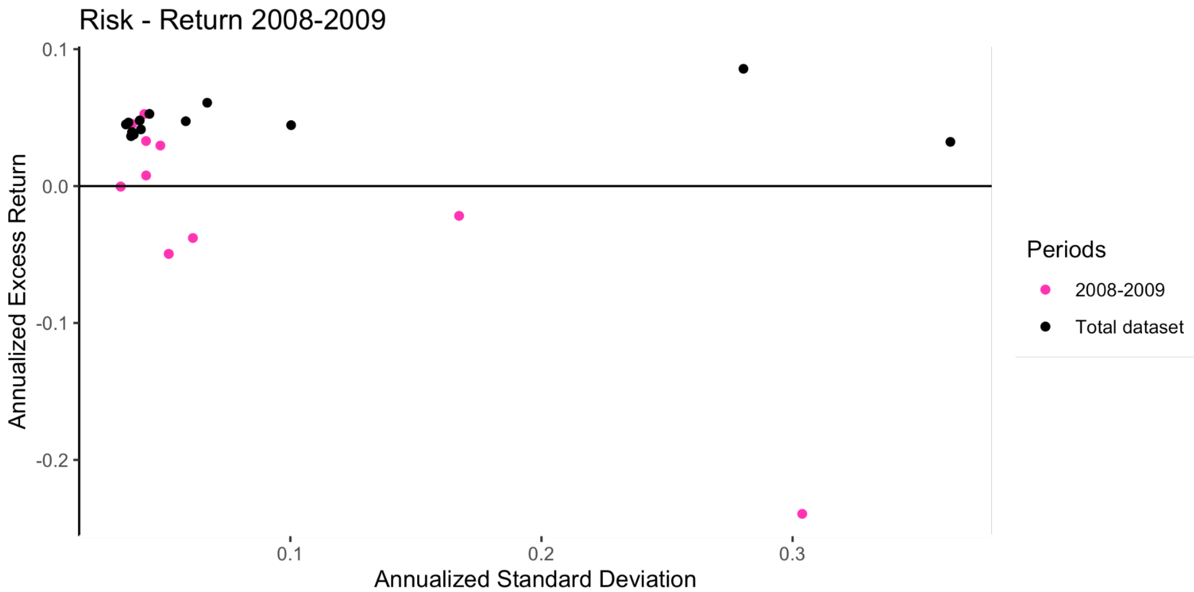


Figure 15: Risk-reward plot for all portfolios between 2008-2009



Covid-19 (2020)

The Covid pandemic prompted a dramatic decline in stock values as markets responded to the virus's unpredictability and the economic ramifications that were unavoidable. In late February 2020, stock prices began to fall as the virus spread worldwide, and the markets began to show signs of stress. Throughout 2020 the market remained turbulent due to lockdowns, restrictions, and uncertainty. The covid-19 crisis was triggered by an unexpected virus, unlike the other crisis, often the consequence of a boom in the economy. There are discussions of how long the crisis lasted; with someone stating that it is still ongoing. In our analysis, we look at the period of the year 2020.

Most portfolios followed the same trend during the crisis in Figures 16 and 17, except for the clear outliers of 2M and 2Y - No constraints. In contrast to the previous crisis, where these two portfolios typically return the lowest value, they are now yielding the highest dollar value of respectively $1.595 and $1.647. All the portfolios, regardless of framework, were able to generate value. Only three of the 13 portfolios performed worse than their overall performance: 2M - Long, 2Y - Long, and 5M - Long.

Figure 16: Dollar investing 2020 for 2-year window portfolios

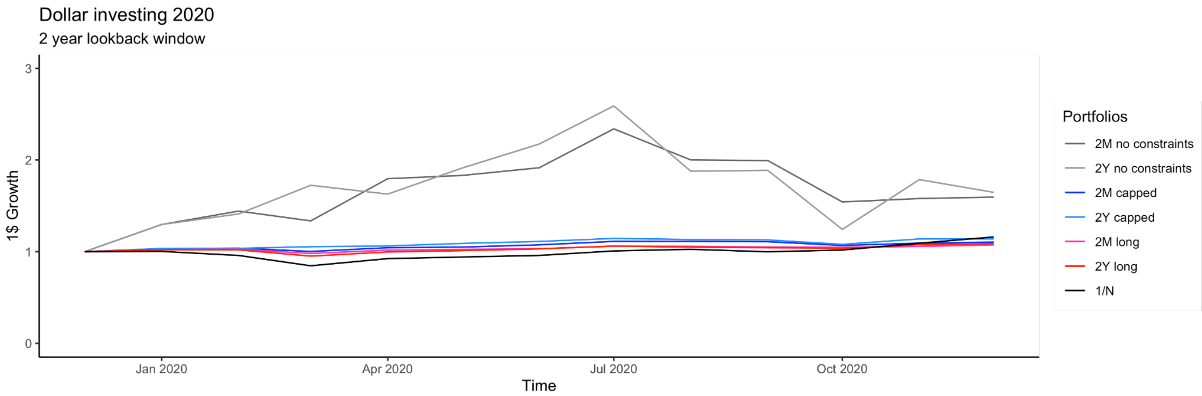


Figure 17: Dollar investing 2020 for 5-year window portfolios

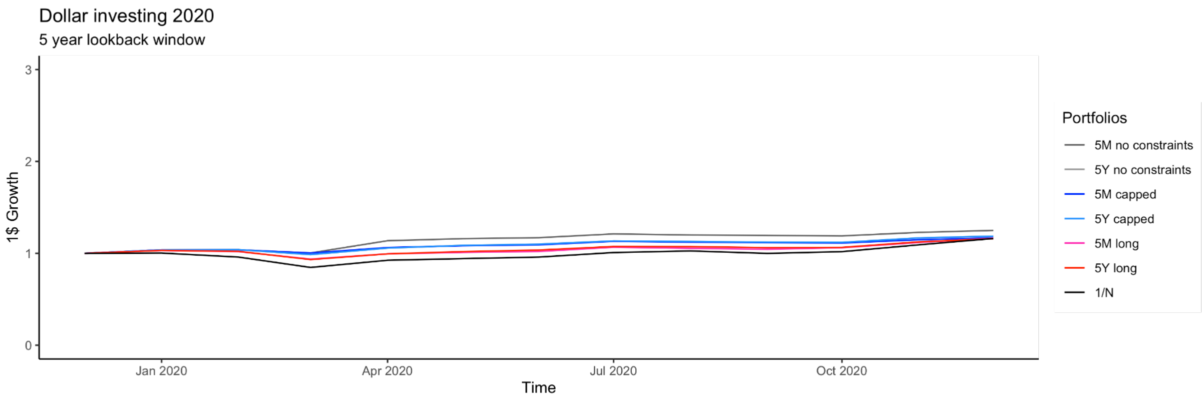
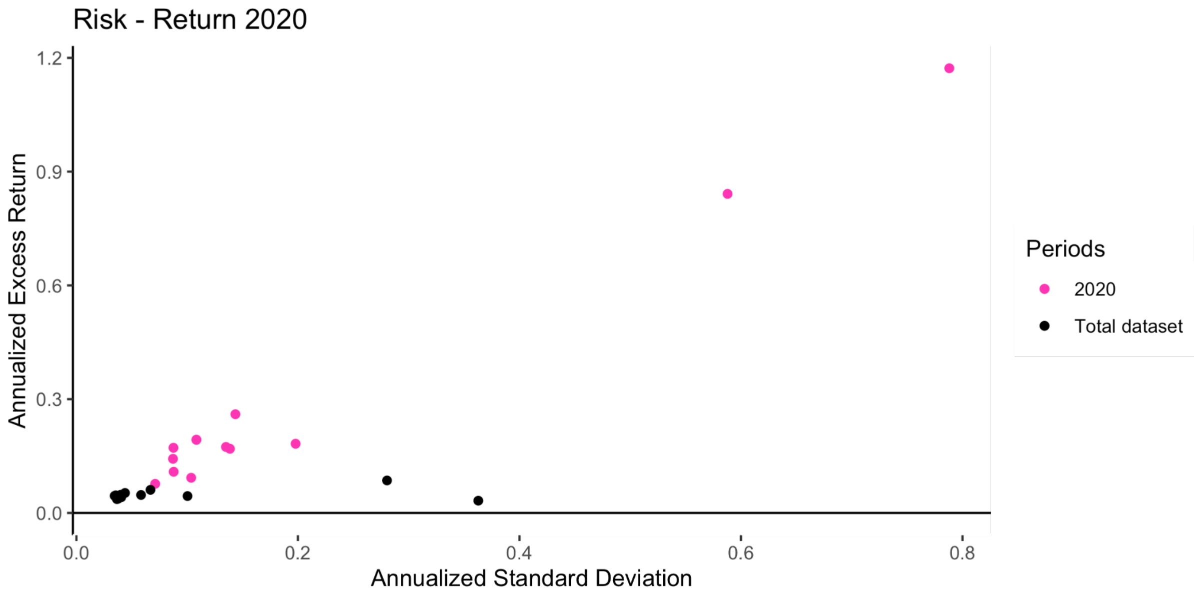


Figure 18: Risk-reward plot for all portfolios in 2020

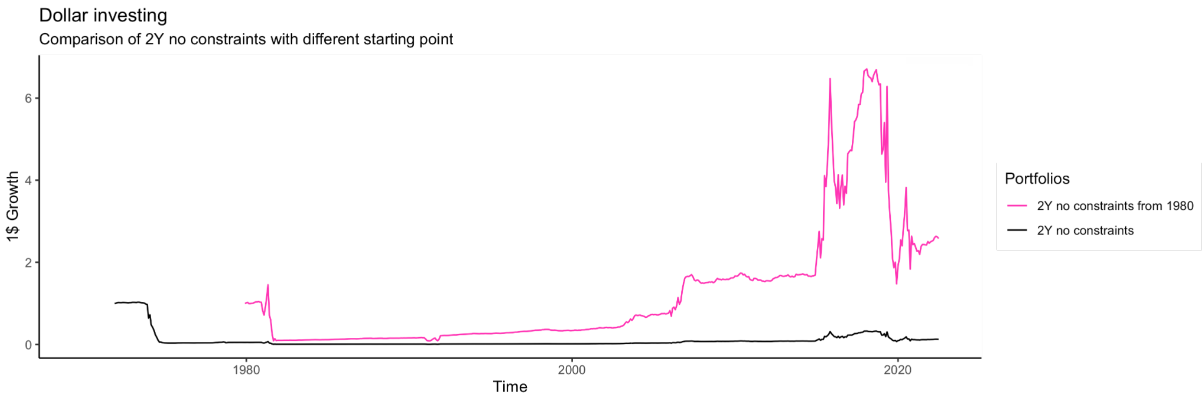


In conclusion, portfolios based on the Markowitz framework tend to generate more value during a crisis than those based on the 1/N framework. Even though the naïve portfolio strategy is diversified in multiple assets, it seems like the Markowitz framework is better at exploiting good diversifiers in the portfolio. This might be one of the reasons why portfolios based on Markowitz perform better during crises. Furthermore, our analysis did not find a clear trend between which lookback window, rebalancing, and constraints performed best.

Beginning of the dataset

In the previous sections, we have seen that the 2Y no-constraints portfolio performs poorly overall during especially the first crisis, the stagflation. The portfolio sticks out as it is never able to make up for the experienced losses. Based on the previous analysis, a strategy with parameters equal to 2Y - No constraints are not favorable for any investors. However, our dataset is available from 1970, which happens to be the start of the stagflation period. The fear is that the start of the dataset is ruining the overall qualities for the 2Y - No constraints portfolio. Therefore, we have analyzed the performance of the same portfolio but excluded the data from 1970-1979.

Figure 19: Dollar investing for 2Y - No constraints



As the diagram clearly shows, the portfolio strategy would have performed much better if the dataset did not start in the middle of a crisis. Therefore, the strategy itself might be better than first anticipated. Table 9 shows that by excluding the dataset before 1980, the annual Sharpe ratio increases from 0.081 to 0.250, an increase of 211%.

Although the portfolio's Sharpe ratio increased, it is still the worst portfolio compared to the rest. Moreover, the rest of the portfolios are still based on the whole dataset, and they still manage to beat the 2Y - No constraint portfolio without the effect of stagflation. This indicates that the 2Y no constraint portfolio is not generating enough value during the rest of the period, making the strategy less favorable than the rest.

Table 9: 2Y - No constraints portfolio performance with different duration

Excess return, standard deviation and Sharpe are all in annualized terms. 2Y no constraints 1980 is calculated with observations from 1980 to July 2022. 2Y no constraints 1972 is calculated with observation from 1972 to July 2022. The table shows the effect of not constructing the portfolio calculations with data from the stagflation.

2Y No constraints Portfolio Performance			
	Excess Return	Standard Deviation	Sharpe
2Y no constraints 1980	9.67 %	36.16 %	0.256
2Y no constraints 1972	3.23 %	36.28 %	0.088

5.3 Incorporating alternative assets

In the previous section, we have seen that investing in a portfolio is almost always better than just investing in a single asset. In other words, diversification as a result of holding a portfolio is preferable in general since these portfolios experience lower standard deviation. We want to investigate if the portfolios performed better due to diversifying into multiple assets or if the alternative assets were the main driver of value. First, we discuss the correlation fluctuations that happen over time in order to see if the diversifying effects of traditional- and alternative assets are stable. Then, we construct two portfolios only investing in traditional assets, with the same parameters as the optimal portfolios, in terms of Sharpe and Sortino, found in the previous subsection.

5.3.1 Rolling correlation

Previously in the analysis, we investigated the correlation between the single assets, indicating which assets would be great diversifiers. Even though the full sample correlation matrix indicates the asset's correlation, it will not explain the co-movements over time. The market forces affect the assets differently over time, meaning that also the correlation changes over time. Two assets negatively correlated in one period can change to be strongly positively correlated in another. The diversification effect will be non-apparent if the correlation quickly changes from negative to strongly positive. The idea is that increasing the number of assets invested in a portfolio will decrease the probability of a strong positive correlation.

To analyze the co-movements between assets correlation over time, we have computed the rolling correlation between the assets. The lookback window used to compute the rolling correlations is the same as for the optimal portfolios, in terms of Sharpe and Sortino, found in the previous subsection, which turned out to be five years.

At first glance, Figure 20 below displays that if a portfolio invests in Equities and all of the traditional assets, it would be relatively diversified. This is due to the fact that there are positive and negative correlations present between these assets. However, if we take a close look at the correlation between CMA and Equity, we can see that this is an example of how a correlation between to assets can go from a negative correlation of -0.736 in June 2001 to a positive correlation of 0.451 in September 2007. On a total level, these two assets had a negative correlation of 0.290. Therefore, only investing on the basis of the total correlation might be misleading, as there are, in fact, periods they could pull risk movements in a completely different way.

One key to minimizing the overall risk is ensuring a well-diversified portfolio. The correlation between the portfolio's assets should be as low as possible, especially before a crash occurs. This can be implemented as a strategy by also rebalancing the correlations, using rolling correlations, regularly to update the risk movements between the assets.

Furthermore, the lowest correlation among traditional assets exists between the Fama-French factors and Equity. A typical portfolio to construct consists of equities and bonds. However, in terms of diversification, we see that Bonds and Equity are relatively highly correlated. This indicates that if one should only hold a portfolio of traditional assets, one should also include holding an asset like RMW, which have a low correlation measured against Equity.

Figure 20: Rolling correlation between Equity and Traditional assets

The rolling correlation, based on a five-year lookback, consists of all traditional assets plotted against Equity (MSCI World Index) over time.

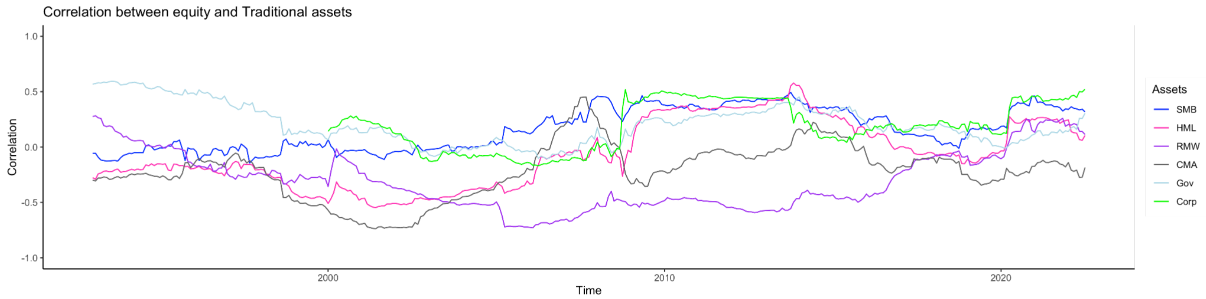


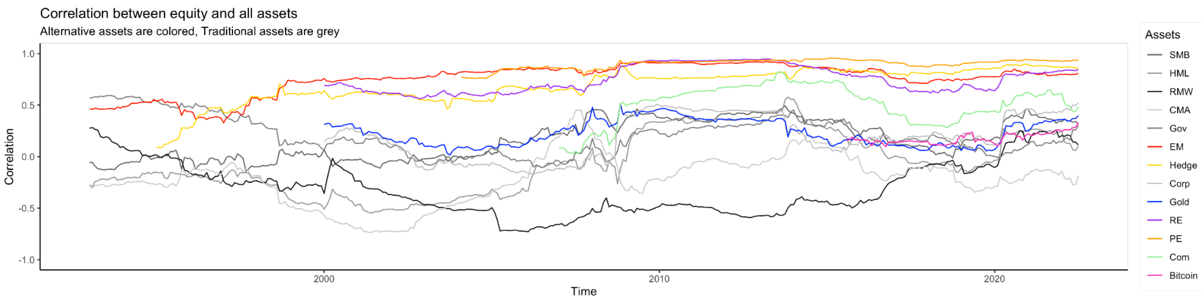
Figure 21 displays the same rolling correlation method, plotted with against all other assets. The theory is that since conventional asset groups, like Equities and Bonds, are highly regulated and are driven by similar market drivers they should be higher correlated. On the other hand, alternative assets, which operate in widely different sectors, have fewer correlations with the larger markets and one another. The idea is that alternative assets would increase the diversification in a portfolio as it is affected by different market forces than a traditional asset.

From the figure it visualizes that most alternative assets are highly correlated with Equity. Drawing the parallel that a portfolio holding Equity as the only traditional asset in combinations with numerous alternative assets not being the most protected portfolio, in terms of lower correlation between assets. This contradicted the hypothesis that disparities between alternative assets and traditional assets should result in a weaker correlation.

However, as discussed earlier, the MSCI World Index contains stocks in sectors similar to some of these alternatives. Of the alternative assets, Bitcoin and Gold is seemingly the best diversifier, throughout the entire period, to obtain a well-diversified portfolio.

Figure 21: Rolling correlation between Equity and all other assets

The rolling correlation, based on a five-year lookback, consists of all alternative assets plotted against Equity (MSCI World Index) over time. The graph below shows the traditional as the grey lines and alternative assets in colors.



Moreover, we applied the same rolling correlation method with the Government bonds as the underlying asset instead of Equity. The motivation is to look see if alternative assets are better diversifiers included in a portfolio containing Bonds, rather than Equity, since most of them was highly correlated to Equity.
Figure 22 shows that correlation between other traditional assets and Government bonds are relatively diversified, except for Equity and Corporate bonds. Indicating that a portfolio containing only Equity, Government- and Corporate bonds would be vulnerable due to its high correlation over time.

The correlation for all assets plotted against Government bonds is seen in Figure 23. In comparison to Figure 21 that showed alternatives highly correlated to Equity, Figure 23 shows an improved correlation coherence between alternative assets and Bonds. Correlation between Bonds and some alternative alternatives are still positive, but the correlation is not as volatile and stays at a lower level than compared against Equity.

Our results indicate that alternative assets are far more correlated with traditional assets than first anticipated based on the theory. This correlation might be because we are using indices as a proxy for most of our alternative assets, making them more exposed to the same market factors. However, we can conclude that alternative assets are higher correlated with equities than bonds. Our findings imply that an alternative asset portfolio combined with bonds would be better diversified over time. However, if an investor wishes to invest in a combination of Equity and alternative assets, they should also include bonds in their portfolio to lower total unsystematic risk.

Figure 22: Rolling correlation between Government bonds and Traditional assets

The rolling correlation, based on a five-year lookback, consists of all traditional assets plotted against Government bonds (FTSE World Government Bond Index) over time.

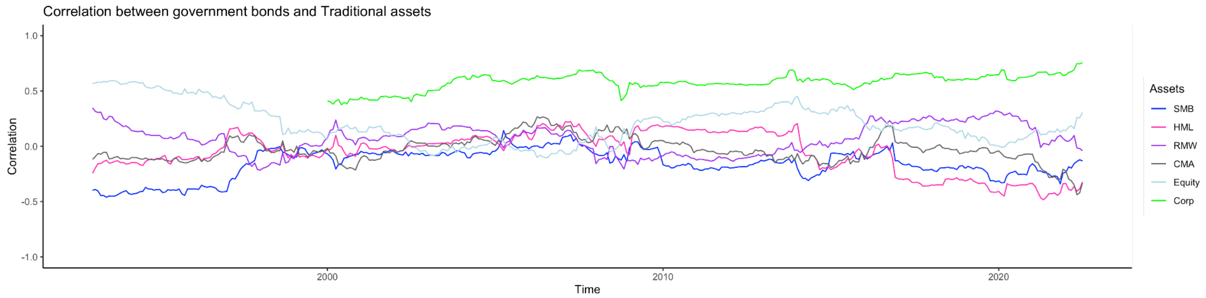
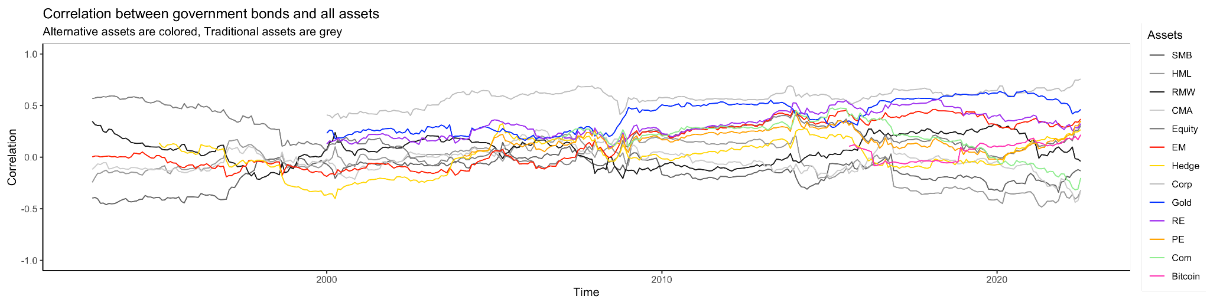


Figure 23: Rolling correlation between Government bonds and all other assets

The rolling correlation, based on a five-year lookback, consists of all other assets plotted against Government bonds (FTSE World Government Bond Index) over time.



5.3.2 Performance of the traditional vs. the optimal

We have calculated an optimal portfolio that only invests in traditional assets for two strategies. The newly constructed traditional portfolios are based on strategies equal to the optimal portfolio regarding Sharpe (5M long) and the optimal portfolio regarding Sortino (5M no constraints). The aim is to see if the alternative assets are the main driver behind the higher value and reduced volatility. We have excluded all observations before 1993, as the first alternative assets are only available after this point, making the portfolios identical until 1993. The portfolio's abbreviation and more detailed description is available in the appendix.

The portfolios excess return, standard deviation and Sharpe is listed in Table 3 Panel C, which was introduced in the beginning of the analysis section. If we compare the portfolios that contain both traditional and alternative assets with the respective portfolio consisting of only traditional assets, we see that they yield a higher excess return. For instance, when alternative assets are incorporated in the “No constraints” models, the excess returns increase by 2.3%, and the excess return increases by 1.41% for the “Long” models. The increased return implies that alternative assets contribute to a higher achieved portfolio return. However, the increased return should not be analyzed separately without looking at the change in risk.

Comparing the two ‘No constraints’ portfolios, we see that the standard deviation decreases from 6.72% to 4.57% when alternative assets are added. The decreased standard deviation indicates that implementing alternative assets also creates a diversification effect that helps lower the portfolio risk. The ‘Alternative - no constraints’ is able to both increase the return and decrease standard deviation compared to the traditional, exhibiting that including alternative assets in this framework will improve the overall performance. This is seen parallel with the Sharpe ratio increasing significantly from 0.400 to 1.075 as alternative assets are incorporated.

Further, looking within the Long framework, we see that even if the portfolio incorporating alternatives receives a higher excess return than the traditional one, it does not reduce the standard deviation. However, when assessing the Sharpe ratio, which considers both risks and returns, it is evident that a portfolio including both traditional and alternative assets yield a better result than a portfolio consisting of only traditional assets. Respectively, the Sharpe increases from 0.950 to 1.171 as alternative assets are included. Therefore, it indicates that by adding alternative assets, the overall performance increases as every increase in return is lesser than the increase in volatility. However, since both risk and return increase, it is more difficult to conclude the diversification effect.

Skewness & Kurtosis

When looking at the additional levels of risk across all portfolios from Table 10, we see that the ‘Traditional - Long’ portfolio returns are the closest to resembling a normal distribution. The only positively skewed portfolio is the ‘All assets-No constraints’, which depicts that allowing for short selling with the use of alternative assets will generate some extremely positive returns. The other portfolios rely more on delivering returns in terms of more recurrent positive returns since they experience a few extreme negative returns. This is especially evident in the extreme increase in both skewness and kurtosis for the ‘Traditional - No constraints’, where the kurtosis is almost ten times as high compared to the “Alternative - No constraints”. Since the portfolio is not allowed to invest in alternatives but allowed to invest freely in terms of shorting, it evidently generates a high number of negative returns, with a skewness of -4.08%. Although the kurtosis is higher in the ‘Alternative - Long’ than the “Traditional - Long” portfolio, respectively 11.25% compared to 3.94%, it generates a higher return without significantly increasing the standard deviation, as seen in Table 3 Panel C, showing that the “bets” actually paid off.

Table 10: Skewness and Kurtosis for ‘Traditional’ and 'Alternative'

Skewness and kurtosis are calculated using Pearson’s standardized third and fourth central moment of distribution. All terms are annualized.

	Alternative - No constraints	Traditional - No constraints	Alternative - Long	Traditional - Long
Skewness	0.56 %	-4.08 %	-0.59 %	-0.61 %
Kurtosis	5.27 %	50.12 %	11.25 %	3.94 %

Sortino

By comparing the portfolios Sortino ratio from Table 11 to the Sharpe ratios presented in Table 3 Panel C, we see that all portfolios experience an increased ratio. For instance, the “Alternative - No constraints” portfolio increases from 1.075 to 2.328, while the “Traditional - No constraints” portfolio only increases from 0.400 to 0.506. The Sortino ratio indicates that the alternative portfolios earn more return per unit of the portfolio's negative risk. Yet, the increase is more apparent for the portfolios including alternatives than the traditional ones, which points to the ‘Alternative’ portfolios' standard deviation, to a greater extent, being penalized for positive returns.

Table 11: Sortino Ratio for 'Traditional' and 'Alternative'

Sortino ratio is annualized

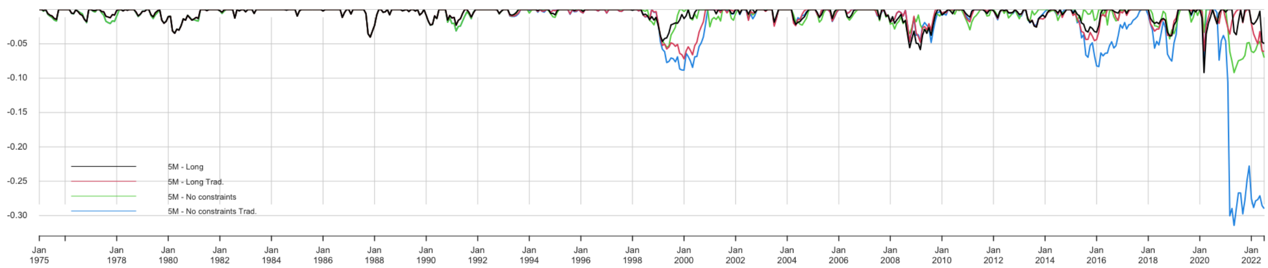
	Alternative - No constraints	Traditional - No constraints	Alternative - Long	Traditional - Long
Sortino	2.328	0.506	1.988	1.544

Drawdown

Additionally, if we compare the drawdown of the portfolios, we see that the alternative portfolios experience a lesser drawdown than the traditional ones. Also, when they first encounter a loss, they are able to quickly recover from them. This is particularly evident in the extensive loss experienced from holding a 'Traditional-No constraints' portfolio during the covid-19 market crash. It is again favoring including alternatives if one wants to short but also ensuring that the portfolio is robust enough during market crashes.

Figure 24: Drawdown of 'Traditional' and 'Alternative' portfolios

The figure contains the drawdown that each portfolio experiences during its holding period. Black represents the "Alternative - Long" portfolio, red represents "Traditional - Long", green represents "Alternative - No constraints" and blue represents the "Traditional - No constraints" portfolio.

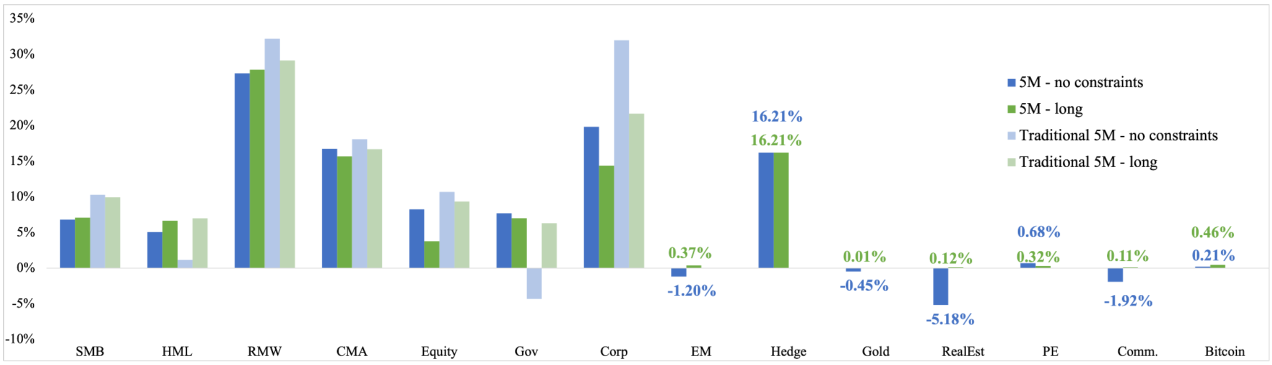


The key takeaway is that the biggest allocations are not heavily switched from traditional to alternatives, as alternative assets become available. Therefore, we can conclude with the "main" driver of value still being the traditional assets and that according to optimizing within the mean-variance framework, the portfolios will not place "all eggs" in the alternative assets' basket. As we discussed in the section about rolling correlation, alternative assets are generally not great diversifiers when combined with equity. However, combining bonds and alternatives is better to lower the correlation. This effect is present in our portfolios as well, as we see that when alternative assets are added, the portfolios allocate less in equity and more in alternatives and government bonds.

In addition, the reduced allocations in equities and corporate bonds is shifted to investing in hedge funds. As we also commented previously, hedge funds are the alternative asset that is least correlated with the traditional ones. Additionally, it is also negatively correlated with the Fama-French factors of RMW, CMA, and HML, which all portfolios are heavily invested in. Hedge funds are also negatively correlated with government bonds, so the increased weights in that traditional asset will enhance diversification.

Figure 25: Asset allocation of 'Traditional' and 'Alternative' portfolios

Average weights allocated in each asset. The darkest colors represent the "Alternative" portfolios, while the lighter colors represent the "Traditional" portfolios.



Furthermore, we examine how the portfolios invest in other alternative assets. In the 'Alternative - No constraints' portfolio, the assets are primarily used as a short bet to generate more money to be placed in other assets. On the other hand, since we do not allow for shorting in the long-only portfolio, the 'Alternative-Long' instead takes smaller long positions to generate a small fraction of the extreme returns some of these alternative asset's experience, which also justifies the higher kurtosis. However, due to the high volatility in these assets, the portfolio does not justify high allocations in them.

6. Discussion and conclusion

Our goal for the thesis was to find the answers to two questions: Will a constructed portfolio including alternative assets be a better investment object than holding any single asset? Furthermore, what is the benefit of holding a portfolio with alternatives assets compared to a traditional equity-bond portfolio?

The first question can be answered in two parts. First, examining if the constructed portfolios outperform the single assets. We compare Sharpe ratios between single assets and portfolios, where we concluded that five portfolios outperform all single assets, and ten portfolios outperform most assets except Bitcoin. Additionally, Bitcoin is the best-performing asset, beating eight portfolios. Diversifying into several assets is preferred, as it reduces the portfolio's standard deviation, resulting in a higher Sharpe ratio. However, there are instances where single assets outperform portfolios, such as the 1/N, 2M - No constraints, and 2Y - No constraints are surpassed by several single assets. Therefore, the findings are not unanimous pointing to constructing portfolios being the best approach.

Our results differ from studies including one single alternative asset, such as the studies done by Amin & Kat (2003), Bond et al. (2007) and Divecha et al. (1992), because they have all deemed it solely beneficial. However, the study conducted by Platanakis et al. (2019) concluded that diversification in five alternative assets is harmful to investors. This is largely due to their study including transaction cost, which we did not address. Therefore, the realized portfolio returns we supposedly achieve might not reflect the actual returns we would gain from holding the constructed portfolios. For example, the majority of our portfolios invest heavily in hedge funds, which often have high commissions that are not deducted from the portfolio returns.

An additional limitation of our thesis is that we only process in-sample data, in contrast to Platanakis et al. (2019) who draw conclusions based on out-of-sample data. Bringing up the potential problem of overfitting because we set an assorted range of data specifications. This can lead to the constructed models being overly tailored, leading to poor generalization that may not be applicable to unseen data.

The second part of the initial research question requires a look at which constructed portfolio, including alternative assets, is optimal. Because as our analysis regarding single assets and constructed portfolios showed, the portfolios are not always beneficial. The takeaway is that the mean-variance model is relatively sensitive to the selected parameters.

Our analysis revealed that portfolios with long-only strategies usually have the highest Sharpe ratios. The analysis showed that the differences between lookback windows become less prominent when portfolios are imposed stricter constraints. The five-year lookback window always performed better than the two-year lookback window in terms of Sharpe ratios. Moreover, we also observed that the two-year lookback window generally yielded higher returns while the five-year lookback window produced lower standard deviations.
A limitation of our analysis is that we only build our portfolios on two different lookback windows, making it hard to conclude the true optimal window. However, the results indicate that the optimal lookback window lies between 5 and 10 years. Even though Jobsons' (1981) paper did not explicitly evaluate the choice of lookback window in alternative portfolios, our analysis is in line with their findings.
Lastly, our analysis showed that monthly rebalancing is the best frequency regardless of the other parameters. The 5M-Long portfolio, consisting of all the ideal parameters, was the highest-performing portfolio.

Our additional analysis of the portfolio strategies associated risk confirmed that the five-year lookback window, Long weight strategy, and frequent rebalancing make portfolios more robust for losses. We found the 1/N framework not being as resilient as the Markowitz framework. This disagrees with the study conducted by DeMiguel et al. (2009), as they found the 1/N consistently outperforming the Markowitz models. The Markowitz framework is better at exploiting good diversifiers in the portfolio, making it more capable of minimizing losses.

Continuing to our second question, we compared the performance of a portfolio containing only traditional assets against the equivalent portfolios consisting of all assets. We found that while alternative assets are generally highly correlated with traditional assets, they can offer better diversification and higher returns when combined with Bonds. Moreover, the portfolios investing in alternative assets can generate higher returns with lower overall risk and better recover from losses. Additionally, the constructed portfolios show that traditional assets are the primary value driver and that alternative assets are primarily used to generate short-term returns or enhance diversification.

To conclude the thesis, we can establish that the saying of not placing all eggs in one basket still stands - just be careful in how you place them.

References

Amin, G. S., & Kat, H. M. (2003). Stocks, bonds, and hedge funds. *The journal of portfolio Management*, *29*(4), 113-120.

Bessler, W., & Wolff, D. (2015). Do commodities add value in multi-asset portfolios? An out-of-sample analysis for different investment strategies. *Journal of Banking & Finance*, *60*, 1-20.

Bloomberg L.P. (2022, September 1). Price Data of. Retrieved from Bloomberg Terminal

Bodie, Z., Kane, A., & Marcus, A. J. (2021). Investments (12th ed., p. 139). Mcgraw-Hill Education.

Bond, S. A., Hwang, S., Mitchell, P., & Satchell, S. E. (2007). Will private equity and hedge funds replace real estate in mixed-asset portfolios?. *The Journal of Portfolio Management*, *33*(5), 74-84.

Buffett, W. (2020, June 2). Warren Buffett says diversification is still important — here's how to do it. CNBC. https://www.cnbc.com/2020/06/02/warren-buffett-says-diversification-is-still-important-heres-how-to-do-it.html

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?. *The review of Financial studies*, *22*(5), 1915-1953.

Divecha, A. B., Drach, J., & Stefek, D. (1992). Emerging markets: a quantitative perspective. *Journal of Portfolio Management*, *19*(1), 41.

French, K. (2021). Current Research Returns. Retrieved September 2, 2022, from Kenneth R. French: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

FTSE Russell ((2022, November). FTSE World Government Bond Index – Developed Markets Capped Select Index Ground Rules. Retrieved from https://research.ftserussell.com/products/downloads/FTSE_World_Government_Bond_Index_-_Developed_Markets_Capped_Select_Index_Ground_Rules.pdf

Jobson, J. D., & Korkie, R. M. (1981). Putting Markowitz theory to work. *The Journal of Portfolio Management*, *7*(4), 70-74.

Kirby, C., & Ostdiek, B. (2012). It's all in the timing: simple active portfolio strategies that outperform naive diversification. *Journal of Financial and Quantitative Analysis*, *47*(2), 437-467.

Kiyosaki, R. T. (1997). The Warren Buffett Way. John Wiley & Sons.

Kuhle, J. (1987). Portfolio diversification and return benefits—common stock vs. real estate investment trusts (REITs). *Journal of Real Estate Research*, *2*(2), 1-9.

Liu, Y., Sun, S., Huang, R., Tang, T., & Wu, X. (2017). The Rise of Alternative Assets and Long-Term Investing. *Boston, MA: Boston Consulting Group*.

Markowitz, H. (1952). Portfolio Selection. The Journal of Finance, pp. 77-91.

MSCI Inc. (2022, November 30). MSCI World Index - Factsheet. Retrieved from: https://www.msci.com/documents/10199/178e6643-6ae6-47b9-8be-e1fc565ededb

MSCI Inc. (2022, November 30). MSCI Emerging Market Index - Factsheet. Retrieved from: https://www.msci.com/documents/10199/c0db0a48-01f2-4ba9-ad01-226fd5678111

Platanakis, E., Sakkas, A., & Sutcliffe, C. (2019). Harmful diversification: Evidence from alternative investments. *The British Accounting Review*, *51*(1), 1-23.

Platanakis, E., Sutcliffe, C., & Ye, X. (2021). Horses for courses: Mean-variance for asset allocation and 1/N for stock selection. *European Journal of Operational Research*, *288*(1), 302-317.

Platanakis, E., & Urquhart, A. (2020). Should investors include bitcoin in their portfolios? A portfolio theory approach. *The British accounting review*, *52*(4), 100837.

Potrykus, M. (2019). Markowitz’s portfolio theory–optimal length of lookback window for gold and the biggests companies on the Warsaw Stock Exchange. *Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu*, *63*(6), 86-100.

Sharpe, W. F. (1966). Mutual fund performance. *The Journal of business*, *39*(1), 119-138.

U.S. Bureau of Economic Analysis. (2022, October 18). 3-Month Treasury Bill Secondary Market Rate, Discount Basis. Retrieved from Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/series/TB3MS

Appendix

Abbreviation	Description	Type
2M - No constraints	Portfolio model with a two-year lookback window, monthly rebalancing and the weights are not imposed any constriction	Markowitz
2Y - No constraints	Portfolio model with a two-year lookback window, yearly rebalancing and the weights are not imposed any constriction	Markowitz
5M - No constraints	Portfolio model with a five-year lookback window, monthly rebalancing and the weights are not imposed any constriction	Markowitz
5Y - No constraints	Portfolio model with a five-year lookback window, yearly rebalancing and the weights are not imposed any constriction	Markowitz
2M - Capped	Portfolio model with a two-year lookback window, monthly rebalancing and the weights are capped at +/- 100%	Markowitz
2Y - Capped	Portfolio model with a two-year lookback window, yearly rebalancing and the weights are capped at +/- 100%	Markowitz
5M - Capped	Portfolio model with a five-year lookback window, monthly rebalancing and the weights are capped at +/- 100%	Markowitz
5Y - Capped	Portfolio model with a five-year lookback window, yearly rebalancing and the weights are capped at +/- 100%	Markowitz
2M - Long	Portfolio model with a two-year lookback window, monthly rebalancing and the weights are not allowed to be negative	Markowitz
2Y - Long	Portfolio model with a two-year lookback window, yearly rebalancing and the weights are not allowed to be negative	Markowitz
5M - Long	Portfolio model with a five-year lookback window, monthly rebalancing and the weights are not allowed to be negative	Markowitz
5Y - Long	Portfolio model with a five-year lookback window, yearly rebalancing and the weights are not allowed to be negative	Markowitz
1/N	The portfolio allocates equally in all assets	Equally weighted

Abbreviation	Description	Type
Alternative - No constraints	Portfolio model with a five-year lookback window, monthly rebalancing and the weights are not imposed any constriction. The portfolio consist of both traditional and alternative assets	Markowitz
Traditional - No constraints	Portfolio model with a two-year lookback window, yearly rebalancing and the weights are not imposed any constriction. The portfolio only contains traditional assets	Markowitz
Alternative - Long	Portfolio model with a five-year lookback window, monthly rebalancing and the weights are not allowed to be negative. The portfolio consist of both traditional and alternative assets	Markowitz
Traditional - Long	Portfolio model with a five-year lookback window, monthly rebalancing and the weights are not allowed to be negative. The portfolio consist of only traditional assets	Markowitz