Relational incentive contracts for teams of multitasking agents

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Relational incentive contracts for teams of multitasking agents

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Abstract

We analyze optimal relational contracts for a group (team) of multitasking agents with hidden actions. Contracts are based on noisy signals that may be correlated across agents and between tasks. The optimal contract defines a performance measure in the form of an index (a scorecard) for each agent, and awards a bonus to the highest-performing agent, provided his or her index exceeds a hurdle. An optimal index generally involves benchmarking against other agents, and this may, in combinanation with the hurdle requirement, introduce a cooperative element in the otherwise competitive incentive structure.

For agents with separate tasks and normally distributed signals, we find that strong correlation (either positive or negative) across agents is beneficial, while larger correlation within each agent's tasks is detrimental for efficiency, and that this has implications for optimal organization of tasks. For agents with common tasks the optimal contract may have features of both tournament and team incentives. The tournament aspect incentivizes an agent to exert effort on his own task, while the hurdle necessary to receive a bonus also incentivizes an agent to help his peers. In our setting this hybrid scheme can only be optimal if signals from agents' tasks are negatively correlated. Otherwise pure team incentives are optimal.

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1 Introduction

Most employment relationships can be characterized as follows: 1. They are long term. Employer and workers engage in repeated interaction. 2. Workers do many tasks, and so their performance is not only measured along one dimension. 3. Performances are not always easy to measure in a clear and transparent way, which implies that one cannot (solely) rely on legally binding incentive contracts. And 4, workers have peers, who they sometimes compete against and sometimes cooperate with.

In the by now rich literature on incentives in organizations, all these ingredients have been extensively analyzed, either in isolation, but sometimes also partly in combinations. To our knowledge, however, all these ingredients have not been analyzed in the same model. We think this is important. Incentive design is notoriously complicated, and the empirical support for many theoretical models is limited. While models analyzing single agent and/or single task and/or verifiable outputs can highlight important dimensions of incentive design, they also miss out on several key characteristics of modern employment relationships.

Hence, in this paper we make an attempt to incorporate all four dimensions listed above. We consider a repeated interaction between a principal and multiple agents who each take a number of actions which affect a number of noisy non-verifiable performance measures (signals). Obviously, we need to make some simplifying assumptions in order to make the model tractable. We assume risk neutrality both for principal and agents, and for the most part, we assume a multinormal distribution of the signals. We also sometimes restrict attention to two agents and two tasks and consider explicitly the case where one of the tasks is to provide help to a peer.

Interestingly, the model delivers a variation of incentive regimes that fits well with the rich practices observed in the field.[1] The optimal contract can entail team incentives or tournament incentives, and even a combination of the two. Moreover, the bonuses are typically based on an index, i.e. a weighted sum of performance outcomes on the various tasks, similar to

[1]As documented by e.g. Bloom and van Reenen (2010) and Bloom et al (2019) there is a rich variation in management practices, including differences inincentive design, across countries and companies.

so-called balanced scorecards.[2]

A key ingredient in the analysis is stochastic properties of the available signals, including stochastic dependencies which make one agent's performance informative about other agents' actions. Such properties have been of central importance in the classical agency literature with verifiable outputs (e.g. Holmström 1979, 1982), but they have received relatively little attention in the relational contracting literature. This may be due to the fact that relational contracting models often avoid risk averse agents - making stochastic properties seemingly less important. However, as we demonstrate in Kvaløy and Olsen (2019, 2022), such properties – like signal variances and correlations – are important even without risk considerations because they affect marginal effort incentives. Analyzing stochastic properties is also interesting from a more practical viewpoint, as firms are getting better access to high quality performance data. Knowledge about the implications of statistical properties of performance data can both guide practitioners in implementing optimal incentives, and also help researchers who want to better understand real world incentive contracts.

The main driver of the incentive variation in our model is noise, and how the noisy signals are correlated. Generally, for agents with separate tasks strong correlation (either positive or negative) across agents strengthens incentives, while larger correlation within each agent's tasks weakens incentives. The latter effect is due to lower precision of the agent's performance index, as this dampens incentives. The former effect comes from the effects of stronger correlations on the competition among agents in the optimal incentive scheme. This competition is generated by the fact that for agents with separate tasks (where helping effort is not possible), the optimal scheme is a tournament based on the agents' performance indexes, and where only one agent (the winner) is paid a bonus, provided his performance index exceeds a hurdle.

We point out that these effects can have interesting implications for the organization of tasks. For example, if task allocation is flexible, and if signals from tasks are positively correlated, it will be advantageous to assign

[2] The concept of a balanced scorecard was introduced by Kaplan and Norton (1992) as a means to measure and evaluate performance based on non-verifiable relevant information. It has been highly influential (see e.g.Hogue, 2014) and scorecards in various forms are extensively used (see e.g. Kvaløy-Olsen 2022 and references therein).

tasks such that “internal” correlation (within an agent’s tasks) is low and “external” correlation (across agents’ tasks) is high. In some cases it may also be optimal to abandon tasks that would not be abandoned in a first-best setting.

When agents have common tasks, and hence helping effort is possible, the tournament scheme incentivizes an agent to exert effort on his own task, and neglect helping effort, while the hurdle necessary to receive a bonus also incentivizes the agent to help his peers. We show that the demotivating element of the tournament scheme necessarily dominates in situations with positive correlation, while this is not necessarily the case for negative correlation. For positive correlation, the optimal incentive system will therefore be a pure team incentive scheme, while for negative correlation, a combination of both tournament and team incentives may be optimal.

Our paper builds on a rich literature on the economics of relational contracting, starting with Klein and Leffler (1981), Shapiro and Stiglitz (1984) and Bull (1987). MacLeod and Malcomson (1989) generalize the case of symmetric information, while Levin (2003) provides a general treatment of relational contracts with asymmetric information. These papers restrict attention to the single agent / single task case.

Relational contracting papers considering multiple agents also restrict attention to the single task case. They include Levin (2002) Kvaløy and Olsen (2006, 2019), Rayo (2007), Glover and Xue (2016) Baldenius et al. (2016) and Deb et al, (2016). The few papers analyzing relational contracts with multitasking agents include Baker, Gibbons and Murphy (2002), Budde (2007) Schottner (2008) Mukerjee and Vasconcelos (2011), Ishihara (2016) and Kvaløy and Olsen (2022). However, these papers do not consider multiple agents. In this paper we combine and extend Kvaløy and Olsen (2008, 2019, 2022) to analyze the more realistic case with multiple agents having multiple tasks.

The rest of the paper is organized as follows. Section 2 presents a general model and a result establishing the structure of the optimal incentive system. An agent’s performance is optimally assessed by an index; a function of the signals, and the agents compete for a bonus on the basis of these indexes. An agent’s index is a weighted sum of the likelihood ratios for her actions. The remaining sections confine attention to normal distributions for the signals, where the relevant indexes are linear functions, and hence quite tractable analytically. Section 3 deals with agents having separate tasks, where no signal is affected by more than one agent's actions. Section 4 considers common tasks, where signals are affected by the actions of more than one agent, which is typically the case when agents can help each other. Section 4 concludes.

2 Model

We consider an ongoing relationship between a principal and $n \geq 2$ multi-tasking agents. Each period agent i performs a number $n^i \geq 2$ of actions $(a^i_1, .., a^i_{n_i}) = a^i$ which affect a number $m \geq 2$ of signals $(x_1, ..., x_m) = x$ that are informative about the actions.[3] The joint distribution of signals is given by a density $f(x; a)$ with $a = (a^1, ..., a^n)$, and the density is assumed to be positive on the set of feasible realizations. The signals, some of which may be outputs, are observable, but not verifiable.They are stochastically independent across periods. Actions are hidden, in the sense that only agent i observes a^i.

Action a^i generates a private cost $c^i(a^i)$ for agent i, and the ensemble a of actions generates a gross value $v(a)$ for the principal. The gross value $v(a)$ is not observed (as is the case if this is e.g. expected revenue for the principal, conditional on the agents' actions). We assume $v(a)$ to be increasing in each variable $(a^i_k \geq 0)$ and concave, and $c^i(a^i)$ to be increasing in each variable and strictly convex with $c^i(0) = 0$ and gradient (marginal costs) $\nabla c^i(0) = 0$ All parties are risk neutral, and the total surplus (per period) in the relationship is $v(a) - \Sigma^n_{i=1} c^i(a^i)$. Outside options are normalized to zero. All parties have the same discount factor $\delta \in (0, 1)$.

Given observable (but not verifiable) signals, the principal offers in each period a non-conditional payment (w_i) and a discretionary bonus $(\beta_i(x))$ conditional on the signals to agent i, $i = 1, ..., n$.. As in Levin (2002) we consider a multilateral punishment structure where a deviation by one party

[3] For notational simplicity we drop a time index on signals, actions and payments, as optimal contracts in our setting are wlog stationary, see below.

triggers a punishment by all parties in the form of breaking up the relationship and taking the respective outside options. In this setting optimal contracts can be taken to be stationary (Levin 2002, 2003).

An optimal contract maximizes the total surplus subject to incentive constraints (IC) for the agents' choice of actions and self-enforcement constraints (EC) for payments of discretionary bonuses. Agent $i's$ IC for choice of action is

$$a^i \in \arg\max_{\tilde{a}^i} E(\beta_i(x)|\, \tilde{a}^i, a^{-i}) - c^i(\tilde{a}^i)$$

with first-order conditions (FOCs)

$$\nabla_{a^i} E(\beta_i(x)|\, a^i, a^{-i}) - \nabla c^i(a^i) = 0.$$

As is well known (Levin 2002, 2003) all enforcement constraints can be satisfied iff the following constraint for the aggregate bonus payment is satisfied:

$$\Sigma_{i=1}^n \beta_i(x) \leq \frac{\delta}{1-\delta}(v(a) - \Sigma_{i=1}^n c^i(a^i)), \qquad \beta_i(x) \geq 0,\ i = 1, ..., n$$

We confine attention to situations where the first-best is not attainable, and hence the (upwards) EC constraint is binding. We also assume that the first-order approach (FOA) is valid, and thus consider only the FOCs for incentive compatibility. The incentive constraints are then linear in the bonuses, and since the EC constraint is also linear in those variables, it follows that the optimal bonuses have a bang-bang structure.

Specifically, from the Lagrangian for the problem we see that there are indexes of the form $\Sigma_{k=1}^{n^i} \mu_k^i \frac{\partial}{\partial a_k^i} f(x, a)$, one for each agent, such that the the agent who realizes the highest index value will be awarded a bonus, provided her index value is positive. If equal index values is a zero probability event, then at most one agent will be awarded a bonus, and this occurs when at least one index is positive.

The bonus scheme is then a form of a tournament, where only a "winner" is awarded a bonus, but the bonus is also conditional on the winner's performance index exceeding a hurdle (here zero). Since the index may in principle depend on all signals, the hurdle requirement may entail a form of benchmarking vis-a-vis the other agents. If equal and positive index values can occur with positive probability, then more than one agent can be awarded a bonus in such an event. We have the following result.

Lemma 1 *There are vectors (multipliers) $\mu^i \in R^{n^i}$, such that the optimal solution entails $\beta_i(x) > 0$ if and only if the expression*

$$\mu^i \nabla_{a^i} f(x,a) \equiv \Sigma_{k=1}^{n^i} \mu_k^i \frac{\partial}{\partial a_k^i} f(x,a),$$

evaluated at the optimal action, is positive, and this expression for agent i is maximal among the corresponding expressions for all agents:

$$\mu^i \nabla_{a^i} f(x,a) \geq \mu^j \nabla_{a^j} f(x,a), \quad j \neq i$$

If the inequality is strict, then only agent i receives a bonus, while otherwise an agent j whose expression is also maximal may also receive a bonus.. In any case, if at least one bonus is positive, then: $\Sigma_i \beta_i(x) = \frac{\delta}{1-\delta}(v(a) - \Sigma_i c^i(a^i))$.

Since we have assumed $f(x,a) > 0$ for all feasible realizations, we may equivalently state the conditions in the lemma in terms of likelihood ratios $\frac{\partial}{\partial a_k^i} f(x,a)/f(x,a)$. Thus there is for each agent i an index

$$y_i(x) = \mu^i \nabla_{a^i} \ln f(x,a) \equiv \Sigma_{k=1}^{n^i} \mu_k^i \frac{\partial}{\partial a_k^i} \ln f(x,a), \tag{1}$$

i.e. a weighted sum of the likelihood ratios for the actions of agent i, such that the agent is paid a bonus for an outcome x iff this index value at x is (i) positive and (ii) is maximal among the agents. This index can be interpreted as a (optimal) performance index, and agents are then rewarded in a tournament-like scheme based on these indexes. Note that the indexes are defined with a being the optimal (equilibrium) action vector.

This characterization of the optimal bonus scheme has similar features to that given by Levin (2002) for the single-task case with stochastically independent signals. In his case the equality condition for the indexes is a zero probability event. In such cases only one agent is given a bonus, namely the agent with the highest "performance index", provided that this index in addition exceeds a hurdle (zero). The scheme outlined here has similar features as Levin's, but covers multi-dimensional actions and stochastic dependencies. Kvaløy and Olsen (2019) analysed a case of single-task agents with correlated signals, while Kvaløy and Olsen (2022) analysed a single multi-tasking agent, and pointed out that the performance index can be seen as an optimal balanced scorecard for the agent. In light of this, the bonus structure oulined in the lemma can be seen as a form of a tournament based on these scorecards.

In the remainder of this paper we will (as in Kvaløy-Olsen 2019, 2022) invoke the assumption of a *multinormal distribution* for the signals. This leads to a tractable model which enables us to derive some interesting properties of the optimal bonus scheme, in particular with respect to how it depends on correlations among the signals. So in the following we will assume that x is multinormal with a given covariance matrix (Σ) and expectations that depend linearly on actions, thus

$$x \sim N(Qa, \Sigma)$$

Here Q is a matrix of dimension $m \times N$, where $N = n^1 + ... + n^n$ is the total number of actions. An analytically convenient property of the normal distribution is that the likelihood ratios are linear functions of the signals, and hence it follows that the performance indexes defined above are also linear in the signals. The linear form of the likelihood ratios follows because $\ln f(x, a)$ is a quadratic form; $\ln f(x, a) = -(x - Qa)'\Sigma^{-1}(x - Qa)/2 + const$, and any derivative $\frac{\partial}{\partial a_k^i} \ln f(x, a)$ is then linear, implying that the index, being a linear combination of linear expressions, is also linear.

The index $y_i(x)$ in (1) will thus here be of the form

$$y_i(x) = \tau^i(x - Qa) \equiv \Sigma_{k=1}^m \tau_k^i (x_k - (Qa)_k), \tag{2}$$

for some vector τ^i, and where $(Qa)_k$ is the k'th element of the vector of expectations. We may note that the incentive scheme can then equivalently be formulated in terms of indexes

$$\tilde{y}_i(x) = \tau^i x \equiv \Sigma_{k=1}^m \tau_k^i x_k,$$

where agent i then gets a bonus only if her (scorecard) index $\tilde{y}_i(x)$ ecxeeds its expectation $E(\tilde{y}_i(x)|\, a) \equiv \tilde{y}_{i0}$, and her "overperformance" $\tilde{y}_i(x) - \tilde{y}_{i0}$ is maximal among the agents. The expected performance $\tilde{y}_{i0} = E(\tilde{y}_i(x)|\, a)$ is here evaluated at the targeted (optimal) action a.

In addition to the index here being linear, the form of the likelihod ratio will generally also have implications for the coefficients τ^i that define the index. In the following we will explore this in more detail for two environments: (i) where agents multitask, but on separate tasks, and (ii) where multitasking involves common tasks.

3 Separate tasks.

We will say that the agents work on separate tasks when it is the case that the signals that are influenced by one agent's actions are not influenced by any of the other agents' actions. There is thus for each agent i a subset of signals x^i that are influenced by agent $i's$ own actions a^i, but not by any other agent's actions a^j. For instance, x^i may consist of two variables that measure quantity and quality aspects, respectively, of the output delivered by agent i, and which she may affect by her actions a^i, but which is not affected by the actions of other agents. Under the multinormal specification we then have

$$E(x^i \big| \, a) = E(x^i \big| \, a^i) = Q_i a^i$$

where Q_i is a matrix of dimension $m^i \times n^i$, where m^i is the dimension of vector x^i.

It now turns out that the indexes derved in Lemma 1, and which are the basis for an optimal bonus scheme, take a form where each agent's performance is benchmarked against all other agent's performances, as specified in the following proposition.

Proposition 1 *When signals are multinormal and agents have separate tasks, there are vectors θ^i such that the optimal index derived in Lemma 1 for agent i takes the form*

$$\theta^i(x^i - E(x^i \big| \, x^{-i}, a)) = \Sigma_{k=1}^{m^i} \theta_k^i (x_k^i - E(x_k^i \big| \, x^{-i}, a)), \qquad i = 1, ..., n. \quad (3)$$

The index for agent i is thus a weighted sum of her performance signals (x_k^i), adjusted for the expected perfomance, conditional on the performances of *other agents. Agent i gets a bonus* $\beta_i(x) > 0$ *only if (i) her index value is maximal among all agents at the signal outcome x, and (ii) the index value is positive.*

In this incentive scheme the agents compete on the basis of their individual indexes (scorecards) to obtain the bonus.The scheme has thus clear elements of relative performance evaluation, in the sense that an agent is made worse off if, all else equal, another agent performs better.

The scheme may also, however, have elements of joint (or collective) performance evaluation, where an agent is made better off if, all else equal, another agent performs better. This element may be present when signals from different agents are negatively correlated. To see this, suppose signal x_k^i for agent i is negatively correlated with signal x_l^j for agent j, and consider a signal realization x where $i's$ index is stricly maximal among the agents, but its value is slightly negative, so that i doesn't get a bonus. A higher realization of signal x_l^j from agent j will then *reduce* agent $i's$ expected performance $E(x_k^i \,|\, x^{-i}, a)$ on signal x_k^i, and hence *increase* the index value for agent i (when $\theta_k^i > 0$). If this increase resulting from a higher x_l^j is enough to make $i's$ index value positive, but without affecting the ordering of index values, then i will get the bonus. In such a case a better performance from agent j makes agent i better off, and hence there is then an element of joint performance evaluation in the incentive scheme.

Some signals from an agent's tasks may be negatively correlated with signals from other agents' tasks, and some signals from his or her tasks may be positively correlated with other agents' signals. Thus there may be elements of joint as well as relative performance evaluation in an agent's performance index. This doesn't occur in the case of single-tasking agents, but the overall incentive scheme may nevertheless contain both elements if signals are negatively correlated. This was pointed out in the analysis of the single-task case in Kvaløy-Olsen (2019), and further discussed in Kvaløy-Olsen (2022b), also for the single-task case.

3.1 Two agents and two tasks

To simplify and gain some specific insights, we will now consider the case of two agents working on two tasks each (e.g. teaching and research) with efforts denoted $a^i = (e_i, h_i)$, $i = 1, 2$. We will here moreover assume that there are 4 signals, one for each activity. Index the signals such that $x = (x_{11}, x_{12}, x_{21}, x_{22})$, and assume $x \sim N(.; \Sigma)$ with

$$Ex_{i1} = e_i, \qquad Ex_{i2} = h_i.$$

Thus x_{i1} is a noisy signal about effort e_i, and x_{i2} is a noisy signal about effort h_i for agent i. We assume that the agents are symmetrically placed from the outset, and hence that the signal structure is symmetric for the two agents (to be specified below). In line with this, all signals are assumed to have the same variance: $var(x_{ij}) = s^2$. Given symmetry among agents, we consider also symmetric incentive schemes, and hence symmetric indexes in the bonus scheme.

Before specifying the structure of correlations among signals, we exploit the fact that indexes in the current setting are in any case linear (as in (2)), and hence that (symmetric) indexes can be written in the following form

$$y_1 = \tau_{11}(x_{11} - e_1) + \tau_{12}(x_{12} - h_1) + \tau_{21}(x_{21} - e_2) + \tau_{22}(x_{22} - h_2)$$

$$y_2 = \tau_{11}(x_{21} - e_2) + \tau_{12}(x_{22} - h_2) + \tau_{21}(x_{11} - e_1) + \tau_{22}(x_{12} - h_1)$$

Here τ_{11} is the weight on agent 1's "e_1 -signal", τ_{12} the weights on her "h_1- signal" etc. By symmetry agent 2 has the same weights on her corresponding signals. Both indexes are defined with efforts set at their equilibrium levels.

Provided the coefficients τ_{kl} are such that $\Pr(y_1 = y_2 | \, e, h) = 0$, agent 1 will get a bonus ($\beta_i(x) = b > 0$) iff $y_1 > y_2$ and $y_1 > 0$. We will subsequently invoke assumptions which ensure that this property (of y_1 and y_2 almost surely being unequal) holds.

Remark. As observed in Section 2, the bonus scheme can equivalently be defined in terms of indexes which for agent 1 takes the form $\tilde{y}_1(x) = \Sigma_{kl}\tau_{kl}x_{kl}$, and with a similar expression $\tilde{y}_2(x)$ for agent 2. Since by symmetry equilibrium efforts will be equal across agents here ($e_1 = e_2, h_1 = h_2$), these indexes have equal expectations for equilibrium efforts: $E(\tilde{y}_1(x) | \, e, h) =$

$E(\tilde{y}_2(x)|\, e, h) \equiv \tilde{y}_0$. Agent i will then get a bonus iff $\tilde{y}_i(x) > \tilde{y}_{-i}(x)$ and $\tilde{y}_i(x) > \tilde{y}_0$.

We now turn to a characterization of the incentives provided by the bonus scheme. We then return to the FOCs for optimal efforts, of which there are two for each agent. In particular for agent i:

$$c_{e_i}(e_i, h_i) = \int \beta_i(x) f_{e_i}(x, e, h) dx$$

$$c_{h_i}(e_i, h_i) = \int \beta_i(x) f_{h_i}(x, e, h) dx$$

where by symmetry $e_1 = e_2, h_1 = h_2$, and the agents are symmetric in all respects. Using again the multinormal distribution, we can then show that the FOCs for agent 1's efforts, and symmetrically for agent 2's efforts, are given as follows

Lemma 2 *Provided* $\Pr(y_1 = y_2|\, e, h) = 0$, *the bonus scheme yields the following FOCs for (symmetric and positive) efforts*

$$c_{e_i}(e_i, h_i)_{i=1} = (\tau_{11} + \frac{\tau_{11} - \tau_{21}}{\sqrt{2(1-\tilde{\rho})}}) b \frac{\phi_0}{2\tilde{s}}$$

$$c_{h_i}(e_i, h_i)_{i=1} = (\tau_{12} + \frac{\tau_{12} - \tau_{22}}{\sqrt{2(1-\tilde{\rho})}}) b \frac{\phi_0}{2\tilde{s}}$$

where $\tilde{s}(\tau) = SD(y_i)$, $\tilde{\rho}(\tau) = corr(y_1, y_2)$, $\phi_0 = \frac{1}{\sqrt{2\pi}}$ *and* b *is the optimal bonus level.*

If agent 1 deviates to some effort e_1' she will affect the signal x_{11} (by changing its mean) and by that affect her own index y_1, but also affect the other agent's index y_2 if $\tau_{21} \neq 0$. The terms in the first formula in the lemma account for these two effects. The second formula has a similar interpretation.

The principal's problem can now be seen as to maximize the surplus $v(e, h) - \Sigma_i c(e_i, h_i)$, given the incentive constrants in the lemma and the enforcement constraint. This will endogenously determine efforts, bonus level b, and the weights τ in the bonus scheme.

We see immediately from the formulas in the lemma that a lower standard deviation $\tilde{s}$ for the indexes will, all else equal, strenghten incentives, and that a lower variance $s^2 = var(x_{kl})$ will have this effect. More precise signals

x_{kl} will thus boost incentives This will, moreover, increase the achieveable surplus and thus be overall beneficial. The latter effect follows because, if a certain surplus S can be achieved with the combination τ, b, s, then the same surplus can be achieved with a combination τ, b', s' where $b' < b$ if $s' < s$. The lower bonus b' will yield slack in the enforcement constraint under s', and thus allow for adjustments to further increase the surplus.

From the lemma we see that incentives will also be affected by the correlation ($\tilde{\rho}$) between the indexes. These effects cannot, however, be assessed without knowing more about the structure of the (optimal) weights τ. Proposition 1 gives us some information of this structure, and we will now exploit this insight.

To simplify somewhat, but yet illustrate some basic points, we will in the following analysis *assume that there is pairwise correlation between the "e-tasks" (x_{11}, x_{21}), between the "h-tasks" (x_{12}, x_{22})* and *between the two tasks operated by each agent, but that the signals are otherwise uncorrelated.* The relevant correlation coefficients are thus[4]

$$\rho_1 = corr(x_{11}, x_{21}), \quad \rho_2 = corr(x_{12}, x_{22}), \tag{4}$$

$$\rho_a = corr(x_{11}, x_{12}) = corr(x_{21}, x_{22}) \tag{5}$$

The last equality reflects that agents are supposed to be symmetric, and hence must have the same correlation between the signals from "internal" tasks. Recall moreover that all signals have the same variance, $var(x_{kl}) = s^2$.

Given these assumptions about correlations, we can now see that the result in Proposition 1 implies the following relations between the coefficients τ_{kl} in the indexes:

$$\tau_{21} = -\rho_1 \tau_{11} \qquad \text{and} \qquad \tau_{22} = -\rho_2 \tau_{12} \tag{6}$$

This is so because with these coefficients we can write the index for e.g agent

[4] The coefficients must satisfy restrictions ensuring that the covariance matrix is positive definite. These are stated in the appendix where needed.

1 as

$$\begin{aligned} y_1 &= \tau_{11}(x_{11} - e_1 - \rho_1(x_{21} - e_2)) + \tau_{12}(x_{12} - h_1 - \rho_2(x_{22} - h_2)) \\ &= \tau_{11}(x_{11} - E(x_{11}|\, x_{21})) + \tau_{12}(x_{12} - E(x_{12}|\, x_{22})), \end{aligned} \tag{7}$$

where the last equality follows because the conditional expectation ot two normally distributed variables with identical variances is of the form $E(x_{ij}|\, x_{kl}) = Ex_{ij} + \rho(x_{kl} - Ex_{kl})$, with $\rho = corr(x_{ij}, x_{kl})$. For coefficients satisfying (6) the index can thus be written in the form with conditional expectations given in (3), with $\theta_1^1 = \tau_{11}$ and $\theta_2^1 = \tau_{12}$.

The index y_1 is thus a weighted sum of two terms, where the first measures agent 1's performance on her e-task compared to what would be expected, given agent 2's performance on her e-task The latter is informative about agent 1's e-effort, since the signals are correlated, and it is here taken into the index as a benchmark, to which agent 1's performance is compared. The second term in the index is similarly a measure of agent 1's perfomance on the h-task, compared to its conditional expectation, given agent 2's performance on her h-task. The index for agent 2 will have a similar representation,

It is straightforward to verify that we now have $corr^2(y_1, y_2) < 1$, and hence that Lemma 2 applies. The marginal effort incentives are thus given by the formulas in the lemma, and with coefficients τ_{kl} satisfying (6). We can then state the following proposition.

Proposition 2 *With noisy signals from separate tasks, and signals from each type of effort being correlated as specified in (4) the optimal perfomance indexes are of the form given in (7) for agent 1, and a similar expression for agent 2. An agent's performance on each task is then benchmarked against the other agent's performance on the correlated task, and the index is a weighted sum of these benchmark-adjusted performances. The agent that realizes the highest index value is awarded the maximal bonus (b), provided that the index value exceeds zero. This incentive scheme leads to marginal effort incentives (at equilibrium efforts) given by the formulas in Lemma 2, and with coefficients τ_{kl} satisfying (6).*

We will now examine the effort incentives in more detail. First note that a scheme that awards a bonus for $y_i > \max\{y_j, 0\}$ is equivalent to a scheme that awards a bonus for $ky_i > \max\{ky_j, 0\}$ with $k > 0$, hence we may normalize and set $\tau_{11} = 1$. To simplify further, we will invoke the assumtion of *identical correlations*, i.e. that $\rho_1 = \rho_2 \equiv \rho$. The expressions in Lemma 2 are evaluated (with $\tau_{21} = -\rho\tau_{11}$, $\tau_{22} = -\rho\tau_{12}$ and $\tau_{11} = 1$) and displayed in the appendix. Here we will first consider the case $\rho_a = 0$, i.e. the case where the two tasks operated by the same agent are uncorrelated. We then obtain the following

$$\tilde{s}^2 = var y_i = s^2(1 + \tau_{12}^2)(1 - \rho^2), \qquad cov(y_1, y_2) = s^2\rho(\rho^2 - 1)(1 + \tau_{12}^2),$$

and hence $\tilde{\rho} = corr(y_1, y_2) = -\rho$. Inserting these expressions into the formulas in Lemma 2, we find by straightforward calculations that they yield the following conditions

$$c_{e_i}(e_i, h_i)_{i=1} = R_0(\rho)\frac{\phi_0 b}{s}/\sqrt{1 + \tau_{12}^2} \tag{8}$$

$$c_{e_i}(e_i, h_i)_{i=1} = R_0(\rho)\frac{\phi_0 b}{s}(\tau_{12}/\sqrt{1 + \tau_{12}^2}), \tag{9}$$

where

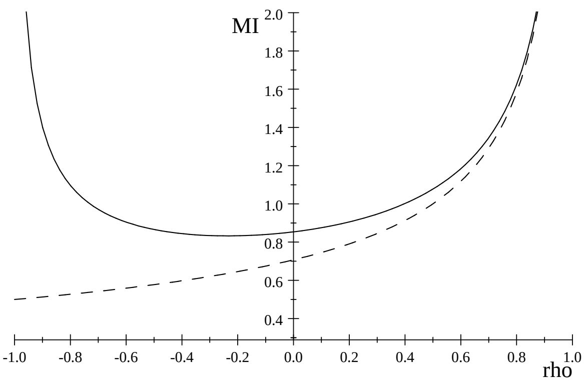
$$R_0(\rho) = (1/\sqrt{1 - \rho^2} + 1/\sqrt{2(1 - \rho)})/2 \tag{10}$$

The principal's problem can now be seen as to maximize the surplus $v(e, h) - \Sigma_i c(e_i, h_i)$, given the incentive constrants in (8)-(9) and the enforcement constraint. This will endogenously determine efforts, bonus level b, and the weight τ_{12} in the bonus scheme.

We see that, all else equal, the weight τ_{12} in the index motivates h-effort and demotivates e-effort, which is as expected of course. For given τ_{12}, the effect of correlation on the incentives for effort are captured in the function $R_0(\rho)$ This function is U-shaped, as shown in the figure below. This implies that stronger correlation (both positive and negative, if the latter is sufficiently strong[5]) boosts incentives. A contributing factor to this effect is that increased correlation reduces the variance of each index (as can be seen in the formula for $var y_i$ above), and hence makes the indexes more precise.

[5] The function $R_0(\rho)$ has a minimum at $\rho = -0.236$. The same function actually also appears in the single-task case in Kvaløy-Olsen 2019.

This is however, not the whole story, since the same effect operates in a simple tournament based on the indexes y_1 and y_2. In such a tournament, agent 1 gets the bonus whenever $y_1 > y_2$, and it is straightforward to see that the marginal incentive for effort e_1 is then (in the present case of $\rho_a = 0$) equal to $R_T(\rho)\phi_0 b/s\sqrt{1+\tau_{12}^2}$, where $R_T(\rho) = 1/\sqrt{2(1-\rho)}$. The graph of $R_T(\rho)$ is depicted by the dashed curve in the figure, and clearly illustrates that incentives in a simple tournament are lower than those in the modified (and optimal) tournament with hurdles.[6] This illustrates the importance of the hurdle requirement in the tournament scheme.

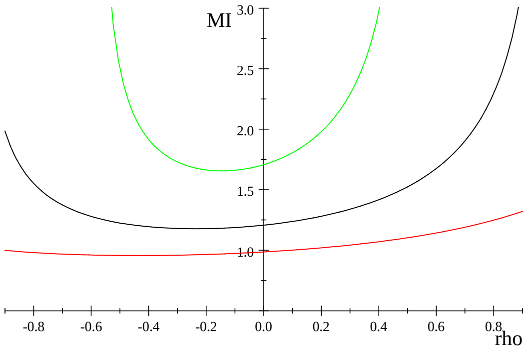


Since incentives are U-shaped in ρ, the optimal surplus will also be U-shaped. This follows because if efforts (e, h) are optimal for a given ρ, then it will be possible to implement the same efforts for a ρ' with $R_0(\rho') > R_0(\rho)$, namely by just reducing the bonus b accordingly.This will yield slack in EC, and thus room for a higher bonus to induce efforts that will increase the surplus.

So far we have analysed the case where pairwise correlations between signals from corresponding tasks "across" agents are equal ($\rho_1 = \rho_2$), and signals from "internal" tasks are uncorrelated ($\rho_a = 0$). We will now briefly consider the modifications that follow from $\rho_a \neq 0$. Precise formulas are given in the appendix, and show that higher ρ_a reduces marginal incentives for efforts. This is illustrated in the figure below, which shows, for given weight τ_{12} and bonus b, the marginal incentives for effort as functions of ρ, for 3 values of ρ_a, namely $\rho_a = -\frac{1}{2}$ (top), $\rho_a = 0$ (middle) and $\rho_a = \frac{1}{2}$ (bottom)[7].

[6] A similar comparison was made in the single-task case analysed in Kvaløy-Olsen 2019.

[7] The graphs in this figure are defined and drawn such that the case $\rho_a = 0$ corresponds to $R_0(\rho)\sqrt{2}$, hence the levels depicted for this case are higher than those in the previous



The figure clearly illustrates that higher correlation between signals from "internal" tasks reduces incentives. A contributing factor for this is that the higher correlation increases the variance of each index, and thus makes the two performance indexes less precise. This in turn weakens incentives.

As outlined for the previous case of $\rho_a = 0$, it will more generally be the case that any change in the correlation structure that strengthens incentives, will also enable a larger surplus to be achieved. The figure tells us that large negative "internal" correlation combined with strong pairwise correlation (negative or positive) "between" agents' tasks are beneficial in this respect.

3.1.1 Task organization.

The structure of signal correlations will have implications for the optimal organization of tasks. If tasks are flexible, then an alternative to the task allocation considered so far would be to allocate the "e-tasks" to one agent and the "h-tasks" to the other agent. Assuming additive cost functions for efforts, the best allocation would then be the one generating the strongest incentives for (symmetric) efforts. The analysis above tells us that large negative "internal" correlation combined with strong pairwise correlation (negative or positive) "between" agents' tasks are beneficial in this respect.

If ρ_a is strong and negative and ρ is strong and positive, then the current organization would be best, since the alternative would yield a strong and positive internal correlation (then ρ), combined with strong and negative

figure. (The graphs in the figure are drawn for $\tau_{12} = 1$ and depicts a function $\tilde{R}(\rho, \rho_a)$ such that $c_{e_1} = \tilde{R}(\rho, \rho_a)\frac{\phi_0}{2s}b$, see (20) in the appendix. Comparing with (8) we thus have $\tilde{R}(\rho, 0) = R_0(\rho)\sqrt{2}$.)

correlation (then ρ_a) "between" agents. If, however, ρ_a is strong and positive and ρ is strong and negative, the alternative organization would be better. If both ρ and ρ_a are positive, then the allocation that yields the smallest "internal" correlation would be best. A full analysis is outside the scope of this paper, but these esamples illustrate the issues involved in such an analysis.

Another aspect of task organization is the possible abandonment of some tasks. We have seen that increasing correlation (ρ_a) between an agent's "internal" tasks is detrimental for efficiency, because such increasing correlation weakens incentives on all tasks. If one task, say the "h-task" is abandoned, then this effect is eliminated, and the agents will here compete as single-tasking agents. This abandonment (or shutting down) of the task can be achieved by setting $\tau_{12} = 0$, and the question is then whether this may in fact be optimal in some situations. Apart from eliminating the detrimental effect of internal (positive) correlation, there is here also an additional beneficial effect on incentives for the e-task, namely the effect that follows from those incentives being decreasing in τ_{12}, and thus maximal for $\tau_{12} = 0$. This is readily seen in (8), and holds also more generally for $\rho_a > 0$.

There will be a loss of output when shutting down one task, but the question is now whether this detrimental effect can be more than compensated by the beneficial effects that we have just descibed. We show in the appendix that there are situations where the answer is affirmative, and hence that shutting down one task from each agent is optimal, although this would not be optimal in a first-best world. The proof considers the case $\rho = 0$ (which simplifies the anlysis), and shows that for $\rho_a > 0$ there will be value and cost configurations that make $\tau_{12} = 0$ optimal for a range of discount factors. As could be expected, this occurs when the cost of providing h-effort is relatively high, so that the value loss from shutting down this task is relatively small. We can then summarize this discussion in the following proposition.

Proposition 3 *For $\rho_a > 0$ there are configurations of cost and value functions such that shutting down one task for each agent is optimal for a range of discount factors, although this would not be optimal in a first-best world.*

4 Common tasks.

In this section we also consider two agents who exert two types of effort (e_i, h_i) each, but now we will consider a setting with common tasks, where we can think of e_i as effort on "own task" and h_i as help on the other agent's task. There are two signals $x = (x_1, x_2) \sim N(., \Sigma)$, and

$$Ex_1 = e_1 + h_2, \qquad Ex_2 = e_2 + h_1$$

We will also assume that the agents are symmetric, and then consider only symmetric solutions, i.e. where the levels of own effort and help, respectively, are in equilibrium equal across agents $(e_1 = e_2, h_1 = h_2)$.

From the multinormal distribution we have that the indexes in Lemma 1 take the following form under symmetry:

$$y_1 = \tau_1(x_1 - e_1 - h_2) + \tau_2(x_2 - e_2 - h_1)$$

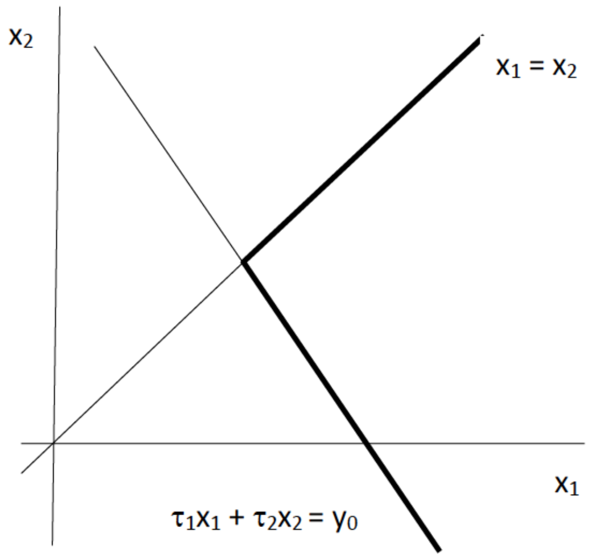
$$y_2 = \tau_1(x_2 - e_2 - h_1) + \tau_2(x_1 - e_1 - h_2)$$

Again, by symmetry the weights on "own task" and "the other's task" should be equal across agents, and this leads to the two expressions for the two indexes.

There are thus performance indexes $y_1 = \tau_1 x_1 + \tau_2 x_2 - y_0$ for agent 1, $y_2 = \tau_1 x_2 + \tau_2 x_1 - y_0$ for agent 2, where (under symmetry) $y_0 = (\tau_1 + \tau_2)(e_1 + h_1)$; such that agent i gets a bonus iff $y_i \geq \max\{0, y_j\}$. If the inequality is strict, only agent i is paid a bonus, and the bonus is then maximal $(\beta_i = \frac{\delta}{1-\delta} Surplus \equiv b)$. If $y_1 = y_2$ is a zero probability event, then almost surely at most one agent is paid a bonus, and if so, the bonus is maximal.

We see that, provided $\tau_1 > \tau_2$, we have $y_1 > y_2$ iff $x_1 > x_2$. So one way to interpret this scheme is that agent 1 is awarded the bonus iff (i) the performance on his own task exceeds that of his partner $(x_1 > x_2)$, and (ii) a "team performance index" $\tau_1 x_1 + \tau_2 x_2$ exceeds a hurdle y_0. The former aspect strongly motivates "own effort" and de-motivates "help", but the latter aspect may ensure that incentives for help are also in place. We will refer to this scheme, which contains both a tournament element as well as a team incentive element, as a hybrid scheme.

The scheme is illustrated in the figure below. Agent 1 gets the bonus for outcomes in the region between the heavy lines. Agent 2 will get the bonus in a symmetric region above the line $x_1 = x_2$.



Our next step is to characterize the incentives provided by this bonus scheme. Assuming the weights τ_1, τ_2 are such that y_1, y_2 are not perfectly correlated (i.e. $corr(y_1, y_2)^2 < 1$), we can, as verified in the appendix, use a procedure completely analogous to the proof of Lemma 2 to see that the scheme yields the following FOCs for symmetric efforts ($e_1 = e_2 > 0$, $h_1 = h_2 > 0$):

$$c_{e_i}(e_i, h_i)_{i=1} = (\tau_1 + \frac{\tau_1 - \tau_2}{\sqrt{2(1-\tilde{\rho}(\tau))}})\frac{\phi_0}{2\tilde{s}(\tau)}b \tag{11}$$

$$c_{h_i}(e_i, h_i)_{i=1} = (\tau_2 + \frac{\tau_2 - \tau_1}{\sqrt{2(1-\tilde{\rho}(\tau))}})\frac{\phi_0}{2\tilde{s}(\tau)}b \tag{12}$$

where $\tilde{s}(\tau_1, \tau_2) = SD(y_i)$, $\tilde{\rho}(\tau_1, \tau_2) = corr(y_1, y_2)$, $\phi_0 = \frac{1}{\sqrt{2\pi}}$ and b is the optimal bonus level.

It is straightforward to see that $corr(y_1, y_2)^2 < 1$ if and only if $\tau_1 \neq \tau_2$. For $\tau_1 = \tau_2$ the two indexes y_1 and y_2 will be perfectly correlated (in fact identical for symmetric efforts), and the formulas above will not apply. In that case the tournament aspect of the overall incentive scheme vanishes, and only the team aspect remains, implying hat both agents can be awarded a bonus only when the conmmon team index $y_1 = y_2 = \tau_1(x_1 + x_2) - y_0$ exceeds zero. Keeping the scheme symmetric, each agent is then awarded a bonus $b/2$, and it is straightforward to verify that this scheme yields the following FOCs for efforts

$$c_{e_i}(e_i, h_i)_{i=1} = \tau_1 \frac{\phi_0}{2\tilde{s}(\tau_1)} b = c_{h_i}(e_i, h_i)_{i=1}, \tag{13}$$

where again $\tilde{s}(\tau_1) = SD(y_1)$.

From the expressions for marginal incentives in the FOCs we see that they depend on τ_1, τ_2 only via the ratio of the two components, say τ_2/τ_1. (This is true for the correlation coefficient $\tilde{\rho}(\tau)$ as well as for $\tilde{s}(\tau)/\tau_1$.) So we may *normalize* by e.g. setting $\tau_1 = 1$. In the following we will (with a slight abuse of notation) let τ denote this (scalar) ratio; i.e. $\tau = \tau_2/\tau_1$.

Keeping τ, b constant, we see that the expressions for margianl incentives are larger when $\tilde{s}$ is smaller, which can be due to smaller variance in the signals x_i. Such conditions will thus, for given b, τ make incentives stronger, which in the end will allow for higher levels of both types of efforts to be implemented.

Taking account of the normalization, we have $cov(y_1, y_2) = cov(x_1 + \tau x_2, x_2 + \tau x_1) = (\rho + 2\tau + \tau^2\rho)s^2$ and

$$\tilde{s}^2 = var y_i = var(x_i + \tau x_j) = s^2(1 + \tau^2 + 2\tau\rho) \tag{14}$$

$$\tilde{\rho} = corr(y_1, y_2) = \frac{2\tau + (1 + \tau^2)\rho}{1 + \tau^2 + 2\tau\rho} \tag{15}$$

We see that both $\tilde{s}$ and $\tilde{\rho}$ increase with increasing x-correlation ρ. All else equal, the higher $\tilde{s}$ reduces incentives for both efforts. Assuming $\tau = \tau_2/\tau_1 < 1$, we see that the higher $\tilde{\rho}$ increases incentives for own effort (e_i), and reduces incentives for helping effort (h_i). Higher x-correlation ρ thus makes it "harder" to induce helping effort.

In fact, it turns out that for $\rho \geq 0$ it becomes *impossible to induce helping effort* when $\tau = \tau_2/\tau_1 < 1$: the marginal incentive on the RHS of (12) then becomes negative. Moreover, for $\rho \geq 0$ it becomes impossible to induce own effort if $\tau > 1$, as the marginal incentive in (11) then becomes negative.. Thus, as we verify below, for $\rho \geq 0$ it is impossible to induce positive levels of both effort types when $\tau \neq 1$.

So, when the signals are non-negatively correlated, the hybrid incentive scheme that involves both a tournament element and a team element cannot provide positive incentives for both "own effort" and helping effort. If both types of effort are vital, this implies that a pure team based incentive scheme ($\tau = 1$) becomes optimal, with marginal effort incentives given in (13),

Under negative signal correlation ($\rho < 0$) it turns out that the hybrid scheme can provide positive incentives for both types of effort for a range of values of $\tau \neq 1$. In this case the hybrid scheme may thus dominate a pure team based scheme, and we verify below that it does indeed do so under some conditions.

To verify these assertions we start by substituting for $\tilde{s}$ and $\tilde{\rho}$ into conditions (11) - (12), which then take the following forms for $\tau \neq 1$ (as verfied in the appendix):

$$c_{e_i}(e_i, h_i)_{i=1} = \left(\frac{1}{\sqrt{1+\tau^2+2\tau\rho}} + \frac{1-\tau}{|1-\tau|}\frac{1}{\sqrt{2(1-\rho)}}\right)\frac{\phi_0}{2s}b \tag{16}$$

$$c_{h_i}(e_i, h_i)_{i=1} = \left(\frac{\tau}{\sqrt{1+\tau^2+2\tau\rho}} - \frac{1-\tau}{|1-\tau|}\frac{1}{\sqrt{2(1-\rho)}}\right)\frac{\phi_0}{2s}b \tag{17}$$

From these expressions we see by straightforward calculations that for $\tau < 1$ the last one is positive iff (for $\tau > 0$) we have $\tau(1-2\rho) > 1$, and for $\tau > 1$ that the first one is positive iff $1-\tau-2\rho > 0$. These conditions can obviously only hold if $\rho < 0$. For $\rho \geq 0$ they cannot hold, and the hybrid scheme (with both a tournament and a team element) cannot provide positive incentives for both types of efforts. If both efforts are essental, e.g. in the sense that the gross value $v(e, h)$ is positive only if both types of efforts are provided, then the hybrid scheme cannot be optimal. A pure team incentive scheme (with $\tau = 1$) is then optimal. In this scheme each agent gets bonus $b/2$ if team output $x_1 + x_2$ exceeds its expected value, given equlibrium efforts, and no agent gets a bonus otherwise.

For $\rho < 0$ we see that there is a range of values $\tau \neq 1$ where the hybrid scheme induces positive incentives for both types of efforts. Whether such a scheme (with $\tau \neq 1$) dominates a pure team scheme (with $\tau = 1$) depends on the primitives of the model, in particular the form of the gross value and cost functions. In the appendix we identify some conditions where the hybrid scheme dominates and hence is optimal. We summarize this in the following proposition

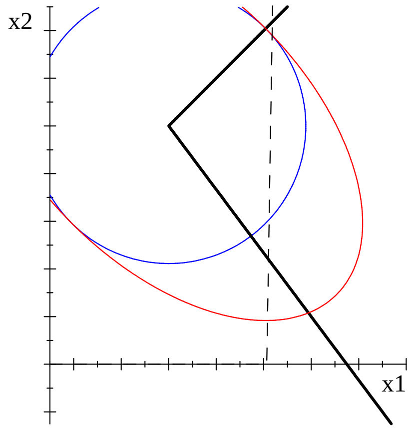
Proposition 4 *For $\tau \equiv \tau_2/\tau_1 \neq 1$ the hybrid incentive scheme generates effort incentives given by the right-hand sides of (16) - (17), and these cannot both be positve if the signals (x_1, x_2) are non-negatively correlated ($\rho \geq 0$). If both effort types are essential, the hybrid scheme can then not be optimal, and the optimal scheme is a pure team based scheme ($\tau = 1$), under which symmetric equilibrium efforts ($e_1 = e_2, h_1 = h_2$) satisfy*

$$c_{e_i}(e_i, h_i)_{i=1} = \frac{\phi_0}{2s\sqrt{2(1+\rho)}} b = c_{h_i}(e_i, h_i)_{i=1},$$

For negatively correlated signals ($\rho < 0$) the hybrid scheme generates positive incentives for both effort types if $\frac{1}{1-2\rho} < \tau < 1$ or if $1 < \tau < 1 - 2\rho$. There are conditions under which this scheme dominates a pure team based scheme, and hence is optimal.

The incentives for a team based scheme displayed in the proposition follow from (13) and (14) with $\tau_1 = \tau = 1$. Observe that this gives identical incentives for both types of effort, which may be quite different from optimal incentives in a first best solution. For example, if helping effort is less productive than effort on the agent's own task, the first-best solution may require lower incentives for the latter type than for the former one. If $\rho < 0$, a hybrid scheme can generate a certain such difference with $\tau < 1$. We show in the appendix that the hybrid scheme may in fact dominate, and hence be optimal, under some such conditions.

We now provide some intuition for why the hybrid scheme exhibits the properties stated in the proposition, and in particular why it can only generate positive incentives for both types of effort if the signals are negatively correlated. To this end, consider the figure below.



Recall that the hybrid scheme (normalized with $\tau_1 = 1$) awards the bonus to agent 1 if $x_1 > x_2$ and the index $x_1 + \tau x_2$ exceeds a hurdle given by its expectation (y_0) for equilibrium efforts. This is represented by by outcomes in the area between the heavy lines in the figure, where the upward sloping line has slope 1 (representing $x_2 = x_1$), and the downward sloping one has slope $-1/\tau$ (representing $x_1 + \tau x_2 = y_0$) The figure illustrates a case where $\tau < 1$.

The figure also depicts isocontours for the joint density of (x_1, x_2), given equilibrium efforts, for two cases, namely for $\rho = 0$, represented by the blue circle and for $\rho < 0$, represented by the red curve.[8] The probability for agent 1 to win the bonus is given by the total probability mass in the bonus region for each of the two cases.

Consider now a small increase of helping effort, say $\Delta h_1 > 0$ for agent 1. This will shift the probability distribution vertically upwards, and hence imply a loss of probability mass in the bonus region along the upper border (where $x_1 = x_2$), and a gain of probability mass along the lower border (where $x_1 + \tau x_2 = y_0$). Letting $g(x_1, x_2)$ represent the density, given equilibrium efforts (and suppressing here its dependence on ρ), the marginal net gain associated with increased helping effort is thus, for given x_1, represented by $-g(x_1, x_1) + g(x_1, (y_0 - x_1)/\tau)$. This net gain is in the figure represented by the difference in the density values at the two intersections between the vertical (dashed) line and the border lines for the bonus region.

[8]The figure implicitly assumes that equilibrium efforts ae the same for the two cases, but this is just a simplification to be able to illustrate both cases in one figure, and is not of importance for the argument.

It follows from the geometry of the isocontours that for $\rho = 0$ (the blue circle), the lower intersection is at a lower isocontour compared to the upper intersection, and hence that the relevant difference in density values is negative. Since this is true for every x_1 in the bonus region, the total net gain obtained by summing (integrating) over all relevant x_1 is also negative. The overall net gain associated with a marginal increase in helping effort is thus negative for the case $\rho = 0$.

We now see that the opposite conclusion holds for the case $\rho < 0$ illustrated in the figure. For given x_1 the lower intersection is at a higher isocontour compared to the upper intersection, and the net gain is thus positive for each relevant x_1, implying that the total net gain is also positive. The overall net gain associated with a marginal increase in helping effort is thus positive for the case of $\rho < 0$ and $\tau < 1$ illustrated in the figure. This explains the differences between incentives for helping effort for the case $\rho = 0$ versus a case with $\rho < 0$.

We finally note that for $\rho > 0$ the relevant isocontour for the probability density will be tilted upwards (rather than downwards, as was the case for $\rho < 0$), and a conclusion similar to the conclusion for the case $\rho = 0$ will then follow. The marginal incentive for helping effort in the hybrid scheme is thus negative also for $\rho > 0$. .

5 Conclusion

Workers are often evaluated along many dimensions. The evaluation of a professor, for instance, will typically be based on his or her publications, citations, student evaluations, research disseminations and services for the department. The performances on each of the tasks will typically be correlated, and at least some of the performance measures will typically be non-verifiable to a third party. Moreover, workers operate in environments where they interact with other workers, and performances are often correlated. The aim of this paper is to study this environment: Optimal incentives for a group or team of multitasking agents whose performance measures are non-verifiable and potentially correlated.

So we analyze optimal relational contracts for a group (team) of multitasking agents with hidden actions, where contracts are based on non-verifiable noisy signals that may be correlated across agents and between tasks. We show that the optimal contract defines a performance measure in the form of an index (a scorecard) for each agent, and awards a bonus to the highest-performing agent, provided his or her index exceeds a hurdle. An optimal index generally involves benchmarking against other agents, and this may, in combinantion with the hurdle requirement, introduce a cooperative element in the otherwise competitive incentive structure.

For agents with separate tasks and normally distributed signals, we found that strong correlation (either positive or negative) across agents is beneficial, while larger correlation within each agent's tasks is detrimental for efficiency, and that this has implications for optimal organization of tasks. For agents with common tasks we find that the optimal contract may have features of both tournament and team incentives. The tournament aspect incentivizes an agent to exert effort on his own task, while the hurdle necessary to receive a bonus also incentivizes an agent to help his peers. In our setting this hybrid scheme can only be optimal if signals from agents' tasks are negatively correlated. Otherwise pure team incentives are optimal.

Our analysis has been confined to settings where all information signals are non-verifiable. In reality there is often a mix of verifiable and non-verifiable signals, which then allows the parties to use formal, externally enforced contracts in addition to self-enforced relational contracts. A general framework for analyzing such contracting has been developed by Watson et al (2020), and an important insight from that framework is that optimal contracts are no longer generally stationary. This makes the analysis of multitasking agents with noisy signals quite challenging, and certainly outside the scope of this paper, but we hope to return to this in future work.[9]

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APPENDIX

Proof of Lemma 1. Letting μ^i be a (row) vector of multipliers for agent i's IC constraint for action a^i (represented by its FOC), and $\lambda(x)$ a multiplier for the EC constraint, the Lagrangean for the problem is

$$L = v(a) - \Sigma_{i=1}^n c(a^i) + \Sigma_{i=1}^n \mu^i (\int \beta_i(x) \nabla_{a^i} f(x,a) - \nabla c(a^i))$$
$$+ \int \lambda(x) (\tfrac{\delta}{1-\delta}(v(a) - \Sigma_{i=1}^n c(a^i)) - \Sigma_{i=1}^n \beta_i(x))$$

Optimal bonuses entail, for all x:

$$\tfrac{\partial L}{\partial \beta_i(x)} = \mu^i \nabla_{a^i} f(x,a) - \lambda(x) \leq 0, \ \ \beta_i(x) \geq 0,$$

with complementary slackness. Moreover, given that the first-best allocation cannot be attained, the EC constraint will bind and $\lambda(x) > 0$. Hence: if bonuses for two agents i and j are positive, then

$$\mu^i \nabla_{a^i} f(x,a) = \lambda(x) = \mu^j \nabla_{a^j} f(x,a)$$

This outcome (for the $x's$) may or may not be a positive probability event. If it is, then in this event of positive probability, both agents are awarded a bonus. Otherwise at most one agent will be awarded a bonus. The bonus will then optimally be assigned to that agent for which the expression $\mu^i \nabla_{a^i} f(x,a)$ is (a) positive and (b) largest among the agents.

Proof of Proposition 1. Consider agent 1, and partition the entire signal vector x such that we can write $x = (x^1, x^2)$, where *in this proof only* $x^2 \equiv x^{-1}$ consists of all signals in x not contained in x^1. (We use this notation here to avoid confusion with notation for inverses.) Aso define

$$e^1 = Q_1 a^1 = E(x^1 | \, a^1) \ \text{ and } \ e^2 = E(x^2 | \, a),$$

where now e^2 does not depend on agent 1's actions a^1. Observe that the normal distribution implies that we can write

$$\ln f(x,a) = -\frac{1}{2}((x^1-e^1)' M_{11}(x^1-e^1) + 2(x^1-e^1)' M_{12}(x^2-e^2) + (x^2-e^2)' M_{22}(x^2-e^2)) + k,$$

where the matrices M_{ij} are partitions of the inverse covariance matrix $M \equiv \Sigma^{-1}$ such that we can write the quadratic form $(x-e)'\Sigma^{-1}(x-e)$ as just displayed. This implies, since $e^1 = Q_1 a^1$ and e^2 does not depend on a^1 that we have

$$\nabla_{a^1} \ln f(x,a) = Q_1'(M_{11}(x^1 - e^1) + M_{12}(x^2 - e^2)).$$

The partition matrices M_{ij} can be expressed in terms of the corresponding partitions Σ_{ij} of the covariance matrix Σ, and we have in particular $M_{12} = -M_{11}\Sigma_{12}\Sigma_{22}^{-1}$. (This follows from $M\Sigma = I$, which implies $M_{11}\Sigma_{12} + M_{12}\Sigma_{22} = 0$.) The formula for M_{12} implies that we now have

$$\mu^1 \nabla_{a^1} \ln f(x,a) = \mu^1 Q_1' M_{11}(x^1 - e^1 - \Sigma_{12}\Sigma_{22}^{-1}(x^2 - e^2)).$$

Moreover, it is a well known fact that conditional expectations for the multinormal distribution have the following form

$$E(x^1 \big| \, x^2) = e^1 + \Sigma_{12}\Sigma_{22}^{-1}(x^2 - e^2),$$

where $e^i = Ex^i$ are the unconditional expecations.Defining vector $\theta^1 = \mu^1 Q_1' M_{11}$, it then follows that the formula for the index $\mu^1 \nabla_{a^1} \ln f(x,a)$ can be written in the form given in the proposition. The same applies for all agents $j \neq 1$..

Proof of Lemma 2. By assumption we consider a case where the coefficients τ_{ij} imply $corr(y_1, y_2)^2 < 1$; ensuring that $\Pr(y_1 = y_2) = 0$. Note that by symmetry of the coefficients in the expressions defining y_1, y_2, and due to identical variances for the $x-$ signals, we have $var(y_1) = var(y_2)$.

Fix equilibrium symmetric efforts e, h, and let $g(y, e', h', \tau)$ be the (joint) distribution for vector y, when agents exert efforts e', h'. This distribution is multinormal with expectations

$$Ey_1 = \tau_{11}(e_1' - e_1) + \tau_{12}(h_1' - h_1) + \tau_{21}(e_2' - e_2) + \tau_{22}(h_2' - h_2)$$

$$Ey_2 = \tau_{11}(e_2' - e_2) + \tau_{12}(h_2' - h_2) + \tau_{21}(e_1' - e_1) + \tau_{22}(h_1' - h_1),$$

and covariance matrix, say $\tilde{\Sigma} = [\tilde{n}_{ij}]$, where the elements depend on τ (and on variance and covariance elements for x). In terms of y the FOCs for agent 1's choice of efforts can be written

$$c_{e_1}(e_1, h_1) = \int \beta_1(y) g_{e_1}(y, e, h, \tau) dy = b \int_{\beta_1(y) > 0} g_{e_1}(y, e, h, \tau) dy$$

$$c_{h_1}(e_1,h_1)=\int \beta_1(y)g_{h_1}(x,e,h,\tau)dy = b\int_{\beta_1(y)>0} g_{h_1}(x,e,h,\tau)dy$$

where b is the magnitude of the bonus, and

$$g_{e_1}(y,e,h,\tau)=\tfrac{\partial}{\partial e_1'}\, g(y,e',h,\tau)|_{e'=e},\;\; g_{h_1}(y,e,h,\tau)=\tfrac{\partial}{\partial h_1'}\, g(y,e,h',\tau)|_{h'=h}$$

Since the normal density is of the form $g(y,e',h',\tau)=K\exp(-\frac{1}{2}\left[(y-Ey)^T\tilde{\Sigma}^{-1}(y-Ey)\right]),$ and Ey depends linearly on e' and h', we have

$$\tfrac{\partial}{\partial e_1'}g(y,e',h,\tau)=-\tfrac{\partial}{\partial y_1}g(y,e',h,\tau)\tfrac{\partial Ey_1}{\partial e_1'}-\tfrac{\partial}{\partial y_2}g(y,e',h,\tau)\tfrac{\partial Ey_2}{\partial e_1'}$$

$$\tfrac{\partial}{\partial h_1'}g(y,e,h',\tau)=-\tfrac{\partial}{\partial y_1}g(y,e',h,\tau)\tfrac{\partial Ey_1}{\partial h_1'}-\tfrac{\partial}{\partial y_2}g(y,e',h,\tau)\tfrac{\partial Ey_2}{\partial h_1'}$$

Since here $\frac{\partial Ey_1}{\partial e_1'}=\tau_{11}$, $\frac{\partial Ey_2}{\partial e_1'}=\tau_{21}$, $\frac{\partial Ey_1}{\partial h_1'}=\tau_{12}$, and $\frac{\partial Ey_2}{\partial h_1'}=\tau_{22}$ we obtain

$$\int_{\beta_1(y)>0}g_{e_1}(y,e,h,\tau)dy=-\tau_{11}\int_{\beta_1(y)>0}g_{y_1}(y,e,h,\tau)dy-\tau_{21}\int_{\beta_1(y)>0}g_{y_2}(y,e,h,\tau)dy$$

$$\int_{\beta_1(y)>0}g_{h_1}(y,e,h,\tau)dy=-\tau_{12}\int_{\beta_1(y)>0}g_{y_1}(y,e,h,\tau)dy-\tau_{22}\int_{\beta_1(y)>0}g_{y_2}(y,e,h,\tau)dy$$

We verify below the following claim

Claim.

$$\int_{\beta_1(y)>0} g_{y_1}(y,e,h,\tau)dy=-\frac{\phi_0}{2\tilde{s}}(1+\frac{1}{\sqrt{2(1-\tilde{\rho})}}) \tag{18}$$

$$\int_{\beta_1(y)>0} g_{y_2}(y,e,h,\tau)dy=\frac{\phi_0}{2\tilde{s}}\frac{1}{\sqrt{2(1-\tilde{\rho})}} \tag{19}$$

where $\tilde{s}=SD(y_i)$ and $\tilde{\rho}=corr(y_1,y_2)$.

Given this claim, the FOCs can thus be written as

$$c_{e_1}(e_1,h_1)=b\tau_{11}(\tfrac{\phi_0}{2\tilde{s}}+\tfrac{\phi_0}{2\tilde{s}}\tfrac{1}{\sqrt{2(1-\tilde{\rho})}})-b\tau_{21}\tfrac{\phi_0}{2\tilde{s}}\tfrac{1}{\sqrt{2(1-\tilde{\rho})}}$$

$$c_{h_1}(e_1,h_1)=b\tau_{12}(\tfrac{\phi_0}{2\tilde{s}}+\tfrac{\phi_0}{2\tilde{s}}\tfrac{1}{\sqrt{2(1-\tilde{\rho})}})-b\tau_{22}\tfrac{\phi_0}{2\tilde{s}}\tfrac{1}{\sqrt{2(1-\tilde{\rho})}}$$

This verifies the formulas given in the lemma.

It remains to verify the claim. To this end, note first that for $(e',h')=(e,h)$ we have $Ey=0$ and hence the density $g(.)$ doesn't depend on e,h. Simplifying notation we therefore now write $g(y,\tau)$ for this density; thus

$$g(y,\tau)=K\exp(-\tfrac{1}{2}\left[y'\tilde{\Sigma}^{-1}y\right]=K\exp(-\tfrac{1}{2}\left[\tilde{m}_{11}y_1^2+\tilde{m}_{22}y_2^2+2\tilde{m}_{12}y_1y_2\right]),$$

where the elements $\tilde{m}_{ij}$ depend on τ.

Since $\beta_1(y) > 0$ iff $y_1 > y_2$ and $y_1 > 0$, we have now

$$\begin{aligned}\int_{\beta_1(y)>0} g_{y_1}(y,\tau)dy &= (\int_{-\infty}^{0} dy_2 \int_0^\infty dy_1 + \int_0^\infty dy_2 \int_{y_2}^\infty dy_1) g_{y_1}(y,\tau)\\ &= \int_{-\infty}^{0} dy_2(-g(0,y_2,\tau)) + \int_0^\infty dy_2(-g(y_2,y_2,\tau))\\ &= -\int_{-\infty}^{0} dy_2 K \exp(-\tfrac{1}{2}\left[\tilde{m}_{22}y_2^2\right]) - \int_0^\infty dy_2 K \exp(-\tfrac{1}{2}\left[\tilde{m}_{11}+\tilde{m}_{22}+2\tilde{m}_{12}\right]y_2^2)\\ &= -\frac{K\sqrt{2\pi}}{2\sqrt{\tilde{m}_{22}}} - \frac{K\sqrt{2\pi}}{2\sqrt{\tilde{m}_{11}+\tilde{m}_{22}+2\tilde{m}_{12}}}\end{aligned}$$

The last line follows since (for $k > 0$)

$$\int_{-\infty}^{0} dy_2 \exp(-\tfrac{1}{2}ky_2^2) = \int_0^\infty dy_2 \exp(-\tfrac{1}{2}ky_2^2) = \frac{1}{\sqrt{k}}\int_0^\infty dz_2 \exp(-\tfrac{z^2}{2}) = \frac{\sqrt{2\pi}}{2\sqrt{k}}$$

Similarly we have

$$\begin{aligned}\int_{\beta_1(y)>0} g_{y_2}(y,\tau)dy &= \int_0^\infty dy_1 \int_{-\infty}^{y_1} dy_2 g_{y_2}(y,\tau) = \int_0^\infty dy_1 g(y_1,y_1,\tau)\\ &= \int_0^\infty dy_1 K \exp(-\tfrac{1}{2}\left[\tilde{m}_{11}+\tilde{m}_{22}+2\tilde{m}_{12}\right]y_1^2)\\ &= \frac{K\sqrt{2\pi}}{2\sqrt{\tilde{m}_{11}+\tilde{m}_{22}+2\tilde{m}_{12}}}\end{aligned}$$

From the properties of the normal distribution we have

$$\tilde{m}_{11} = \frac{1}{\tilde{s}_1^2(1-\tilde{\rho}^2)},\ \tilde{m}_{22} = \frac{1}{\tilde{s}_2^2(1-\tilde{\rho}^2)},\ \ \tilde{m}_{12} = -\frac{\tilde{\rho}}{\tilde{s}_1\tilde{s}_2(1-\tilde{\rho}^2)},\ \ K = \frac{1}{2\pi\tilde{s}_1\tilde{s}_2\sqrt{1-\tilde{\rho}^2}}$$

where $\tilde{s}_i^2 = var y_i, \tilde{\rho} = corr(y_1, y_2)$. As noted above we have (by symmetry) $\tilde{s}_1^2 = \tilde{s}_2^2 = \tilde{s}^2$, so

$$\tilde{m}_{11} = \tilde{m}_{22} = \frac{1}{\tilde{s}^2(1-\tilde{\rho}^2)}, \qquad \tilde{m}_{12} = -\tilde{m}_{11}\tilde{\rho}, \qquad K = \frac{1}{2\pi\tilde{s}}\sqrt{\tilde{m}_{11}}$$

Hence

$$\frac{K\sqrt{2\pi}}{2\sqrt{\tilde{m}_{22}}} = \frac{1}{2\pi\tilde{s}}\sqrt{\tilde{m}_{11}}\frac{\sqrt{2\pi}}{2\sqrt{\tilde{m}_{22}}} = \frac{1}{2\sqrt{2\pi}\tilde{s}} = \frac{\phi_0}{2\tilde{s}}$$

$$\frac{K\sqrt{2\pi}}{2\sqrt{\tilde{m}_{11}+\tilde{m}_{22}+2\tilde{m}_{12}}} = \frac{1}{2\pi\tilde{s}}\sqrt{\tilde{m}_{11}}\frac{\sqrt{2\pi}}{2\sqrt{2\tilde{m}_{11}-2\tilde{m}_{11}\tilde{\rho}}} = \frac{1}{\sqrt{2\pi}\tilde{s}}\frac{1}{2\sqrt{2(1-\tilde{\rho})}} = \frac{\phi_0}{2\tilde{s}}\frac{1}{\sqrt{2(1-\tilde{\rho})}}$$

This verifies (18) - (19), hence verifies the *Claim*, and thus completes the proof.

Verification of equations (8)-(9). We verify here equations (8)-(9), and do so by first deriving the corresponding equations for the more general case of a correlation ρ_a not necessarily zero. Observe first that positive definiteness of the correlation matrix requires $1-\rho^2-\rho_a^2>0$ and $(1-\rho^2)^2-(2\rho^2-\rho_a^2+2)\rho_a^2>0$. Now, for $\tau_{11}=1$, and thus from (6) $\tau_{21}=-\rho$ and $\tau_{22}=-\rho\tau_{12}$ we find, by straightforward calculations:

$$cov(y_1,y_2)/s^2=((1+\tau_{12}^2)(\rho^2-1)-4\tau_{12}\rho_a)\rho$$

$$var(y_i)/s^2=(1+\tau_{12}^2)(1-\rho^2)+2\tau_{12}(1+\rho^2)\rho_a$$

Hence for $\tilde{\rho}=corr(y_1,y_2)$ and $\tilde{s}^2=var(y_i)$ we get

$$(1-\tilde{\rho})\tilde{s}^2/s^2=((1+\tau_{12}^2)(1-\rho)+2\tau_{12}\rho_a)(1+\rho)^2$$

Inserting this (and $\tau_{11}=1$, $\tau_{21}=-\rho$, $\tau_{22}=-\rho\tau_{12}$) in the first formula in Lemma 2 yields

$$(\tau_{11}+\tfrac{\tau_{11}-\tau_{21}}{\sqrt{2(1-\tilde{\rho})}})\tfrac{1}{2\tilde{s}}=(1+\tfrac{1+\rho}{\sqrt{2((1+\tau_{12}^2)(1-\rho)+2\tau_{12}\rho_a)(1+\rho)}}\tfrac{\tilde{s}}{s})\tfrac{1}{2\tilde{s}},$$

and hence we have

$$c_{e_i}(e_i,h_i)_{i=1}=(\frac{1}{\sqrt{(1+\tau_{12}^2)(1-\rho^2)+2\tau_{12}(1+\rho^2)\rho_a}}+\frac{1}{\sqrt{2((1+\tau_{12}^2)(1-\rho)+2\tau_{12}\rho_a)}})\frac{\phi_0 b}{2s} \tag{20}$$

This reduces to the formula in (8) when $\rho_a=0$.

We note that the expression is decreasing in ρ_a, which implies that the strength of incentives are decreasing in this parameter.

From the second formula in Lemma 2 we now have

$$(\tau_{12}+\tfrac{\tau_{12}-\tau_{22}}{\sqrt{2(1-\tilde{\rho})}})\tfrac{1}{2\tilde{s}}=(\tau_{12}+\tfrac{\tau_{12}(1+\rho)}{\sqrt{2(1-\tilde{\rho})}})\tfrac{1}{2\tilde{s}}=(1+\tfrac{(1+\rho)}{\sqrt{2(1-\tilde{\rho})}})\tfrac{1}{2\tilde{s}}\tau_{12},$$

and hence this expression is equal to the expression in (20) multiplied by τ_{12}. This verifies formula (9).

Proof. of Proposition 3. Let $R(\tau_{12},\rho_a,\rho)$ be the function defined by the expression in (20), so that the FOCs for equilibrium (symmetric) efforts are given by

$$c_{e_i}(e_i,h_i)=R(\tau_{12},\rho_a,\rho)\tfrac{\phi_0}{2s}b \qquad \textit{and} \qquad c_{h_i}(e_i,h_i)=\tau_{12}R(\tau_{12},\rho_a,\rho)\tfrac{\phi_0}{2s}b$$

These conditions define, for given b, efforts as functions of τ_{12}, and the optimal τ_{12} is then found by choosing it to maximize the total surplus

$v(e,h) - \Sigma_i c(e_i,h_i)$. Letting $S^T(b)$ be the resulting surplus, the equilibrium b is then found as the largest solution to $b = \frac{\delta}{1-\delta}S^T(b)$.

We will show that $\tau_{12} = 0$ can be optimal. Consider a case of quadratic costs $c(e_i,h_i) = e_i^2/2 + h_i^2/2\kappa$ and linear gross value per agent $v \cdot (e_i + h_i)$. To simplify further, consider the case $\rho = 0$. From the expression for $R(\tau_{12},\rho_a,\rho)$ in (20) with $\rho = 0$ we then have FOCs for equilibrium efforts for agent 1 (and symmetrically for agent 2) as follows

$$e_1 = R(\tau_{12},\rho_a,0)\tfrac{\phi_0}{2s}b = (\tfrac{1}{\sqrt{(1+\tau_{12}^2)+2\tau_{12}\rho_a}} + \tfrac{1}{\sqrt{2((1+\tau_{12}^2)+2\tau_{12}\rho_a)}})\tfrac{\phi_0}{2s}b,$$

$$h_1/\kappa = \tau_{12}R(\tau_{12},\rho_a,0)\tfrac{\phi_0}{2s}b$$

The condition for optimal $\tau_{12} > 0$ is, for given b:

$$(v - c_{e_1})\tfrac{\partial e_1}{\partial \tau_{12}} + (v - c_{h_1})\tfrac{\partial h_1}{\partial \tau_{12}} = 0,$$

and $\tau_{12} = 0$ is optimal if the expression on the LHS is negative at $\tau_{12} = 0$. Noting that $c_{h_1} = h_1/\kappa = 0$ when $\tau_{12} = 0$, the condition is then

$$(v - R(0,\rho_a,0)\tfrac{\phi_0}{2s}b)R_{t_{12}}(0,\rho_a,0)\tfrac{\phi_0}{2s}b + (v-0)\kappa R(0,\rho_a,0)\tfrac{\phi_0}{2s}b < 0$$

We have here

$$R(0,\rho_a,0) = 1 + 1/\sqrt{2} \qquad \textit{and} \qquad R_{\tau_{12}}(0,\rho_a,0) = -(1+1/\sqrt{2})\rho_a,$$

where the last equality follows from differentiating the expression for $R(\tau_{12},\rho_a,0)$. The condition for $\tau_{12} = 0$ to be optimal is then

$$-(v - Kb)\rho_a + v\kappa < 0, \quad \text{where } K = R(0,\rho_a,0)\tfrac{\phi_0}{2s} = (1+1/\sqrt{2})\tfrac{\phi_0}{2s},$$

i.e.

$$b < v(1 - \kappa/\rho_a)/K \tag{21}$$

We see that this condition is feasible when $0 < \kappa < \rho_a$

It now remains to verify that the condition can hold for the equilibrium bonus b. This bonus must satisfy $b \leq \frac{\delta}{1-\delta}2S^F$, where S^F is the first-best surplus per agent. the latter is achieved for $e_1 = v$ and $h_1 = \kappa v$ and equals $v^2(1+\kappa)/2$, hence b must satisfy

$$b \leq \tfrac{\delta}{1-\delta}v^2(1+\kappa)$$

Condition (21) will therefore certainly hold if

$$\frac{\delta}{1-\delta}v^2(1+\kappa) < v(1-\kappa/\rho_a)/K \tag{22}$$

Let $S(b)$ be the surplus per agent when $\tau_{12}=0$ and thus $e_1 = R(0,\rho_a,0)\frac{\phi_0}{2s}b \equiv Kb$ and $h_1 = 0$. We then have $S(b) = vKb - (Kb)^2/2$, and the equilibrium bonus is then given by $b = \frac{\delta}{1-\delta}2S(b)$. This equaton has a positive solution iff $1 < \frac{\delta}{1-\delta}2S'(0) = \frac{\delta}{1-\delta}2vK$. This condition and condition (22) will now hold if

$$\frac{1}{2vK} < \frac{\delta}{1-\delta} < \frac{(1-\kappa/\rho_a)}{v(1+\kappa)K}, \tag{23}$$

where $K = (1+1/\sqrt{2})\frac{\phi_0}{2s}$ as defined above. Condition (23) is feasible for κ sufficiently small, more precisely for $\kappa < (1+2/\rho_a)^{-1}$. For parameters satisfying (23) and $\rho = 0$ we have thus shown that $\tau_{12} = 0$ is optimal in the relational contract.

Verification of equations (11)-(12). As in the proof of Lemma 2, fix optimal and symmetric efforts e, h, and consider the distribution for vector y when agents exert efforts e', h'. This distribution is here multinormal with expectations

$$Ey_1 = \tau_1(e_1' + h_2' - e_1 - h_2) + \tau_2(e_2' - e_2 + h_1' - h_1)$$

$$Ey_2 = \tau_1(e_2' + h_1' - e_2 - h_1) + \tau_2(e_1' - e_1 + h_2' - h_2),$$

By comparison with the expressions in the proof of Lemma 2, namely

$$Ey_1 = \tau_{11}(e_1' - e_1) + \tau_{12}(h_1' - h_1) + \tau_{21}(e_2' - e_2) + \tau_{22}(h_2' - h_2)$$

$$Ey_2 = \tau_{11}(e_2' - e_2) + \tau_{12}(h_2' - h_2) + \tau_{21}(e_1' - e_1) + \tau_{22}(h_1' - h_1),$$

we see that the present case is one where

$$\tau_1 = \tau_{11} = \tau_{22}, \qquad \textit{and} \qquad \tau_2 = \tau_{21} = \tau_{12}$$

From the FOCs following formulas (18)-(19) in the proof of Lemma 2 we thus have

$$c_{e_1}(e_1,h_1) = b\tau_{11}\left(\frac{\phi_0}{2\tilde{s}} + \frac{\phi_0}{2\tilde{s}}\frac{1}{\sqrt{2(1-\tilde{\rho})}}\right) - b\tau_{21}\frac{\phi_0}{2\tilde{s}}\frac{1}{\sqrt{2(1-\tilde{\rho})}} = b\frac{\phi_0}{2\tilde{s}}\left(\tau_1 + \frac{\tau_1-\tau_2}{\sqrt{2(1-\tilde{\rho})}}\right)$$

$$c_{h_1}(e_1,h_1) = b\tau_{12}(\frac{\phi_0}{2\tilde{s}} + \frac{\phi_0}{2\tilde{s}}\frac{1}{\sqrt{2(1-\tilde{\rho})}}) - b\tau_{22}\frac{\phi_0}{2\tilde{s}}\frac{1}{\sqrt{2(1-\tilde{\rho})}} = b\frac{\phi_0}{2\tilde{s}}(\tau_2 + \frac{\tau_2-\tau_1}{\sqrt{2(1-\tilde{\rho})}})$$

This verifies the formulas (11)-(12) for the case of common tasks.

Verification of equations (16)-(17). From the formulas for $\tilde{s}$ and $\tilde{\rho}$ in (15) and (14) we have

$$1-\tilde{\rho} = (1-\tau)^2\frac{1-\rho}{\tau^2+2\rho\tau+1} = (1-\tau)^2(1-\rho)s/\tilde{s}$$

So, from (11) with $\tau_1 = 1$ and $\tau_2 = \tau$ we then get

$$(\tau_1 + \frac{\tau_1-\tau_2}{\sqrt{2(1-\tilde{\rho})}})\frac{1}{\tilde{s}} = (1 + \frac{1-\tau}{\sqrt{2(1-\rho)(1-\tau)^2}}\frac{\tilde{s}}{s})\frac{1}{\tilde{s}} = \frac{1}{\tilde{s}} + \frac{1-\tau}{|1-\tau|}\frac{1-\tau}{\sqrt{2(1-\rho)}}\frac{1}{s}$$

Substituting for $\tilde{s} = s\sqrt{\tau^2+2\rho\tau+1}$ we see that this verifies formula (16). The formula (17) is similarly derived from (12).

Proof of Proposition 4. It remains to verify the assertion that a hybrid scheme may dominate a pure team scheme. Consider a case of quadratic costs $c(e_i,h_i) = e_i^2/2 + h_i^2/2\kappa$ and linear gross value per agent $v\cdot(e_i+h_i)$.

Observe that for $\tau < 1$ and $\tau \to 1^-$, the incentives given in (16) - (17) for the hybrid scheme converge to the following limits

$$c_{e_1} = (\frac{1}{\sqrt{2+2\rho}} + \frac{1}{\sqrt{2(1-\rho)}})\frac{\phi_0}{2s}b = (1+\sqrt{\frac{1+\rho}{1-\rho}})\frac{\phi_0}{2s\sqrt{2+2\rho}}b,$$

$$c_{h_1} = (\frac{1}{\sqrt{2+2\rho}} - \frac{1}{\sqrt{2(1-\rho)}})\frac{\phi_0}{2s}b = (1-\sqrt{\frac{1+\rho}{1-\rho}})\frac{\phi_0}{2s\sqrt{2+2\rho}}b$$

These are both positive for $\rho < 0$. Define $\chi = \sqrt{\frac{1+\rho}{1-\rho}}$ and $\xi = \frac{\phi_0}{2s\sqrt{2+2\rho}}$. For quadratic costs we have then

$$e_1 = (1+\chi)\xi b, \qquad h_1/\kappa = (1-\chi)\xi b$$

If the associated surplus for these efforts is $S(b,\chi)$, the hybrid scheme can (for $\tau < 1$) do no worse than $S(b,\chi)-\varepsilon$, for arbitrary $\varepsilon > 0$.

Now observe that the incentives and associated efforts for a pure team scheme are, according to the formula in Proposition 4, obtained by setting $\chi = 0$. We can therefore compare the two schemes by comparing $\chi > 0$ and $\chi = 0$.

For efforts $e_1 = (1+\chi)\xi b$ and $h_1 = \kappa(1-\chi)\xi b$ the social surplus per agent is

$$S(b,\chi) = b^2(\tfrac{v}{b}((1+\chi)\xi + \kappa(1-\chi)\xi) - \tfrac{1}{2}(1+\chi)^2\xi^2 - \tfrac{\kappa}{2}(1-\chi)^2\xi^2)$$

With EC binding, the equilibrium bonus $b = b_\chi$ satisfies $b = \frac{\delta}{1-\delta}2S(b,\chi)$. For this equation to have a positive solution we must have $\frac{\delta}{1-\delta}2\frac{\partial}{\partial b}S(0,\chi) > 1$. Suppose now that $\kappa < 1$ and moreover

$$v\xi(1+\kappa) = \tfrac{\partial}{\partial b}S(0,0) \le \tfrac{1-\delta}{\delta}\tfrac{1}{2} < \tfrac{\partial}{\partial b}S(0,\chi) = v\xi((1+\kappa) + (1-\kappa)\chi))$$

Then by the first inequality there is no positive bonus that can be sustained under the pure team incentive scheme ($\chi = 0$), while by the second inequality such a positive bonus exists under the hybrid scheme. The hybrid scheme then generates a higher surplus than does the pure team scheme.

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