A Distributional Robust Analysis of Buyback and Cap-and-Trade Policies

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Abstract

This study delves into a dynamic Stackelberg game comprised of a manufacturer and a retailer, operating in an environment with fluctuating demand and price-dependent consumer behavior. The multi-period optimization challenges the manufacturer to strategically set wholesale and buyback prices, while the retailer determines the retail price and order quantities within a single contract. In this dynamic framework, the players operate under the constraints of a cap-and-trade policy, with limited knowledge of demand distributions, characterized only by mean and standard deviation parameters. To address this inherent uncertainty, we employ a distributionally robust approach. Additionally, we explore the enduring effects of historical decisions on present-day demand, reflecting a memory-like market behavior. Through numerical examples, we illuminate the influence of buyback contracts and cap-and-trade policies on decision-making processes within this setting.

JEL classification: C61, C62, C63, C72, C73, D81, Q52.

Keywords: Cap-and-Trade Policy, Multi-Period Stackelberg Game, Distributional-Robust Demand, Single Contract, Buyback Contract, Sustainability.
1 Introduction

1.1 Cap-and-Trad Policy

Rapid industrialization and urbanization have accelerated environmental pollution and incurred purification costs in many countries. Indeed, economic growth has come at the expense of increased environmental pollution. China, one of the largest polluters, has performed an assessment that reveals the damaging effects on various sectors such as agriculture, global temperature, and life expectancy (Du et al., 2016). The primary cause of global warming has been carbon dioxide and this prompts governments to consider the urgent need to reduce this pollution. In recent years, authorities have concentrated on measuring pollution levels and investigating potential mechanisms for carbon emissions reduction and addressing associated risks (Wang et al., 2021; Taleizadeh et al., 2021; Cao et al., 2017; Xu et al., 2017; Du et al., 2015, 2013).

While efforts to combat global climate change have been initiated worldwide, environmental policies and sustainability initiatives have become a competitive advantage for manufacturers. Furthermore, customers are increasingly aware of the importance of low-carbon products and are willing to pay more for products that have a minimal environmental impact (Wang et al., 2021; Tong et al., 2019). This growing awareness has led supply chain stakeholders to incorporate sustainable development and low-carbon environmental policies into their operations and decision-making processes to align with market changes (Wang et al., 2021). Companies such as HP, Dell, and Acer are actively working to reduce e-waste, energy consumption, and carbon emissions, while others like Ford and Volkswagen are exploring the production of vehicles powered by alternative energy sources. For example, Siemens adopted cleaner technologies in 2016, leading to a reduction of 521 million tons of carbon emissions, which accounted for over 60% of Germany’s annual (Tong et al., 2019). Retailers such as Walmart and Tesco have also engaged in green activities and implemented carbon footprint labeling for their products (Mondal and Giri, 2022).

The first international agreement addressing greenhouse gas emissions and the reduction of carbon emissions footprint was the UNFCCC (United Nations Framework Convention on Climate Change) It aimed to establish official obligations and support measures to reduce emissions impact on the environment (Du, 2016).
et al., 2015, 2013). In the UK, fiscal policies such as the Climate Change Agreement (CCA), Climate Change Tax (CCT), and Carbon Price Support (CPS) have been implemented to control greenhouse gas emissions (Xu et al., 2018). Also, the European Union’s Emissions Trading System (EU ETS) lowered the emissions cap by 15% in 2015 since its inception in 2005 (Mondal and Giri, 2022).

The Kyoto Protocol was introduced to guide decision-makers in regulating companies’ activities related to carbon pollution. The cap-and-trade (C&T) policy is the main framework mechanism of this protocol and is considered one of the most effective policies (Mondal and Giri, 2022a; Du et al., 2016). Although, among all carbon reduction policies, the common carbon policies have usually been introduced as C&T and carbon tax (Feng et al., 2021), the carbon tax does not limit the emissions by an emissions cap. Under the C&T regulation, manufacturers are allocated a maximum allowance of free emission credits. If this allowance capacity is insufficient to achieve optimal results, manufacturers can purchase emission credits or implement greener production methods to reduce their carbon emissions. They can also sell the surplus quotas to generate profit (Taleizadeh et al., 2021; Li et al., 2021). Accordingly, companies that actively reduce their emissions are rewarded economically and environmentally (Mondal and Giri, 2022b). According to a report by the European Commission in 2013, the EU Emissions Trading Scheme covered 31 countries and limited nearly 50% of carbon emissions. In this system, the government establishes the necessary policies for emission trading quotas, while companies are responsible for regulating their allocated quotas (Cao et al., 2017; Xu et al., 2017; Du et al., 2015). For example, in 2013, Foxconn invested less than 50 million RMB in energy-saving retrofits but made a profit of 10 million RMB (60 million RMB in revenue) by selling surplus carbon credits (Tong et al., 2019).

It is argued that the C&T policy offers more profit potential compared to other environmental policies and it has been widely adopted in recent years in countries such as Norway, Netherlands, Sweden, Denmark, and China (Chen et al., 2020). A well-designed C&T system can improve the efficiency of emissions reduction goals when the regulator sets appropriate emissions cap and trading price (Du et al., 2015; Chen et al., 2020).

To evaluate the effectiveness of the C&T policy, Mondal and Giri examined a supply channel with green activities and price-dependent deterministic demand. They stud-
ied four models: centralized, decentralized, bargaining revenue sharing, and retailer-led revenue sharing under the C&T policy. Their findings indicate that a higher carbon emissions allowance price motivates manufacturers to improve their green operations, leading to a reduction in carbon dioxide emissions (Mondal and Giri, 2022a). Feng et al. investigated cooperation in a supply chain using a joint replenishment game, where two or more dependent or independent firms cooperate horizontally under the C&T system to identify the optimal joint ordering strategy. They found that the retailer with the highest altruistic parameter value benefits from the surplus of carbon emissions allowance (Feng et al., 2021).

Zhao et al. propounded a remanufacturing problem under the C&T policy and proposed three production decision models: single-product remanufacturing with fixed carbon emissions, extended single-product remanufacturing with variable carbon emissions, and multi-product remanufacturing models (Zhao et al., 2021). Wang et al. combined the C&T policy with the customers’ low-carbon preferences with a differential game model, considering three different scenarios: non-cooperation (coop) scenario where the manufacturer is the leader of the two-echelon supply chain, the supplier’s emissions reduction efforts are supported by the manufacturer and a two-way coop contract when both channel members support each other’s emissions reduction efforts (Wang et al., 2021).

Taleizadeh et al. examined a supply chain problem involving a retailer and a manufacturer, under C&T, where they could either compete or cooperate in pricing and production decisions. Their model suggests that cooperation yields greater benefits, considering environmental concerns (Taleizadeh et al., 2021). Li et al. investigated two types of subsidy policies based on fixed green technology investment cost (FC subsidy) and the amount of emissions reduction (ER subsidy) under the C&T mechanism, using Stackelberg game models. The results indicated that government subsidy policies alone cannot guarantee green technology investment and total carbon emissions reduction (Li et al., 2021).

Chen et al. compared the effects of carbon emissions tax policy with a C&T system using a static optimal model. They found that the C&T system is more efficient for emission reduction than the carbon tax. However, the impact of the C&T system on a manufacturer’s profit is uncertain and dependent on the carbon cap. Therefore, selecting
an appropriate emissions cap and carbon trading price is crucial for ensuring the efficiency of this policy (Chen et al., 2020). Even though former research by Wittneben implies the opposite argument stating that a carbon emissions tax might be a faster and economically more beneficial approach to reducing greenhouse gas emissions. They believe that a carbon emissions tax can generate more income for the government to invest in green projects, while income from C&T policy is more uncertain. Additionally, implementing a new tax is less complex than the process required for implementing a C&T system (Wittneben, 2009).

Wang and Han proposed a dual mechanism of C&T and subsidies/penalties for a (re)manufacturing problem with stochastic return and random yield rates, considering four different distribution functions (Wang and Han, 2020). Similarly, Mondal and Giri studied competition and cooperation among retailers and a manufacturer under government invention and the C&T policy. They developed a centralized policy and three manufacturer-led decentralized policies viz. Collusion, Cournot (Nash), and Stackelberg (Mondal and Giri, 2022b). Aghaie et al. concentrated on the application of the C&T policy in groundwater extraction management with four different monitoring scenarios. They simulated this model using agent-based modeling addressing interactions between social, institutional, economic, and groundwater systems (Aghaie et al., 2020).

Kushwaha, et al. proposed a mixed-integer linear programming model for a remanufacturing system. They determined the optimal combination of channels for collecting used products from different regions in a finite multi-period setting under C&T regulation (Kushwaha et al., 2020). Tong, et al. employed the evolutionary game and the C&T policy considering customer preference for low-carbon products, to examine the behavior of a powerful retailer in a retailer-led supply chain. They used system dynamics to simulate and analyze dynamic and transient behaviors. Their results indicate that the emissions cap, market price of carbon credits, and consumers’ preferences for low-carbon products are key factors affecting retailer and manufacturer behaviors (Tong et al., 2019).

Li et al. applied a Stackelberg game between the government and the manufacturer. They indicated that the manufacturer is more incentivized to upgrade its purification technology in a high-carbon preference market compared to a low-carbon preference market (Li et al., 2018). Turki et al. investigated a (re)manufacturing plan considering
the differences between new and remanufactured items, random machine failures, the 
C&T policy, and distinct random customer demands for both types of products. Their 
results revealed that a lower carbon cap and/or a high price of carbon trading, impel 
the producer to collect and remanufacture used items and limit carbon emissions (Turki 
et al., 2018). Xia et al. incorporated reciprocal preferences and consumers’ low-carbon 
awareness (CLA) into a dynamic supply chain where the manufacturer plays a Stackel-
berg game with a retailer. Their results demonstrate that the optimal wholesale price 
increases with CLA, while the optimal emissions level decreases with CLA (Xia et al., 
2018). Xu et al. studied the decision-making and coordination of a centralized and 
decentralized supply chain under C&T regulation and the Stackelberg game. They in-
vestigated pricing and carbon emissions abatement decisions, considering the preferences 
for low-carbon products (Xu et al. 2018). Cao et al. investigated the impacts of the 
C&T policy and low-carbon subsidy policy on the production and level of carbon emis-
sions reduction of a manufacturer under the Stackelberg game. Their findings indicated 
that the level of carbon emissions reduction is positively related to the carbon trading 
price. They also discussed that a low-carbon subsidy policy is more beneficial for society 
when the environmental damage coefficient is below a certain threshold; otherwise, the 
C&T policy is preferred (Cao et al. 2017).

Ji et al. studied three decision models: one without C&T regulation, one based on 
grandfathering mechanism and C&T regulation, and one with C&T regulation based 
on benchmarking mechanism. They concluded that the benchmark model, compared 
to grandfathering, can more effectively incentivize manufacturers to produce low-carbon 
products and motivate retailers to promote low-carbon products (Ji et al. 2017).

Xu et al. addressed the coordination problem of a make-to-order (MTO) supply 
chain, which includes a manufacturer and a retailer, with wholesale price and cost-sharing 
contracts under C&T policy. Their findings indicated that the manufacturer and retailer 
optimal profits decrease (increase) by buying (selling) prices of emissions allowance (Xu 
et al. 2017). Du, et al. assessed the trade-off between reducing and incrementing carbon 
emissions while considering economic considerations in a single period. They studied the 
factors that could impact the optimal production strategy and profit where customers 
prefer low-carbon products. Their analysis assumed equal buy and sell prices in an 
oligopoly market (Du et al. 2016).
Du, et al. conducted studies in 2013 and 2015 to investigate the impact of C&T emissions regulation on a single-period supply chain problem. In their models, the channel follows a Stackelberg game between an emission permit supplier and an emission-dependent manufacturer. The supplier and manufacturer made decisions regarding permit pricing and production quantity, respectively (Du et al., 2013, 2015). Du, et al. considered a supplier and a manufacturer in a Stackelberg game framework, where the emission cap is allocated to the manufacturer by the government. Their findings revealed that optimal production and the manufacturer’s profit had a positive relationship with the emissions cap increment, while the supplier’s profit had a negative relation to the emission cap increment (Du et al., 2013). Du, et al., On the other hand, illustrated that the supplier began the first step considering the high permit price inspired the manufacturer to reduce the production quantity to satisfy the imposed emissions cap which resulted in the supplier’s profit deduction (Du et al., 2015).

This paper investigates the impact of C&T on players’ decisions and profits. The channel consists of a manufacturer and a retailer in a Stackelberg game, and the manufacturer is the leader. In our market, the demand for a perishable commodity is stochastic and dynamic and a function of historical retail prices. The demand function has a distribution that may change over time. However, it is often improbable to have complete information about the distribution either because comprehensive information is not available, or it is too costly to obtain. A distributional-robust (DR) approach assists in coping with this kind of incomplete information. The expected profit for the retailer is replaced by a lower (weak) bound relative to the obtainable value with complete knowledge of the distribution. In our proposed framework, future demands are influenced by historical price choices. This effect operates as a kind of market memory, and it adds a property to dynamic demand models reflecting a fundamental aspect of many real-world markets. Consequently, there are opportunities for strategic pricing aimed at shaping demand in subsequent periods.

The players sign a single contract covering all associated decisions for all periods. Compared to multi-periodic contracts, a single contract optimization requires monitoring all decision variables and their effect on each period simultaneously. The difference between the value of single and periodic contracts provokes the players to select a single contract over periodic ones (Fakhrabadi and Sandal, 2023a).
If the retailer faces over-ordering, the leftovers might be salvaged or discarded. It means the retailer carries the demand stochasticity and the manufacturer only feels it through the quantity ordered. Even though, after supplying this order, the manufacturer does not observe any risk. Hence, to split the risk of overordering, the manufacturer offers a non-negative buyback value at each period for unsold items. This transfers part of the risk to the manufacturer. Hence, the manufacturer decides the wholesale and buyback prices, and the retailer decides the retail price and the order quantities. In short, the contributions of this paper include:

- addressing the multi-period DR supply chain with a single contract under C&T regulation.
- determining wholesale and buyback prices by the manufacturer, and retail prices and order quantities by the retailer for all periods.
- employing price-dependent and dynamic demands where current demand depends on the price history as well as the current price.
- obtaining optimal buyback values and risk sharing in the presence of strategic pricing opportunities.

1.2 Buyback Contracts

Buyback contract is prevalent in many commodities such as fashion apparel, books, and CDs. The mechanism operates such that the channel members deal in a single contract wherein the manufacturer provides all wholesale and buy-back prices. Contingent upon this information, the retailer decides on all retailer/market prices and order quantities. This may encourage the retailer to order more while sharing the demand uncertainty with the manufacturer (Qin et al., 2021; Xue et al., 2019). Otherwise, with no buyback contract, only the retailer is directly facing the uncertainty of demand, while the manufacturer only senses it through the order quantity (Azad Gholami et al., 2019). The buyback contract shares the risk of demand stochasticity between the upstream (manufacturer) and downstream (retailer) of the channel and improves the efficiency of the channel (Qin et al., 2021).

For a perishable good, at the end of each period, the unsold items are to be salvaged at a lower price, bought back by the manufacturer, or sent to the destruction center at
manufacter cost (at buyback price). When the manufacturer offers a buyback price, the retailer is incentivized to order more, and this may increase the manufacturer’s profit. Inversely, without a buyback contract, the retailer may order less. Finding optimal buyback prices for a multi-periodic problem can be a challenge due to the nestedness caused by the price history-dependent demand (Azad Gholami et al., 2019).

Hou et al. studied coordination between one manufacturer and two suppliers in the presence of demand uncertainty and supply risks. They study a firm with two sources of the same product, a main and a backup, where the former is cheaper but is accompanied by disruption risks. They argued that the buyer benefits from a backup supplier through a buyback policy to deal with the risks (Hou et al., 2010). Wu examined the effect of the buyback contract (as a parameter) on retail price, order quantity, and wholesale price in a vertical integration case (chain optimizing) and a Stackelberg game. Their single-period formulation revealed that buyback contracts can yield a higher profit in both approaches (Wu, 2013). Wei and Tang analyzed the buyback contract as a risk-sharing tool in a single-period Stackelberg game and compared it with the chain maximizing output and found that supply chain profit enhanced while using the buy-back strategy (Wei and Tang, 2013). One manufacturer and two competing retailers in the Xu et al. study illustrated the value of buyback contracts. They created three scenarios as a buyback contract is offered to neither one, one, or both retailers with a price-dependent static demand. They indicated that offering a buyback contract to both retailers benefits all channel members even in high-level competition (Xue et al., 2019). In another attempt to optimize the supply channels with buyback contracts, Azad Gholami et al. considered a multi-periodic channel with delayed information. Their Stackelberg game compromised a manufacturer and a retailer in a multi-periodic setting. They found that too generous a buyback price can decrease the expected profit for the retailer and create a sub-optimal profit for the manufacturer as well (Azad Gholami et al., 2019).

Qin et al. built a supply chain with buyback contracts and fairness concerns under stochastic demand and employed the Bayesian theorem. Their findings indicated that both the retailer’s first order quantity and total order quantity decreased with the wholesale price and increased with the buyback price (Qin et al., 2021). Momeni et al. investigated a buyback coordination mechanism to encourage the channel to participate in operations regeneration to reuse the expired products in other productions. Their
results illustrated that the optimal solution could happen only if the revenue of a reused product in addition to the saving on its disposing cost, was greater than its reproducing cost (Momeni et al., 2022). Gong et al. analyzed inventory management where the demand arrives continuously with a drifted Brownian motion and buyback contract. They found that the supplier usually does not benefit from a low buyback price because the optimal policy is conservative when the buyback price is low. It leads to a lower chance of the products to be expired and hence the rate of profit is not affected by the buyback price (Gong et al., 2022). We embed the buyback contract as a decision variable into the manufacturer optimization problem to share the demand risks and increase fairness. The manufacturer’s decision variables then are wholesale and buyback prices and those of the retailer are the retail price and order volume.

1.3 Demand Structure

Our demand is structured as a dynamic function in a multi-periodic setting. It can be of different forms in each period. The time horizon consists of \( n \) discrete intervals (referred to as periods). Considering an arbitrary period \( k \) when \( k \in \{1, \ldots, n\} \), the general form of demand is given as

\[
D_k(\mathbf{r}_k) = \mu_k(\mathbf{r}_k) + \sigma_k(\mathbf{r}_k)\varepsilon_k
\]

where \( \mathbf{r}_k = (r_1, \ldots, r_n) \), \( \forall i \in \{1, \ldots, n\} \).

The mean \( \mu_k \) and standard deviation \( \sigma_k \) are known functions of retail price history. The stochastic part of the demand \( \varepsilon_k \) is normalized to have a mean and standard deviation of 0 and 1 and they are independent of each other (between periods). This problem can be solved when the distribution of demand is known (Fakhrabadi and Sandal, 2023b).

This paper investigates situations with incomplete demand information because it is either impossible to obtain all the information or it is too costly. The distributional-robust (DR) approach for a multi-periodic price history-dependent problem is introduced in a seminal paper by Fakhrabadi and Sandal, 2023 (Fakhrabadi and Sandal, 2023b). We provide more information regarding DR approach formulation in section 2.
2 Model Formulation

Notation

\[ w = \{w_1, \ldots, w_n\} \] Wholesale price (decision variable)
\[ b = \{b_1, \ldots, b_n\} \] Buyback price (decision variable)
\[ r = \{r_1, \ldots, r_n\} \] Retail price (decision variable)
\[ q = \{q_1, \ldots, q_n\} \] Order quantity (decision variable)
\[ c^m = \{c^m_1, \ldots, c^m_n\} \] Manufacturer cost
\[ c^r = \{c^r_1, \ldots, c^r_n\} \] Retailer cost
\[ \beta = \{\beta_1, \ldots, \beta_n\} \] Discount factor over individual periods
\[ D = \{D_1, \ldots, D_n\} \] Demand \( (D_k = \mu_k(r) + \sigma_k(r)\varepsilon_k) \)
\[ \mu = \{\mu_1, \ldots, \mu_n\} \] Mean of demand
\[ \sigma = \{\sigma_1, \ldots, \sigma_n\} \] Standard deviation of demand
\[ \varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\} \] Stochastic and independent drivers of the demand
\[ s = \{s_1, \ldots, s_n\} \] Salvage price/discard cost
\[ k \in \{1, \ldots, n\} \] Time or period
\[ q^c = \{q^c_1, \ldots, q^c_n\} \] Maximum allowance for production
\[ u = \{u_1, \ldots, u_n\} \] The unit cost of buying allowances for producing extra
\[ v = \{v_1, \ldots, v_n\} \] The unit price selling unused allowances
\[ \pi^m = \{\pi^m_1, \ldots, \pi^m_n\} \] Manufacturer’s profit (present value)
\[ \pi^r = \{\pi^r_1, \ldots, \pi^r_n\} \] Retailer’s profit (present value)

We have adopted the short notation in this paper: For any vectors \(A\) and \(B\), we define \(AB = BA = [A_iB_j]_{i=1}^n\).

To address our proposed model, the algorithm is built for a perishable product in a multi-period Stackelberg game. In this game, the upstream (manufacturer) is the leader and the downstream (retailer) follows him. The manufacturer, first, declares the wholesale and buyback prices, and then the retailer decides on the retail prices and order quantities. The unsold items cannot be restored at the end of each period and sold at the next period. Therefore, for the retailer, any unsold item is discarded at cost \(s\), salvaged at price \(s\), bought back by the manufacturer at price \(b\), or the manufacturer pays cost \(b\) to the retailer to discard/salvage the unsold items at cost/price \(s\). All variables and parameters remain constant within each period but may vary between periods. The players agree on
a single contract where they can observe their decisions and the consequences across all
periods simultaneously and improve their decisions. The nucleus’s objective is to ensure
the attainment of the highest possible value, and a single contract creates a higher value
for the channel compared to a periodic contract (Fakhrabadi and Sandal, 2023).

The C&T policy structures this channel where the manufacturer is constrained with
a maximum allowance of pollution generating, but he is permitted to buy the extra
allowance required or sell the surplus allowance he has not consumed. This trade can
be categorized either as an income (when selling surplus allowance) or as an additional
cost (when buying extra allowance) which may increase or decrease the channel’s profit.
The prices of buying and selling the allowance can be unequal.

For simplicity in exposition, we drop the time index $k$ whenever an equation is held
by just adding subscript $k$ to all quantities involved. Since the channel consists of a
manufacturer and a retailer, the bilevel optimization algorithm maximizes the manufac-
turer’s value subject to the retailer’s value maximization. The algorithm allows only
non-negative values and variables; however, the profit may be negative for a period.
The parameters and functions can vary at each period. The manufacturer’s operation
is constrained to a maximum production allowance, $q^c$, where he can trade it. The
manufacturer’s profit function is

$$\pi^m = (w - c^m)q - b(q - D)^+ - u(q - q^c)^+ + v(q^c - q)^+,$$  \hspace{1cm} (2)

where $u$ is the purchase price and $v$ the selling price of the production allowance. The
manufacturer purchases production allowance when $(q^* - q^c)^+$ is non-zero and sells when
$(q^c - q^*)^+$ is non-zero ($q^*$ denotes the optimal order quantity). In the first case, $u$ is a
unit cost and in the second case, $v$ is a unit income. The manufacturer then expects to
make a profit of

$$E[\pi^m] = (w - c^m - b)q + b\mu - bE(D - q)^+ - u(q - q^c)^+ + v(q^c - q)^+.$$  \hspace{1cm} (3)

When the distribution of the demand is known, Eq. (3) can be simplified by $E(D - q)^+ = \int_\Omega (x - q)f(x)dx$, where $f(x)$ is the probability density function of demand $D$
with compact support on $\Omega$.

With a buyback contract, the demand stochasticity permeates the manufacturer’s
profit in addition to the retailer’s profit. The manufacturer decides on wholesale price and buyback values. Even though a high buyback price may encourage the retailer to order more, a too-generous buyback price is detrimental to the manufacturer’s expected profit.

The inner level optimization occurs with the retailer’s profit function as,

$$\pi_r = r \min(D, q) + (b + s)(q - D)^+ - wq - c'q. \quad (4)$$

The terms in Eq. (4) depict the revenue, unsold items income, the purchase cost of the order, and the retailer’s other costs for units ordered, respectively. The retailer’s expected profit is

$$E[\pi_r] = (r - s - b)\mu - (w + c' - b - s)q - (r - s - b)E(D - q)^+, \quad (5)$$

where $r > b + s$ due to economic feasibility. The key conclusions are summarized in the following propositions.

**Proposition 2.1.** The bi-level optimization in general is (from Eqs. (3) and (5))

$$\max_{(\vec{w}, \vec{b}) \in \mathcal{W}} JD^m \quad s.t. \quad (\vec{r}, \vec{q}) = \arg \max_{(\vec{r}, \vec{q}) \in \mathcal{R}} JD^r,$$

where

$$JD^x = \alpha_1 E[\pi^x_r] + \alpha_2 E[\pi^x_r] + \ldots + \alpha_n E[\pi^x_r] \quad \text{for} \quad x \in \{m, r\}, \quad (6)$$

and $\alpha_k = \beta_1 \cdot \beta_2 \cdot \ldots \cdot \beta_k$.

$\beta_k$ represents the discounting factor for the period $k$, and $m$ and $r$ correspond to the manufacturer and retailer, respectively. $\mathcal{W}$ and $\mathcal{R}$ are constraints on the manufacturer and retailer.

The distributionally robust (DR) bi-level optimization is

$$\max_{(\vec{w}, \vec{b}) \in \mathcal{W}} J^m \quad s.t. \quad (\vec{r}, \vec{q}) = \arg \max_{(\vec{r}, \vec{q}) \in \mathcal{R}} J^r,$$

where

$$J^x = \alpha_1 \Pi^x_r + \alpha_2 \Pi^x_r + \ldots + \alpha_n \Pi^x_r \quad \text{for} \quad x \in \{m, r\}, \quad (7)$$

and $\Pi^m$ and $\Pi^r$, where $r > b + s$, are players’ expected profits’ tight lower bounds for the case with full information;
\[ E(\pi^m(q, w, b)) \geq (w - c^m - b)q + b\mu - \frac{b}{2}\left(\sqrt{\sigma^2 + (q - \mu)^2} - q + \mu\right) - u(q - q^c)^+ + v(q^c - q)^+ \equiv \Pi^m \tag{8} \]

\[ E(\pi^r(q, w, b, \mathbf{r})) \geq (r - s - b)\mu - (w + c^r - b - s)q - \frac{(r - b - s)}{2}\left(\sqrt{\sigma^2 + (q - \mu)^2} - q + \mu\right) \equiv \Pi^r. \tag{9} \]

Hence, \( J^x \leq JD^x \), i.e., both DR players payoffs are a tight lower bound for the case with full information.

**Proof.** See Appendix A. \( \square \)

**Proposition 2.2.** The following holds in a DR framework:

For any feasible decision set \((w_k, \mathbf{r}_k, b_k)\) at period \(k\), the optimal order quantity is

\[ q_k(w_k, \mathbf{r}_k, b_k) = \mu_k(\mathbf{r}_k) + \sigma_k(\mathbf{r}_k)\Lambda_k(w_k, r_k, b_k), \]

\[ \Lambda_k = \frac{2\eta_k - 1}{2\sqrt{\eta_k(1 - \eta_k)}}, \quad \eta_k = \frac{r_k - w_k - c_k}{r_k - s_k - b_k}. \tag{10} \]

**Proof.** See Appendix B \( \square \)

**Proposition 2.3.** The optimal order quantity is increasing in buyback price.

**Proof.** Following from Eq. (10),

\[ \frac{\partial q}{\partial b} = \frac{\sigma\eta}{4(r - s - b)(\eta(1 - \eta))^{3/2}}. \tag{11} \]

\( \square \)

### 3 Numerical Implementation

In this section, we offer illustrative instances of the solution algorithm expounded in Section 2. From Eq. (1), \( D_k(\mathbf{r}_k) = \mu_k(\mathbf{r}_k) + \sigma_k(\mathbf{r}_k)\varepsilon_k \) we exemplify a price history-dependent demand where the retail price of period \(k\) influences periods \(k, k + 1,\) and \(k + 2\), i.e., \( D_k = D_k(r_{k-2}, r_{k-1}, r_k) \). In our numerical illustrations, we choose the following form of the demand.
\[ D_k(r_{k-2}, r_{k-1}, r_k) = \Phi_k(r_{k-2}, r_{k-1})\mu_k(r_k) + \Phi_k(r_{k-2}, r_{k-1})\sigma_k(r_k)\varepsilon_k, \]
\[ \Phi_1 \equiv 1, \quad \Phi_2 = e^{\gamma(R-r_1)}, \quad \Phi_k = e^{\gamma_k(r_{k-2}-r_{k-1})} \quad \text{for} \quad k \in \{3, \ldots, n\}, \]
\[ \hat{\mu}_k(r_k) = 100 - 2r_k, \quad \text{and} \quad \hat{\sigma}_k(r_k) = 0.2\hat{\mu}_k(r_k). \]

The parameters \( \gamma \) and \( R \) represent the strength of a current deviation to the future demand and reference retail price respectively. This choice aims to streamline complexity while enabling a comprehensive exploration of the independent role also the interplay between buyback, \( C&T \), and the effective price history.

A multi-period model sans the price history effect examines a recurring game scenario; to this extent, all periods adhere to the same optimal policy, leaving no opportunity for strategic pricing maneuvers. In contrast, the model incorporating the influence of the historical prices not only steers the channel towards outcomes that mirror reality but also exhibits the potential to enhance the channels’ value. This elevation is facilitated by its ability to stimulate future demand through the strategic reduction of current prices.

The parameters set of \( c^m_k = 10, c^r_k = 2, \beta_k = 0.97, R = 40, \gamma_k = 0.02, s_k = 0, k \in \{1, \ldots, n\} \) and \( n = 12 \) is used in upcoming cases. More information about the parameters and other functions, employed in examples, are provided in the next sections.

### 3.1 The Effect of Buyback Contracts

We initiate our numerical exploration by introducing unconstrained models that encompass both scenarios with and without a buyback contract (the model with buyback is named the base model later in this paper). In this context, we operate under the assumption of an absence of environmental constraints while the participating entities remain engaged in a buyback contract (and non-buyback) that accounts for historical price influences. As outlined in section 2, our approach encompasses dependent bilevel optimization, including manufacturer optimization at the outer level and retailer optimization at the inner level. For this example, the player’s expected profits with a buyback contract from the expressions in Eqs. (8) and (9) are

\[ \Pi^m_k(w_k, b_k, \bar{r}_k) = (w_k - c^m_k)q_k(w_k, b_k, \bar{r}_k) - \]
\[ \frac{b_k}{2} \left( \sqrt{\sigma^2_k(\bar{r}_k) + (q_k(w_k, b_k, \bar{r}_k) - \mu_k(\bar{r}_k))^2} + q_k(w_k, b_k, \bar{r}_k) - \mu_k(\bar{r}_k) \right). \]
\[ \Pi_k^r(w_k, b_k, \bar{r}_k) = (r_k - s_k - b_k)\mu_k(\bar{r}_k) - (w_k + c_k^r - s_k - b_k)q_k(w_k, b_k, \bar{r}_k) - \frac{(r_k - s_k - b_k)}{2} \left( \sqrt{\sigma_k^2(\bar{r}_k) + (q_k(w_k, b_k, \bar{r}_k) - \mu_k(\bar{r}_k))^2} - q_k(w_k, b_k, \bar{r}_k) + \mu_k(\bar{r}_k) \right). \] (14)

The non-buyback results are derived from,

\[ \Pi_k^m(w_k, \bar{r}_k) = (w_k - c_k^m)q_k(w_k, \bar{r}_k), \] (15)

\[ \Pi_k^f(w_k, \bar{r}_k) = (r_k - s_k)\mu_k(\bar{r}_k) - (w_k + c_k^f - s_k)q_k(w_k, \bar{r}_k) - \frac{(r_k - s_k)}{2} \left( \sqrt{\sigma_k^2(\bar{r}_k) + (q_k(w_k, \bar{r}_k) - \mu_k(\bar{r}_k))^2} - q_k(w_k, \bar{r}_k) + \mu_k(\bar{r}_k) \right). \] (16)

Using Eqs. (13) and (14) for the model with buyback contract and Eqs. (15) and (16) for the non-buyback model, and parameter set \{c_k^m, c_k^f, \beta_k, R, \gamma_k, s_k, n\}, the players’ profits are illustrated in Figure 1.

![Figure 1: Optimal profits, the models with and without buyback contracts](image)

In this example, while the manufacturer obtains a higher value through a buyback contract.
contract, the retailer pays the cost of carrying lower risk;

\[ J^m_{\text{Buyback}} = 3105, \quad J^r_{\text{Buyback}} = 1611, \]
\[ J^m_{\text{Non-buyback}} = 2988, \quad J^r_{\text{Non-buyback}} = 1666. \]

The manufacturer observes a 4% gain, while the retailer experiences a 3.3% loss. This outcome becomes evident upon examining the decisions illustrated in Figure 2 denoted as \((r^*, w^*, b^*)\). The manufacturer strategically introduces a non-zero buyback price, which is paired with a higher wholesale price. This approach compensates for the additional incurred risk due to the buyback arrangement promoting the retailer to respond by raising the retail price and the order quantity (Figure 3). This dynamic reveals that the buyback pricing in this scenario stimulates the retailer to ramp up their order volume.

In a model with the buyback contract, the wholesale prices operate within the range of \([26.3, 28.2]\) while the corresponding buyback prices fall within the span of \([16.3, 18.2]\). Initially, this buyback price- equivalent to 64 – 65% of the wholesale price- might appear overly generous or surprising. However, when considering the progression of wholesale price increments compared to the model lacking the buyback contract, it becomes evident that the cost associated with the buyback is offset by an average wholesale price increase of \(\approx 7\%\).
3.2 Buyback and Cap-and-Trade Policy versus only Buyback

Within the framework of the C&T policy, the cost of procuring a production allowance commonly surpasses its sales price. We have assumed that any excess production allowance cannot be rolled over or utilized in subsequent periods. Hence, when the manufacturer encounters an excess allowance situation and determines that the optimal solution falls below the allowed capacity, the prudent course of action is to sell the surplus. Failing to do so would result in the forfeiture of potential revenue.

To illustrate this example, Eqs. (8) and (9) are considered. The parameter configuration \( \{c_m^k, c_r^k, s_k, \beta_k, R, \gamma_k, n\} \) is the same as in the previous section (the base model, only with buyback). The buying and selling prices used for this case are \( u_k = 1.5, v_k = 1 \). The insights drown from this example are embodied in Figure 4, which elucidates the profit trajectories of two distinct models: the model subject to the constraints of the C&T policy and buyback contract and the model only with buyback contract.
Within this scenario, the application of the $C&T$ policy results in a reduction of the players’ values leading to $J_{m-CT}^m = 3096$, $J_{r-CT}^r = 1481$. In contrast, without the influence of the $C&T$ policy, the values are different, with $J_{m-no-CT}^m = 3105$, $J_{r-no-CT}^r = 1611$.

Interestingly, despite the overall diminishment in value, the manufacturer secured higher profits during period 4 under the $C&T$ policy. However, the shift in strategy translates to a marginal 0.29% decrease in the manufacturer’s overall value, while the retailer experiences a more substantial decline of 8.1%. These values are rooted in the price dynamics and order quantity delineated in Figures 5, 6.

Figure 4: Optimal profits, buyback with $C&T$ policy model (CT) vs. buyback only (BB)

Figure 5: Optimal prices with and without $C&T$ constraint
where the imposition of the capacity constraint is met with heightened prices for both players. The manufacturer price spectrum which initially ranged from $[26.3, 28.3]$ in the model only with a buyback contract, undergoes a shift to $[27.4, 29]$ in the presence of the C&T policy’s constraints in addition to the buyback contract. Similarly, the retailer price span, initially $[39.8, 41]$ in the model only with buyback, adjusted to $[40.2, 41.1]$ under the influence of the C&T policy plus buyback contract.

Observing the combined insights offered by both plots in Figure 6, the optimal channel behavior becomes evident. This optimal configuration emerges when the manufacturer chooses to sell the surplus proportion of their production allowance during periods 1, 3, and 5-7 while opting to buy during the remaining periods. The dynamic is depicted in the right plot of Figure 6, where positive values correspond to the selling volume and negative numbers denote the buying volume.

### 3.3 Price Sensitivity of the Cap-and-Trade Policy

The pricing structure within the C&T policy yields a substantial influence over the strategic choices undertaken by players in the channel. One significant implication emerges when the selling price surpasses $w - c^m - b$. In this scenario, if production is not mandated, the manufacturer may opt to sell production allowance more than engaging in production activities. Conversely, the impact of a low marginal purchase price lies in its potential to stimulate heightened production levels within the channel provided
this aligns with optimality. Employing the base parameters set \( \{c_{k}, c_{k}^{r}, s_{k}, \beta_{k}, R, \gamma_{k}, n\} \), profits are illustrated in Figure 7.

![Figure 7: Optimal profits](image)

The scenarios mentioned yield values

\[
\begin{align*}
    &J_{m} & J_{r} \\
    a: & v = 1, u = 1.5 & 3095.7 & 1481 \\
    b: & v = 2, u = 2 & 3096.4 & 1412 \\
    c: & v = 1, u = 8 & 3094 & 1444 \\
    d: & v = 10, u = 12 & 3318 & 763 \\
    \text{Base model:} & v = 0, u = 0 & 3105 & 1611
\end{align*}
\]

Within this context, our base model serves as the reference point, characterized by \( J_{m} = 3105 \) and \( J_{r} = 1611 \).

Scenario ‘d’ emerges as advantageous for the manufacturer, although it conversely diminishes the retailer’s value to the lowest point (compared to the other scenarios). In contrast, scenario ‘a’ presents the manufacturer with the lowest value while elevating the retailer’s position within other scenarios. To provide a visual understanding, we refer to Figures 8, 9, where the optimal order quantities and prices are depicted.
The maximum allowance is depicted by a black line in the optimal order quantity figure (left). Following the pattern of ordering, the right figure showcases the trading type.

Notably, in scenario ‘d’ the allowance trading prices act as an incentive for the manufacturer to adopt a higher pricing strategy throughout each period. This strategic move effectively curtails the retailer’s order volume, thereby aligning with the intent to limit capacity allowance consumption. Consequently, the new optimal decisions \((r^*, w^*, q^*)\) along with the selling profit of production allowance make a higher profit for the manufacturer (in all scenarios and baseline model). Elevating the trading prices inevitably leads to a corresponding increase in the manufacturer’s payoff while concurrently diminishing the retailer’s payoff.
4 Concluding Remark

Our study delves into a multi-period Stackelberg game imbued with distributional-robust price-history dependent demand, unraveling intricate dynamics within the context of our proposed model. This innovative framework encapsulates a unified contract strategy (single contract) that effectively addresses all periods’ decisions simultaneously. Additionally, the introduction of a buyback contract represents a strategic risk-sharing mechanism, where the manufacturer also undertakes the uncertainty inherent in demand fluctuations. To further enhance its environmental impact, our model embraces a Cap-and-Trade (C&T) policy, serving to regulate pollution.

The exploration unfolds through numerical examples that illuminate the profound impact of a buyback contract on channel results. Moreover, we delve into the intricate interplay of trading prices within the C&T policy on channel behavior.

For instance, the model elucidates that the viability of a generous buyback price can hinge on the probability of leftover inventory—referring to the scenario where the retailer orders less than the demand mean. In this context, a seemingly high buyback price, which comes initially along with an elevated wholesale price and order volume, can ultimately generate heightened profits for the manufacturer.

Importantly, the interplay between the players through a buyback contract doesn’t universally induce increased order volume from the retailer, because a lower risk for the
retailer is fulfilled by a higher wholesale price. Thus, optimizing the buyback price is a crucial strategic consideration.

The model also illuminates that the production capacity constraints enforced by the C&I policy do not uniformly impose restrictions. Instead, their impact shifts based on the prevailing prices of production allowances. When the selling price remains below the manufacturer’s profit from production, the allowance trade-off fails to yield higher profits for the players compared to the baseline model. However, as the selling price of the production allowance aligns with and exceeds the manufacturer’s profit from production, the manufacturer reaps amplified profits from production allowance trading, while the retailer consistently faces a disadvantageous position.

Appendix A

To compute the expected value of the retailer and manufacturer profits, from Eqs. (3) and (5), the value of $E(D - q)^+$ is required. Referring to the paper of Fakhrabadi and Sandal (Fakhrabadi and Sandal, 2023),

$$(D-q)^+ = \frac{1}{2} \{ |D-q| + (D-q) \}, \quad (17)$$

$$E(D-q)^+ = \frac{1}{2} \{ E[|D-q|] + E(D-q) \}. \quad (18)$$

From Cauchy-Schwartz inequality

$$E[|D-q|] \leq \sqrt{E[(D-q)^2]} = \sqrt{\sigma^2 + (q-\mu)^2}. \quad (19)$$

Therefore

$$E(D-q)^+ \leq \frac{\sqrt{\sigma^2 + (q-\mu)^2} - q + \mu}{2}. \quad (20)$$

The inequality in Eq. (20) introduces a tight lower bound on expected players’ profits for any distribution with the same $\mu$ and $\sigma$. Hence Eqs. (3) and (5) are recast as
\[ E[\pi^m] \geq (w - c^m - b)q + b\mu - \frac{b}{2} \left( \sqrt{\sigma^2 + (q - \mu)^2} - q + \mu \right) - u(q - q^-)^+ + v(q^- - q^+)^+ \equiv \Pi^m, \]  
\[ (21) \]

\[ E[\pi^r] \geq (r - b - s)\mu - (w + c^r - b - s)q - \frac{(r - b - s)}{2} \left( \sqrt{\sigma^2 + (q - \mu)^2} - q + \mu \right) \equiv \Pi^r. \]  
\[ (22) \]

There is at least one distribution (namely the worst distribution) that Eqs. \((21)\) and \((22)\) hold with equality.

Appendix B

Notice that for the economic feasibility \((r > b + s)\), \(\Pi^r_k\) is strictly concave in \(q_k\). For any feasible set of \((w_k, \bar{r}_k, b_k)\), the unique nonnegative solution of \(\frac{\partial \Pi^r_k}{\partial q_k} = 0\) is the global maximum given by

\[ q_k = \mu_k(\bar{r}_k) + \sigma_k(\bar{r}_k)\Lambda_k, \quad \Lambda_k = \frac{2\eta_k - 1}{2\sqrt{\eta_k(1 - \eta_k)}}, \quad \eta_k = \frac{r_k - w_k - c^r_k}{r_k - s_k - b_k}. \]  
\[ (23) \]

Since \(\max_q (\alpha_1 \Pi^r_1 + \alpha_2 \Pi^r_2 + \cdots + \alpha_n \Pi^r_n) \leq \max_{q_1} (\alpha_1 \Pi^r_1) + \cdots + \max_{q_n} (\alpha_n \Pi^r_n)\) holds with equality by Eq. \((23)\), the result is guaranteed to yield the maximum.

References


