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Winning by a Mile

A Mathematical Programming Approach to Reducing Distance and Ensuring Fairness in Travel in the FIFA World Cup

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Abstract

The FIFA World Cup presents a complex logistical challenge, featuring an intricate tournament schedule that requires teams to frequently move between their base camps and match venues. In this thesis, we explore the potential of mathematical programming as a tool to devise an optimal tournament schedule that reduces extensive travel and ensures fair travel distribution.

We create a FIFA World Cup scheduling framework consisting of mixed integer linear programming models. The framework consists of a series of individual optimization models, crafted from the guidelines of the World Cup of 2014 and 2018. All models yield significantly improved objectives relative to the historical benchmarks of 2014 and 2018. For the models minimizing total distance traveled throughout the group stage, the results range from a decrease of 25% to 48% in distance covered compared to historical distances. For the models minimizing the distance between the least and most traveling teams among all teams, the results range from a decrease of 81% to 96% for this inner range compared to historical differences. For the models minimizing the distance between the least and most traveling teams within each group, the results range from a decrease of 83% to 98% for the sum of the groupwise inner ranges compared to the sum of the historical groupwise inner ranges.

We further combine the individual objectives into a multi-objective model using the ϵ-constraint method, thereby showcasing a Pareto front of candidate solutions that all yield results that surpass the historical benchmarks for both objectives simultaneously.

Our findings strongly indicate that utilizing mathematical programming for the World Cup match scheduling process offers the potential to reduce the overall distances traveled while concurrently ensuring a more balanced distribution of travel burdens among the participants. We highlight the 2026 World Cup as an ideal prospect for implementing this approach.

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1. Introduction and Scope of Research

1.1 Introduction

The FIFA World Cup stands unparalleled as the premier global sporting event captivating audiences with its broadcasting reach. Estimates indicate that more than three and a half billion people watched a least one minute of the coverage in the recent 2018 World Cup (Richter, 2022), Given the multitude of stakeholders invested in the outcomes for each national team, crafting an optimal tournament schedule is undoubtedly an essential part of a successful World Cup.

A critical component under the control of tournament organizers is the travel itinerary for each team during the group stage. As studies have shown that excessive travel, particularly through different time zones, can affect athletes' physical performance, FIFA runs the risk that a skewed match schedule may negatively impact team performance, potentially influencing the outcomes of the tournament. Furthermore, factors such as environmental impact and spectator travel plans are also significantly affected by the distances each team covers in the final schedule. With the expansion from 32 to 48 teams in the 2026 FIFA World Cup, ensuring a balanced schedule that reduces unnecessary travel is more crucial than ever.

1.2 Scope of Research

This thesis aims to develop a framework of optimization models for the FIFA World Cup group stage scheduling using mathematical programming. We base the construction of our foundational model on the schedules of the FIFA World Cup 2014 and 2018, striving for the closest replication achievable in terms of regulations and restrictions. In the foundational model, the objective is to minimize travel distances while adhering to significant factors for replication identified in these previous tournaments. We also address important fairness aspects of equitable distribution of travel burden for the participating teams in the subsequent development of the foundational model. Furthermore, we explore the possibilities of concurrently addressing both the minimization of travel distance and equitable distribution of travel burden using multi-objective modeling. These candidate models are finally discussed in the context of how they could be adapted to the scheduling needs of the FIFA 2026 World Cup group stage.

We concentrate our modeling efforts on the group stage of the World Cup, due to the significant pre-planning required and the array of potential outcomes, which allows for the selection of an optimal schedule from numerous other feasible options. The knock-out stage, on the other hand, operates with a pre-defined progression of teams, a modification simplifying the task of efficiently assigning matches to venues. Consequently, modeling this phase is ultimately a simplified adaptation of the group stage model, with reduced constraints and fixed pairings of teams. Achieving a robust model for the group stage directly implies that transitioning to a model for the knock-out rounds would be a straightforward and uncomplicated step – and therefore not problematized.

In other words, we aim to design a framework that can assist decision-makers in the advanced planning of a World Cup, and we believe that the group stage offers the most significant opportunities for scheduling optimization. Therefore, our research will concentrate exclusively on this phase, with the final models having the possibility of easy adaptation for the knock-out stages as the tournament progresses.

In the official bid book to host the 2026 FIFA World Cup, the bid committee briefly reported that they had developed software to generate a match schedule to minimize the travel distance for teams given some predefined criteria (United Bid Committee, 2018). This is not similar nor highly relevant to our work, as their algorithm only considered distances between venues without factoring in base camps, in addition to the underlying assumptions for the match schedule that have changed since that time. For instance, the change in size of groups from three to four. Besides other pre-determined criteria, further details of the software or methods it employs are not elaborated on. To the best of our knowledge, this software – or its findings – has not been particularly communicated or promoted at a later time, which may be an indication that the software was not ready to be used when the bid book was published back in 2018. Nevertheless, it does indicate that FIFA and the host organizers consider the matter of travel distances to be an important factor when developing the match schedule.

1.3 Related Work

The problem at hand relates to the typical Traveling Tournament Problem (TTP) as a logistical timetabling problem. The objective of minimizing distance, in addition to some of the constraints, resembles the TTP. The TTP is a widely studied topic in the realm of sports scheduling literature, see for instance Easton et al. (2003), Frohner et al. (2023), and Ribeiro & Urrutia (2007). Typically, the context revolves around a double round-robin format, spanning an extended period, where teams compete against each other twice - once at home and once away. The primary goal of these models is to minimize the collective travel distance for all participating teams subject to constraints characteristic for tournaments – like no team plays the same opponent in two consecutive rounds. There have been various adaptations of this problem, each modified to fit their respective purpose in developing a framework for better decision-making in practical scenarios. For instance, integer programming was utilized for organizing the South American Qualifiers for the 2018 FIFA World Cup (Durán et al., 2017), demonstrating its application in significant real-world sporting events, and underlining the value of further exploring mathematical programming to enhance the World Cup tournament.

Perceived fairness is a vital part of sports scheduling. Kendall et al. (2010) offer an extensive overview of the most widely recognized fairness standards used in organizing round-robin tournaments. These include reducing the number of breaks and rest differences, handling carryover effects, ensuring group balancing, as well as fair referee assigning. Van Bulck & Goossens (2020) illustrate further implementations of the optimization of metrics for fairness in tournament schedules by e.g. minimizing the *games played difference index* (Suksompong, 2016). Although fairness is a widely recognized criterion in tournament scheduling, our research has revealed limited references to defining fairness in terms of the distribution of travel distances for teams in the earlier literature on sports scheduling.

However, a recent study by Osicka & Guajardo (2023) has introduced cooperative game theory into the tournament scheduling framework and has employed various fair distribution metrics to achieve the dual objective of balancing the conventional minimization of travel distances while simultaneously ensuring a fair distribution of travel burdens among the teams. This multi-objective optimization approach is an important continuation of the traditional TTP and for sports scheduling in general. It holds high relevance to our research, as ensuring equitable distribution of distances becomes crucial in a World Cup format where matches are scheduled within a short timeframe, allowing only a few days of rest between consecutive games. In this format, striving for equality in resting and traveling is considered an important part of creating a fair tournament.

Although we do not face a typical TTP, as the case of the World Cup revolves around a group stage format and thereby turning it into a problem outside the TTP category, the described literature review and earlier findings are still highly relevant to our work. However, to the best of our knowledge, the group stage tournament format is not as widely covered in the existing literature, necessitating the construction of our models from the ground, guided by tournament guidelines used in 2014 and 2018. A more thorough description of the constraining elements is provided in later chapters.

1.4 Structure

This thesis is divided into 8 chapters. This chapter presents a short introduction and the scope of our work. The second chapter introduces relevant background in addition to our motivation for the research. The third chapter describes the underlying problem in more detail, while the fourth chapter outlines the methodological framework used for modeling and solving the problem. The fifth chapter introduces the optimization models and data. In the sixth chapter, we present the results which are then used to discuss the relevance of our models for the upcoming World Cup in chapter seven, in addition to the shortcomings of this thesis. The eighth and final chapter provides a summary of our work and concluding remarks.

2. Background

2.1 Description of Format and Itinerary

The initial phase of the FIFA World Cup is the group stage, where the participating teams are allocated into groups for a series of round-robin matches. During the group stage, every team gets the opportunity to play against the other teams in its group once. Teams earn 3 points for a victory, 1 point for a draw, and no points for a loss, meaning that if a match concludes in a tie after the regular 90 minutes, it is recorded as a draw, with no additional time or penalty shootouts to determine a winner. The teams' positions in the group are finally determined by their total points. In instances where two or more teams accumulate the same number of points, additional criteria such as goal difference, the number of goals scored, and results from head-to-head matches are employed to establish the group rankings. Conventionally, the two highest-ranking teams from each group progress to the knockout stage, transitioning the tournament into a single-elimination format.

Historically, each participating team's travel schedule has involved commuting between their selected base camp and the various match venues each round. Contrary to various football tournament structures such as the Champions League, FA Cup, and others, the World Cup group stage does not feature home and away matches. In former World Cups, teams have typically selected a specific base camp to reside in between their games. As a result, teams do not exclusively travel to away matches, nor do they adopt a continuous "on the road" travel pattern throughout the tournament. Rather, teams follow a back-and-forth schedule, moving between their base camps and a new match venue each round.

In the past, the group stage of the FIFA World Cup has featured 32 teams, sorted into 8 groups of 4. This format is set to change in the 2026 World Cup, with an expansion that will see the inclusion of 48 teams. Originally, FIFA had contemplated a significant departure from tradition, proposing to organize the 48 teams into 16 groups of 3. This arrangement quickly came under scrutiny due to concerns about its level of fairness, including a heightened risk of match-fixing and an imbalance in the scheduling, as highlighted by Guyon (2022).

Prompted by these issues, FIFA reconsidered its approach and decided to divide the teams into 12 groups of 4 instead. Beyond the question of how many teams should be in each group, the expansion to 48 teams introduces a range of additional complexities. Research on this topic has provided various alternative tournament formats to effectively accommodate the increased number of participating teams (Krumer & Guajardo, 2023). Upon further consideration of the new format, it becomes apparent that the scheduling of the group stage and the planning of travel logistics for each team in the 2026 FIFA World Cup will also be characterized by the ultimate tournament structure. When writing this thesis, the latest statement from FIFA reveals that the two best teams in each group, in addition to the eight best third places advance to the round of 32, indicating a classical knockout structure to the final (FIFA, 2023).

2.2 Previous Scheduling Practices

Our research into previous scheduling and planning practices for the World Cups reveals, as far as we know, a lack of focus on reducing travel distances. Limited information is available about FIFA's methodology for arranging match schedules, but considering the massive position, it can be reasonably conjectured that commercial considerations play a predominant role. For example, prioritizing the facilitation of fan attendance and simplifying broadcasting procedures. In the 2022 World Cup, due to the exceptionally compact nature in Qatar, FIFA stated that group-stage games were assigned to stadiums to accommodate the comfort of spectators, teams, and media (FIFA, 2022). For the 2014 and 2018 World Cups, which were spread over much larger geographical areas, we have not found similar statements.

While recognizing the importance of these commercial elements and their understandable continued influence on future World Cups, we think it is essential to consider them in greater conjunction with the elements examined in this thesis, especially for tournaments spanning over great geographical areas, such as the 2026 World Cup which is hosted by Canada, the US, and Mexico together.

2.3 Motivation

2.3.1 Traveling Affecting Athlete Performance

As mentioned in the introductory part, a schedule that ensures fairness in travel distances is in part motivated by concerns over travel fatigue and its potential impact on the tightly packed schedule of tournaments like the World Cup. While studies vary regarding the extent of the impact, the article by Janse van Rensburg et al. (2021) provides a comprehensive analysis of the reasons behind travel fatigue and jet lag, as well as their effects.

When discussing the burden of travel fatigue on athletes, the article lists performance influential factors, which include the total distance traveled, the time of travel, and frequency. There is varying evidence on the potential consequences of travel fatigue alone and the acute effects of air travel without crossing time zones are limited to negative influence on perceptual measures. However, after rapidly crossing 3 or more time zones the circadian system cannot immediately adjust to the light-dark cycle in the new time zone. As a rule of thumb, the duration of natural alignment is 0.5 days per time zone crossed in a westerly direction and 1 day per time zone crossed in an easterly direction. Until the realignment between the circadian system and the new local time zone is complete, performance could be impacted.

The assessment emphasizes the intricate nature and insufficient empirical studies on the evaluation and handling of travel fatigue and jet lag in athletes, limiting the ability to provide solid advice. It agrees that travel fatigue and jet lag can pose significant challenges to athletes in terms of their performance and increased susceptibility to sickness or injury, depending on the competition time.

Another comparative study measuring the circadian advantage in Major League Baseball found results claiming that teams with a three-hour time zone advantage won 61% of their games, while those with one- and two-hour advantages had a 52% winning percentage (Winter et al., 2009). These findings are pertinent as the structure and demands of the World Cup are akin to the scenarios examined in the literature, indicating that such travel-related challenges are likely to be encountered by participants in future World Cups, including the 2026 World Cup spanning over 4 time zones across North America, thereby highlighting the need for a fair and considerate scheduling approach.

2.3.2 Environmental and Fan Implications of Travel Reduction

Reducing the overall travel distance during the tournament undoubtedly yields numerous additional benefits. From an environmental standpoint, less travel translates to reduced emissions and a smaller carbon footprint. Furthermore, for fans wishing to support their national team, shorter travel distances mean a more accessible and budget-friendly tournament, fostering a more inclusive atmosphere for everyone interested. Elaborating on these perspectives and quantifying the positives are outside the scope of this thesis, but we consider it to be self-evident that reducing the overall travel distance during the tournament is a vital element in a more distance-efficient framework for organizing the World Cup schedule.

2.3.3 Fairness Perspectives

Concentrating exclusively on minimizing the total travel distance might however lead to a tournament schedule that falls short of being optimal when considered from an overall perspective. As discussed by Osicka and Guajardo, an important aspect of a successful tournament execution from a sporting perspective is the degree of perceived fairness among the participating teams. As stated in the paper:

> While minimizing all the traveling between games is efficient from the overall perspective, it overlooks the distribution of the travel among the teams. Consequently, some teams may end up better than others with respect to their individual goals, an imbalance which may affect teams' often-limited resources or preparedness for the games (Osicka & Guajardo, 2023).

This point is notably relevant to the World Cup, specifically the case of 2014 when some teams had a mere two days of rest in between consecutive matches. For instance, Nigeria and Bosnia-Herzegovina both played on the 22nd of June and then again on the 25th of June, resting only on the 23rd and 24th. Likewise, Portugal and the United States both played on the 23rd and the 26th, also resulting in only two full rest days in between matches. Research done by Scoppa (2013) shows that a difference in rest days between opponents can be an advantage when the rest time is equal to or less than three days. With the tight timeframe for debriefing, rest, and further strategic planning present in former World Cups, the duration spent in transit can profoundly impact a team's preparedness for their upcoming game. If a team is subjected to a notably more demanding travel itinerary compared to its competitors, the eventual outcome of the match could be a direct consequence of the tournament's pre-planned scheduling decisions.

This said, in the context of the World Cup, various dimensions of fairness are already integral to the process of scheduling and planning. For example, the seeding of teams into different groups considers their historical performance, as reflected in their FIFA rankings. Additional literature has introduced further suggestions for improvement of the fairness in the group draws in tournaments, for instance, Cea et al. (2020), Guyon (2014), and Csató (2023). Moreover, it is a common practice for tournament organizers to ensure a diverse representation of teams from different continents within each group (Organising Committee for FIFA Competitions, 2022). Efforts are also made to distribute match tickets equitably among the nations, aiming to mitigate any potential "home venue" advantage that might arise from a random allocation of tickets – and thus an overrepresentation of fans from one of the nations.

Though these fairness perspectives are already factored in during the scheduling of the World Cup group stage, there seems to have been less focus on fairness in terms of the distribution of traveling burden. As shown in **Figure 2.1** and **Figure 2.2** for the 2014 World Cup, and **Figure 2.3** and **Figure 2.4** for the 2018 World Cup, our analysis reveals significant disparities in the travel demands placed on different teams in past World Cups. The graphs effectively depict the cumulative distance each team traveled back and forth between their respective base camps to match venues.

The graphs are complemented by detailed representations of a pair of travel routes shown on maps, highlighting the most extreme cases of the longest and shortest journeys, thereby clearly demonstrating the unequal travel burdens borne by various teams. Considering the impact that total travel distance can have on team performance, the disparities observed in previous World Cups provide a compelling rationale for refining the balance in travel distances among the participating teams, ultimately contributing to an enhanced sense of fairness throughout the tournament. It can be noted that since the teams have picked their base camps themselves, they could probably have chosen base camps differently if the travel distance was their primary concern – e.g. the United States as shown in **Figure 2.2**. However, this would only help to some degree and is only applicable when the objective is to minimize the *total distance* traveled.

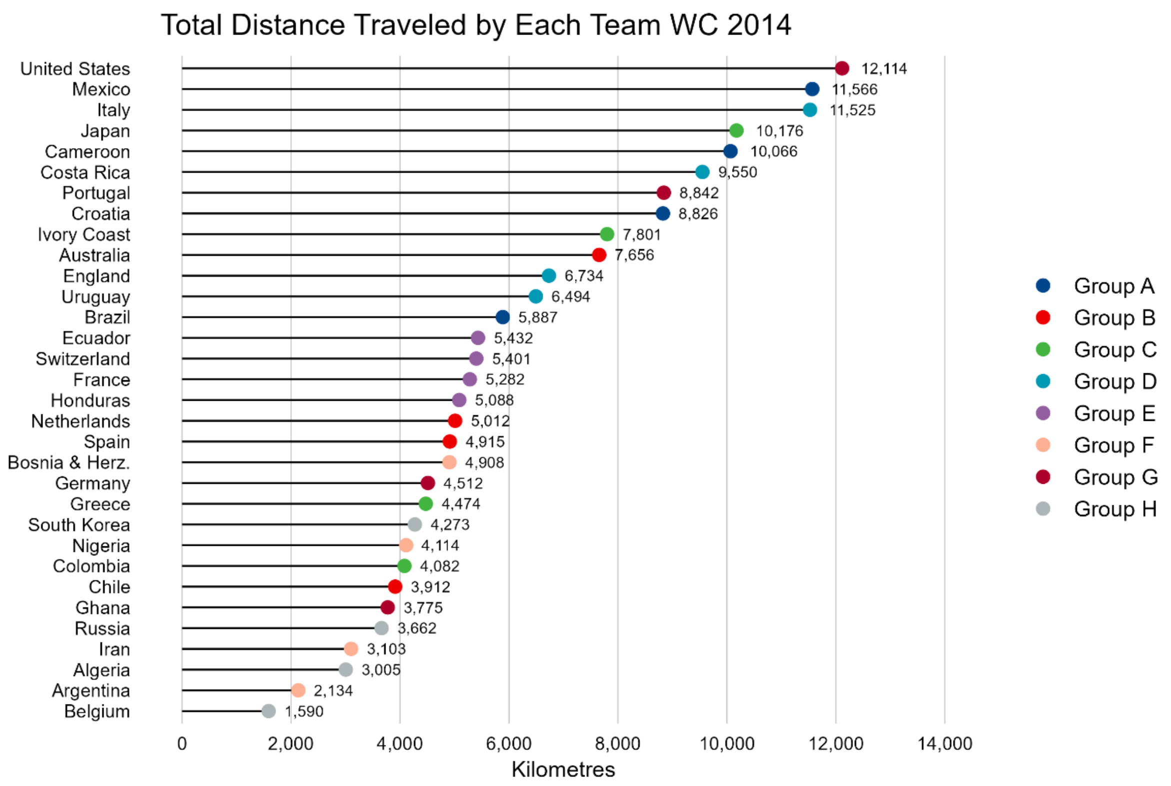

***Figure 2.1**: Total distance traveled by each team in WC 2014 during the group stage*

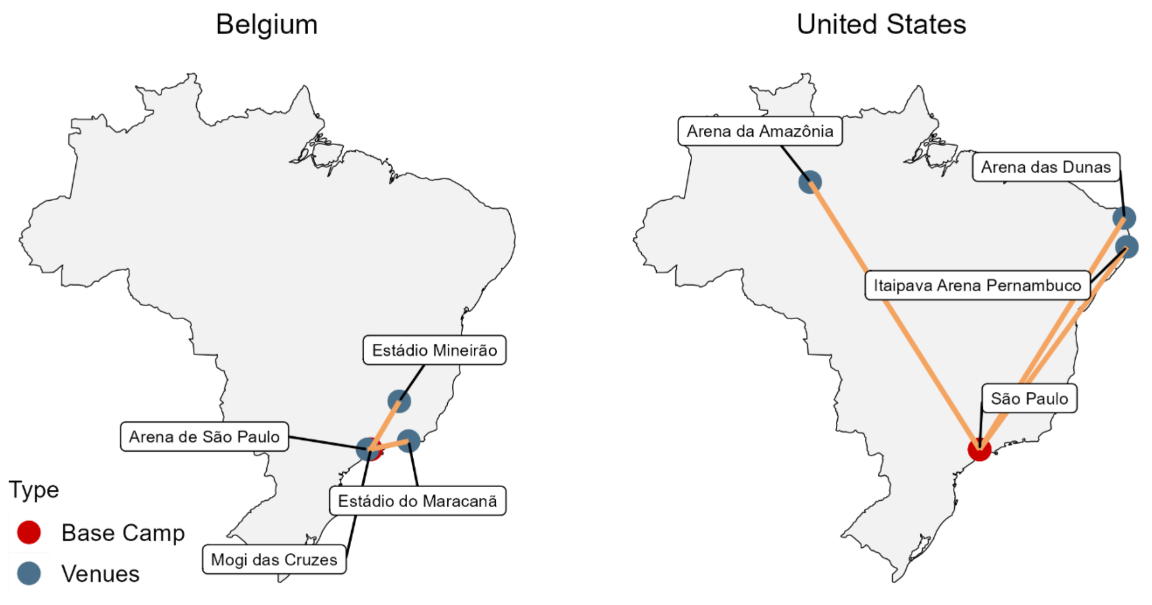

***Figure 2.2**: Base camp and venues for Belgium and the United States.*

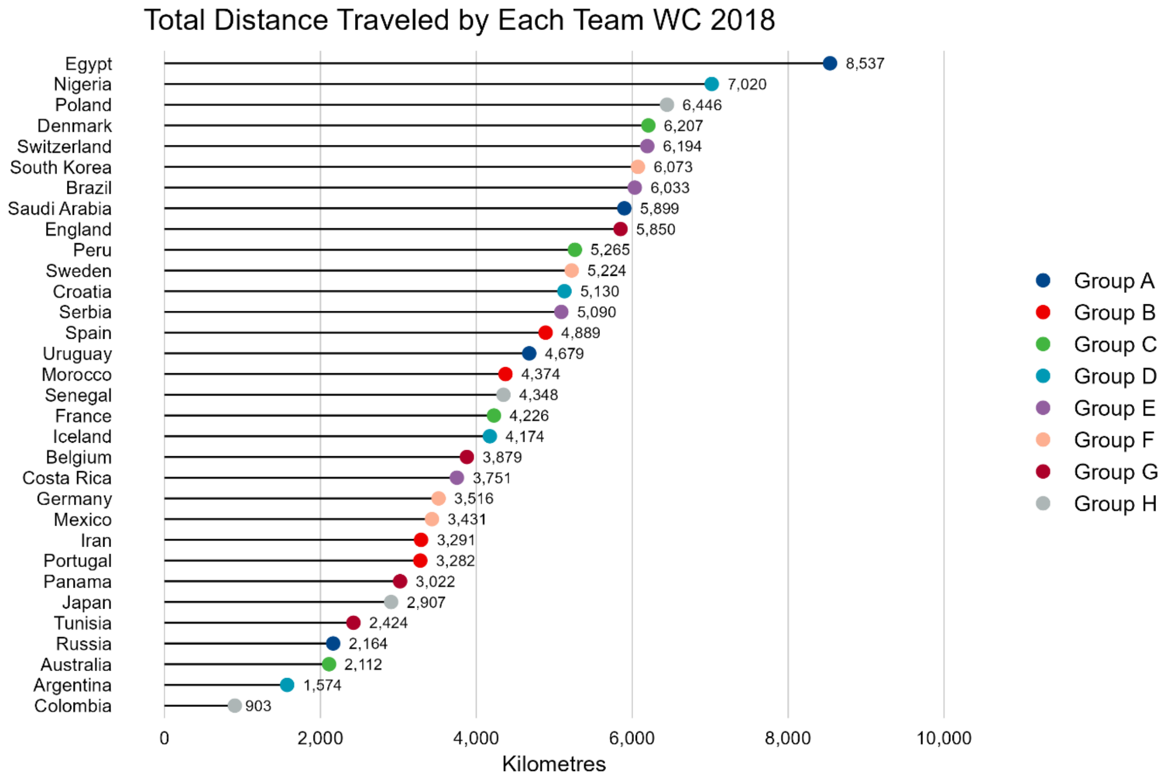

Figure 2.3: *Total distance traveled by each team in WC 2018 during the group stage*

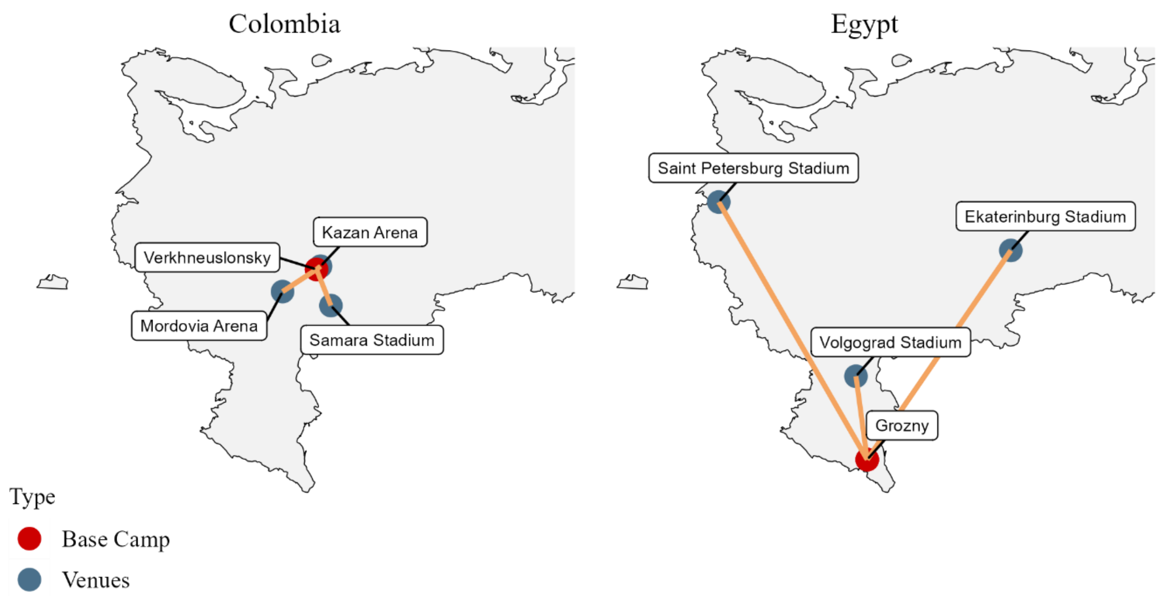

Figure 2.4: *Base camp and venues for Colombia and Egypt.*

3. Description of the Problem

As established, the main goal of this thesis is to enhance the strategic planning framework for the World Cup, thereby aiding those in charge of crafting the event's schedule. Our hypothesis posits that previous World Cup events have undervalued the importance of reducing travel distances and maintaining fair travel distribution among the participants. In response to this, we focus on developing a mathematical programming model for enhanced tournament scheduling in terms of itinerary coordination. This model will not only highlight these past travel distance oversights by presenting improved retrospective schedules but will also serve as a suggested framework for devising more efficient and fair scheduling solutions for future World Cups.

Consequently, the problem at hand entails not only creating a mathematical model designed to streamline the schedule by reducing the overall distance traveled but also involves the crucial step of translating principles of fairness into mathematical expressions for subsequent improvements to the model. This chapter is dedicated to exploring the ideas behind the mathematical formulations of the objectives, the selected fairness principles, and the tournament constraints, and addressing the complexities encountered throughout the modeling process.

3.1 Minimization of Total Distance

The challenge of minimizing the total distance traveled revolves around creating a model able to evaluate all feasible routes within the boundaries of tournament guidelines, and then, subsequently, selecting the combination of routes that yields the minimum cumulative distance. Therefore, the mathematical formulation of the objective function must allow for assessing every possible route while efficiently identifying those that are optimal for achieving the least total distance.

A route is defined as the travel from a base camp to a venue. For the two initial rounds, the routes are duplicated in the final sum to factor in the return from the venue to the base camp. For the final round, the return route is not included in the objective function. This is because sports-related activities in the group stage conclude after the last match. However, if we prioritize factors other than sports performance, like reducing carbon footprints, as key benefits of minimized travel, it could be logical to count the return journey in our calculations.

This inclusion should not significantly impact the results, as the return route is identical to the initial journey to the venue.

The solution to the problem is an objective function that computes the aggregate of all feasible distances, each multiplied by an assigned set of binary decision variables determined by the model. These variables represent a series of *yes* or *no* choices - essentially, a binary determination of whether a route is included in the summation of total distance. Every potential combination within the set of feasible solutions is considered by the model. When a team's potential route is identified as efficient for minimizing total distance, its corresponding binary variable is assigned a value of 1, signifying *yes*. This inclusion integrates the route into the total distance calculation by multiplying its distance by 1. Conversely, if a route is deemed sub-optimal, the binary variable is set to 0, denoting *no*. This setting effectively excludes the route from the final solution by multiplying its distance by zero, resulting in a contribution of zero to the overall distance sum. Though highly computationally demanding, the model should ultimately be able to find the single best solution of chosen routes among the range of possible solutions, thereby solving the problem at hand.

3.2 Incorporating Fairness

Another central focus of our research question is to investigate methods for embedding fairness within our optimization model for World Cup scheduling. Establishing the parameters for measuring this fairness is a vital consideration. In the literature on sports scheduling, several metrics for ensuring fairness in tournament schedules have been already explored, for instance, Van Bulck & Goossens (2020) focus on fairness within the fixture list and Osicka & Guajardo (2023) explore a game theory approach to distance distribution among the teams. We want to expand on this idea of incorporating fairness in terms of distance distribution in a mathematical model for tournament scheduling.

We recognize that World Cup scheduling already incorporates various aspects of fairness criteria, discussed in section 2.3. Additionally, we bear in mind the predetermined tournament guidelines and time frames that result in a relatively inflexible fixture list of match dates. Consequently, our primary focus is on creating models that effectively and fairly distribute travel distances among teams within the demanding constraints of a tight and strict match fixture, as well as the other established tournament regulations of a FIFA World Cup.

Our chosen objective for enhancing fairness is minimizing the inner range between the maximum and minimum value of travel distance among the teams. The inner range is derived by subtracting the minimum distance from the maximum distance, guaranteeing that every team's travel routes fall within these boundaries. This method is designed to achieve a fairer allocation of travel distances across teams in the sense of equitable distribution. While the mathematical model - incorporating a range of possible distances combined with binary decision-making to identify optimal routes - remains constant, the objective has shifted. The selection of binary variables is now focused on minimizing the variation in total travel distances, aiming for the closest possible parity, rather than a sole focus on minimizing the sum of all distances.

As far as we know, this is the first paper to address this metric for fair travel distance distribution in a mathematical programming model for tournament scheduling. Naturally, there are other allocation methods worth exploring in future research, each potentially offering unique insights and solutions to the challenge of ensuring fairness. This expansion of metrics is however beyond the scope of this thesis and case study.

3.2.1 Fairness Metrics

After determining our primary strategy to reduce the internal range of travel distances, further discussions are necessary within the context of the World Cup format. When we view the equitable distribution of travel distance through a hierarchical order, it becomes apparent that the perceived fairness can be categorized into at least two distinct metrics: 1) optimized equality between *all* teams in the tournament, and 2) optimized equality between all teams within each *group*.

Metric 1: Total Inner Range

This goal is centered around ensuring a comprehensive sense of fairness across all participating teams regarding the total distance traveled during the group stage. By striving to minimize the range between the maximum and minimum distance traveled among the set of all teams, the resulting schedule is deemed fair from a holistic viewpoint, as it ensures that all teams advancing to the knockout stage have experienced comparable travel demands in the preliminary rounds.

Metric 2: Groupwise Inner Ranges

The goal of this objective is to establish equitable conditions for all teams within a particular group. As previously discussed, ensuring fairness at this stage is crucial for the overall success of the tournament, given that the outcomes of these group-stage matches determine which teams advance to the subsequent knockout rounds. By concentrating our efforts on each group, the model could find a better result in terms of equality within those specific groups, while staying within the bounds of what the model can feasibly accomplish.

Comparative Analysis of Selected Fairness Metrics

When considering the two distinct metrics of fairness, it becomes apparent that both present compelling cases for being favored. Ideally, both metrics would be satisfied to an acceptable degree by the model since, in theory, achieving one does not necessarily have to compromise the other. However, the problem arises when conflicts emerge during the resolution of a model that focuses exclusively on one of the objectives; the metric of fairness which is deemed most critical by the decision maker. To illustrate, we consider two hypothetical cases:

Case 1: Minimization of Total Inner Range

In this case, the objective is to minimize the total inner range of travel distances to ensure overall fairness. We have the minimization of the total inner range as the objective. We assume a situation where the final value of the total inner range is 10,000 km (ranging from a maximum of 20,000 to a minimum of 10,000). Further, we assume values of groupwise inner ranges for two groups, Group A and Group B, as shown in **Table 3.1**.

Table 3.1: *First hypothetical case for inner range*

	Max	**Min**	**Inner Range**
Total Inner Range	20 000	10 000	10 000
Groupwise Inner Ranges			
Group A	20 000	18 000	2 000
Group B	20 000	10 000	10 000

Though the final objective value could be deemed satisfactory narrow, when examining the inner ranges on a group basis, there is however a notable variance in their inner ranges. Teams in Group A exhibit an inner range of 2,000 km (from 20,000 to 18,000), whereas the inner range for Group B is 10,000 km (from 2,000 to 10,000). Comparatively, the travel distance variation for Group B is fivefold that of Group A. Even though the final value of the total inner range could be satisfactory, there is still a marked difference in travel demands within the individual groups which could be considered undesirable. The team in Group B that travels five times further than the least-traveling team in the same group arguably has a greater disadvantage in proceeding to the knockout stage. For Group A, this variation is of far less significance.

Case 2: Minimization of Groupwise Inner Ranges

In this case, the objective is to minimize the sum of all groupwise inner ranges. In this hypothetical example, the difference between the maximum and minimum for both groups is 2,000 km: (8,000 – 6,000) and (30,000 – 28,000) respectively, giving a final sum of 4,000. Conversely, the total inner range has expanded to 24,000 km. While there is an increased sense of fairness in terms of equitable travel distances within each group, this benefit is offset by an increase in the total travel range, thereby diminishing the perceived fairness across *all* teams in the tournament.

Table 3.2*: Second hypothetical case for inner range*

	Max	Min	Inner Range
Total Inner Range	30 000	6 000	24 000
Groupwise Inner Ranges			
Group A	8 000	6 000	2 000
Group B	30 000	28 000	2 000

In practice, as in theory, it may be that minimizing the overall inner range does not conflict with reducing the inner range within groups. Yet, given the intricacies of a mathematical model suited for tournament scheduling, prioritizing one aspect of fairness could compromise another in the search for the optimal solution. Our initial analysis of these models has indeed demonstrated such trade-offs, and we find it crucial to address the matter. Ultimately, the selection of the appropriate fairness metric will rest with the decision-maker, but it is important to be aware of the pitfalls in the search for a fair schedule.

Balancing Fairness and Total Distance

Regardless of the perceived relative importance of the different fairness objectives discussed, aiming exclusively to minimize the inner range of travel distances - either in totality or within specific groups - may initially appear to be sufficient for a scheduling model. However, it quickly becomes evident that this method could unintentionally result in an increased total travel distance – with all the negative effects this could impose, discussed in section 2.3. To illustrate this point, again consider two hypothetical cases: in the first, the minimum and maximum values among all teams are 10,000 and 20,000 respectively, resulting in an inner range of 10,000 (20,000 – 10,000). In the second case, these values are 110,000 and 120,000 respectively, also yielding an inner range of 10,000 (120,000 – 110,000). Despite the identical inner ranges, the total distance traveled in the second scenario is significantly higher, making the first scenario the preferable option.

This modeling fallacy, where different optimal scenarios are incorrectly treated as equivalent, introduces the need for a multi-objective approach, where we introduce a trade-off between the traditional minimization of total distance and a sole focus on the chosen fairness criterion.

Tournament Guidelines and Conventions as Model Constraints

Another crucial aspect in the development of a World Cup scheduling framework is ensuring its broad applicability to various editions of the tournament, thereby requiring minimal customization. Thus, a thorough feature of all the models in this thesis is that they are adapted for both the 2018 and 2014 World Cup. Incorporating past World Cup tournament guidelines into the development of our model allows us to construct our framework such that it aligns with the fundamental standards and norms of a traditional World Cup format. Additionally, by applying the constructed framework to past tournaments, we can conduct a retrospective assessment to evaluate its improvements relative to historical benchmarks.

However, in our research of former World Cup schedules, we find that there have been differences in tournament guidelines, which further calls for a need for individual customization of the hard constraints in a replicating mathematical model. Simply by examining the match schedule, it is challenging to discern what is guided by established principles, sport-specific factors, commercial considerations, or mere randomness. To illustrate, during the 2018 FIFA World Cup, there were no instances of more than four matches scheduled on a single day in the group stage. However, in the 2014 World Cup, there was one occasion in the group stage where five matches were played on the same day. Another example from the 2018 World Cup is that Russia had their second match on the same day that the last playing teams in round 1 played their first match, but between the second and third rounds, the transitions between rounds were not within the same day.

Consequently, the final mathematical model will face certain limitations in fully replicating the former schedules, and it is necessary to include some minor adjustments to create a more "universal" World Cup scheduling model. This makes it less tailored to the actual schedules than it potentially could be by creating one customized model per tournament edition. Nevertheless, most of the constraints - and arguably all the most important aspects as well - are mathematically formulated such that they stay in line with both former World Cups researched. The constraints that differentiate between the two former World Cups in terms of right-hand side values, such as the minimum number of rest days, are easily modified without changing the mathematical formulation of the constraint itself.

Additionally, we differentiate between two types of approaches for each adaption of the single objective models: one that incorporates pre-allocated base camps, equal to the setup observed in the World Cups we aim to emulate, and another that utilizes the same set of base camps but allows the model itself to assign teams to a base camp based on optimal outcomes for the objective at hand. The second model represents a more relaxed problem, which is expected to produce superior results – though it might exactly align with the restricted approach. Nonetheless, this model-driven approach does not align as closely with the actual distances traveled from the nation's self-chosen base camps in the former World Cups and is thus not as fit for ex-post match fixture evaluation. However, we include this approach in our analysis to facilitate a comparison between the two versions of the model, highlighting a potential future direction for World Cup schedules where team allocations to base camps are determined by a model rather than being selected by the nations themselves, as has been convention. The difference represents a minor adjustment in the mathematical formulation of the constraint and does not compromise the model's versatility.

4. Methodological Framework

This chapter lays the foundation for the modeling process by introducing key methodological concepts. It offers a concise overview of critical mathematical programming techniques employed, including linear programming, (mixed) integer linear programming, and multi-objective optimization. Understanding these concepts is vital for grasping the methodologies that underlie our analysis, as well as acknowledging the strengths and limitations of the mathematical methods used concerning real-world problems.

4.1 Mathematical Programming

Mathematical programming is a branch of operations research that involves finding the best solution to a practical problem. It encompasses various methods for solving mathematical problems that involve determining the values of decision variables to optimize a particular objective function, within a well-defined set of linear or nonlinear equations and inequalities representing constraints. The objective function, which is to be maximized or minimized, can be linear, nonlinear, or discrete. The area of mathematical programming consists of a broad range of techniques that are widely used in, and customized to, various fields such as economics, engineering, logistics, and finance for decision-making and problem-solving.

4.1.1 Linear Programming

Linear programming (LP) is a specific subset of mathematical programming that deals exclusively with linear equations and inequalities. It is designed to optimize a linear objective function, subject to a set of linear constraints. Formulating a problem as an LP is advantageous due to its simplicity, ensuring clarity in the relationships between variables. It guarantees solvability through efficient algorithms like the Simplex method, even for problems with numerous variables and constraints. The convex nature of these problems means any local optimum is a global optimum, offering certainty in the optimality of solutions.

Integer Linear Programming (ILP) and Mixed Integer Linear Programming (MILP) refine the linear programming framework to handle problems where some or all decision variables are integers, crucial for scenarios demanding discrete decisions. MILP permits a combination of integer and continuous variables, enabling the modeling of complex real-world situations with both divisible and indivisible resources. These methods also frequently incorporate binary variables - specific integers limited to values of 0 or 1 - to represent binary choices like yes-or-no decisions.

4.1.2 Multi-Objective Optimization

In a multi-objective optimization problem, several objectives need to be optimized simultaneously. This contrasts with single-objective optimization, where the goal is to identify the optimal solution for a singular criterion. In mathematical terms, a multi-objective optimization problem is defined by a vector-valued objective function that aims to optimize two or more objectives simultaneously. For minimization, this can be formulated as:

Minimize: $\{f_1(x), f_2(x), \dots f_k(x)\}$

s.t: $x \in \Omega$

Where Ω is the feasible region of the problem, meaning that the decision variables of the individual objectives, x, are defined within what is feasible for the overall problem. $f_i(x)$ is the i-th objective function to be minimized or maximized, and k is the total number of singular objectives.

When optimizing a set of objective functions instead of a singular objective, these objectives are often in conflict with each other, meaning that improving one objective may lead to the deterioration of another (Chang, 2015). In other words, there may be no unique solution that optimizes all objectives simultaneously. The conflict of multiple objectives to be optimized introduces thus the use of a Pareto front (Bing, 2022), which contains all solutions that are Pareto optimal; meaning that no single objective in an optimized multi-objective solution may be enhanced without deteriorating another objective. In other words, a solution is Pareto optimal if there is no other solution that dominates it, where domination means being better in at least one objective and no worse in the others. The Pareto front thus provides a trade-off curve, illustrating the extent to which one objective can be improved at the expense of another.

The challenge in multi-objective optimization lies not only in finding the Pareto front but also in selecting the most appropriate solution from a practical perspective. Mathematically, each Pareto-optimal solution in multi-objective optimization is considered equally valid, while in practical applications, only one solution is ultimately to be selected. This final selection process requires the involvement of a decision-maker who possesses a deep understanding of the practical aspects of the problem and can reflect upon and express the relative importance of each objective. For a comprehensive numerical example, the reader is directed to the paper by Romanko et al. (2006).

In conclusion, it is the process of delineating the Pareto front and the subsequent qualitative evaluation of candidate solutions by the decision maker that culminates in the selection of a *final solution* for the multi-objective optimization problem.

ϵ-constraint method

In the literature on multi-objective optimization, the ϵ-constraint method is widely used due to its simplicity and effectiveness. See for instance Becerra & Coello (2006), Mavrotas (2009), and Romanko et al. (2006). The method is a scalarization method, which effectively turns a multi-objective optimization problem into a single-objective problem.

The ϵ-constraint method involves selecting one objective to optimize while converting the other objectives into constraints with specified bounds, referred to as ϵ-levels. By systematically varying these ϵ-levels within the possible range of the constrained objective values, a series of single-objective optimization problems are solved, each yielding a solution that reflects a different trade-off between the objectives. This method effectively traces out the Pareto front, showcasing the trade-offs inherent in the problem. The method is particularly useful for finding solutions on non-convex Pareto fronts, where traditional weighted sum approaches may fail to represent all the trade-offs between objectives.

The method stands out for its intuitiveness in the decision-making process, catering to users of all levels of modeling expertise. If a hierarchy among the different objectives is predetermined, the process can intuitively be described as a method of saying “f1 is more important than f2 and we do not want to sacrifice more than 20% (or 30% or 50%) of the optimal value of f1 to improve f2.” (Romanko et al., 2006). Given this context, this method is about optimizing the secondary objective within the confines of a maximum allowable deterioration to the primary one. However, in the absence of an established hierarchy, the methodology remains consistent. The distinction lies in the inherent understanding of the frontier of solutions, which shifts to an evaluation of the relative trade-offs between the objectives rather than a focus on the sacrifice from the top priority objective.

Formulation

Let f_1 and f_2 be two objectives to be optimized. Given a set of decision variables x belonging to the feasible region Ω, the ϵ-constraint method can be expressed as follows:

For each ϵ_i in the set $\{\epsilon : \min f_1(x) \leq \epsilon \leq \max f_1(x)\}$, solve the following optimization problem:

Minimize: $f_2(x)$

s.t: $x \in \Omega,\ f_1(x) \leq \epsilon_i$

For each iteration, the value of f_1 and f_2 are registered as a combined optimal solution given the current constraint. This process is repeated until the full range of epsilon values has been imposed as constraints, thus revealing the Pareto front for the trade-off between f_1 and f_2.

Appropriate Epsilon Boundaries

A limitation of the ϵ-constraint method is the necessity for the modeler to preselect a spectrum of ϵ-levels. In a multi-objective optimization problem, it is not certain that the solution space for a combined objective aligns with that of the individual objectives. A common misstep is selecting an epsilon that falls outside the feasible solution space of the combined objectives. In the literature on multi-objective optimization, the use of Utopia and Nadir bounding has proven to be the most effective in mitigating this issue (Romanko et al., 2006; see also Blank et al., 2019; Yeung & Zhang, 2023). The process involves an initial estimation of suitable bounds by identifying the extremities of the Pareto front prior to its complete delineation. This helps in establishing a range of epsilon values that are likely to yield feasible and relevant solutions (Chaudhuri et al.). In the search for the outer boundaries, two points are estimated; 1) the Utopia point z_i^U, and 2) the Nadir point z_i^N.

The Utopia point represents the ideal point where all objectives achieve their best possible values simultaneously within the feasible Pareto set. This point is, however, usually not attainable due to conflicting objectives, but works as an upper bound of the best theoretical solution possible and serves as a benchmark evaluating the performance of the Pareto-optimal solutions. The Utopia point of each objective is formulated as:

$$z_i^U = \text{minimize } \{f_i(x) : x \in \Omega\}$$

And the combined Utopia point is a vector of the estimated ideal values:

$$Z_U = (z_1^U, z_2^U, \ldots, z_k^U)$$

The Nadir point on the other hand is an estimation of a worst-case scenario, represented by the worst values of the objectives from the Pareto-optimal set. The Nadir point is however not as straightforward to find as the Utopia, as the definition yields that this should be the worst points within the Pareto-optimal set, and not the individually worst points available – meaning the individually maximized objective solution points are not the Nadir point, see **Figure 4.1** (Wang et al., 2017).

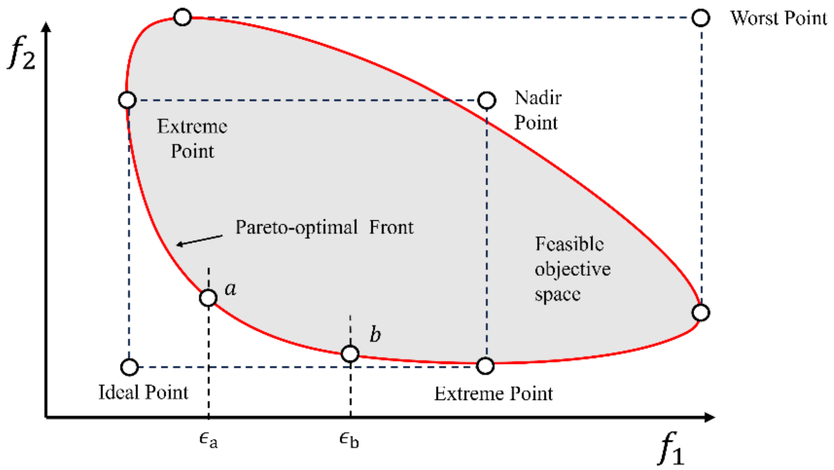

Figure 4.1*: Illustration of a Pareto-optimal front*

Thus, the Pareto front, or at least the extreme points of the Pareto front, must be known in advance. Formally, the finding of the Nadir point can be stated as:

$$z_i^N = \text{maximize } \{f_i(x) : x \in \mathrm{P}\}$$

Where P indicates the Pareto-optimal set of solutions for the decision variables. The combined Nadir point is a vector of the single Nadir point values:

$$Z_N = (z_1^N, z_2^N, \ldots, z_k^N)$$

Essentially, the distinction between calculating the Utopia and Nadir points is based on the domain in which the decision variables x are specified: respectively Ω for Utopia and P for Nadir, where $P \subseteq \Omega$. For the Utopia point, x is defined within the broader set Ω, because the optimal value of $f_i(x)$ found in Ω will inherently be included in P too – it is the best possible value and must thus lie as an extreme on the Pareto front. As Ω is known in advance, while P is not, it is easier to estimate the Utopia than the Nadir. **Figure 4.1** illustrates this distinction.

An adequate *estimation* of the Nadir point can be found by identifying the coordinate of the j-th objective given that the i-th objective is at its ideal value (Audet et al., 2020). This is shown in **Figure 4.1**, where if you "constrain" f_1 to be equal to its minimum in isolation, the Ideal single point, and "maximize" f_2, you end up at the Extreme Point in the upper left corner of the Pareto front. At this point, f_2 reaches a coordinate that can be considered an estimate of the Nadir point for the second objective, as it is the most unfavorable (highest) value for f_2 given that f_1 is at its minimum, and thus the worst value of f_2 within the set of Pareto optimal solutions. The same holds for the coordinates at the other extreme of the Pareto front.

The awareness of the Nadir and Utopia objective values also helps the decision-maker to set realistic expectations by being aware of the entire range of possible outcomes for each objective (Chaudhuri et al.). Consequently, the set of solutions available on the Pareto front can be effectively compared with both the best-case (Utopia) and worst-case (Nadir) scenarios – helping the decision-maker in concluding a *final solution* with more information on the performance of each solution relative to these extremes.

Having identified the appropriate boundaries, it follows that for each ϵ_i in the set $\{\epsilon : z_1^U \leq \epsilon \leq z_1^N\}$, the following optimization problem is solved:

Minimize: $f_2(x)$

s.t: $x \in \Omega, f_1(x) \leq \epsilon_i$

4.2 Solver and Software

Within the field of mathematical programming, several solvers and algorithms have been developed to tackle different types of problems, and the last couple of decades have seen an impressive improvement in runtime. Problems deemed impossible to solve at the beginning of the decade are now solved within seconds (Koch et al., 2022). For a list of leading solvers and their specific areas of application, see the article from Mann (2022).

A leading technique for solving complex optimization problems is the cut-and-branch method. The method is a sophisticated algorithm that combines cutting planes and branching strategies to solve MILP problems. Initially, it generates cutting planes to tighten the linear relaxation of the MILP problem, effectively excluding non-integer solutions without eliminating any integer solutions. This step helps in finding a closer integer-feasible solution space by adding linear constraints. After applying a sufficient number of cutting planes, the method shifts to a branching phase, where it systematically splits the problem into subproblems by rounding continuous (relaxed) variables to integer values. These steps are repeated, solving sub-problems of linear relaxations, adding cuts, and branching until the optimal integer solution is found (Mann, 2022).

When selecting a solver for ILP/MILP problems, Gurobi Optimization's solver software emerges as an excellent option. The solver consistently achieves top performance in benchmarks, showing year-on-year improvements in solve times (AMPL, 2023) by taking advantage of modern technical architecture and multicore processors (MathWorks, 2023). This proven track record has led to Gourbi's widespread adoption by over 2,500 companies in over 40 industries worldwide (Business Wire, 2023).

Dealing with the complexities of large integer programming problems requires significant computational resources and solving time. To navigate this challenge, the Mixed Integer Programming (MIP) Gap acts as an effective measure for evaluating the quality of solutions found before the solvers' self-termination at a global optimum (Miltenberger, 2023). It measures the discrepancy between the best feasible solution found so far, called the Upper Bound, and the optimal solution of a relaxed version of the problem, where integer constraints are ignored. The solution to the relaxed problem provides a so-called Lower Bound since it is guaranteed to be equal to or better than the globally optimal solution to the integer-constrained problem. As the solver progresses and identifies improved integer-constrained solutions, the Upper Bound is updated accordingly. The MIP gap is thus a dynamic measure, reflecting the solver's progress towards global optimality. Formally, it is defined as:

$$MIPgap = \frac{|Upper\ Bound - Lower\ Bound|}{|Upper\ Bound|}$$

For example, a reported MIP gap of 1% indicates that the current solution is within 1% of the theoretically optimal solution. While there might exist even better solutions that are closer to the Lower Bound, the present solution's deviation from the optimal is no more than 1%.

For modelers facing constraints on computing resources, the MIP gap can be employed in a couple of strategic ways. One approach is to impose a time cap on the solver's operation and use the MIP gap at the end of this period to assess the quality of the solution obtained. Alternatively, the MIP gap can serve as a stopping threshold itself - for instance, deciding in advance to accept solutions that fall within 1% of the Lower Bound. However, there is a possibility that the MIP gap of the globally optimal solution might exceed the decision-maker's preset threshold. This can lead to a scenario where the solver continues its search for a better solution even after the optimal value has been reached (Miltenberger, 2023). Nevertheless, it is important to keep a pragmatic attitude when solving complex integer programming problems. As stated by Matthias Miltenberger from Gurobi Optimization:

> For real-world applications, it is always sensible to set a positive MIP Gap tolerance to manage the tradeoff between having a feasible solution that is good enough for the use case and the computation time required to explore the branch-and-bound tree (many times it's prohibitive to exhaust that search) (Miltenberger, 2023)

Beyond choosing a solver, it is necessary to use software that facilitates mathematical modeling and integrates the selected solver. For this thesis, the software AMPL is utilized to serve this function.

5. Optimization Model

5.1 Introduction

In pursuit of validating our hypothesis regarding the triviality of travel distance distribution in the scheduling of the 2018 and 2014 World Cups, we perform independent modeling of these specific tournament scheduling problems. The primary goal is to construct models that adhere to the essential tournament constraints, and subsequently, tailor the models to the specific requirements of the tournament we replicate and the objective to optimize. Consequently, we initiate the modeling process by establishing a foundational binary ILP model geared towards minimizing the cumulative travel distance of all teams combined, named the “Total Distance Model”. This base model is further refined to incorporate specific tournament constraints and the selected fairness considerations, resulting in the “Total Inner Range Model” and the “Groupwise Inner Range Model”. The similarities and differences are described in detail in the following sections.

The results from the single objective models lead to the creation of a multi-objective optimization model: the ϵ-constraint model. The core of our multi-objective optimization approach lies in the creation of a Pareto front, illustrating the range of compromises among the conflicting objectives from the single objective models. This approach is described in section 5.4.

5.2 Data

For this research, we require data detailing the actual distances between base camps and venues, alongside group stage information from previous World Cups. We describe below the methodology for data extraction and outline considerations of the chosen approach, including necessary compromises.

5.2.1 Previous Tournament Schedules

Data regarding both the group stage matches and venue and base camp locations of the World Cups is sourced from Wikipedia due to its accessibility and ease of download (2014 FIFA

World Cup venues, 2023; 2014 FIFA World Cup, 2023; 2018 FIFA World Cup, 2023). The data is further cross-checked with details from FIFA's official records to ensure accuracy and reliability. The final group tables and match-related data are imported into R where they undergo a thorough cleaning process. Subsequently, this refined dataset is integrated with the additional data on venue and base camp locations. The final dataset facilitates a straightforward extraction of overall, group-specific, and single-team travel itineraries and the corresponding distances.

5.2.2 Distances

We adopt a structured approach for calculating distances by leveraging geospatial data. Specifically, we utilize the Google Maps API in conjunction with the *geocode* function in R to extract geographical coordinates (longitude and latitude). Subsequently, these coordinates facilitate the construction of a distance matrix. This matrix interconnects base camp sites with venues, thereby enabling the computation of inter-site distances utilizing the *distm* function. These distances in matrix form are further implemented as a parameter in the mathematical programming software AMPL.

It is however important to note that the distances derived in the distance matrix are transit approximations and do not reflect the precise travel paths undertaken by teams. This is because our model does not account for the mode of transportation - whether by bus, airplane, or a combination. Instead, our calculations are based on geodesic measurements, yielding a “highly accurate estimate of the shortest distance between two points on an ellipsoid” (distGeo: Distance on an ellipsoid (the geodesic), 2023)

While acknowledging the limitations of geodesic distance as an exact measure of the transits, we posit that substantial reductions in geodesic distance will presumably correspond to decreased actual travel distance as well. Consequently, we rationalize that employing geodesic distance measurements represents a pragmatic balance between the extensive resource expenditure required to obtain exact travel data and the utility of geodesic distances as a reasonable proxy for actual distances.

It is also important to note that the precision of these distances, with respect to base camp locations, is not absolute in terms of meter-level accuracy as we rely solely on city names for base camps. However, we employ an approximation method by considering the geographical area of the base camp cities and extracting the distances from the actual coordinates of the venues. Given the scope of the distances involved, often spanning several thousand kilometers, this method of estimation is deemed sufficiently accurate. The relevance of meter-level precision naturally diminishes when considering the magnitude of the relevant distances.

5.3 Single Objective Models

In this chapter, we outline the mathematical formulation of the single objective models.

5.3.1 Total Distance Model

Sets

The number of sets will be constant in each model, but the values must of course be modified to fit the exact tournament. There are 6 sets of data in the model:

$V = Venues$

$B = Base\ camps$

$G = Groups$

$T = Teams$

$M = Match\ days$

$R = Rounds$

Parameters

The number of parameters will be constant in each model, besides the last set of T_i^B. This parameter will be in use for the models that have a *pre-assigning* of base camps. If the model is to allocate base camps in line with the best objective, then this parameter is dropped from the model.

$T_i^G \in G, \forall\ i \in T$:	The group of team i
$Dist_{b,v}$, $\forall\ b \in B, v \in V$:	The distance from base camp b to venue v
$M_m^R \in R, m \in M$:	The round for match day m

$T_i^B \in B, i \in T$: The base camp for team i

Decision Variables

$$X_{i,b,v,m} = \begin{cases} 1 \text{ if team i travels from base camp b to venue v on matchday m} \\ 0 \text{ otherwise} \end{cases}$$

$$B_{i,b} = \begin{cases} 1 \text{ if team i is assigned to base camp b} \\ 0 \text{ otherwise} \end{cases}$$

$$Z_{m,i,v} = \begin{cases} 1 \text{ if team i is assigned to venue v on match day m} \\ 0 \text{ Otherwise} \end{cases}$$

$$Q_{m,v} = \begin{cases} 1 \text{ if venue v is used on match day m} \\ 0 \text{ Otherwise} \end{cases}$$

$$T_{i,j,m,v} = \begin{cases} 1 \text{ if team i is playing against team j on match day m at venue v} \\ 0 \text{ Otherwise} \end{cases}$$

$$Y_{g,m} = \begin{cases} 1 \text{ if the whole group g is playing on matchday m} \\ 0 \text{ Otherwise} \end{cases}$$

$$P_{m_1,m_2,g,r} = \begin{cases} 1 \text{ if group g plays within the consecutive match days } m_1 \text{ and } m_2 \text{ in round r} \\ 0 \text{ Otherwise} \end{cases}$$

Objective Function

As the scope of research is limited to the group stage travel, the objective function of the model is a total aggregation of the distance traveled from the base camp area to venue 1 and back, then from the base camp area to venue 2 and back, and finally the distance from the base camp area to venue 3, for all teams. It is important to note that travel distances beyond the conclusion of the final game were not included, as it no longer exerted any influence on performance during the group stage. This is factored in by subtracting the last term of the equation.

Minimize total distance:

$$2 \cdot \sum_{i \in T} \sum_{b \in B} \sum_{v \in V} \sum_{m \in M} Distance_{b,v} \cdot X_{i,b,v,m} - \sum_{i \in T} \sum_{b \in B} \sum_{v \in V} \sum_{m \in M:\, M_m^R = 3} Distance_{b,v} \cdot X_{i,b,v,m}$$

Constraints

Base camps

The following constraints are base camps specific. Constraint (1) is divided into two parts: Constraint (1.1) applies to the scenario where base camps are *pre-assigned*, and Constraint (1.2) addresses the situation where base camps have not been pre-allocated. Constraint (1.1) ensures that each team can only be assigned to one base camp during the whole group stage (pre-allocated, but we need the variable to be equal to 1 for the correct base camp) and (1.2) that each team can only be assigned to one base camp during the whole group stage. Constraint (2) ensures that if a team is assigned to base camp b, then it must always travel from the same base camp b to any venue v it plays at. It is divided into two parts: (2.1) applies to the case with *pre-assigned* base camps, where base camp b is given by the T_i^B parameter and (2.2) applies to the case without *pre-assigned* base camps. Constraint (3) takes care of the fact that a team that plays at the venue must make the actual travel from base camp b.

$$\sum_{b \in B:\, T_i^B = b} B_{i,b} = 1, \quad \forall \ i \in T \tag{1.1}$$

$$\sum_{b \in B} B_{i,b} = 1 \tag{1.2}$$

$$\sum_{v \in V} \sum_{m \in M} X_{i,b,v,m} = 3 \cdot B_{i,b}, \quad \forall \ i \in T, b \in B: T_i^B = b \tag{2.1}$$

$$\sum_{v \in V} \sum_{m \in M} X_{i,b,v,m} = 3 \cdot B_{i,b}, \quad \forall \ i \in T, b \in B \tag{2.2}$$

$$\sum_{b \in B} X_{i,b,v,m} \leq Z_{m,i,v}, \qquad \forall\, m \in M, v \in V, i \in T \tag{3}$$

Venues

Constraint (4) ensures that each venue is assigned exactly four matches in total during the group stage, constraint (5) that each venue should be used a maximum of 2 times per round, and (6) that each venue should be used at least one time per round. Constraint (7) ensures that either 0 or 2 *teams* must be present at a venue during a match day to ensure that a team is not assigned to a match without an opponent. In addition, a venue cannot host more than one match per matchday. Constraint (8) ensures that a team is not to be assigned to the same venue more than once, and (9) decides the minimum *n* rest days for a venue in between hosting a match. For the 2014 World Cup in Brazil and the 2018 World Cup in Russia, the minimum number of rest days, *n*, is 1 and 2 respectively. Here, the number of rest days corresponds to the number of whole days *between* two matchdays, the period where the venue should not be in use.

$$\sum_{m \in M} Q_{m,v} = 4, \qquad \forall\, v \in V \tag{4}$$

$$\sum_{m \in M: M_m^R = r} Q_{m,v} \leq 2, \qquad \forall\, v \in V, r \in R \tag{5}$$

$$\sum_{m \in M: M_m^R = r} Q_{m,v} \geq 1, \qquad \forall\, v \in V, r \in R \tag{6}$$

$$\sum_{i \in T} 0.5 \cdot Z_{m,i,v} = Q_{m,v}, \qquad \forall\, v \in V, m \in M \tag{7}$$

$$\sum_{m \in M} Z_{m,i,v} \leq 1, \qquad \forall\, v \in V, i \in T \tag{8}$$

$$\sum_{i \in T} \sum_{j=0}^{n} Z_{m+j,i,v} \leq 2, \qquad \forall\, v \in V, m \in M : m \leq |\, M \,| - n \tag{9}$$

Matches

Constraint (10) ensures that a team can play a maximum of one match per day, in the sense of being assigned to a venue maximum one time per match day, and (11) that each team should play one game each round. Constraint (12) says that every team should play exactly three matches during the group stage and (13) that they must have at least *n* rest days between two consecutive matches. For the 2014 World Cup in Brazil and the 2018 World Cup in Russia, the minimum number of rest days for a team, n, is 2 and 3 respectively. The number of rest days corresponds to the number of whole days *between* two matchdays. Constraint (14) secures that the maximum number of games at any given matchday should be 4, implied by a maximum of 8 teams playing, and (15) that the total matches per matchday in *round 3* should be exactly equal to 4, also implied by 8 teams playing. Further, constraint (16) ensures that the minimum number of games at any given matchday, except for the opening day, should be 2, indicating a minimum of 4 teams playing. On the opening day, the first match is the only match, as ensured by constraint (17), and constraint (18) ensures that the host nation is set to play in that match.

$$\sum_{v \in V} Z_{m,i,v} \leq 1, \qquad \forall\ i\ \in T, m \in M \tag{10}$$

$$\sum_{v \in V} \sum_{m \in M : M_m^R = r} Z_{m,i,v} = 1, \qquad \forall\ r \in R, i \in T \tag{11}$$

$$\sum_{m \in M} \sum_{v \in V} Z_{m,i,v} = 3, \qquad \forall\, i\ \in T \tag{12}$$

$$\sum_{v \in V} \sum_{j=0}^{n} Z_{m+j,i,v} \leq 1, \qquad \forall\, i \in T, m \in M : m \leq |\, M \,| - n \tag{13}$$

$$\sum_{v\in V}\sum_{i\in T} Z_{m,i,v} \leq 8\,, \qquad \forall\ m \in M \tag{14}$$

$$\sum_{v\in V}\sum_{i\in T} Z_{m,i,v} = 8\,, \qquad \forall\ m \in M{:}\, M_m^R = 3 \tag{15}$$

$$\sum_{v\in V}\sum_{i\in T} Z_{m,i,v} \geq 4\,, \qquad \forall\ m \in M{:}\, m \neq 1 \tag{16}$$

$$\sum_{v\in V}\sum_{i\in T} Z_{m,i,v} = 2, \qquad \forall m \in M{:}\, m\ = 1 \tag{17}$$

$$\sum_{v\,\in\,V} Z_{m,i,v} = 1, \qquad \forall\ i \in T, m \in M{:}\, v = Opening\ venue, i = Host\ nation\ and\ m = 1 \tag{18}$$

Specific Constraints for Round 1

The three following constraints are specified for round 1. These constraints aim to stipulate that all teams within a group must play within two consecutive days in round 1. This introduces two different scenarios. Both matches can be held on the same day, or one match is held the day before the other in that group. To formulate this into the model, we have constructed constraint (19) which allows for the possibility that the two matches are scheduled over two days and constraint (20) which forces both matches to be held within a single day. Since only one of the two scenarios can be true, constraint (21) assures that only one of these sequences will occur.

$$\sum_{i\in T:T_i^G=g}\ \sum_{v\in V}\sum_{j=0}^{1} Z_{m+j,i,v} \geq 4\cdot P_{m,m+1,g,r}\,, \qquad \forall\ r \in R, g \in G, m \in M\ :\ m \in \{1..4\}\ \&\ r = 1 \tag{19}$$

$$\sum_{i\in T:T_i^G=g}\ \sum_{v\in V} Z_{mj,i,v} \geq 4\cdot P_{m,m,g,r}\,, \qquad \forall\ r \in R, g \in G, m \in M\ :\ m \in \{1..5\}\ \&\ r =\ 1 \tag{20}$$

$$\sum_{m\in\{1..4\}}\ \sum_{j=0}^{1} P_{m,m+j,g,r} + P_{5,5,g,r} = 1, \qquad \forall\ g \in G, r \in R\ :\ r = 1 \tag{21}$$

Specific Constraints for Round 2

The three constraints implemented in Round 2 mirror those established in Round 1, reflecting the identical scheduling practices applied to both rounds. However, the constraints are divided to guarantee that every group completes their first matches by the end of Round 1, which is defined by a specific number of match days. Under the current mathematical logic, combining these constraints would unintentionally allow for a possibility where teams within a group play their first matches in Round 2. This situation is impractical and contradicts the intended scheduling design. Constraint (22) allows for the possibility that the two matches are scheduled within two consecutive days, and constraint (23) forces both matches to be held within a single day. Since only one of the two scenarios can be true, constraint (24) assures that only one of these sequences will occur.

$$\sum_{i \in T: T_i^G = g} \sum_{v \in V} \sum_{j=0}^{1} Z_{m+j,i,v} \geq 4 \cdot P_{m,m+1,g,r}, \quad \forall\, r \in R, g \in G, m \in M : m \in \{6..10\}\ \&\ r = 2 \tag{22}$$

$$\sum_{i \in T: T_i^G = g} \sum_{v \in V} Z_{mj,i,v} \geq 4 \cdot P_{m,m,g,r}, \quad \forall\, r \in R, g \in G, m \in M : m \in \{6..11\}\ \&\ r = 2 \tag{23}$$

$$\sum_{m \in \{6..10\}} \sum_{j=0}^{1} P_{m,m+j,g,r} + P_{11,11,g,r} = 1, \quad \forall\, g \in G, r \in R :\ r = 2 \tag{24}$$

Specific Constraints for Round 3

The following constraints are only specified for round 3 and ensure that all teams within a group should play on the same matchday in round 3. The round-specific constraints for the third round are consequently different than for the other two rounds. Constraint (25) ensures that all four teams – or both matches – in a group g must play on the same day, while constraint (26) guarantees that all groups are assigned to a single match day in round 3.

$$\sum_{v \in V} \sum_{i \in T: T_i^G = g} Z_{m,i,v} \geq 4 \cdot Y_{g,m}, \quad \forall\ g \in G, m \in M: M_m^R = 3 \tag{25}$$

$$\sum_{m \in M:\, M_m^R = 3} Y_{g,m} = 1, \qquad \forall\, g \in G \tag{26}$$

Logical Constraints

Constraint (27) ensures that the teams compete exclusively with other teams in their group by prohibiting two teams belonging to different groups from being assigned to the same venue on the same day. Constraint (28) verifies that a situation in which team i competes against team j is equivalent to the situation where team j faces team i, and constraint (29) guarantees that team i plays against team j only once during the group stage. Finally, constraint (30) links the Z variable and T variable together. If team i is allocated to venue v on matchday m, their opponent - team j – is assigned to the same venue on the same match day.

$$Z_{m,i,v} + Z_{m,j,v} \leq 1, \qquad \forall\, i \in T, j \in T, g \in G, m \in M, v \in V: T_i^G = g \text{ and } T_i^G \neq g \tag{27}$$

$$T_{i,j,m,v} = T_{j,i,m,v}, \qquad \forall\, m \in M, v \in V, i \in T, j \in T, g \in G : i \neq j \tag{28}$$

$$\sum_{m \in M, v \in V} T_{i,j,m,v} = 1, \qquad \forall\, i \in T, j \in T, g \in G: i \neq j \tag{29}$$

$$Z_{m,i,v} = \sum_{j \in T: T_i^G = g \text{ and } i \neq j} T_{i,j,m,v}, \qquad \forall\, i \in T, g \in G, m \in M, v \in V : T_i^G = g \tag{30}$$

5.3.2 Total Inner Range Model

In the new model for minimizing the total inner range, the sets, parameters, and constraints from the Total Distance Model are unchanged. We address the same problem but shift our focus to a different objective. In addition to the existing variables, we introduce a new decision variable D_i to monitor and decide each team's cumulative distance traveled:

$$D_i = 2 \cdot \sum_{b \in B} \sum_{v \in V} \sum_{m \in M} Distance_{b,v} \cdot X_{i,b,v,m} - \sum_{b \in B} \sum_{v \in V} \sum_{m \in M} Distance_{b,v} \cdot X_{i,b,v,m}\,, \qquad \forall\ i \in T$$

This is similar to the objective function in the Total Distance Model, but instead of performing a sum over the distance of each team, the variable tracks this distance for each team instead.

Moreover, we employ auxiliary variables $MaxDistance$ and $MinDistance$ to keep a record of the extreme values of D_i across all teams. These variables are adjusted dynamically to reflect the new extremes as they occur. The new problem can be formulated as:

Minimize total inner range:

$$MaxDistance - MinDistance$$

s.t

$$MaxDistance \geq D_i, \qquad \forall\ i \in T \tag{31}$$

$$MinDistance \leq D_i, \qquad \forall\ i \in T \tag{32}$$

All other constraints (1) – (30)

5.3.3 Groupwise Inner Range Model

In the model aimed at minimizing the inner range *within groups*, the sets, parameters, and constraints from the Total Distance Model are still retained, but with another shift in the objective. The decision variable D_i, which measures the distance each team travels, is consistent with the one used in the Total Inner Range Model.

In this version, the auxiliary variables *MaxDistance* and $MinDistance$ are used to track the longest and shortest distances traveled by the teams within their respective groups. These variables are regularly updated to the latest maximum or minimum values of D_i within each group. The updated problem can be formulated as:

Minimize the sum of the groupwise inner ranges:

$$\sum_{g \in G} MaxDistance_g - MinDistance_g$$

s.t

$$MaxDistance_g \geq D_i, \qquad \forall\, i \in T, g \in G :\ T_i^G = g \tag{33}$$

$$MinDistance_g \leq D_i, \qquad \forall\, i \in T, g \in G :\ T_i^G = g \tag{34}$$

All other constraints (1) – (30)

By focusing on reducing the aggregate of all inner ranges, we naturally encourage the minimization of the individual inner ranges within each group.

5.4 Multi-Objective Model

5.4.1 The ϵ-Constraint Model

We have chosen the minimization of the total distance to be the objective to be minimized, given a maximum limit constraint on the total inner range. This limit is adjusted within established Utopia and Nadir boundary points. The rationale behind having total distance as our objective lies in the ability to validate solution quality through the observable MIP Gap. Due to our limited computational resources, we cannot ascertain a single, optimal solution and must rely on approximations. In an ideal scenario, where solutions are optimally derived, the choice of which objective to minimize or constrain becomes irrelevant. However, under conditions of early termination, we are settling on sub-optimal solutions that might yield unanticipated outcomes and fail to accurately demonstrate the pattern of trade-offs involved. Therefore, having access to a significant MIP Gap is crucial for identifying such deviations.

Reducing the total inner range to a satisfactorily low level should concurrently ensure that the groupwise inner range is maintained at an acceptable maximum. This indicates that for a multi-objective model addressing all three conflicting objectives, it is sufficient to keep track of the value of the total inner range to ensure the solution stays within a chosen upper limit of possible groupwise inner ranges. This simplifies the optimization process, as we deal with a dual-objective problem where one objective is to be optimized subject to an allowable deterioration of the other as a trade-off.

Model Formulation

The objective is to minimize total distance, with the upper limit on the total inner range being included as a constraint. The maximum allowable value of the total inner range is denoted ϵ. Due to the total inner range now being a constraint and not an objective, we must introduce some new auxiliary variables to make sure the $MaxDistance$ and $MinDistance$ accurately take the value of the maximum and minimum distance traveled, while maintaining linearity. This is accomplished by a sophisticated application of the Big M method.

We create two additional binary variables:

$$U_i = \begin{cases} 1 \text{ if the MaxDistance is taking the value of } \mathrm{D_i} \\ 0 \text{ otherwise} \end{cases}$$

$$L_i = \begin{cases} 1 \text{ if the MinDistance is taking the value of } \mathrm{D_i} \\ 0 \text{ otherwise} \end{cases}$$

To determine the range of epsilon values, we evenly divide the span between the Utopia point and the Nadir point into equal segments. Each segment represents an incremental from its preceding value resulting in a set of epsilons. Then, for each ϵ_i, we solve the optimization problem:

Minimize total distance:

$$2 \cdot \sum_{i \in T} \sum_{b \in B} \sum_{v \in V} \sum_{m \in M} Distance_{b,v} \cdot X_{i,b,v,m} - \sum_{i \in T} \sum_{b \in B} \sum_{v \in V} \sum_{m \in M:\, M_m^R = 3} Distance_{b,v} \cdot X_{i,b,v,m}$$

s.t

$$MaxDistance \geq D_i, \qquad \forall\, i \in T \tag{35}$$

$$MaxDistance \leq D_i + M \cdot (1 - U_i), \qquad \forall\, i \in T, M = 60{,}000\ km \tag{36}$$

$$MinDistance \leq D_i, \qquad \forall\, i \in T \tag{37}$$

$$MinDistance \geq D_i - M \cdot (1 - L_i), \qquad \forall\, i \in T, M = 60{,}000\ km \tag{38}$$

$$\sum_{i \in T} U_i = 1 \tag{39}$$

$$\sum_{i \in T} L_i = 1 \tag{40}$$

$$MaxDistance - MinDistance \leq \epsilon \tag{41}$$

All other constraints (1) – (30)

The Big M method ensures that the model selects $MaxDistance$ as the greatest distance traveled, and $MinDistance$ as the minimum distance traveled, among all teams. The value of M is thus decided such that it does not restrict this selection, being larger than the absolute maximum of distances possible traveled by any team. Yet it remains within a reasonable range to prevent approximation issues during the solving process. The final set of solutions constitutes the Pareto front, offering a spectrum of possible non-dominated solutions for the decision-maker to choose from.

6. Results

We will in this chapter present the results from the optimization models. Initially, we explore the outcomes derived from the single objective models. To facilitate an easy comparison of the objective values, the results are organized into detailed tables, accompanied by a discussion of the insights gathered. We divide the presentation of our findings into two tables, one for 2014 and one for 2018. Each table is structured to present the historical record as a benchmark, with the corresponding optimization results for each model – covering the two different scenarios with *pre-assigned* and *unassigned* base camps. Additionally, we offer detailed graphical representations of each team's covered distances to enhance comparative analysis, alongside visualizations of selected teams' travel routes on maps. These graphics underscore the model's capability in not just obtaining a final objective value, but also in crafting detailed tournament schedules that are immediately actionable for the decision-makers of the World Cup.

Secondly, we present the results from the multi-objective model. As we anticipated, the insights obtained from the single-objective models did necessitate the adoption of a multi-objective approach. We showcase the Pareto front and provide an extensive discussion of our findings.

Finally, we summarize the outcomes of the analysis of the past tournaments before we shift our attention toward a discussion regarding the significance of our findings for the planning of the World Cup 2026.

Regarding the outcomes of the Single Objective Models, the comprehensive data on each team's travel distances across all model solutions is also available in a tabulated format in Appendix 10.1 and 10.2. For the Multi-Objective Model, an equivalent table is presented in Appendix 10.3. Moreover, each model inherently produces comprehensive match fixtures derived from the outcomes of the decision variables, outlining the round number, date, opposing teams, and venue for each match. Interested readers looking to review the alternative match fixtures, specific to each model, can find these in Appendix 10.5.

To formulate the mathematical models, we have used the software AMPL and chosen Gurobi as our solver engine. The AMPL-files are attached in an external appendix and are briefly explained in Appendix 10.4. While conducting the final runs of our models and obtaining the best possible results presented in this thesis, we take advantage of the NEOS Server, which is an internet-based service for solving numerical optimization problems. The server is hosted by the Wisconsin Institute for Discovery. For information about the solver, see Czyzyk et al., Dolan (2001), and Gropp & Moré (1997). The server has a limitation of 8 hours in run time, meaning that all our results are retrieved after the problem has been iterating for 8 hours. To be able to find a feasible solution within this time restriction, we modified the Gurobi solver to focus on finding feasible solutions quickly rather than focusing on proving optimality for the models. This has led to sub-optimal solutions at termination.

Furthermore, when evaluating the MIP Gaps of the solutions presented, it is worth noting that for the models without *pre-assigned* base camps, the Lower Bound for the integer-relaxed problem is exceedingly low, making it considerably distant from the feasible solutions of the integer-constrained counterparts. This is due to the allowance for teams to be partially assigned across multiple base camps, which in turn affects the calculated distances. As a result, the MIP Gap is expected to be high for these models. However, once base camps are *pre-assigned*, the variables are constrained to integer values even in the relaxed problem, leading to a more accurate and potentially achievable Lower Bound for the optimal constrained solution.

6.1 Results Single Objective Models

6.1.1 Results and Discussion 2014 Models

Table 6.1 presents a comprehensive comparison of each model's performance across various metrics. The models are listed on the left, and the three primary objectives of the models are found on the right side with their respective main objective outcomes emphasized in bold for clarity. We have included the full spectrum of objective values for every model to facilitate a thorough comparison.

To ensure the reliability of our results, “Solution Limits” are incorporated in the table. Where applicable, the MIP Gap is reported. However, for models aimed at minimizing the inner range, the MIP Gap - calculated as the current solution value minus the Lower Bound, divided by the current solution value - consistently registers as 100% since the Lower Bound is zero, demonstrating a relaxed feasible solution where the Inner Range indeed is zero. To prevent any misinterpretation of the models' efficiency, we have marked “N/A” in the table where the

MIP Gap calculation does not yield meaningful insights into the solution's performance. We know that the solutions are sub-optimal, but an exact evaluation of the results at termination is challenging without a valid MIP Gap. Nonetheless, given the positive outcomes observed so far, having access to a longer runtime or tuning parameters only holds the potential for improvements.

Table 6.1: *Results for single objective models: 2014 World Cup*

		2014 Models					
		Pre-assigned Basecamps			*Unassigned Basecamps*		
	Historical Benchmark	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**
Objectives							
Total Distance	195 909	**146 964**	208 731	200 665	**114 282**	253 437	232 720
Total Inner Range	10 524	7 708	**1 973**	7 075	6 223	**468**	4 750
Total Groupwise Inner Range	34 688	30 018	9 413	**3 699**	25 774	2 609	**808**
Solution Limits							
MIP Gap	-	*0.07 %*	*N/A*	*N/A*	*40.40 %*	*N/A*	*N/A*
Cutoff Time	-	*8h*	*8h*	*8h*	*8h*	*8h*	*8h*

Beginning with the *pre-assigned* base camps category, the performance of the Total Distance Model is evident. Its primary objective is to minimize the total distance traveled by all teams, and it achieves this prominently, reducing the overall distance by approximately 25%. This substantial reduction underscores the model's effectiveness in its core goal. Moreover, the model demonstrates a MIP Gap of only 0.07%, which is indicative of a highly reliable solution. When we turn our attention to the fairness metrics, the model continues to show improvements over historical benchmarks, despite these objectives not being explicitly targeted for minimization in the model's formulation. However, it is important to note that there are still noticeable variations both within individual groups and in the total range of distances traveled. These variations, while diminished, highlight areas for potential further refinement in balancing all objectives.

Upon analyzing the remaining two models, the Total Inner Range Model and the Groupwise Inner Range Model, we observe a parallel trend. These models perform exceedingly well with respect to their primary goals, demonstrating notable improvements over historical benchmarks. Respectively, an improvement of approximately 81% for the total inner range and 89% for the total groupwise inner range objectives. Yet, when looking at the corresponding value of total distance, the outcomes are not as favorable, though the increases observed relative to the benchmark are not overly substantial. Nonetheless, the rise in total distance underscores an area for improvement, and balancing the dual objectives of minimizing distance and enhancing fairness remains a key focus for the further development of these models.

Transitioning to the *unassigned* base camps category, a similar pattern emerges. In direct comparison to the historical benchmark, we find substantial performance when the main objectives of the models are evaluated independently of the other metrics. In numbers, the Total Distance Model achieves a 42% reduction, the Total Inner Range Model registers a 96% reduction, and the groupwise inner range exhibits an impressive 98% reduction. The models continue to exhibit significant enhancements over the benchmark in their respective objectives, albeit with a great necessity for improvement in terms of objective balancing.

Another interesting output of this analysis is the relative difference between the *unassigned* and the *pre-assigned* base camps categories. The relaxation of the problem, where the model itself allocates the teams to a base camp according to the objective criteria, is showing notable results compared to the constrained approach that adheres to historical allocations. In sequential order, as displayed in the table, we observe improvements in objective values by 22%, 76%, and 78% upon relaxing the base camp constraint. These results suggest that adopting a model-driven strategy for base camp allocation could be a strategic move for future World Cups worth serious consideration.

Graphical Demonstration of Results 2014

To visually illustrate our findings, we present each team's cumulative travel distance through graphical plots. These plots not only showcase the variation in travel distances among teams but also highlight the comparative differences between the *pre-assigned* base camp solutions and the more dynamic model-driven base camp allocation approach. Additionally, for the Total Distance Model and the Total Inner Range Model, we identify the teams with the shortest and longest routes and exhibit their complete itineraries on maps – the shortest route on the top and the longest route underneath. In the case of the Groupwise Inner Range Model, we rather focus on identifying the groups with the shortest and longest routes combined and collectively map these routes, aligning more with the model's objective.

Total Distance Model

Figure 6.1 displays the distribution of travel distances. Given *pre-assigned* base camps, we obtain a distribution ranging from Spain with 9,191 kilometers to Bosnia and Herzegovina with 1,483 kilometers. The distribution is highly uneven, however, the total distance covered by any team is reduced relative to the historical benchmark, where the United States covered over 12,000 kilometers, as depicted in section 2.3. Further, comparing the two base camp allocation approaches, the model-driven allocation excels both in total distance and relative distribution, having a range from Japan's 7,429 kilometers to Honduras's 1,206 kilometers.

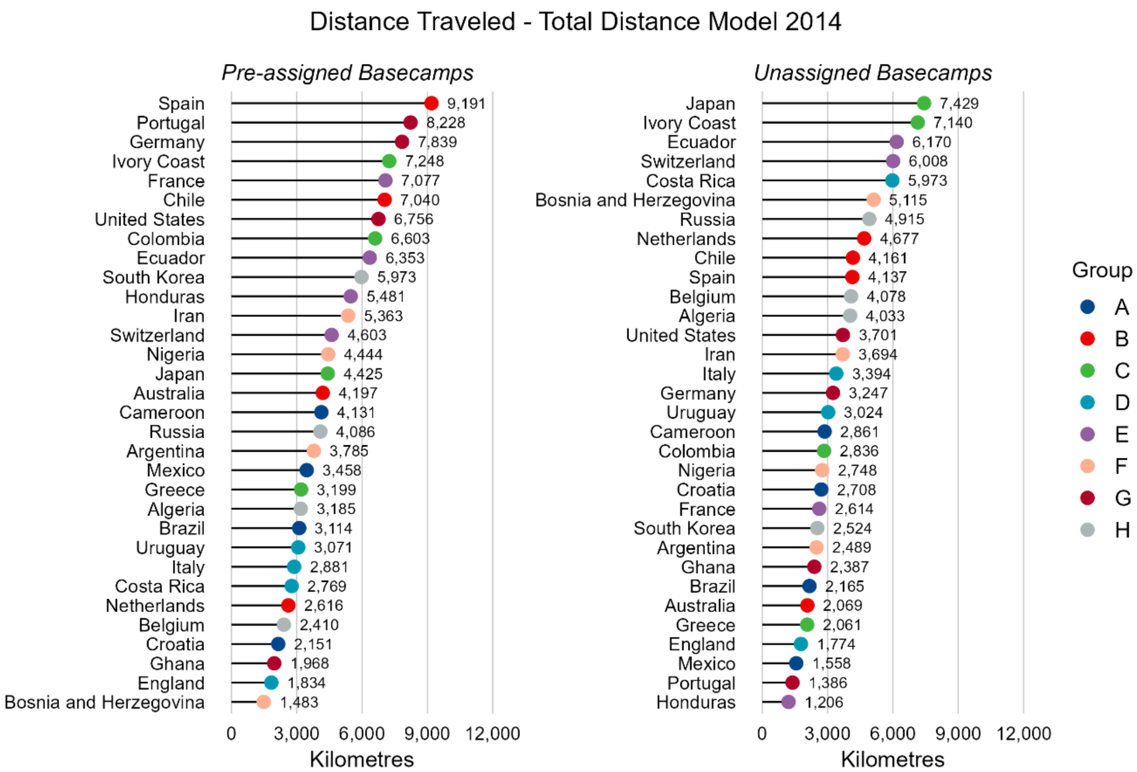

Figure 6.1: *Distance traveled for all teams from the Total Distance Model 2014*

When illustrated on maps, the outcomes are not as clearly depicted. There are still major differences in the routes between the longest and shortest traveling teams. However, these maps should carefully be evaluated with the knowledge and insights from the tables and graphical plots in mind, knowing that the total distance covered by all teams combined is indeed an improvement relative to the historical benchmark.

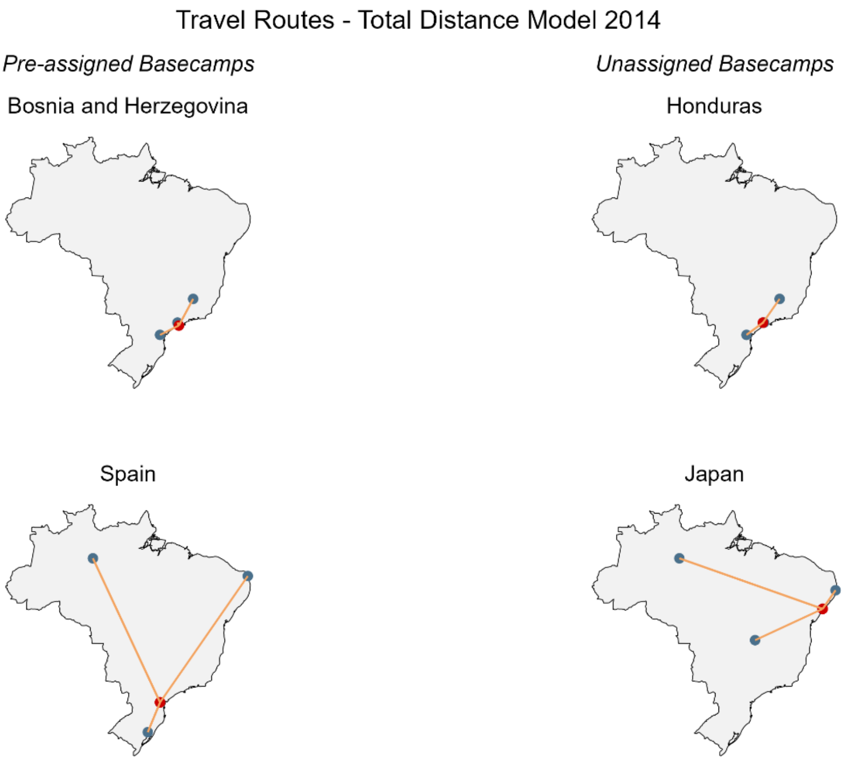

Figure 6.2: *Travel routes from the Total Distance Model 2014*

Total Inner Range Model

Moving to the Total Inner Range Model, we observe a significantly more uniform distribution of travel distances. Every team's travel distance falls within the total inner range, resulting in a more balanced and, consequently, fairer distribution of the travel burden.

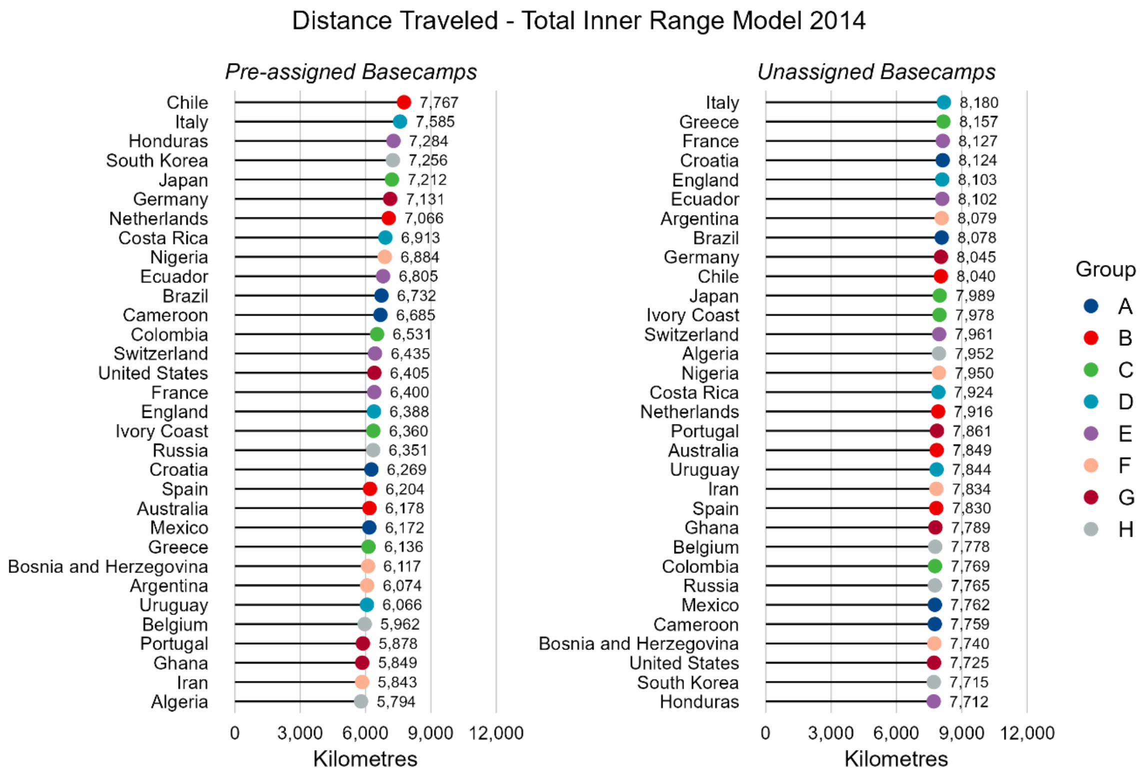

***Figure 6.3**: Distance traveled from the Total Inner Range Model 2014*

Upon examining the maps, the sense of fairness in the tournament schedule becomes even more apparent. The travel routes of the shortest (top map) and the longest (bottom map) traveling teams are remarkably similar when compared to both the historical benchmark and the Total Distance Model.

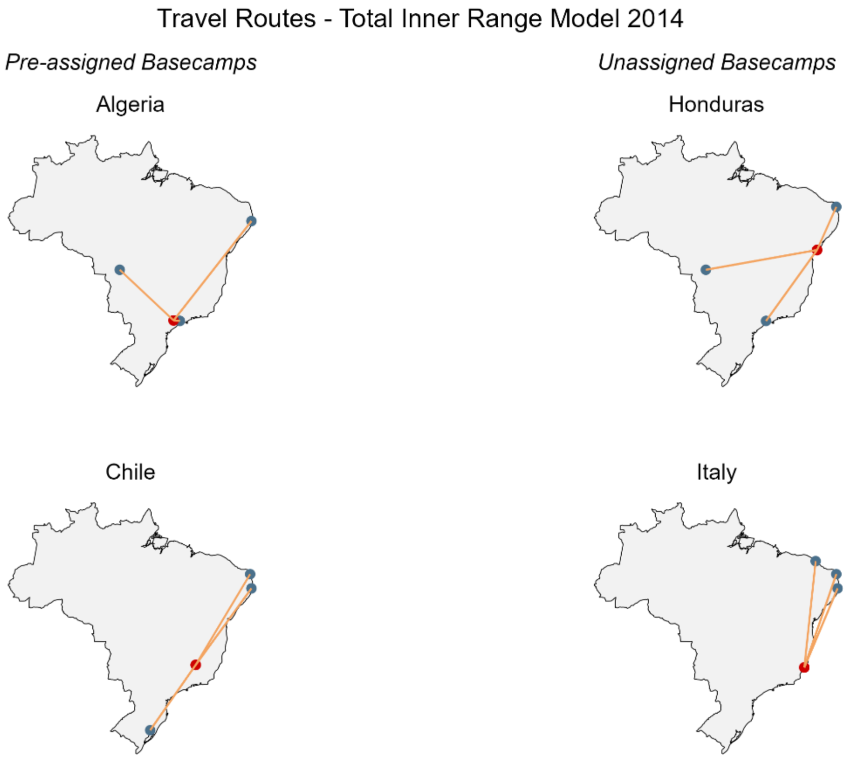

***Figure 6.4**: Travel routes from the Total Inner Range Model 2014*

Groupwise Inner Range Model

Finally, the distribution of distances from the Groupwise Inner Range Models depicts a near uniform distribution of distances within each group, but with a high range between the longest traveling and the shortest traveling groups. There is a high level of fairness within each group, but a decrease in the perceived fairness from the more holistic perspective.

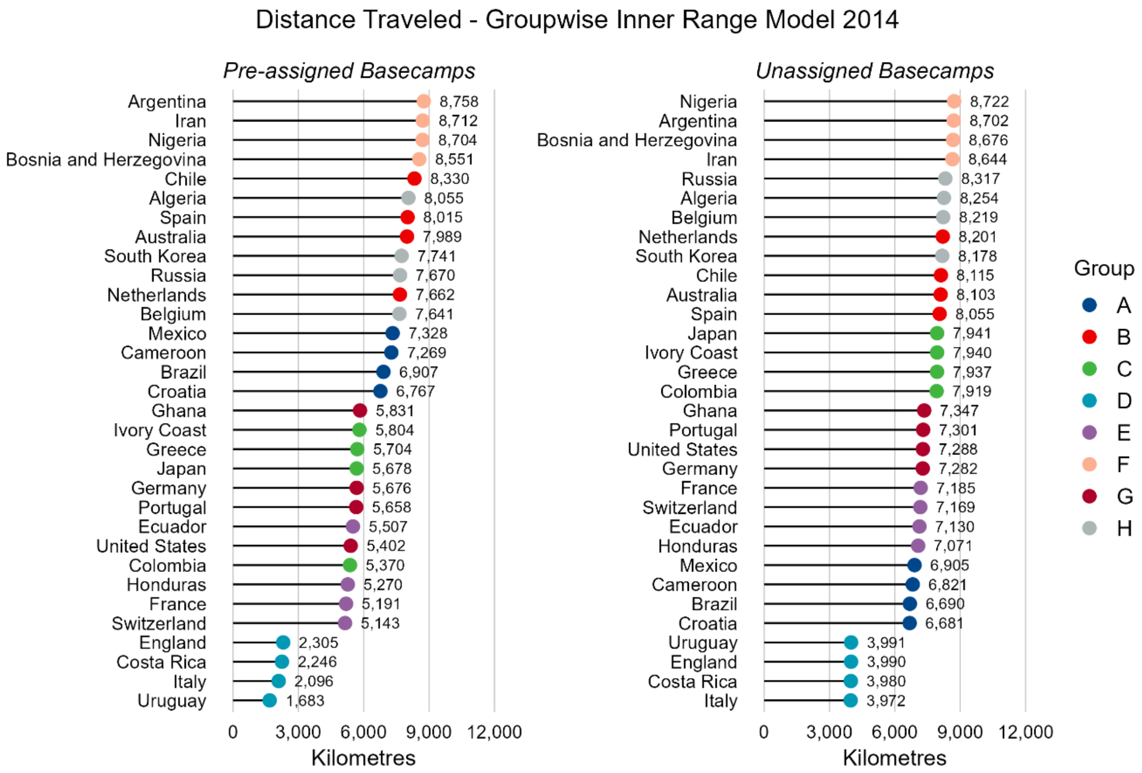

***Figure 6.5**: Distance traveled from the Groupwise Inner Range model 2014*

These observations are also reflected in the travel routes illustrated on the maps. We notice that teams with the shortest travel distances have their base camps and venues clustered within a tight radius, whereas teams in the longest traveling group traverse greater distances across Brazil. When these routes are plotted collectively, they reveal a fair distribution of travel burden within each group. However, this also highlights an increased disparity between the groups, making the collective difference in travel burden more pronounced.

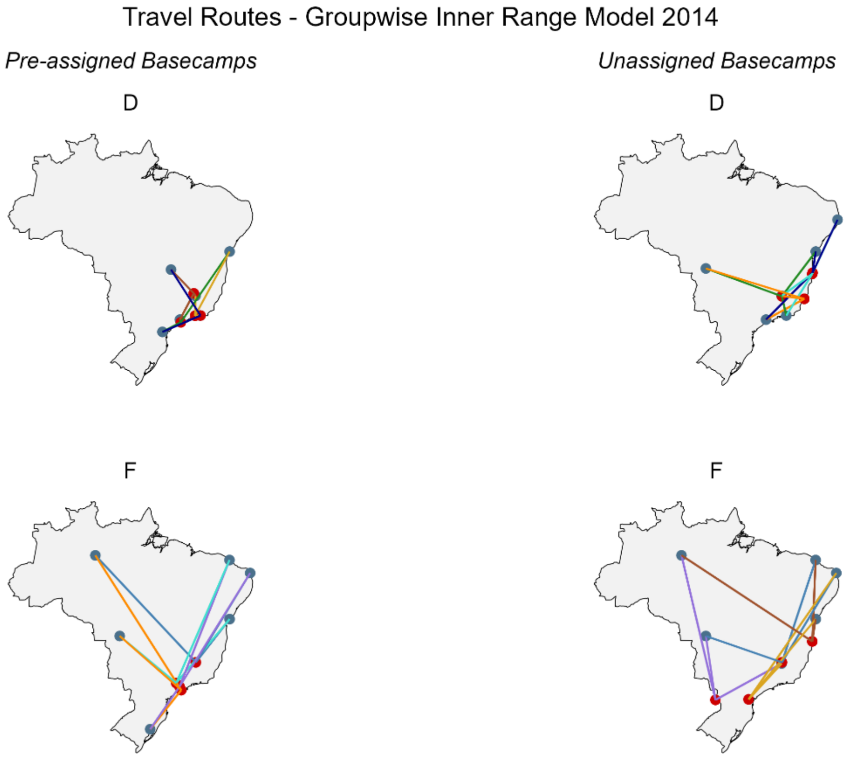

Figure 6.6: *Travel routes from the Groupwise Inner Range Model 2014*

Insights on the Balancing of the Selected Fairness Metrics

The combined quantitative and graphical analysis reveals a significant finding regarding the objective of balancing fairness criteria. It is demonstrated that by focusing on minimizing the total inner range, an adequately low groupwise inner range is inherently achieved. This is due to the nature of the total inner range encompassing the maximum possible variance within a group. Though evident in **Table 6.1**, the graphical depiction of the total inner range (**Figure 6.3**) further clarifies this notion, as demonstrated by the moderate slope of the curve spanning from its lowest to highest points, which is particularly noticeable in the *unassigned model*, but also apparent in the *pre-assigned model*.

In Section 3.2, we explored a hypothetical situation in which an optimal total inner range of 10,000 could lead to significant imbalance, especially since, in this hypothetical case, this range was concentrated within a single group. Therefore, the concern about the potential imbalance of fairness criteria in an optimal solution was indeed well-founded. However, upon implementing the proposed model, the total inner range was successfully reduced to values of 1,972 in the *pre-assigned model* and 468 in the *unassigned model*. Such substantial decreases guarantee that the largest potential difference between any teams in a group is limited to these values – 1,972 and 468 respectively – which can be deemed sufficiently low to satisfy both fairness criteria under consideration.

Consequently, this finding suggests that the allocation of resources towards specifically minimizing the groupwise inner range might be redundant, given that the total inner range has reached an acceptable threshold. Focusing solely on the total inner range not only simplifies the optimization procedure but also offers a more cost-effective method for ensuring equitable distribution among groups – given that the threshold is sufficiently low during the first model runs. Such conclusions will however be up to the decision maker, depending on the relative perceived importance of the different metrics of fairness.

6.1.2 Results and Discussion 2018 Models

Examining **Table 6.2**, we see the same patterns as for the 2014 results. Each model notably surpasses its primary objective, demonstrating significant enhancements over historical benchmarks. As we progress from left to right in the table, the *pre-assigned* base camp models show remarkable improvements, achieving respective gains of 33%, 82%, and 83%. Furthermore, as anticipated, the *unassigned models* perform even better than the *pre-assigned models*, yielding improvements over the historical benchmark of 48%, 93%, and 91%. Direct comparisons reveal that *unassigned models* exhibit improvements of 22%, 63%, and 87% over their *pre-assigned* counterparts. The improvements are illustrated graphically in the next sub-section.

***Table 6.2**: Results for single objective models: 2018 World Cup*

		2018 Models					
		Pre-assigned Basecamps			*Unassigned Basecamps*		
	Historical Benchmark	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**
Objectives							
Total Distance	141 914	**95 728**	136 361	137 086	**74 470**	137 089	156 659
Total Inner Range	7 635	6 499	**1 387**	3 376	3 108	**520**	5 275
Total Groupwise Inner Range	31 576	24 774	8 315	**5 326**	11 270	3 182	**699**
Solution Limits							
MIP Gap	-	*0.07 %*	*N/A*	*N/A*	*72.30 %*	*N/A*	*N/A*
Cutoff Time	-	*8h*	*8h*	*8h*	*8h*	*8h*	*8h*

Despite the proven advancements over the historical benchmark with the use of our replicating model for the 2018 edition of the World Cup, there is still room for improvement of objective balancing in all versions of this single objective model. We therefore conclude that the insights gained from the 2018 models are fully in line with the results from the 2014 edition.

Graphical Demonstration of Results 2018

The graphical representation of the 2018 models, including both plots and mapped routes, closely mirrors the results from 2014. Given this similarity, an exhaustive explanation is considered unnecessary. However, we have included these graphical results to visually illustrate and substantiate the model's capabilities in creating improved tournament schedules and travel routes beyond just the satisfactory final objective values.

Total Distance Model

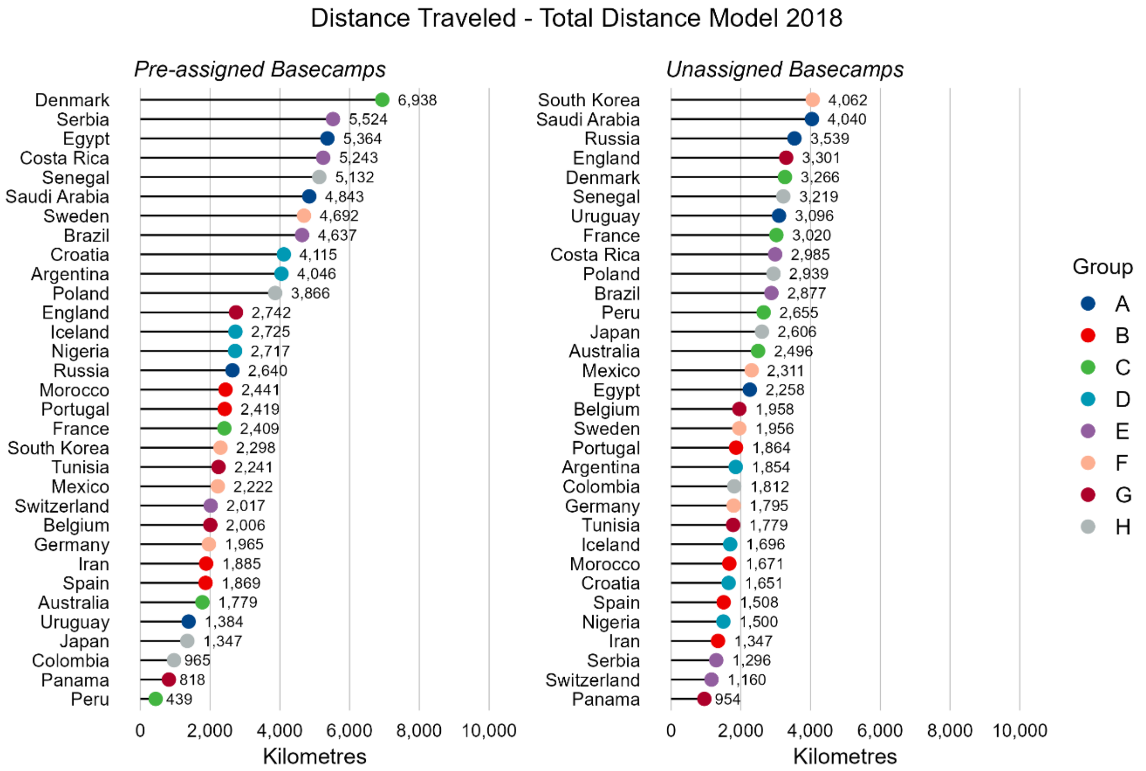

Figure 6.7: *Distance traveled from the Total Distance Model 2018*

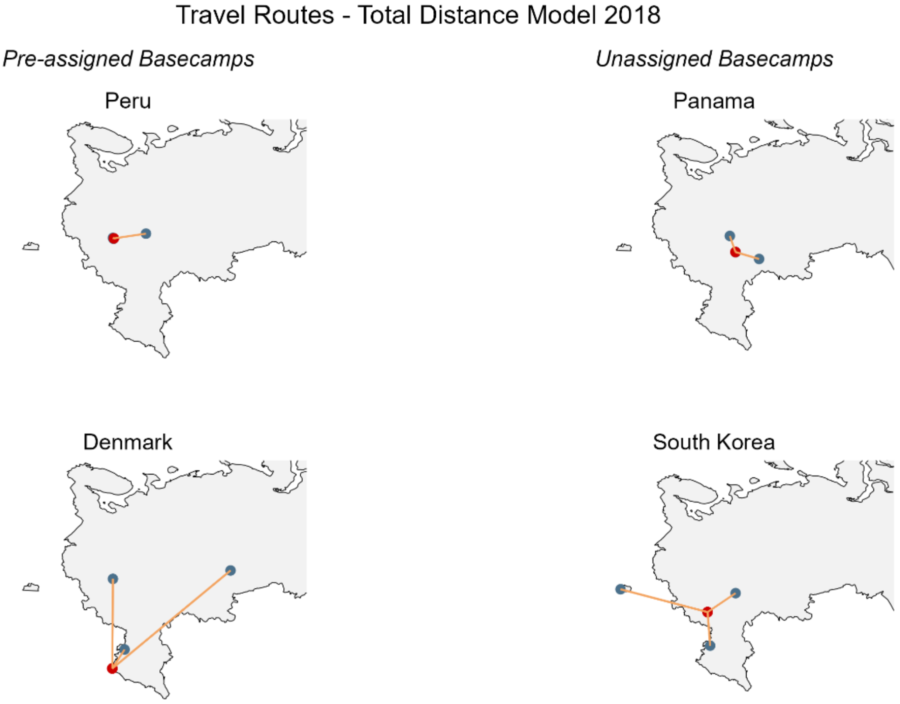

Figure 6.8: *Travel routes from the Total Distance Model 2018*

Total Inner Range Model

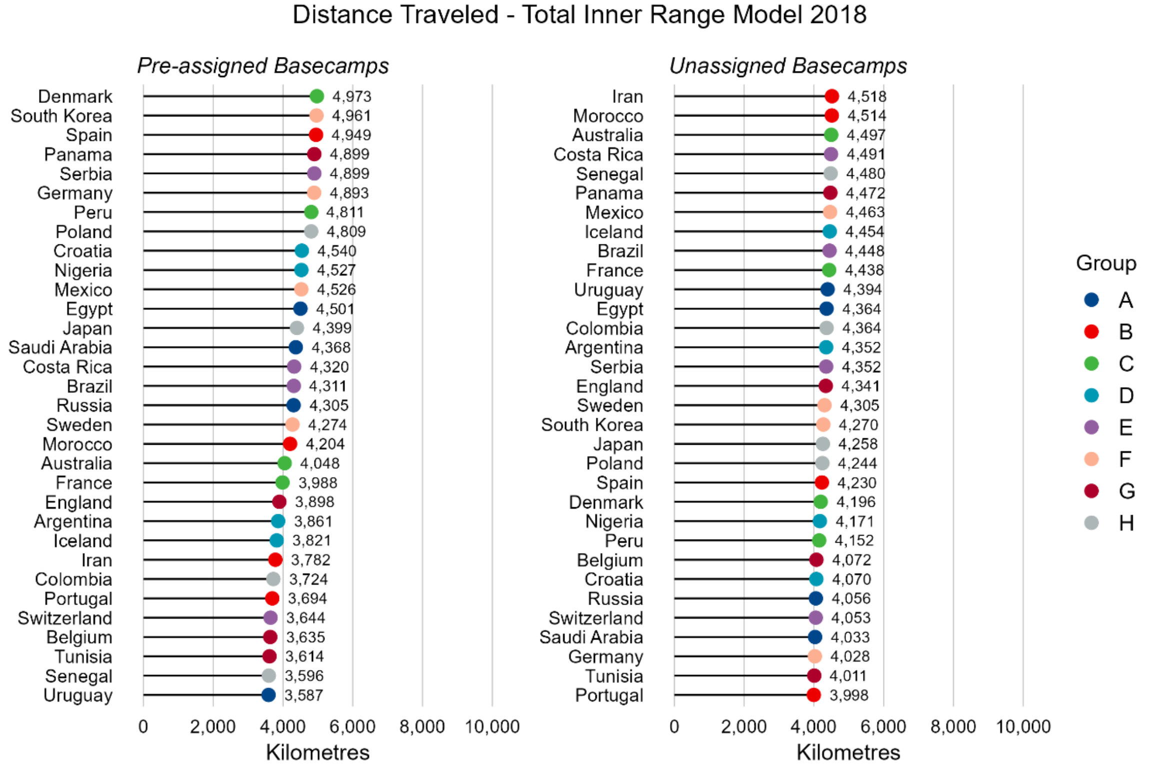

Figure 6.9: *Distance traveled from the Total Inner Range Model 2018*

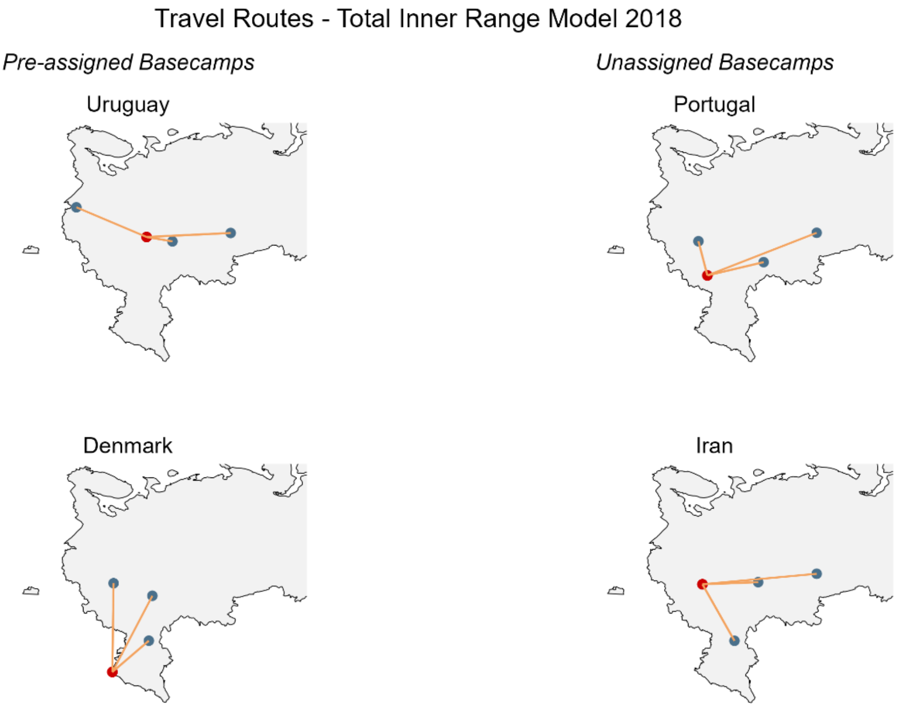

Figure 6.10: *Travel routes from the Total Inner Range Model 2018*

Groupwise Inner Range Model

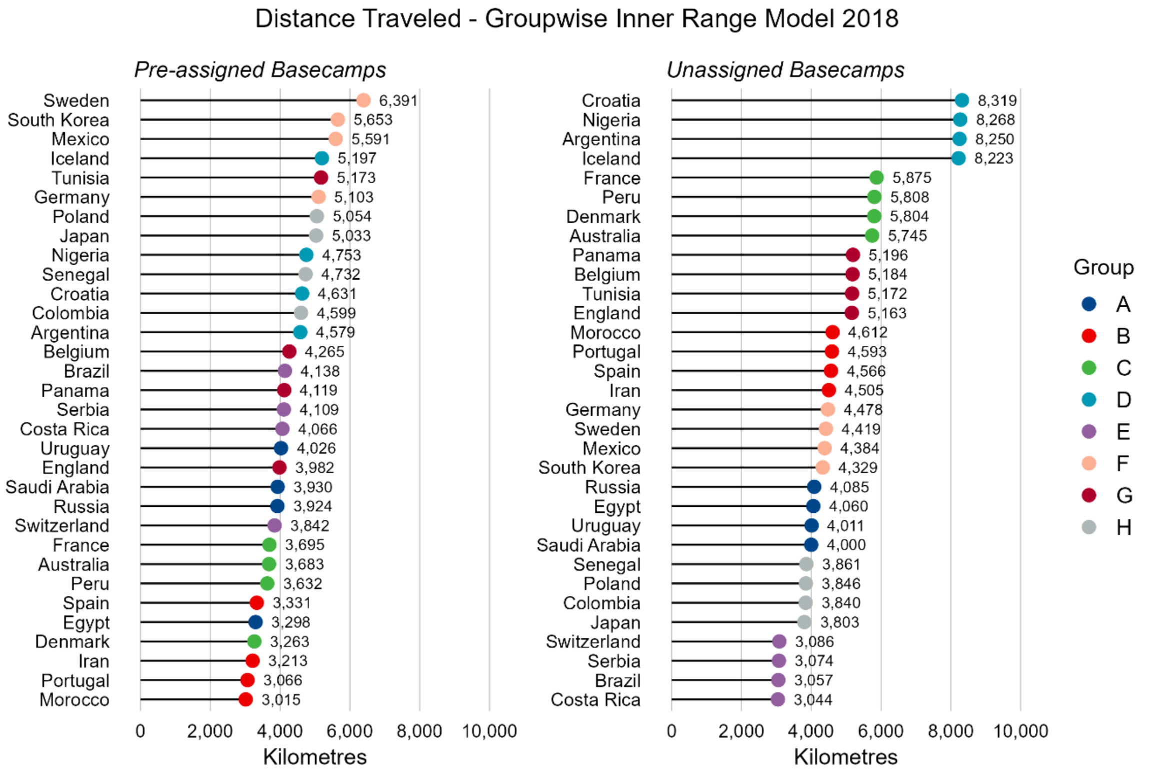

***Figure 6.11**: Distance traveled from the Groupwise Inner Range Model 2018*

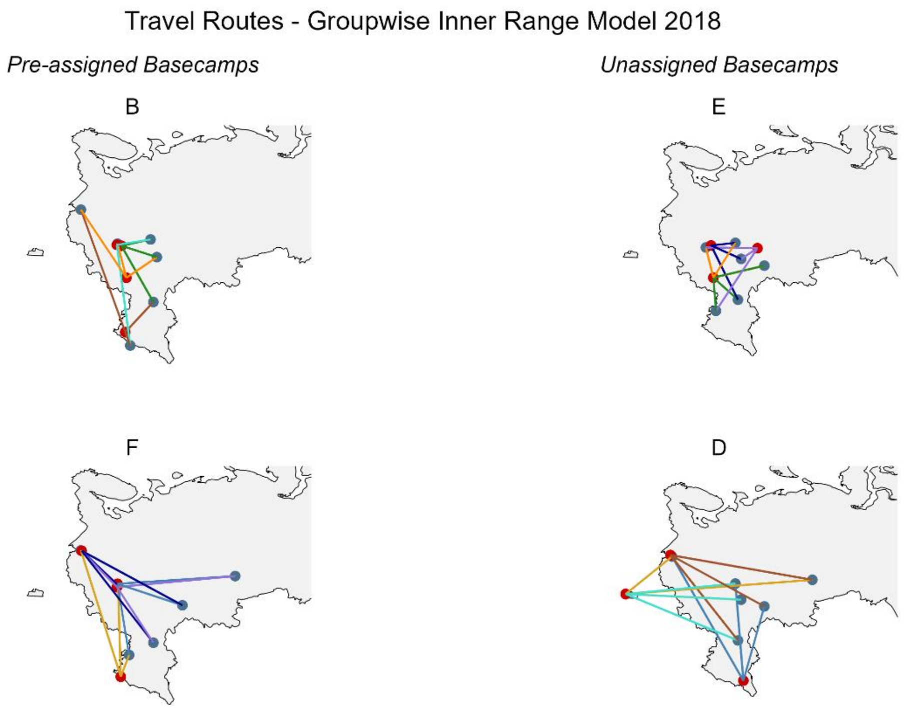

***Figure 6.12:** Travel routes from the Groupwise Inner Range Model 2018*

6.1.3 Conclusions from the Single Objective Model Analysis

The analysis of our Single Objective Models indicates a significant improvement over historical benchmarks across all objectives. Notably, the Total Distance Model demonstrates advancements in all objectives, including those not explicitly targeted by its objective function, whether *pre-assigned* or *unassigned*, and for both the 2014 and 2018 data. This model serves as compelling evidence of our ability to enhance the existing framework for reducing travel distances in the World Cup group stage.

Moreover, the models designed to enhance a fair and equitable distribution of travel burdens among teams have notably improved these metrics. However, this advancement is often accompanied by a trade-off of a rise in the overall travel distance, a rise that on several occasions has exceeded the historical benchmark. While these models excel in achieving their primary objective, there is a clear need for refinement in balancing multiple objectives.

Summarized, the analysis suggests there exists a spectrum of desirable solutions for tournament schedules, each presenting a trade-off between minimizing total travel distance and enhancing perceived fairness among teams. In advancing the World Cup scheduling framework, exploring this trade-off curve seems a logical step. For decision-makers, the ideal solution might well lie at a midpoint between the extremes of these objectives.

6.2 Results Multi-Objective Model

Upon executing the multi-objective ϵ-constraint Model, we have been faced with a resource shortfall of available runtime of the models. This limitation necessitates a downsizing of our analytical scope, requiring us to concentrate on a more selective assortment of models. The selection of this narrowed scope is outlined in the following.

6.2.1 Narrowed Scope

We have decided to focus exclusively on the models from the case of 2014, due to limited software resources preventing us from running the large number of models that an expanded analysis would require within a reasonable time frame. This choice is justified by an observation that the performance patterns for both years are virtually identical when compared to the benchmark and in their relative trade-off between the different objectives. Further, *unassigned* base camps yield the same pattern of relative objective values as *pre-assigned* base camps, but with consistently superior numerical results in all versions. In addition, by focusing on reducing the total inner range within our historical models, we attain desirable results for both the primary objective and the related groupwise inner ranges.

In summary, due to limited solving resources, we scope the analysis down to the 2014 *assigned* models. This is ultimately justified by the fact the primary objective of this thesis is to evaluate the effectiveness of the chosen methodology in enhancing World Cup planning in general, not to conduct in-depth retrospective analyses of the historical events.

6.2.2 Results and Discussion

To determine the range of epsilon values to use in the model formulation, we evenly divide the span between 7,753 (the Single Nadir Point) and 1,972 (the Single Utopia Point) into five equal segments. Each segment represents an incremental increase of $\frac{7753-1972}{5} = 1156$ from its preceding value. This gives the set of epsilons: $\{\epsilon : \epsilon_i = 1972 + 1156 \cdot i,\ i \in \{0 \dots 5\},\ 1972 \leq \epsilon \leq 7753\}$. Then, for each ϵ_i, we solve the optimization problem. **Table 6.3** below summarizes the findings:

Table 6.3*: Results for the ϵ-constraint model*

	Single Utopia	Epsilon levels - Maximum Total Inner Range				Single Nadir
	1 972	*3 128*	*4 284*	*5 440*	*6 596*	*7 753*
Objectives						
Total Distance	208 967	159 734	150 360	147 441	147 062	146 957
Total Inner Range	1 972	3 070	4 251	5 339	6 377	7 753
Solution limit						
MIP Gap	-	*8.17 %*	*2.66 %*	*0.64 %*	*0.37 %*	-
Cutoff Time	-	*8h*	*8h*	*8h*	*8h*	-

Figure 6.13 displays the graphical representation of the results. The total inner range is measured along the x-axis, while the y-axis measures the total distance traveled. At the extremes of the front, we find the solutions derived from the single-objective models, where the point with the lowest value along the x-axis represents the solution from the single-objective model minimizing the total inner range, and the point with the lowest value along

the y-axis represents the single objective model minimizing total distance. The actual 2014 schedule is included as well, marked as Historical Benchmark.

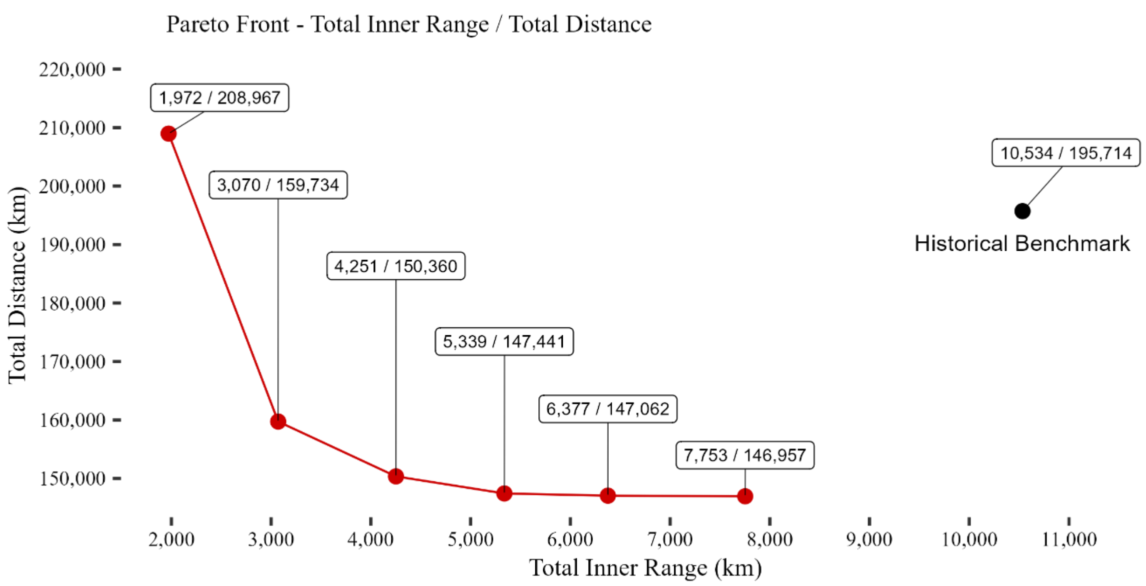

Figure 6.13: *Pareto front of results from the ϵ-Constraint Model*

The figure serves as a good illustration of the trade-off between the two objectives and forms a Pareto front outlined by the ϵ-constrained method. The figure also illustrates that the estimated Pareto front yields numerous solutions superior to the historical benchmark. An interesting finding is the possibility of reducing the total inner range significantly, almost without compromising the total distance traveled, as demonstrated by a reduction in the total inner range from 7,753 km to 5,339 km by increasing the total distance traveled from 146,957 km to just 147,441 km. That is; by just increasing the total distance traveled by 0,3%, the inner range decreased by over 31%. Beyond this point, the trade-off between the two objectives demands more careful consideration, as the further reduction in total inner range comes at the cost of a proportionally higher increase in total distance.

In mathematical terms, the proposed solutions are all equally optimal as they represent the non-dominated solutions within the given constraints. However, when a decision maker selects the final solution, it is important to weigh these trade-off points considering broader tournament considerations. For instance, reducing the total inner range from its Nadir point by 60% (from 7,753 to 3,070) results in a concurrently modest increase of about 9% in the total distance. This balance might thus be seen as the optimal choice for the final solution, securing fairness at a relatively low cost of increasing total distance. The further reduction of the total inner range towards its absolute minimum, the Utopia point, might not offer significant additional practical benefits, given that the tournament already exhibits a fair distribution at the preceding point. Moreover, this additional reduction would lead to a nearly 50,000-kilometre increase in total distance. Such an increase would significantly add to the collective travel demands, amplify the environmental impact, and make the tournament less accessible to fans who travel. Furthermore, understanding the extremes of this front enables the decision maker to make more nuanced decisions, with a clear insight into the extent of possible trade-offs. Understanding that the total inner range can at most be reduced to 1,972 may render the compromise point at 3,070 more appealing compared to a scenario where the minimum achievable inner range remains uncertain.

While the graphical results demonstrate how this trade-off between objectives may look like when being presented to the decision-maker, there are a few things to keep in mind. Firstly, as was the case with the single objective models, these exact numerical results only represent the best achievable solutions within the constraints of our computational capacity for this thesis. Furthermore, because the extreme points selected as inputs, originating from the Single Objective Models, lack absolute optimality, the endpoints of the Pareto front consequently fail to represent the true extremities. These limitations therefore affect the shape of the presented Pareto front, resulting in an approximation rather than an exact representation of the true front. Nonetheless, the set of solutions that achieve true optimality would be delineated even more distinctly from the historical benchmark. Thus, despite these limitations, the presented Pareto front effectively demonstrates the interplay between the two objectives, fulfilling its purpose as an intuitive representation of candidate solutions.

6.3 Summary of Analysis

The findings reveal substantial opportunities for enhancement in previous World Cup schedules, aligned with the studied objectives. Both the 2014 and 2018 World Cups exhibit comparable results. A summary of the numerical improvements in objective values relative to the historical benchmark is given in the following.

In the 2014 World Cup, the Total Inner Range model achieved a 25% reduction in total distance using *pre-assigned* base camps, while the model-driven assigning approach improved this measure by 42%. For the Total Inner Range model, the respective improvement percentages are 81% and 96%. Meanwhile, for the Groupwise Inner Range model, these improvements are 89% and 98%, respectively. For the 2018 schedule, the improvements are 33% and 48% for the Total Distance Models, 82% and 93% for the Total Inner Range Models, and 83% and 91% for the Groupwise Inner Range Models.

The ϵ-constrained model demonstrates a range of trade-offs between the objectives of the Total Inner Range Model and Total Distance Model, resulting in a variety of solutions that resemble a Pareto-optimal front. For instance, given the ϵ_3-interval, the model proposes a schedule that leads to an approximate 25% decrease in total distance while concurrently achieving a 31% reduction in total inner range. This dual improvement significantly surpasses the historical benchmarks for both objectives.

In conclusion, our analysis underscores the substantial improvements achieved through the implementation of the optimization framework for the tournament schedules of the 2014 and 2018 World Cups. This approach thereby has the potential to concurrently reduce both the total distance covered by all teams and the disparities in travel distances among them in future editions of the tournament.

7. Extensions and Discussion

7.1 Heading Towards the FIFA 2026 World Cup

In this section, we leverage the insights from our analysis to discuss the applicability of our methodology for the planning of the 2026 World Cup. As outlined in the introduction, this thesis aims to analyze the scheduling practices in past World Cups and explore how optimization techniques can enhance these processes. The goal is to leverage these insights to refine and contribute to the logistical planning framework for future World Cups. Until this point, we have introduced a range of readily applicable models for potential World Cup organizers, each crafted to provide detailed tournament schedules optimized for their respective objectives. Following the successful demonstration of these models' effectiveness on previous World Cup scheduling, we now turn our attention towards the next edition of the tournament.

The upcoming tournament will span a broad region, extending from the west coast of Canada to the US east coast and into the central parts of Mexico. Unfortunately, details on the format of the 2026 World Cup are limited. Aside from the 16 designated host cities (FIFA, 2023), much remains undisclosed, including the locations of potential base camp cities. Given this lack of information, our approach is not to apply our models directly. Instead, we briefly discuss how a similar methodology could be effectively employed to structure the match schedule when more information becomes available.

7.1.1 New Considerations and Constraints

Considering the extensive geographical coverage of the 2026 World Cup, the findings and insights derived from the analysis of the 2014 and 2018 editions are particularly relevant as these tournaments similarly covered vast distances. Consequently, in terms of the match scheduling, both the objectives and many of the mathematical constraints derived from previous tournaments are likely to remain relevant. This said, since the upcoming World Cup will deviate from tradition in terms of the increased number of participants, in addition to matches taking place in three countries, there will be some distinct differences as well as potential new constraints unique to the 2026 edition.

As noted, the 2026 World Cup will utilize sixteen stadiums for the matches. With 48 teams and a total of 72 group-stage matches, it is impossible to allocate an equal number of matches to each stadium. Given that 72 divided by 16 equals 4.5, one could expect that each venue will host either 4 or 5 matches to make it as evenly distributed as possible. The bid book (Organising Committee for FIFA Competitions, 2022) states a maximum utilization of seven matches per stadium throughout the whole tournament. Consequently, the constraints designed for assigning 4 matches to each venue will need to be revised. We consider changes such as this a minor modification to the existing constraints. Other mathematically formulated constraints will presumably require some adjustments as well depending on the final guidelines for 2026. However, as it is too soon to conclude what these will entail, further discussion on this matter will not be elaborated on at this time.

The sixteen host cities in the upcoming World Cup are spread across an area that covers four different time zones. We have already discussed the issue of traveling across time zones and how it can negatively affect the performance of athletes. Hence, it is considerable to prioritize the minimization of such travel. To address this, we suggest introducing a new constraint that takes this into consideration. The constraint can be modeled in different ways, such as prohibiting teams from traveling across more than two or three time zones, which would presumably curtail the longest travel routes. Although further details were not disclosed, in the earlier mentioned bid book for the 2026 World Cup it is stated that regional clusters for teams and groups are to be prioritized which makes it reasonable to expect some form of restriction on extensive traveling in an east-west direction (United Bid Committee, 2018). **Figure 7.1** illustrates the clustering of venues based on time zones serving as a grouping criterion.

To conclude, despite the limited availability of information, our brief review still emphasizes the necessity of making tournament-specific modifications to our mathematical models. These adaptations will be essential to ensure that our models can provide a detailed and actionable tournament schedule that meets the unique requirements of the 2026 edition. Upon receiving finalized tournament guidelines and potential base camp locations, as well as accessing additional resources for the further refinement of the models, we hold an optimistic outlook regarding our capability to employ our methodology and deliver an optimal tournament schedule for the 2026 World Cup, should the organizers express interest.

Figure 7.1: *Possible regions for the 2026 World Cup*

7.2 Limitations and Pending Decisions

7.2.1 Solving Limitations

As mentioned throughout Chapter 6, we have used the external NEOS server for solving the final solutions. Although this has provided us with adequate computational capacities for finding workable solutions for our analysis, the problems are complex and demanding, and some of our solutions unfortunately exhibit a large MIP Gap. Gurobi offers numerous options for tuning parameters and solving methods that can help find better solutions. We have employed a few of these tools, albeit to a limited extent due to time shortage. Nevertheless, as established throughout the thesis, the exact optimality of the numerical results themselves is not pivotal to the analysis conducted, thereby diminishing the significance of this limitation.

7.2.2 Refinement of Model Formulations

We remain confident in our model's ability to generate valid and comprehensive tournament schedules, as demonstrated in our results and the match fixtures presented in Appendix 10.5. However, we also acknowledge the potential for refining the mathematical formulation of the model, which could lead to reduced running times, more interpretable MIP Gaps, and simplify the process of identifying ultimate optimal solutions.

7.2.3 Discussion on Allocation of Base Camps

An additional question for the organizers is to address the pre-allocation of base camps. In earlier World Cups, the national teams and federations have decided their location for base camp themselves. On the other hand, models without pre-allocated base camps will in most instances yield better objective values. Both have their advantages and disadvantages, accompanied by differences in terms of practical implementation.

Beginning with the pre-allocated models, the order of venue allocation and base camp assignment does not entirely align with historical practices. In previous World Cups, participating nations typically unveiled their chosen base camp locations *after* the fixtures were finalized. This allowed teams to make informed decisions regarding their choice of base camp considering the locations of their matches. Thus, the traditional approach involved formulating the fixture schedule independently of base camp considerations. This stands in contrast to our models, which optimize the schedule and venue allocation based on predetermined base camp assigning. Hence, our specific evaluation of the numerical enhancements provided by our models in comparison to the traditional scheduling process might be somewhat overstated, given that the base camp locations are an important aspect of the model. While this underscores a limitation in our analysis of these models' performance, they nonetheless provide valuable insights into the logistical aspects of the World Cup group stage travel.

Conversely, the models employing model-driven base camp allocation do not seek to optimize while being constrained by predetermined base camp locations. Instead, the model deals with a defined set of possible base camp locations to allocate among the teams, which may be more in line with the convention of not restraining nations to base camps before the match fixturing and corresponding venue allocation. The outcome of this more flexible approach tends to yield superior objective values, as evidenced by the results we have presented, at least in most cases.

Nevertheless, a notable drawback of these models is that participating nations do not have a say in determining the location of their base camp, they are simply assigned the base camp that yields the optimal objective value. It is reasonable to presume that implementing such a rigid system would pose challenges, given the various other factors and considerations that come into play when nations decide on their preferred base camp locations.

Therefore, a compromise can provide a beneficial solution, where the teams are allocated to a base camp by the model – using the *unassigned* models – but with the flexibility to switch to an alternative base camp within the same cluster or region as the model-assigned base camp. This enables the organizer to plan the group stage according to the chosen objective to optimize, utilizing the optimization model, while also providing teams with some mobility in selecting their preferred base camp location after the match fixtures are determined. Certainly, this will result in a departure from the proposed solution found by the model. However, the degree of this deviation relies on the extent of flexibility granted to teams for relocating from their assigned base camp locations, and maintaining some level of rigidity will help ensure it remains closely aligned with the optimal schedule.

7.2.4 Discussion on the Allocation of Itineraries

Ultimately, even for models that only focus on fair travel distributions, some teams will inevitably face longer travel routes than others. Achieving a perfect balance would demand a scenario in which all teams cover the same distance, but this is unattainable within the boundaries of the model's feasibility boundaries, dictated by other critical constraints. This concern holds less significance when teams choose their base camps with knowledge of the venue locations, but it becomes highly pronounced in the model-driven base camp allocation approach. When base camps are determined by the model, the team assigned the longest itinerary may perceive this as deeply unfair since they had no influence over the selection of their base camps; they have essentially ended up with the "least favorable" route by chance.

This said, when the inner range decreases to a certain level, one could argue that the consequences and impact of this disparity would become practically indifferent, thus allocating who travels most and least negligible in practice. However, in the results we have presented earlier, there remains a significant divergence in several of the solutions. Therefore, especially in models without pre-allocated base camps, it is crucial to engage in a more in-depth discussion about how to determine which teams and groups should be granted the "best" allocation and who should be assigned the "least favorable" allocation. This is particularly pertinent since the model operates symmetrically, allowing teams and/or groups to swap itineraries without affecting the objective value.

Addressing this additional fairness concern extends beyond the scope of this thesis. Nonetheless, it stands as an issue demanding attention in the event of potential implementation. Once again, the compromise of permitting base camp changes within a restricted distance radius after the venue allocation emerges as a relevant component of a possible final solution.

8. Conclusion

In this thesis, we have developed a scheduling optimization framework for the FIFA World Cup using mathematical programming. Our ultimate objective was to refine the framework of tournament scheduling of the FIFA World Cup by creating travel itineraries that substantially reduce travel distances while ensuring a more balanced distribution of travel burdens among the participating teams. We constructed a series of Mixed Integer Linear Programming models with the respective objectives: (1) minimizing the total distance traveled, (2) minimizing the inner range between the most and least traveling teams, and (3) minimizing the inner range between the most and least traveling teams within groups. We also demonstrated a set of trade-off solutions between the conflicting objectives using the ϵ-constriant method for multi-objective optimization. The models were implemented in AMPL and solved using the Gurobi Optimizer.

Our findings emphasize that the application of the optimization framework significantly enhanced the tournament schedules of the researched World Cups of 2014 and 2018. Notably, there is substantial potential for reducing both the total distance covered by all 32 teams and the variations in travel distances among these teams. These improvements are apparent whether teams are *pre-assigned* to respective base camps, or the model is employed to allocate these base camps as well. While the degree of relative improvement varies depending on specific objectives and years examined, there is a consistent and noteworthy reduction in the objective values across all models when compared to historical benchmarks. Importantly, models without *pre-assigned* base camps consistently outperform those with *pre-assigned* base camps.

As for the multi-objective ϵ-constrained model, our research reveals a nuanced trade-off between minimizing the total distance and minimizing the total inner range. This model generates a spectrum of candidate schedules, each illustrating different levels of compromise between the objectives. Nevertheless, all solutions demonstrate enhancements by concurrently reducing the overall distance traveled and ensuring a fairer distribution of travel when compared with the scheduling approach used in the 2014 World Cup in Brazil.

These results culminate in the main finding of this thesis; applying mathematical programming to the World Cup match scheduling process can significantly reduce the collective travel burden while concurrently ensuring a fair and equitable distribution of travel distances among the participating teams. While crafting the overall tournament program is a multifaceted challenge, shaped by numerous considerations that extend beyond this study's focus, our insights strongly advocate for a reconsideration of conventional scheduling strategies in upcoming tournaments. These findings aim to inspire and guide the planning of future events, with the 2026 World Cup emerging as a particularly promising opportunity for leveraging these advancements.

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10. Appendix

10.1 Extended Table 2014 Models

		2014 Models					
		Pre-assigned Basecamps			*Unassigned Basecamps*		
	Historical Benchmark	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**
Objectives							
Total Distance	195 909	**146 964**	208 731	200 665	**114 282**	253 437	232 720
Total Inner Range	10 524	7 708	**1 973**	7 075	6 223	**468**	4 750
Total Groupwise Inner Range	34 688	30 018	9 413	**3 699**	25 774	2 609	**808**
Distances of each team							
Group A							
Brazil	5 887	3 114	6 732	6 907	2 165	8 078	6 690
Cameroon	10 066	4 131	6 685	7 269	2 861	7 759	6 821
Croatia	8 826	2 151	6 269	6 767	2 708	8 124	6 681
Mexico	11 566	3 458	6 172	7 328	1 558	7 762	6 905
Group B							
Australia	7 656	4 197	6 178	7 989	2 069	7 849	8 103
Chile	3 912	7 040	7 767	8 330	4 161	8 040	8 115
Netherlands	5 012	2 616	7 066	7 662	4 677	7 916	8 201
Spain	4 915	9 191	6 204	8 015	4 137	7 830	8 055
Group C							
Colombia	4 082	6 603	6 531	5 370	2 836	7 769	7 919
Greece	4 474	3 199	6 136	5 704	2 061	8 157	7 937
Ivory Coast	7 801	7 248	6 360	5 804	7 140	7 978	7 940
Japan	10 176	4 425	7 212	5 678	7 429	7 989	7 941
Group D							
Costa Rica	9 550	2 769	6 913	2 246	5 973	7 924	3 980
England	6 734	1 834	6 388	2 305	1 774	8 103	3 990
Italy	11 525	2 881	7 585	2 096	3 394	8 180	3 972
Uruguay	6 494	3 071	6 066	1 683	3 024	7 844	3 991
Group E							
Ecuador	5 432	6 353	6 805	5 507	6 170	8 102	7 130
France	5 282	7 077	6 400	5 191	2 614	8 127	7 185
Honduras	5 088	5 481	7 284	5 270	1 206	7 712	7 071
Switzerland	5 401	4 603	6 435	5 143	6 008	7 961	7 169
Group F							
Argentina	2 134	3 785	6 074	8 758	2 489	8 079	8 702
Bosnia and Herzegovina	4 908	1 483	6 117	8 551	5 115	7 740	8 676
Iran	3 103	5 363	5 843	8 712	3 694	7 834	8 644
Nigeria	4 114	4 444	6 884	8 704	2 748	7 950	8 722
Group G							
Germany	4 512	7 839	7 131	5 676	3 247	8 045	7 282
Ghana	3 775	1 968	5 849	5 831	2 387	7 789	7 347
Portugal	8 842	8 228	5 878	5 658	1 386	7 861	7 301
United States	12 114	6 756	6 405	5 402	3 701	7 725	7 288
Group H							
Algeria	3 005	3 185	5 794	8 055	4 033	7 952	8 254
Belgium	1 590	2 410	5 962	7 641	4 078	7 778	8 219
Russia	3 662	4 086	6 351	7 670	4 915	7 765	8 317
South Korea	4 273	5 973	7 256	7 741	2 524	7 715	8 178
Solution Limits							
MIP Gap	-	*0.07 %*	*N/A*	*N/A*	*40.40 %*	*N/A*	*N/A*
Cutoff Time	-	*8h*	*8h*	*8h*	*8h*	*8h*	*8h*

10.2 Extended Table 2018 Models

		2018 Models					
		Pre-assigned Basecamps			*Unassigned Basecamps*		
	Historical Benchmark	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**	**Total Distance Model**	**Total Inner Range Model**	**Groupwise Inner Range Model**
Objectives							
Total Distance	141 914	**95 728**	136 361	137 086	**74 470**	137 089	156 659
Total Inner Range	7 635	6 499	**1 387**	3 376	3 108	**520**	5 275
Total Groupwise Inner Range	31 576	24 774	8 315	**5 326**	11 270	3 182	**699**
Distances of each team							
Group A							
Egypt	8 537	5 364	4 501	3 298	2 258	4 364	4 060
Russia	2 164	2 640	4 305	3 924	3 539	4 056	4 085
Saudi Arabia	5 899	4 843	4 368	3 930	4 040	4 033	4 000
Uruguay	4 679	1 384	3 587	4 026	3 096	4 394	4 011
Group B							
Iran	3 291	1 885	3 782	3 213	1 347	4 518	4 505
Morocco	4 374	2 441	4 204	3 015	1 671	4 514	4 612
Portugal	3 282	2 419	3 694	3 066	1 864	3 998	4 593
Spain	4 889	1 869	4 949	3 331	1 508	4 230	4 566
Group C							
Australia	2 112	1 779	4 048	3 683	2 496	4 497	5 745
Denmark	6 207	6 938	4 973	3 263	3 266	4 196	5 804
France	4 226	2 409	3 988	3 695	3 020	4 438	5 875
Peru	5 265	439	4 811	3 632	2 655	4 152	5 808
Group D							
Argentina	1 574	4 046	3 861	4 579	1 854	4 352	8 250
Croatia	5 130	4 115	4 540	4 631	1 651	4 070	8 319
Iceland	4 174	2 725	3 821	5 197	1 696	4 454	8 223
Nigeria	7 020	2 717	4 527	4 753	1 500	4 171	8 268
Group E							
Brazil	6 033	4 637	4 311	4 138	2 877	4 448	3 057
Costa Rica	3 751	5 243	4 320	4 066	2 985	4 491	3 044
Serbia	5 090	5 524	4 899	4 109	1 296	4 352	3 074
Switzerland	6 194	2 017	3 644	3 842	1 160	4 053	3 086
Group F							
Germany	3 516	1 965	4 893	5 103	1 795	4 028	4 478
Mexico	3 431	2 222	4 526	5 591	2 311	4 463	4 384
South Korea	6 073	2 298	4 961	5 653	4 062	4 270	4 329
Sweden	5 224	4 692	4 274	6 391	1 956	4 305	4 419
Group G							
Belgium	3 879	2 006	3 635	4 265	1 958	4 072	5 184
England	5 850	2 742	3 898	3 982	3 301	4 341	5 163
Panama	3 022	818	4 899	4 119	954	4 472	5 196
Tunisia	2 424	2 241	3 614	5 173	1 779	4 011	5 172
Group H							
Colombia	903	965	3 724	4 599	1 812	4 364	3 840
Japan	2 907	1 347	4 399	5 033	2 606	4 258	3 803
Poland	6 446	3 866	4 809	5 054	2 939	4 244	3 846
Senegal	4 348	5 132	3 596	4 732	3 219	4 480	3 861
Solution Limits							
MIP Gap	-	*0.07 %*	*N/A*	*N/A*	*72.30 %*	*N/A*	*N/A*
Cutoff Time	-	*8h*	*8h*	*8h*	*8h*	*8h*	*8h*

10.3 Extended Table ϵ-constraint Model

	Historical Benchmark	Single Utopia	Epsilon levels - Maximum Total Inner Range				Single Nadir
	2014 Model - Pre-assigned Basecamps						
	-	*1 972*	*3 128*	*4 284*	*5 440*	*6 596*	*7 753*
Objectives							
Total Distance	195 909	208 967	159 734	150 360	147 441	147 062	146 957
Total Inner Range	10 524	1 972	3 070	4 251	5 339	6 377	7 753
Total Groupwise Inner Range	34 688	9 658	15 965	20 030	30 256	29 581	30 015
Distances of each team							
Group A							
Brazil	5 887	6 732	4 362	5 094	5 070	3 114	3 114
Cameroon	10 066	6 685	6 361	4 163	5 476	4 131	4 131
Croatia	8 826	6 269	4 175	3 823	2 301	2 151	2 151
Mexico	11 566	6 172	5 202	3 154	3 154	3 458	3 458
Group B							
Australia	7 656	6 178	4 197	4 275	2 674	4 197	4 197
Chile	3 912	7 767	3 815	3 107	6 945	7 040	7 040
Netherlands	5 012	7 066	5 347	4 672	4 343	1 721	2 616
Spain	4 915	6 204	6 886	6 355	6 555	8 097	9 191
Group C							
Colombia	4 082	6 531	4 956	4 705	6 603	6 603	6 603
Greece	4 474	6 136	5 068	6 740	3 199	3 199	3 199
Ivory Coast	7 801	6 360	5 312	6 515	7 248	7 248	7 248
Japan	10 176	7 212	4 086	6 718	4 425	4 425	4 425
Group D							
Costa Rica	9 550	6 913	4 355	4 675	6 556	2 769	2 769
England	6 734	6 388	6 837	3 554	1 910	1 834	1 834
Italy	11 525	7 585	4 759	2 800	2 204	2 881	2 881
Uruguay	6 494	6 066	5 660	3 029	7 061	3 071	3 071
Group E							
Ecuador	5 432	6 805	4 271	4 073	6 353	6 353	6 353
France	5 282	6 400	4 352	6 402	7 077	7 077	7 077
Honduras	5 088	7 529	4 804	5 883	5 472	5 472	5 472
Switzerland	5 401	6 435	3 919	4 603	4 603	4 603	4 603
Group F							
Argentina	2 134	6 074	4 622	6 057	3 785	3 785	3 785
Bosnia and Herzegovina	4 908	6 117	4 709	6 503	1 972	3 043	1 483
Iran	3 103	5 843	6 255	4 782	5 363	7 077	5 363
Nigeria	4 114	6 884	6 524	3 787	4 979	4 444	4 444
Group G							
Germany	4 512	7 131	3 855	4 149	3 418	7 839	7 839
Ghana	3 775	5 849	4 033	2 489	1 968	1 968	1 968
Portugal	8 842	5 878	5 634	5 588	5 872	7 693	8 228
United States	12 114	6 405	4 764	5 056	5 798	6 111	6 756
Group H							
Algeria	3 005	5 794	4 653	3 185	3 521	3 185	3 185
Belgium	1 590	5 953	3 942	3 543	2 132	2 413	2 413
Russia	3 662	6 351	5 640	4 910	3 429	4 086	4 086
South Korea	4 273	7 256	6 376	5 973	5 973	5 973	5 973
Solution limit							
MIP Gap	-	-	*8.17 %*	*2.66 %*	*0.64 %*	*0.37 %*	-
Cutoff Time	-	-	*8h*	*8h*	*8h*	*8h*	-

10.4 AMPL-files

The AMPL-files are attached in an external zip-file. The zip-file contains four folders for: (1) Total Distance Model, (2) Total Inner Range Model, (3) Groupwise Inner Range Model, and (4) Epsilon-Constraint Model. Each folder contains specified mod-files and dat-files, in addition to one basic run-file that can be used to run the models located in the same folder.

10.5 Match Fixtures

FIFA World Cup 2014 – Match Fixture Minimizing Total Distance (*pre-assigned base camps*)

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Mexico	Arena de Sao Paulo
1	A	2014-06-13	Croatia - Cameroon	Arena Pernambuco
1	B	2014-06-13	Chile - Australia	Arena Fonte Nova
1	C	2014-06-13	Colombia - Japan	Arena da Baixada
1	C	2014-06-13	Greece - Ivory Coast	Estadio Castelao
1	B	2014-06-14	Netherlands - Spain	Estadio Beira-Rio
1	D	2014-06-14	Italy - England	Estadio do Maracana
1	E	2014-06-14	France - Ecuador	Arena Pantanal
1	G	2014-06-14	Germany - Portugal	Arena da Amazonia
1	D	2014-06-15	Costa Rica - Uruguay	Estadio Nacional
1	E	2014-06-15	Switzerland - Honduras	Arena Fonte Nova
1	G	2014-06-15	United States - Ghana	Arena das Dunas
1	H	2014-06-15	Algeria - Russia	Arena da Baixada
1	F	2014-06-16	Argentina - Bosnia and Herzegovina	Estadio Mineirao
1	F	2014-06-16	Nigeria - Iran	Arena Pantanal
1	H	2014-06-16	Belgium - South Korea	Estadio Beira-Rio
2	A	2014-06-17	Brazil - Croatia	Arena Fonte Nova
2	C	2014-06-17	Colombia - Greece	Arena das Dunas
2	A	2014-06-18	Mexico - Cameroon	Estadio Mineirao
2	C	2014-06-18	Ivory Coast - Japan	Estadio Nacional
2	E	2014-06-18	France - Switzerland	Estadio Castelao
2	E	2014-06-18	Ecuador - Honduras	Estadio Beira-Rio
2	G	2014-06-19	Germany - Ghana	Arena Pernambuco
2	H	2014-06-19	Belgium - Russia	Estadio do Maracana
2	H	2014-06-19	Algeria - South Korea	Arena Pantanal
2	D	2014-06-20	Costa Rica - Italy	Arena de Sao Paulo
2	G	2014-06-20	United States - Portugal	Arena da Baixada
2	B	2014-06-21	Netherlands - Australia	Estadio do Maracana
2	B	2014-06-21	Chile - Spain	Arena da Amazonia
2	D	2014-06-21	Uruguay - England	Estadio Mineirao
2	F	2014-06-22	Argentina - Nigeria	Estadio Nacional
2	F	2014-06-22	Bosnia and Herzegovina - Iran	Arena de Sao Paulo
3	A	2014-06-23	Brazil - Cameroon	Estadio do Maracana
3	A	2014-06-23	Mexico - Croatia	Arena das Dunas
3	G	2014-06-23	Germany - United States	Arena Fonte Nova
3	G	2014-06-23	Portugal - Ghana	Estadio Castelao
3	C	2014-06-24	Colombia - Ivory Coast	Arena Pantanal
3	C	2014-06-24	Greece - Japan	Arena Pernambuco
3	H	2014-06-24	Belgium - Algeria	Arena de Sao Paulo
3	H	2014-06-24	Russia - South Korea	Arena da Amazonia
3	B	2014-06-25	Netherlands - Chile	Estadio Mineirao
3	B	2014-06-25	Spain - Australia	Arena das Dunas
3	D	2014-06-25	Costa Rica - England	Estadio Beira-Rio
3	D	2014-06-25	Uruguay - Italy	Estadio Castelao
3	E	2014-06-26	France - Honduras	Estadio Nacional
3	E	2014-06-26	Switzerland - Ecuador	Arena Pernambuco
3	F	2014-06-26	Argentina - Iran	Arena da Amazonia
3	F	2014-06-26	Nigeria - Bosnia and Herzegovina	Arena da Baixada

FIFA World Cup 2014 – Match Fixture Minimizing Total Distance (*unassigned base camps*)

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Mexico	Arena de Sao Paulo
1	A	2014-06-13	Croatia - Cameroon	Estadio Nacional
1	D	2014-06-13	Italy - England	Arena da Baixada
1	F	2014-06-13	Argentina - Iran	Arena das Dunas
1	H	2014-06-13	Belgium - Algeria	Arena Pernambuco
1	B	2014-06-14	Netherlands - Spain	Estadio Castelao
1	D	2014-06-14	Costa Rica - Uruguay	Arena Pantanal
1	F	2014-06-14	Nigeria - Bosnia and Herzegovina	Estadio Mineirao
1	H	2014-06-14	Russia - South Korea	Arena Fonte Nova
1	B	2014-06-15	Chile - Australia	Estadio do Maracana
1	C	2014-06-15	Colombia - Ivory Coast	Arena das Dunas
1	E	2014-06-15	France - Honduras	Arena de Sao Paulo
1	G	2014-06-15	United States - Ghana	Estadio Beira-Rio
1	C	2014-06-16	Greece - Japan	Arena Pernambuco
1	E	2014-06-16	Switzerland - Ecuador	Arena da Amazonia
1	G	2014-06-16	Germany - Portugal	Estadio Mineirao
2	A	2014-06-17	Mexico - Cameroon	Estadio do Maracana
2	D	2014-06-17	Costa Rica - England	Estadio Beira-Rio
2	F	2014-06-17	Argentina - Bosnia and Herzegovina	Estadio Castelao
2	F	2014-06-17	Nigeria - Iran	Arena Fonte Nova
2	A	2014-06-18	Brazil - Croatia	Arena da Baixada
2	B	2014-06-18	Netherlands - Australia	Estadio Mineirao
2	B	2014-06-18	Chile - Spain	Arena Pernambuco
2	D	2014-06-18	Uruguay - Italy	Arena de Sao Paulo
2	C	2014-06-19	Ivory Coast - Japan	Arena da Amazonia
2	H	2014-06-19	Algeria - South Korea	Arena das Dunas
2	C	2014-06-20	Colombia - Greece	Arena Fonte Nova
2	H	2014-06-20	Belgium - Russia	Estadio Nacional
2	E	2014-06-21	Switzerland - Honduras	Arena da Baixada
2	G	2014-06-21	Germany - United States	Arena Pantanal
2	E	2014-06-22	France - Ecuador	Estadio Nacional
2	G	2014-06-22	Portugal - Ghana	Estadio do Maracana
3	A	2014-06-23	Brazil - Cameroon	Arena Fonte Nova
3	A	2014-06-23	Mexico - Croatia	Estadio Beira-Rio
3	F	2014-06-23	Argentina - Nigeria	Arena Pernambuco
3	F	2014-06-23	Bosnia and Herzegovina - Iran	Arena Pantanal
3	C	2014-06-24	Colombia - Japan	Estadio Nacional
3	C	2014-06-24	Greece - Ivory Coast	Estadio Castelao
3	D	2014-06-24	Costa Rica - Italy	Arena da Amazonia
3	D	2014-06-24	Uruguay - England	Estadio do Maracana
3	B	2014-06-25	Netherlands - Chile	Arena das Dunas
3	B	2014-06-25	Spain - Australia	Arena Pantanal
3	G	2014-06-25	Germany - Ghana	Arena de Sao Paulo
3	G	2014-06-25	United States - Portugal	Arena da Baixada
3	E	2014-06-26	France - Switzerland	Estadio Beira-Rio
3	E	2014-06-26	Ecuador - Honduras	Estadio Mineirao
3	H	2014-06-26	Belgium - South Korea	Estadio Castelao
3	H	2014-06-26	Algeria - Russia	Arena da Amazonia

FIFA World Cup 2014 – Match Fixture Minimizing Total Inner Range (pre-assigned base camps)

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Cameroon	Arena de Sao Paulo
1	A	2014-06-13	Mexico - Croatia	Estadio Mineirao
1	G	2014-06-13	Germany - United States	Arena da Amazonia
1	G	2014-06-13	Portugal - Ghana	Arena das Dunas
1	H	2014-06-13	Belgium - South Korea	Arena Fonte Nova
1	D	2014-06-14	Uruguay - England	Arena da Baixada
1	F	2014-06-14	Argentina - Bosnia and Herzegovina	Arena de Sao Paulo
1	F	2014-06-14	Nigeria - Iran	Estadio do Maracana
1	H	2014-06-14	Algeria - Russia	Arena Pernambuco
1	B	2014-06-15	Netherlands - Spain	Arena Fonte Nova
1	B	2014-06-15	Chile - Australia	Estadio Beira-Rio
1	C	2014-06-15	Colombia - Ivory Coast	Arena das Dunas
1	D	2014-06-15	Costa Rica - Italy	Estadio Nacional
1	C	2014-06-16	Greece - Japan	Estadio Castelao
1	E	2014-06-16	France - Switzerland	Arena Pantanal
1	E	2014-06-16	Ecuador - Honduras	Arena Pernambuco
2	F	2014-06-17	Nigeria - Bosnia and Herzegovina	Arena da Amazonia
2	G	2014-06-17	Germany - Portugal	Estadio Mineirao
2	G	2014-06-17	United States - Ghana	Arena da Baixada
2	F	2014-06-18	Argentina - Iran	Estadio Castelao
2	H	2014-06-18	Belgium - Russia	Estadio Nacional
2	H	2014-06-18	Algeria - South Korea	Arena de Sao Paulo
2	A	2014-06-19	Brazil - Mexico	Arena Pantanal
2	E	2014-06-19	France - Ecuador	Estadio Beira-Rio
2	A	2014-06-20	Croatia - Cameroon	Arena das Dunas
2	E	2014-06-20	Switzerland - Honduras	Estadio do Maracana
2	B	2014-06-21	Netherlands - Chile	Arena Pernambuco
2	C	2014-06-21	Colombia - Greece	Estadio Mineirao
2	D	2014-06-21	Costa Rica - Uruguay	Arena Fonte Nova
2	D	2014-06-21	Italy - England	Arena Pantanal
2	B	2014-06-22	Spain - Australia	Arena da Baixada
2	C	2014-06-22	Ivory Coast - Japan	Estadio Nacional
3	F	2014-06-23	Argentina - Nigeria	Estadio Beira-Rio
3	F	2014-06-23	Bosnia and Herzegovina - Iran	Estadio Mineirao
3	H	2014-06-23	Belgium - Algeria	Arena Pantanal
3	H	2014-06-23	Russia - South Korea	Estadio do Maracana
3	D	2014-06-24	Costa Rica - England	Arena Pernambuco
3	D	2014-06-24	Uruguay - Italy	Arena da Amazonia
3	E	2014-06-24	France - Honduras	Estadio Castelao
3	E	2014-06-24	Switzerland - Ecuador	Arena de Sao Paulo
3	B	2014-06-25	Netherlands - Australia	Estadio Nacional
3	B	2014-06-25	Chile - Spain	Arena das Dunas
3	G	2014-06-25	Germany - Ghana	Arena Fonte Nova
3	G	2014-06-25	United States - Portugal	Estadio do Maracana
3	A	2014-06-26	Brazil - Croatia	Arena da Amazonia
3	A	2014-06-26	Mexico - Cameroon	Estadio Castelao
3	C	2014-06-26	Colombia - Japan	Estadio Beira-Rio
3	C	2014-06-26	Greece - Ivory Coast	Arena da Baixada

FIFA World Cup 2014 – Match Fixture Minimizing Total Inner Range (*unassigned base camps*)

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Cameroon	Arena de Sao Paulo
1	A	2014-06-13	Mexico - Croatia	Estadio Castelao
1	F	2014-06-13	Bosnia and Herzegovina - Iran	Estadio Nacional
1	G	2014-06-13	United States - Ghana	Arena da Baixada
1	E	2014-06-14	France - Switzerland	Estadio Mineirao
1	F	2014-06-14	Argentina - Nigeria	Arena da Amazonia
1	G	2014-06-14	Germany - Portugal	Arena Fonte Nova
1	H	2014-06-14	Belgium - Algeria	Arena das Dunas
1	D	2014-06-15	Costa Rica - England	Estadio do Maracana
1	D	2014-06-15	Uruguay - Italy	Arena Pernambuco
1	E	2014-06-15	Ecuador - Honduras	Arena Pantanal
1	H	2014-06-15	Russia - South Korea	Arena da Baixada
1	B	2014-06-16	Netherlands - Chile	Estadio Mineirao
1	B	2014-06-16	Spain - Australia	Estadio Beira-Rio
1	C	2014-06-16	Colombia - Greece	Arena Fonte Nova
1	C	2014-06-16	Ivory Coast - Japan	Arena de Sao Paulo
2	F	2014-06-17	Argentina - Bosnia and Herzegovina	Estadio do Maracana
2	G	2014-06-17	Portugal - Ghana	Estadio Castelao
2	F	2014-06-18	Nigeria - Iran	Arena Fonte Nova
2	G	2014-06-18	Germany - United States	Arena Pantanal
2	D	2014-06-19	Costa Rica - Italy	Arena das Dunas
2	H	2014-06-19	Belgium - Russia	Estadio Beira-Rio
2	H	2014-06-19	Algeria - South Korea	Arena da Amazonia
2	D	2014-06-20	Uruguay - England	Estadio Mineirao
2	E	2014-06-20	France - Ecuador	Estadio Castelao
2	E	2014-06-20	Switzerland - Honduras	Arena de Sao Paulo
2	B	2014-06-21	Netherlands - Spain	Estadio Nacional
2	B	2014-06-21	Chile - Australia	Estadio do Maracana
2	C	2014-06-21	Greece - Japan	Arena da Amazonia
2	A	2014-06-22	Brazil - Croatia	Arena Pantanal
2	A	2014-06-22	Mexico - Cameroon	Arena da Baixada
2	C	2014-06-22	Colombia - Ivory Coast	Arena Pernambuco
3	F	2014-06-23	Argentina - Iran	Estadio Beira-Rio
3	F	2014-06-23	Nigeria - Bosnia and Herzegovina	Estadio Mineirao
3	G	2014-06-23	Germany - Ghana	Arena das Dunas
3	G	2014-06-23	United States - Portugal	Arena de Sao Paulo
3	B	2014-06-24	Netherlands - Australia	Arena Pernambuco
3	B	2014-06-24	Chile - Spain	Arena da Amazonia
3	H	2014-06-24	Belgium - South Korea	Arena Fonte Nova
3	H	2014-06-24	Algeria - Russia	Estadio Nacional
3	D	2014-06-25	Costa Rica - Uruguay	Arena Pantanal
3	D	2014-06-25	Italy - England	Estadio Castelao
3	E	2014-06-25	France - Honduras	Arena das Dunas
3	E	2014-06-25	Switzerland - Ecuador	Arena da Baixada
3	A	2014-06-26	Brazil - Mexico	Arena Pernambuco
3	A	2014-06-26	Croatia - Cameroon	Estadio Nacional
3	C	2014-06-26	Colombia - Japan	Estadio do Maracana
3	C	2014-06-26	Greece - Ivory Coast	Estadio Beira-Rio

FIFA World Cup 2014 – Match Fixture Minimizing Groupwise Inner Ranges (*pre-assigned base camps*)

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Cameroon	Arena de Sao Paulo
1	A	2014-06-13	Mexico - Croatia	Estadio do Maracana
1	F	2014-06-13	Argentina - Nigeria	Arena Fonte Nova
1	G	2014-06-13	Germany - Ghana	Estadio Nacional
1	G	2014-06-13	United States - Portugal	Arena da Baixada
1	B	2014-06-14	Netherlands - Australia	Arena Pernambuco
1	B	2014-06-14	Chile - Spain	Estadio Castelao
1	C	2014-06-14	Colombia - Greece	Estadio Mineirao
1	F	2014-06-14	Bosnia and Herzegovina - Iran	Estadio Beira-Rio
1	C	2014-06-15	Ivory Coast - Japan	Arena Pantanal
1	D	2014-06-15	Costa Rica - Uruguay	Arena de Sao Paulo
1	D	2014-06-15	Italy - England	Estadio do Maracana
1	E	2014-06-16	France - Honduras	Arena Pernambuco
1	E	2014-06-16	Switzerland - Ecuador	Estadio Mineirao
1	H	2014-06-16	Belgium - Algeria	Arena da Amazonia
1	H	2014-06-16	Russia - South Korea	Arena das Dunas
2	G	2014-06-17	Germany - United States	Estadio Castelao
2	G	2014-06-17	Portugal - Ghana	Arena Fonte Nova
2	B	2014-06-18	Netherlands - Spain	Estadio Nacional
2	C	2014-06-18	Colombia - Japan	Estadio Beira-Rio
2	C	2014-06-18	Greece - Ivory Coast	Estadio do Maracana
2	F	2014-06-18	Argentina - Bosnia and Herzegovina	Arena da Amazonia
2	B	2014-06-19	Chile - Australia	Arena Pantanal
2	E	2014-06-19	France - Ecuador	Arena de Sao Paulo
2	F	2014-06-19	Nigeria - Iran	Estadio Castelao
2	A	2014-06-20	Brazil - Croatia	Arena das Dunas
2	E	2014-06-20	Switzerland - Honduras	Arena da Baixada
2	H	2014-06-20	Belgium - South Korea	Estadio Beira-Rio
2	A	2014-06-21	Mexico - Cameroon	Arena Pernambuco
2	H	2014-06-21	Algeria - Russia	Arena Pantanal
2	D	2014-06-22	Costa Rica - England	Arena da Baixada
2	D	2014-06-22	Uruguay - Italy	Estadio Mineirao
3	B	2014-06-23	Netherlands - Chile	Arena das Dunas
3	B	2014-06-23	Spain - Australia	Estadio Beira-Rio
3	E	2014-06-23	France - Switzerland	Estadio Nacional
3	E	2014-06-23	Ecuador - Honduras	Estadio do Maracana
3	C	2014-06-24	Colombia - Ivory Coast	Arena da Amazonia
3	C	2014-06-24	Greece - Japan	Arena Fonte Nova
3	G	2014-06-24	Germany - Portugal	Arena Pernambuco
3	G	2014-06-24	United States - Ghana	Arena de Sao Paulo
3	F	2014-06-25	Argentina - Iran	Arena das Dunas
3	F	2014-06-25	Nigeria - Bosnia and Herzegovina	Arena Pantanal
3	H	2014-06-25	Belgium - Russia	Estadio Mineirao
3	H	2014-06-25	Algeria - South Korea	Arena da Baixada
3	A	2014-06-26	Brazil - Mexico	Estadio Castelao
3	A	2014-06-26	Croatia - Cameroon	Arena da Amazonia
3	D	2014-06-26	Costa Rica - Italy	Arena Fonte Nova
3	D	2014-06-26	Uruguay - England	Estadio Nacional

FIFA World Cup 2014 – Match Fixture Minimizing Groupwise Inner Ranges (*unassigned base camps*)

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Croatia	Arena de Sao Paulo
1	A	2014-06-13	Mexico - Cameroon	Arena Pantanal
1	F	2014-06-13	Bosnia and Herzegovina - Iran	Estadio Castelao
1	G	2014-06-13	Germany - United States	Arena das Dunas
1	F	2014-06-14	Argentina - Nigeria	Estadio Mineirao
1	G	2014-06-14	Portugal - Ghana	Arena Fonte Nova
1	H	2014-06-14	Belgium - Russia	Arena da Amazonia
1	H	2014-06-14	Algeria - South Korea	Arena Pernambuco
1	C	2014-06-15	Ivory Coast - Japan	Arena das Dunas
1	D	2014-06-15	Costa Rica - Uruguay	Estadio do Maracana
1	D	2014-06-15	Italy - England	Arena de Sao Paulo
1	E	2014-06-15	Ecuador - Honduras	Arena da Baixada
1	B	2014-06-16	Netherlands - Chile	Estadio Nacional
1	B	2014-06-16	Spain - Australia	Estadio Mineirao
1	C	2014-06-16	Colombia - Greece	Estadio Beira-Rio
1	E	2014-06-16	France - Switzerland	Arena Fonte Nova
2	G	2014-06-17	United States - Ghana	Arena da Baixada
2	H	2014-06-17	Belgium - South Korea	Arena de Sao Paulo
2	G	2014-06-18	Germany - Portugal	Arena Pernambuco
2	H	2014-06-18	Algeria - Russia	Estadio Beira-Rio
2	E	2014-06-19	Switzerland - Ecuador	Estadio Nacional
2	F	2014-06-19	Nigeria - Iran	Arena da Amazonia
2	D	2014-06-20	Uruguay - Italy	Estadio Mineirao
2	E	2014-06-20	France - Honduras	Estadio Beira-Rio
2	F	2014-06-20	Argentina - Bosnia and Herzegovina	Arena das Dunas
2	C	2014-06-21	Colombia - Ivory Coast	Estadio Nacional
2	C	2014-06-21	Greece - Japan	Arena Pantanal
2	D	2014-06-21	Costa Rica - England	Arena Fonte Nova
2	A	2014-06-22	Brazil - Mexico	Estadio Castelao
2	A	2014-06-22	Croatia - Cameroon	Arena Pernambuco
2	B	2014-06-22	Netherlands - Spain	Arena da Baixada
2	B	2014-06-22	Chile - Australia	Estadio do Maracana
3	F	2014-06-23	Argentina - Iran	Arena Fonte Nova
3	F	2014-06-23	Nigeria - Bosnia and Herzegovina	Arena Pantanal
3	G	2014-06-23	Germany - Ghana	Arena da Amazonia
3	G	2014-06-23	United States - Portugal	Arena de Sao Paulo
3	E	2014-06-24	France - Ecuador	Estadio do Maracana
3	E	2014-06-24	Switzerland - Honduras	Estadio Castelao
3	H	2014-06-24	Belgium - Algeria	Arena das Dunas
3	H	2014-06-24	Russia - South Korea	Arena da Baixada
3	C	2014-06-25	Colombia - Japan	Estadio Mineirao
3	C	2014-06-25	Greece - Ivory Coast	Arena da Amazonia
3	D	2014-06-25	Costa Rica - Italy	Arena Pantanal
3	D	2014-06-25	Uruguay - England	Arena Pernambuco
3	A	2014-06-26	Brazil - Cameroon	Estadio do Maracana
3	A	2014-06-26	Mexico - Croatia	Estadio Nacional
3	B	2014-06-26	Netherlands - Australia	Estadio Castelao
3	B	2014-06-26	Chile - Spain	Estadio Beira-Rio

FIFA World Cup 2018 – Match Fixture Minimizing Total Distance (*pre-assigned base camps*)

Round	Group	Date	Match	Venue
1	A	2018-06-14	Russia - Saudi Arabia	Luzhniki Stadium Moscow
1	A	2018-06-15	Uruguay - Egypt	Samara Stadium
1	C	2018-06-15	Denmark - Peru	Otkrytiye Arena Moscow
1	D	2018-06-15	Argentina - Iceland	Rostov-on-Don Stadium
1	H	2018-06-15	Japan - Poland	Mordovia Arena Saransk
1	C	2018-06-16	France - Australia	Kazan Arena
1	D	2018-06-16	Croatia - Nigeria	Volgograd Stadium
1	E	2018-06-16	Switzerland - Costa Rica	Ekaterinburg Stadium
1	H	2018-06-16	Colombia - Senegal	Nizhny Novgorod Stadium
1	E	2018-06-17	Brazil - Serbia	Fisht Stadium Sochi
1	F	2018-06-17	Sweden - South Korea	Kaliningrad Stadium
1	F	2018-06-17	Mexico - Germany	Luzhniki Stadium Moscow
1	G	2018-06-17	Belgium - England	Saint Petersburg Stadium
1	B	2018-06-18	Spain - Morocco	Rostov-on-Don Stadium
1	B	2018-06-18	Portugal - Iran	Otkrytiye Arena Moscow
1	G	2018-06-18	Tunisia - Panama	Mordovia Arena Saransk
2	A	2018-06-19	Uruguay - Saudi Arabia	Nizhny Novgorod Stadium
2	A	2018-06-19	Russia - Egypt	Volgograd Stadium
2	D	2018-06-20	Croatia - Argentina	Saint Petersburg Stadium
2	D	2018-06-20	Nigeria - Iceland	Fisht Stadium Sochi
2	C	2018-06-21	Denmark - Australia	Ekaterinburg Stadium
2	E	2018-06-21	Brazil - Switzerland	Samara Stadium
2	E	2018-06-21	Serbia - Costa Rica	Kaliningrad Stadium
2	B	2018-06-22	Spain - Iran	Volgograd Stadium
2	B	2018-06-22	Portugal - Morocco	Mordovia Arena Saransk
2	C	2018-06-22	France - Peru	Luzhniki Stadium Moscow
2	H	2018-06-22	Colombia - Japan	Kazan Arena
2	F	2018-06-23	Sweden - Germany	Rostov-on-Don Stadium
2	F	2018-06-23	Mexico - South Korea	Saint Petersburg Stadium
2	H	2018-06-23	Senegal - Poland	Fisht Stadium Sochi
2	G	2018-06-24	Belgium - Tunisia	Otkrytiye Arena Moscow
2	G	2018-06-24	England - Panama	Nizhny Novgorod Stadium
3	A	2018-06-25	Uruguay - Russia	Kazan Arena
3	A	2018-06-25	Saudi Arabia - Egypt	Ekaterinburg Stadium
3	D	2018-06-25	Croatia - Iceland	Kaliningrad Stadium
3	D	2018-06-25	Argentina - Nigeria	Samara Stadium
3	B	2018-06-26	Spain - Portugal	Fisht Stadium Sochi
3	B	2018-06-26	Iran - Morocco	Luzhniki Stadium Moscow
3	E	2018-06-26	Brazil - Costa Rica	Saint Petersburg Stadium
3	E	2018-06-26	Switzerland - Serbia	Mordovia Arena Saransk
3	C	2018-06-27	France - Denmark	Rostov-on-Don Stadium
3	C	2018-06-27	Peru - Australia	Nizhny Novgorod Stadium
3	F	2018-06-27	Sweden - Mexico	Volgograd Stadium
3	F	2018-06-27	South Korea - Germany	Otkrytiye Arena Moscow
3	G	2018-06-28	Belgium - Panama	Kazan Arena
3	G	2018-06-28	England - Tunisia	Kaliningrad Stadium
3	H	2018-06-28	Colombia - Poland	Samara Stadium
3	H	2018-06-28	Japan - Senegal	Ekaterinburg Stadium

FIFA World Cup 2018 – Match Fixture Minimizing Total Distance (*unassigned base camps*)

Round	Group	Date	Match	Venue
1	A	2018-06-14	Russia - Egypt	Luzhniki Stadium Moscow
1	A	2018-06-15	Uruguay - Saudi Arabia	Kaliningrad Stadium
1	D	2018-06-15	Croatia - Nigeria	Mordovia Arena Saransk
1	D	2018-06-15	Argentina - Iceland	Nizhny Novgorod Stadium
1	G	2018-06-15	Belgium - Panama	Samara Stadium
1	C	2018-06-16	France - Australia	Otkrytiye Arena Moscow
1	C	2018-06-16	Denmark - Peru	Fisht Stadium Sochi
1	F	2018-06-16	Mexico - Germany	Saint Petersburg Stadium
1	G	2018-06-16	England - Tunisia	Ekaterinburg Stadium
1	E	2018-06-17	Switzerland - Serbia	Luzhniki Stadium Moscow
1	F	2018-06-17	Sweden - South Korea	Rostov-on-Don Stadium
1	H	2018-06-17	Colombia - Poland	Volgograd Stadium
1	B	2018-06-18	Spain - Portugal	Nizhny Novgorod Stadium
1	B	2018-06-18	Iran - Morocco	Mordovia Arena Saransk
1	E	2018-06-18	Brazil - Costa Rica	Kaliningrad Stadium
1	H	2018-06-18	Japan - Senegal	Kazan Arena
2	A	2018-06-19	Russia - Saudi Arabia	Fisht Stadium Sochi
2	D	2018-06-19	Nigeria - Iceland	Samara Stadium
2	A	2018-06-20	Uruguay - Egypt	Saint Petersburg Stadium
2	C	2018-06-20	France - Peru	Rostov-on-Don Stadium
2	C	2018-06-20	Denmark - Australia	Volgograd Stadium
2	D	2018-06-20	Croatia - Argentina	Luzhniki Stadium Moscow
2	F	2018-06-21	Sweden - Germany	Otkrytiye Arena Moscow
2	G	2018-06-21	Belgium - Tunisia	Kazan Arena
2	F	2018-06-22	Mexico - South Korea	Kaliningrad Stadium
2	G	2018-06-22	England - Panama	Mordovia Arena Saransk
2	H	2018-06-22	Japan - Poland	Ekaterinburg Stadium
2	E	2018-06-23	Brazil - Serbia	Saint Petersburg Stadium
2	E	2018-06-23	Switzerland - Costa Rica	Nizhny Novgorod Stadium
2	H	2018-06-23	Colombia - Senegal	Fisht Stadium Sochi
2	B	2018-06-24	Spain - Iran	Kazan Arena
2	B	2018-06-24	Portugal - Morocco	Otkrytiye Arena Moscow
3	A	2018-06-25	Uruguay - Russia	Samara Stadium
3	A	2018-06-25	Saudi Arabia - Egypt	Rostov-on-Don Stadium
3	D	2018-06-25	Croatia - Iceland	Volgograd Stadium
3	D	2018-06-25	Argentina - Nigeria	Ekaterinburg Stadium
3	C	2018-06-26	France - Denmark	Kaliningrad Stadium
3	C	2018-06-26	Peru - Australia	Saint Petersburg Stadium
3	F	2018-06-26	Sweden - Mexico	Luzhniki Stadium Moscow
3	F	2018-06-26	South Korea - Germany	Mordovia Arena Saransk
3	E	2018-06-27	Brazil - Switzerland	Kazan Arena
3	E	2018-06-27	Serbia - Costa Rica	Otkrytiye Arena Moscow
3	G	2018-06-27	Belgium - England	Fisht Stadium Sochi
3	G	2018-06-27	Tunisia - Panama	Nizhny Novgorod Stadium
3	B	2018-06-28	Spain - Morocco	Volgograd Stadium
3	B	2018-06-28	Portugal - Iran	Ekaterinburg Stadium
3	H	2018-06-28	Colombia - Japan	Rostov-on-Don Stadium
3	H	2018-06-28	Senegal - Poland	Samara Stadium

FIFA World Cup 2018 – Match Fixture Minimizing Total Inner Range (*pre-assigned*)

Round	Group	Date	Match	Venue
1	A	2018-06-14	Russia - Saudi Arabia	Luzhniki Stadium Moscow
1	A	2018-06-15	Uruguay - Egypt	Kazan Arena
1	F	2018-06-15	Sweden - Germany	Fisht Stadium Sochi
1	F	2018-06-15	Mexico - South Korea	Rostov-on-Don Stadium
1	G	2018-06-15	England - Tunisia	Saint Petersburg Stadium
1	B	2018-06-16	Portugal - Iran	Mordovia Arena Saransk
1	C	2018-06-16	France - Denmark	Volgograd Stadium
1	C	2018-06-16	Peru - Australia	Ekaterinburg Stadium
1	G	2018-06-16	Belgium - Panama	Kaliningrad Stadium
1	B	2018-06-17	Spain - Morocco	Samara Stadium
1	E	2018-06-17	Brazil - Costa Rica	Otkrytiye Arena Moscow
1	E	2018-06-17	Switzerland - Serbia	Nizhny Novgorod Stadium
1	D	2018-06-18	Croatia - Argentina	Kazan Arena
1	D	2018-06-18	Nigeria - Iceland	Rostov-on-Don Stadium
1	H	2018-06-18	Colombia - Senegal	Saint Petersburg Stadium
1	H	2018-06-18	Japan - Poland	Fisht Stadium Sochi
2	A	2018-06-19	Uruguay - Russia	Ekaterinburg Stadium
2	F	2018-06-19	Mexico - Germany	Kaliningrad Stadium
2	A	2018-06-20	Saudi Arabia - Egypt	Volgograd Stadium
2	F	2018-06-20	Sweden - South Korea	Otkrytiye Arena Moscow
2	G	2018-06-20	Belgium - Tunisia	Mordovia Arena Saransk
2	G	2018-06-20	England - Panama	Samara Stadium
2	B	2018-06-21	Spain - Portugal	Rostov-on-Don Stadium
2	B	2018-06-21	Iran - Morocco	Saint Petersburg Stadium
2	C	2018-06-21	Denmark - Australia	Luzhniki Stadium Moscow
2	C	2018-06-22	France - Peru	Kazan Arena
2	E	2018-06-22	Brazil - Switzerland	Fisht Stadium Sochi
2	H	2018-06-22	Colombia - Poland	Nizhny Novgorod Stadium
2	E	2018-06-23	Serbia - Costa Rica	Kaliningrad Stadium
2	H	2018-06-23	Japan - Senegal	Mordovia Arena Saransk
2	D	2018-06-24	Croatia - Iceland	Otkrytiye Arena Moscow
2	D	2018-06-24	Argentina - Nigeria	Samara Stadium
3	B	2018-06-25	Spain - Iran	Ekaterinburg Stadium
3	B	2018-06-25	Portugal - Morocco	Volgograd Stadium
3	F	2018-06-25	Sweden - Mexico	Nizhny Novgorod Stadium
3	F	2018-06-25	South Korea - Germany	Luzhniki Stadium Moscow
3	A	2018-06-26	Uruguay - Saudi Arabia	Saint Petersburg Stadium
3	A	2018-06-26	Russia - Egypt	Fisht Stadium Sochi
3	C	2018-06-26	France - Australia	Rostov-on-Don Stadium
3	C	2018-06-26	Denmark - Peru	Mordovia Arena Saransk
3	E	2018-06-27	Brazil - Serbia	Kazan Arena
3	E	2018-06-27	Switzerland - Costa Rica	Samara Stadium
3	H	2018-06-27	Colombia - Japan	Otkrytiye Arena Moscow
3	H	2018-06-27	Senegal - Poland	Kaliningrad Stadium
3	D	2018-06-28	Croatia - Nigeria	Luzhniki Stadium Moscow
3	D	2018-06-28	Argentina - Iceland	Volgograd Stadium
3	G	2018-06-28	Belgium - England	Nizhny Novgorod Stadium
3	G	2018-06-28	Tunisia - Panama	Ekaterinburg Stadium

FIFA World Cup 2018 – Match Fixture Minimizing Total Inner Range (*unassigned*)

Round	Group	Date	Match	Venue
1	A	2018-06-14	Russia - Egypt	Luzhniki Stadium Moscow
1	A	2018-06-15	Uruguay - Saudi Arabia	Samara Stadium
1	G	2018-06-15	Belgium - Panama	Kazan Arena
1	G	2018-06-15	England - Tunisia	Volgograd Stadium
1	H	2018-06-15	Colombia - Poland	Mordovia Arena Saransk
1	D	2018-06-16	Croatia - Argentina	Fisht Stadium Sochi
1	D	2018-06-16	Nigeria - Iceland	Ekaterinburg Stadium
1	E	2018-06-16	Brazil - Costa Rica	Otkrytiye Arena Moscow
1	H	2018-06-16	Japan - Senegal	Rostov-on-Don Stadium
1	E	2018-06-17	Switzerland - Serbia	Kaliningrad Stadium
1	F	2018-06-17	Sweden - Germany	Luzhniki Stadium Moscow
1	F	2018-06-17	Mexico - South Korea	Nizhny Novgorod Stadium
1	B	2018-06-18	Spain - Portugal	Samara Stadium
1	B	2018-06-18	Iran - Morocco	Volgograd Stadium
1	C	2018-06-18	France - Australia	Kazan Arena
1	C	2018-06-18	Denmark - Peru	Saint Petersburg Stadium
2	A	2018-06-19	Uruguay - Russia	Fisht Stadium Sochi
2	A	2018-06-19	Saudi Arabia - Egypt	Mordovia Arena Saransk
2	G	2018-06-20	Belgium - Tunisia	Rostov-on-Don Stadium
2	G	2018-06-20	England - Panama	Otkrytiye Arena Moscow
2	H	2018-06-20	Colombia - Senegal	Luzhniki Stadium Moscow
2	E	2018-06-21	Brazil - Switzerland	Nizhny Novgorod Stadium
2	E	2018-06-21	Serbia - Costa Rica	Volgograd Stadium
2	F	2018-06-21	Sweden - South Korea	Ekaterinburg Stadium
2	H	2018-06-21	Japan - Poland	Samara Stadium
2	C	2018-06-22	France - Peru	Fisht Stadium Sochi
2	D	2018-06-22	Croatia - Nigeria	Saint Petersburg Stadium
2	F	2018-06-22	Mexico - Germany	Kaliningrad Stadium
2	C	2018-06-23	Denmark - Australia	Rostov-on-Don Stadium
2	D	2018-06-23	Argentina - Iceland	Mordovia Arena Saransk
2	B	2018-06-24	Spain - Iran	Kazan Arena
2	B	2018-06-24	Portugal - Morocco	Otkrytiye Arena Moscow
3	G	2018-06-25	Belgium - England	Saint Petersburg Stadium
3	G	2018-06-25	Tunisia - Panama	Kaliningrad Stadium
3	H	2018-06-25	Colombia - Japan	Nizhny Novgorod Stadium
3	H	2018-06-25	Senegal - Poland	Ekaterinburg Stadium
3	E	2018-06-26	Brazil - Serbia	Samara Stadium
3	E	2018-06-26	Switzerland - Costa Rica	Rostov-on-Don Stadium
3	F	2018-06-26	Sweden - Mexico	Fisht Stadium Sochi
3	F	2018-06-26	South Korea - Germany	Mordovia Arena Saransk
3	C	2018-06-27	France - Denmark	Volgograd Stadium
3	C	2018-06-27	Peru - Australia	Otkrytiye Arena Moscow
3	D	2018-06-27	Croatia - Iceland	Luzhniki Stadium Moscow
3	D	2018-06-27	Argentina - Nigeria	Kazan Arena
3	A	2018-06-28	Uruguay - Egypt	Nizhny Novgorod Stadium
3	A	2018-06-28	Russia - Saudi Arabia	Kaliningrad Stadium
3	B	2018-06-28	Spain - Morocco	Saint Petersburg Stadium
3	B	2018-06-28	Portugal - Iran	Ekaterinburg Stadium

FIFA World Cup 2018 – Match Fixture Minimizing Groupwise Inner Ranges (*pre-assigned*)

Round	**Group**	**Date**	**Match**	**Venue**
1	A	2018-06-14	Russia - Saudi Arabia	Luzhniki Stadium Moscow
1	A	2018-06-15	Uruguay - Egypt	Volgograd Stadium
1	C	2018-06-15	France - Peru	Kazan Arena
1	G	2018-06-15	Belgium - England	Saint Petersburg Stadium
1	G	2018-06-15	Tunisia - Panama	Ekaterinburg Stadium
1	C	2018-06-16	Denmark - Australia	Rostov-on-Don Stadium
1	D	2018-06-16	Argentina - Nigeria	Samara Stadium
1	E	2018-06-16	Brazil - Switzerland	Fisht Stadium Sochi
1	E	2018-06-16	Serbia - Costa Rica	Kaliningrad Stadium
1	B	2018-06-17	Iran - Morocco	Otkrytiye Arena Moscow
1	D	2018-06-17	Croatia - Iceland	Mordovia Arena Saransk
1	H	2018-06-17	Senegal - Poland	Nizhny Novgorod Stadium
1	B	2018-06-18	Spain - Portugal	Volgograd Stadium
1	F	2018-06-18	Sweden - South Korea	Luzhniki Stadium Moscow
1	F	2018-06-18	Mexico - Germany	Ekaterinburg Stadium
1	H	2018-06-18	Colombia - Japan	Saint Petersburg Stadium
2	A	2018-06-19	Russia - Egypt	Fisht Stadium Sochi
2	G	2018-06-19	England - Tunisia	Kazan Arena
2	A	2018-06-20	Uruguay - Saudi Arabia	Otkrytiye Arena Moscow
2	C	2018-06-20	France - Denmark	Mordovia Arena Saransk
2	C	2018-06-20	Peru - Australia	Nizhny Novgorod Stadium
2	G	2018-06-20	Belgium - Panama	Rostov-on-Don Stadium
2	D	2018-06-21	Croatia - Argentina	Kaliningrad Stadium
2	D	2018-06-21	Nigeria - Iceland	Volgograd Stadium
2	F	2018-06-22	Sweden - Germany	Saint Petersburg Stadium
2	F	2018-06-22	Mexico - South Korea	Samara Stadium
2	H	2018-06-22	Colombia - Senegal	Ekaterinburg Stadium
2	B	2018-06-23	Spain - Iran	Fisht Stadium Sochi
2	B	2018-06-23	Portugal - Morocco	Mordovia Arena Saransk
2	H	2018-06-23	Japan - Poland	Rostov-on-Don Stadium
2	E	2018-06-24	Brazil - Serbia	Luzhniki Stadium Moscow
2	E	2018-06-24	Switzerland - Costa Rica	Nizhny Novgorod Stadium
3	D	2018-06-25	Croatia - Nigeria	Otkrytiye Arena Moscow
3	D	2018-06-25	Argentina - Iceland	Kazan Arena
3	G	2018-06-25	Belgium - Tunisia	Kaliningrad Stadium
3	G	2018-06-25	England - Panama	Samara Stadium
3	C	2018-06-26	France - Australia	Ekaterinburg Stadium
3	C	2018-06-26	Denmark - Peru	Fisht Stadium Sochi
3	F	2018-06-26	Sweden - Mexico	Rostov-on-Don Stadium
3	F	2018-06-26	South Korea - Germany	Volgograd Stadium
3	B	2018-06-27	Spain - Morocco	Saint Petersburg Stadium
3	B	2018-06-27	Portugal - Iran	Nizhny Novgorod Stadium
3	H	2018-06-27	Colombia - Poland	Luzhniki Stadium Moscow
3	H	2018-06-27	Japan - Senegal	Mordovia Arena Saransk
3	A	2018-06-28	Uruguay - Russia	Kaliningrad Stadium
3	A	2018-06-28	Saudi Arabia - Egypt	Samara Stadium
3	E	2018-06-28	Brazil - Costa Rica	Otkrytiye Arena Moscow
3	E	2018-06-28	Switzerland - Serbia	Kazan Arena

FIFA World Cup 2018 – Match Fixture Minimizing Groupwise Inner Ranges (*unassigned*)

Round	Group	Date	Match	Venue
1	A	2018-06-14	Russia - Saudi Arabia	Luzhniki Stadium Moscow
1	A	2018-06-15	Uruguay - Egypt	Kazan Arena
1	B	2018-06-15	Spain - Morocco	Otkrytiye Arena Moscow
1	B	2018-06-15	Portugal - Iran	Samara Stadium
1	D	2018-06-15	Argentina - Iceland	Volgograd Stadium
1	D	2018-06-16	Croatia - Nigeria	Saint Petersburg Stadium
1	F	2018-06-16	Sweden - Germany	Ekaterinburg Stadium
1	F	2018-06-16	Mexico - South Korea	Fisht Stadium Sochi
1	C	2018-06-17	Peru - Australia	Kaliningrad Stadium
1	G	2018-06-17	Belgium - Panama	Mordovia Arena Saransk
1	H	2018-06-17	Colombia - Japan	Rostov-on-Don Stadium
1	H	2018-06-17	Senegal - Poland	Nizhny Novgorod Stadium
1	C	2018-06-18	France - Denmark	Samara Stadium
1	E	2018-06-18	Brazil - Costa Rica	Volgograd Stadium
1	E	2018-06-18	Switzerland - Serbia	Luzhniki Stadium Moscow
1	G	2018-06-18	England - Tunisia	Otkrytiye Arena Moscow
2	B	2018-06-19	Spain - Iran	Saint Petersburg Stadium
2	B	2018-06-19	Portugal - Morocco	Kazan Arena
2	A	2018-06-20	Saudi Arabia - Egypt	Kaliningrad Stadium
2	D	2018-06-20	Croatia - Iceland	Nizhny Novgorod Stadium
2	D	2018-06-20	Argentina - Nigeria	Ekaterinburg Stadium
2	F	2018-06-20	Sweden - Mexico	Mordovia Arena Saransk
2	A	2018-06-21	Uruguay - Russia	Fisht Stadium Sochi
2	F	2018-06-21	South Korea - Germany	Rostov-on-Don Stadium
2	C	2018-06-22	France - Australia	Kazan Arena
2	C	2018-06-22	Denmark - Peru	Otkrytiye Arena Moscow
2	H	2018-06-22	Japan - Senegal	Luzhniki Stadium Moscow
2	G	2018-06-23	Tunisia - Panama	Saint Petersburg Stadium
2	H	2018-06-23	Colombia - Poland	Volgograd Stadium
2	E	2018-06-24	Brazil - Switzerland	Mordovia Arena Saransk
2	E	2018-06-24	Serbia - Costa Rica	Samara Stadium
2	G	2018-06-24	Belgium - England	Fisht Stadium Sochi
3	A	2018-06-25	Uruguay - Saudi Arabia	Rostov-on-Don Stadium
3	A	2018-06-25	Russia - Egypt	Nizhny Novgorod Stadium
3	B	2018-06-25	Spain - Portugal	Ekaterinburg Stadium
3	B	2018-06-25	Iran - Morocco	Kaliningrad Stadium
3	C	2018-06-26	France - Peru	Luzhniki Stadium Moscow
3	C	2018-06-26	Denmark - Australia	Volgograd Stadium
3	F	2018-06-26	Sweden - South Korea	Saint Petersburg Stadium
3	F	2018-06-26	Mexico - Germany	Kazan Arena
3	D	2018-06-27	Croatia - Argentina	Samara Stadium
3	D	2018-06-27	Nigeria - Iceland	Mordovia Arena Saransk
3	H	2018-06-27	Colombia - Senegal	Otkrytiye Arena Moscow
3	H	2018-06-27	Japan - Poland	Fisht Stadium Sochi
3	E	2018-06-28	Brazil - Serbia	Nizhny Novgorod Stadium
3	E	2018-06-28	Switzerland - Costa Rica	Rostov-on-Don Stadium
3	G	2018-06-28	Belgium - Tunisia	Ekaterinburg Stadium
3	G	2018-06-28	England - Panama	Kaliningrad Stadium

FIFA World Cup 2014: ϵ-constraint model - epsilon-level: 1

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Mexico	Arena de Sao Paulo
1	A	2014-06-13	Croatia - Cameroon	Arena das Dunas
1	D	2014-06-13	Costa Rica - England	Estadio do Maracana
1	D	2014-06-13	Uruguay - Italy	Estadio Mineirao
1	H	2014-06-13	Belgium - South Korea	Estadio Beira-Rio
1	C	2014-06-14	Colombia - Japan	Arena da Baixada
1	G	2014-06-14	Germany - Portugal	Arena Fonte Nova
1	G	2014-06-14	United States - Ghana	Arena Pernambuco
1	H	2014-06-14	Algeria - Russia	Estadio Nacional
1	B	2014-06-15	Netherlands - Spain	Arena Pantanal
1	C	2014-06-15	Greece - Ivory Coast	Estadio Castelao
1	F	2014-06-15	Nigeria - Iran	Arena da Amazonia
1	B	2014-06-16	Chile - Australia	Arena Fonte Nova
1	E	2014-06-16	France - Switzerland	Estadio Nacional
1	E	2014-06-16	Ecuador - Honduras	Arena da Baixada
1	F	2014-06-16	Argentina - Bosnia and Herzegovina	Arena Pernambuco
2	G	2014-06-17	Germany - Ghana	Arena das Dunas
2	H	2014-06-17	Russia - South Korea	Arena Pantanal
2	A	2014-06-18	Brazil - Cameroon	Estadio Nacional
2	G	2014-06-18	United States - Portugal	Arena de Sao Paulo
2	H	2014-06-18	Belgium - Algeria	Arena da Baixada
2	A	2014-06-19	Mexico - Croatia	Estadio Castelao
2	B	2014-06-19	Chile - Spain	Estadio Mineirao
2	E	2014-06-19	France - Ecuador	Estadio Beira-Rio
2	B	2014-06-20	Netherlands - Australia	Estadio do Maracana
2	D	2014-06-20	Uruguay - England	Arena da Amazonia
2	E	2014-06-20	Switzerland - Honduras	Arena Fonte Nova
2	D	2014-06-21	Costa Rica - Italy	Arena Pantanal
2	F	2014-06-21	Argentina - Nigeria	Estadio Mineirao
2	F	2014-06-21	Bosnia and Herzegovina - Iran	Arena de Sao Paulo
2	C	2014-06-22	Colombia - Greece	Arena Pernambuco
2	C	2014-06-22	Ivory Coast - Japan	Estadio do Maracana
3	G	2014-06-23	Germany - United States	Estadio Mineirao
3	G	2014-06-23	Portugal - Ghana	Arena da Amazonia
3	H	2014-06-23	Belgium - Russia	Arena Fonte Nova
3	H	2014-06-23	Algeria - South Korea	Estadio Castelao
3	D	2014-06-24	Costa Rica - Uruguay	Estadio Nacional
3	D	2014-06-24	Italy - England	Estadio Beira-Rio
3	E	2014-06-24	France - Honduras	Arena Pantanal
3	E	2014-06-24	Switzerland - Ecuador	Arena das Dunas
3	A	2014-06-25	Brazil - Croatia	Arena Pernambuco
3	A	2014-06-25	Mexico - Cameroon	Arena da Baixada
3	C	2014-06-25	Colombia - Ivory Coast	Arena de Sao Paulo
3	C	2014-06-25	Greece - Japan	Arena da Amazonia
3	B	2014-06-26	Netherlands - Chile	Estadio Castelao
3	B	2014-06-26	Spain - Australia	Arena das Dunas
3	F	2014-06-26	Argentina - Iran	Estadio Beira-Rio
3	F	2014-06-26	Nigeria - Bosnia and Herzegovina	Estadio do Maracana

FIFA World Cup 2014: ϵ-constraint model - epsilon-level: 2

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Mexico	Arena de Sao Paulo
1	A	2014-06-13	Croatia - Cameroon	Arena Pernambuco
1	C	2014-06-13	Colombia - Ivory Coast	Estadio Nacional
1	E	2014-06-13	Ecuador - Honduras	Estadio Beira-Rio
1	F	2014-06-13	Bosnia and Herzegovina - Iran	Estadio do Maracana
1	C	2014-06-14	Greece - Japan	Arena da Amazonia
1	E	2014-06-14	France - Switzerland	Estadio Castelao
1	F	2014-06-14	Argentina - Nigeria	Estadio Mineirao
1	G	2014-06-14	Germany - Ghana	Arena das Dunas
1	D	2014-06-15	Italy - England	Estadio do Maracana
1	G	2014-06-15	United States - Portugal	Arena de Sao Paulo
1	H	2014-06-15	Belgium - South Korea	Estadio Beira-Rio
1	H	2014-06-15	Algeria - Russia	Arena da Baixada
1	B	2014-06-16	Netherlands - Australia	Arena Fonte Nova
1	B	2014-06-16	Chile - Spain	Arena Pantanal
1	D	2014-06-16	Costa Rica - Uruguay	Estadio Nacional
2	C	2014-06-17	Colombia - Japan	Arena da Baixada
2	C	2014-06-17	Greece - Ivory Coast	Arena Pernambuco
2	A	2014-06-18	Mexico - Cameroon	Estadio do Maracana
2	F	2014-06-18	Argentina - Bosnia and Herzegovina	Arena da Amazonia
2	F	2014-06-18	Nigeria - Iran	Arena Pantanal
2	A	2014-06-19	Brazil - Croatia	Arena das Dunas
2	D	2014-06-19	Uruguay - England	Estadio Mineirao
2	G	2014-06-19	Germany - United States	Arena Fonte Nova
2	B	2014-06-20	Netherlands - Spain	Estadio Beira-Rio
2	D	2014-06-20	Costa Rica - Italy	Arena de Sao Paulo
2	G	2014-06-20	Portugal - Ghana	Estadio Castelao
2	B	2014-06-21	Chile - Australia	Estadio Mineirao
2	E	2014-06-21	Switzerland - Honduras	Arena Fonte Nova
2	H	2014-06-21	Belgium - Russia	Estadio Nacional
2	E	2014-06-22	France - Ecuador	Arena da Baixada
2	H	2014-06-22	Algeria - South Korea	Arena Pantanal
3	D	2014-06-23	Costa Rica - England	Arena da Amazonia
3	D	2014-06-23	Uruguay - Italy	Arena das Dunas
3	G	2014-06-23	Germany - Portugal	Estadio Nacional
3	G	2014-06-23	United States - Ghana	Arena Pernambuco
3	A	2014-06-24	Brazil - Cameroon	Estadio Mineirao
3	A	2014-06-24	Mexico - Croatia	Estadio Castelao
3	F	2014-06-24	Argentina - Iran	Arena Fonte Nova
3	F	2014-06-24	Nigeria - Bosnia and Herzegovina	Arena da Baixada
3	E	2014-06-25	France - Honduras	Arena Pantanal
3	E	2014-06-25	Switzerland - Ecuador	Arena Pernambuco
3	H	2014-06-25	Belgium - Algeria	Arena de Sao Paulo
3	H	2014-06-25	Russia - South Korea	Arena da Amazonia
3	B	2014-06-26	Netherlands - Chile	Estadio do Maracana
3	B	2014-06-26	Spain - Australia	Estadio Castelao
3	C	2014-06-26	Colombia - Greece	Arena das Dunas
3	C	2014-06-26	Ivory Coast - Japan	Estadio Beira-Rio

FIFA World Cup 2014: ϵ-constraint model - epsilon-level: 3

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Mexico	Arena de Sao Paulo
1	A	2014-06-13	Croatia - Cameroon	Arena Pernambuco
1	B	2014-06-13	Chile - Spain	Arena da Amazonia
1	C	2014-06-13	Colombia - Japan	Arena da Baixada
1	G	2014-06-13	Germany - Portugal	Arena Fonte Nova
1	B	2014-06-14	Netherlands - Australia	Estadio do Maracana
1	C	2014-06-14	Greece - Ivory Coast	Estadio Castelao
1	D	2014-06-14	Uruguay - Italy	Estadio Nacional
1	G	2014-06-14	United States - Ghana	Arena das Dunas
1	D	2014-06-15	Costa Rica - England	Estadio Mineirao
1	E	2014-06-15	France - Ecuador	Arena Pantanal
1	E	2014-06-15	Switzerland - Honduras	Arena Fonte Nova
1	H	2014-06-15	Belgium - South Korea	Estadio Beira-Rio
1	F	2014-06-16	Argentina - Nigeria	Estadio Nacional
1	F	2014-06-16	Bosnia and Herzegovina - Iran	Arena de Sao Paulo
1	H	2014-06-16	Algeria - Russia	Arena da Baixada
2	A	2014-06-17	Brazil - Croatia	Arena Fonte Nova
2	B	2014-06-17	Netherlands - Spain	Estadio Beira-Rio
2	A	2014-06-18	Mexico - Cameroon	Estadio do Maracana
2	B	2014-06-18	Chile - Australia	Estadio Mineirao
2	C	2014-06-19	Ivory Coast - Japan	Estadio Nacional
2	E	2014-06-19	France - Switzerland	Estadio Castelao
2	E	2014-06-19	Ecuador - Honduras	Estadio Beira-Rio
2	C	2014-06-20	Colombia - Greece	Arena das Dunas
2	F	2014-06-20	Nigeria - Iran	Arena Pantanal
2	G	2014-06-20	Germany - Ghana	Arena Pernambuco
2	G	2014-06-20	United States - Portugal	Arena da Baixada
2	D	2014-06-21	Italy - England	Estadio do Maracana
2	F	2014-06-21	Argentina - Bosnia and Herzegovina	Estadio Mineirao
2	D	2014-06-22	Costa Rica - Uruguay	Arena da Amazonia
2	H	2014-06-22	Belgium - Russia	Arena de Sao Paulo
2	H	2014-06-22	Algeria - South Korea	Arena Pantanal
3	B	2014-06-23	Netherlands - Chile	Arena das Dunas
3	B	2014-06-23	Spain - Australia	Arena da Baixada
3	G	2014-06-23	Germany - United States	Estadio Mineirao
3	G	2014-06-23	Portugal - Ghana	Estadio Castelao
3	C	2014-06-24	Colombia - Ivory Coast	Arena Pantanal
3	C	2014-06-24	Greece - Japan	Arena Pernambuco
3	F	2014-06-24	Argentina - Iran	Arena da Amazonia
3	F	2014-06-24	Nigeria - Bosnia and Herzegovina	Estadio Beira-Rio
3	A	2014-06-25	Brazil - Cameroon	Arena das Dunas
3	A	2014-06-25	Mexico - Croatia	Estadio Castelao
3	D	2014-06-25	Costa Rica - Italy	Arena de Sao Paulo
3	D	2014-06-25	Uruguay - England	Arena Fonte Nova
3	E	2014-06-26	France - Honduras	Estadio Nacional
3	E	2014-06-26	Switzerland - Ecuador	Arena Pernambuco
3	H	2014-06-26	Belgium - Algeria	Estadio do Maracana
3	H	2014-06-26	Russia - South Korea	Arena da Amazonia

FIFA World Cup 2014: ϵ-constraint model - epsilon-level: 4

Round	Group	Date	Match	Venue
1	A	2014-06-12	Brazil - Mexico	Arena de Sao Paulo
1	A	2014-06-13	Croatia - Cameroon	Arena Pernambuco
1	B	2014-06-13	Netherlands - Spain	Arena da Baixada
1	B	2014-06-13	Chile - Australia	Arena Fonte Nova
1	H	2014-06-13	Belgium - Russia	Estadio do Maracana
1	C	2014-06-14	Greece - Ivory Coast	Estadio Castelao
1	D	2014-06-14	Costa Rica - Italy	Arena de Sao Paulo
1	D	2014-06-14	Uruguay - England	Estadio Mineirao
1	H	2014-06-14	Algeria - South Korea	Arena Pantanal
1	C	2014-06-15	Colombia - Japan	Arena da Baixada
1	F	2014-06-15	Argentina - Nigeria	Estadio Nacional
1	G	2014-06-15	Germany - Portugal	Arena da Amazonia
1	G	2014-06-15	United States - Ghana	Arena das Dunas
1	E	2014-06-16	France - Ecuador	Arena Pantanal
1	E	2014-06-16	Switzerland - Honduras	Arena Fonte Nova
1	F	2014-06-16	Bosnia and Herzegovina - Iran	Estadio Beira-Rio
2	A	2014-06-17	Mexico - Cameroon	Estadio Mineirao
2	D	2014-06-17	Costa Rica - Uruguay	Estadio Nacional
2	A	2014-06-18	Brazil - Croatia	Arena Fonte Nova
2	C	2014-06-18	Colombia - Greece	Arena das Dunas
2	D	2014-06-18	Italy - England	Estadio do Maracana
2	C	2014-06-19	Ivory Coast - Japan	Estadio Nacional
2	E	2014-06-19	Ecuador - Honduras	Estadio Beira-Rio
2	G	2014-06-19	Germany - Ghana	Arena Pernambuco
2	G	2014-06-19	United States - Portugal	Arena de Sao Paulo
2	B	2014-06-20	Netherlands - Australia	Estadio do Maracana
2	B	2014-06-20	Chile - Spain	Arena da Amazonia
2	E	2014-06-20	France - Switzerland	Estadio Castelao
2	H	2014-06-21	Belgium - South Korea	Estadio Beira-Rio
2	H	2014-06-21	Algeria - Russia	Arena da Baixada
2	F	2014-06-22	Argentina - Bosnia and Herzegovina	Estadio Mineirao
2	F	2014-06-22	Nigeria - Iran	Arena Pantanal
3	A	2014-06-23	Brazil - Cameroon	Estadio do Maracana
3	A	2014-06-23	Mexico - Croatia	Arena das Dunas
3	G	2014-06-23	Germany - United States	Arena Fonte Nova
3	G	2014-06-23	Portugal - Ghana	Estadio Castelao
3	C	2014-06-24	Colombia - Ivory Coast	Arena Pantanal
3	C	2014-06-24	Greece - Japan	Arena Pernambuco
3	H	2014-06-24	Belgium - Algeria	Arena de Sao Paulo
3	H	2014-06-24	Russia - South Korea	Arena da Amazonia
3	B	2014-06-25	Netherlands - Chile	Estadio Mineirao
3	B	2014-06-25	Spain - Australia	Arena das Dunas
3	D	2014-06-25	Costa Rica - England	Estadio Beira-Rio
3	D	2014-06-25	Uruguay - Italy	Estadio Castelao
3	E	2014-06-26	France - Honduras	Estadio Nacional
3	E	2014-06-26	Switzerland - Ecuador	Arena Pernambuco
3	F	2014-06-26	Argentina - Iran	Arena da Amazonia
3	F	2014-06-26	Nigeria - Bosnia and Herzegovina	Arena da Baixada