Norwegian School of Economics
Bergen, Fall 2023

Bigger isn’t always better

A twenty-one-year study of the size effect on the Oslo Stock Exchange

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Master thesis, Economics and Business Administration

Major: Financial Economics

NORWEGIAN SCHOOL OF ECONOMICS

Acknowledgements

First and foremost, we wish to express our sincere gratitude to our supervisor, Jørgen Haug, for encouraging us to think creatively and allowing us autonomy in both the selection of research topics and the problem-solving process. His insight and guidance have truly been of great value. We also want to thank the Norwegian School of Economics (NHH) for providing access to databases, which has been vital for our research. Further, we wish to exploit this occasion to also thank friends and family for supporting us during our years in Bergen, which has culminated in this thesis. Lastly, we thank each other for a pleasant and enjoyable partnership in what has been a rewarding learning experience.

Norwegian School of Economics

Bergen, December 2023

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Abstract

Our analysis reveals an average monthly size effect of 1.29 percent on the Oslo Stock Exchange from 2000 to 2020, a significance that persists after adjusting for various systematic risk factors. Despite substantial exposure to the Small Minus Big (SMB) factor, our findings suggest it cannot fully account for the observed effect. Our liquidity analysis indicates a tendency for liquid stocks to outperform illiquid stocks of similar size, although the evidence is inconclusive. Examination of liquidity risk demonstrates that the size effect is exposed to systematic liquidity risk, yet conclusive results are hindered by robustness concerns and potential model biases. Our investigation reveals an average size effect in January of 12.08 percent and no significant returns observed from February to December. This highlights that the size effect is concentrated exclusively in January. Furthermore, indications suggest that market turmoil and uncertainty impact the effect, although conclusive results cannot be ascertained. Lastly, we find a negative correlation between market liquidity and the size effect, posing potential implications for the execution of trading strategies based on the size effect and offering insight into the persistence of the effect.

Keywords: Size effect, systematic risk, liquidity, January effect, flight-to-safety

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1. Introduction

The purpose of this thesis is to analyse the size effect on the Oslo Stock Exchange between 2000 and 2020. The size effect refers to the empirical observation of small stocks, as measured by market capitalization, referred to hereafter as *size*, to outperform large stocks over time. The effect was formally introduced by Rolf W. Banz (1981) and has since been subject to much debate and research in finance literature. However, studies show inconsistent results across time periods and markets. In this thesis, we aim to establish whether the effect is present on the Oslo Stock Exchange, before testing several competing theories to explain its existence.

To establish the existence of the effect we utilize a portfolio sorting procedure. This includes sorting stocks in decile[1] portfolios, based on size, and analysing monthly returns. We examine the size effect by analysing the spread between the smallest size decile portfolio (hereafter SmallCap), and the highest size decile portfolio (hereafter LargeCap), which constitutes the *decile spread portfolio*. The decile spread portfolio is structured as a long-short equity portfolio which is long the SmallCap decile and short the LargeCap decile portfolio. We report a 1.29 percent average monthly return of the decile spread portfolio, significantly different from zero. This indicates that the size effect is present on the Oslo Stock Exchange between 2000-2020.

To further analyse the effect, we seek to adjust the effect for systematic risk factors. For this purpose, we utilize two asset pricing models, the CAPM and the Fama-French three-factor model. We regress the SmallCap, LargeCap, and the decile spread portfolio against the models, to retrieve a systematic risk adjusted size effect. For the CAPM regressions, we report a significant average size effect adjusted for the market risk premium factor of 1.33 percent per month. For the Fama-French three-factor model regression, we report a significant adjusted average size effect of 1.46 percent per month.

Furthermore, we consider liquidity-based perspectives on the size effect. This includes testing if the effect can be explained by variation in liquidity and liquidity risk. In this section, we perform a conditional portfolio sorting procedure to identify variation in return between stocks of similar size, but with different liquidity. To test liquidity-risk theories, we extend the Fama-

[1] A decile describes one of ten equal parts of a dataset. We later refer to both octiles, which is the equivalent for eight parts, and quintiles, which is the equivalent for five parts.

French three-factor model to include a liquidity risk factor. In the conditional portfolio sorting, we fail to identify significant results. The extended Fama-French three-factor model regression shows significant exposure of the decile spread portfolio to the liquidity risk factor, but this cannot explain the effect which remains significant at 1.55 percent per month.

We proceed to adjust for the January effect. This effect refers to the empirical observation of higher stock returns in January than in other months, which is especially evident in small stocks (Rozeff & Kinney, 1976). When adjusting for January, we find no significant size effect remaining. However, in January we find a significant average return of 12.08 percent for the decile spread portfolio.

In the final analysis, we explore the flight-to-safety effect. This includes analysing how the effect behaves in times of significant market turmoil and uncertainty, captured using the VIX[2] and a market liquidity measure. We find indications that the size effect reverts in times of significant market volatility. However, as the results lack significance, the results are inconclusive.

1.1. Motivation

Researching the size effect offers insight into several areas within finance. From a corporate finance perspective, the size effect influences valuation because long-term stock returns serve as a proxy for equity cost of capital (Palys & McNish, 2020). The effect also pertains to investment theory, as market inefficiencies represent profit opportunities. Researching the size effect might therefore be of interest to a wide array of readers.

Furthermore, research has shown that the presence, and magnitude, of the size effect varies greatly across markets (Dijk, 2014). Considering the unique characteristics of the Oslo Stock Exchange, analysing the effect in this market may therefore provide new evidence and subtle nuances about the effect to the existing literature. Lastly, it has been argued that the size effect has diminished over time (Schwert, 2003). Our time frame will therefore provide an updated analysis of the persistence of the size effect.

[2] VIX refers to The Chicago Options Exchange Volatility Index.

1.2. Background

For a comprehensive understanding of the size effect, we first introduce the background for its discovery. The concept of asset pricing models was formally introduced by William F. Sharpe (1964) and John Lintner (1965), with significant contributions from Jack Treynor (1961; 1962) and Jan Mossin (1966), when they introduced the Capital Asset Pricing Model (CAPM). The CAPM is a single-factor model used to price assets based on the risk free-rate and their exposure to the market risk premium factor. Nearly 70 years after its inception, the model remains among the most prevalent asset pricing models used among finance professionals and academics worldwide[3].

Despite being widely recognized, the model eventually proved to have significant shortcomings and was heavily criticized (Roll, 1977; Friend & Blume, 1970). One of the most common critiques was the failure of CAPM to acknowledge the importance of other systematic risk factors impacting expected return, such as size. This critique referred to the empirical observation of small stocks outperforming large stocks over time. This later became known as the size effect, or the size effect anomaly.

[3] Harvey and Graham (2001) found that about 74 percent of chief financial officers use the CAPM. Furthermore, Welch and Goyal (2008) later found approximately 75 percent of finance professors to recommend using the CAPM for calculating equity cost of capital.

2. Literature review

The size effect was originally discovered by the American economist Rolf W. Banz in 1981. Banz (1981) observes an empirical relationship between stock returns and the market value of common stocks, where small stocks earn significantly higher average returns than large stocks over time. He finds the effect to be most prevalent with the smallest stocks and argues that investors are reluctant to hold these stocks, due to less information transparency, resulting in higher expected return. When analysing the return of quintile portfolios, Banz finds the lowest size portfolio to significantly outperform the highest by 0.40 percent per month. Banz's work inspired research on both the presence and explanations of the size effect.

In the decade following Banz's publication, the size effect was extensively researched, primarily on U.S. data, with results generally aligning with Banz's findings. Reinganum (1981) examines size-based decile portfolios between 1963 and 1977, on stocks listed on the NYSE and the Amex, and finds the lowest size decile to outperform the largest by 1.8 percent per month. Keim (1983) extends the dataset to stretch between 1963 and 1979 and find an average size effect of 2.5 percent per month.

Keim (1983) also examines the effect adjusted for market risk exposure using the CAPM. He finds that despite small stocks having significantly higher betas than large stocks, this cannot fully explain the effect (Keim, 1983). Lamoureux and Sanger (1989) include Nasdaq stocks and analyse 20 size-based portfolios between 1973 and 1985. They find an average monthly size effect of 1.7 percent in NYSE and Amex stocks, and a 2.0 percent average monthly size effect in Nasdaq stocks (Lamoureux & Sanger, 1989). Contrary to Keim's findings on the NYSE and the Amex, Lamoureux and Sanger find large firms to have significantly higher betas than small firms on the Nasdaq (Lamoureux & Sanger, 1989). Fama and French (1992) extend the CAPM, by including a size and value factor in their three-factor model. They argue that the size effect represents a systematic risk factor deemed too important to be neglected in an asset pricing model.

Research outside the U.S. markets shows consistent results for the size effect's presence and significance in Europe and other emerging markets in Southeast Asia and South America (Dijk, 2014). Bagella et al. (2000) examine decile portfolios and find an average monthly size effect of 1.2 percent for U.K. stocks between 1971 and 1997. Stehle (1997) also examine decile portfolios and find a 0.5 percent monthly effect in German stocks between 1954 and 1990. Annaert et al. (2002) examine decile portfolios in the European markets collectively on data between 1974 and 1992 and find a 1.5 percent average size effect per month.

Despite research on data prior to the works of Banz consistently showing the presence of the size effect across markets, other research argues that the effect has vanished over time. Neither Horowitz (2000a) nor Amihud (2002) finds a significant effect when examining data between 1982 and 1998 in the U.S. and the European markets. William Schwert (2003) even argues that the size effect has disappeared after its discovery as investors have capitalized on the market inefficiency.

However, more recent studies find that the size effect is still present when adjusting for quality factors. Novy-Marx (2013) finds a connection between size and expected returns after controlling for quality, defined as profitable stocks[4], on data from 1963 to 2010 in the U.S. markets. Asness et al. (2018) find similar results, showing a size effect between 1926 and 2012 in the U.S. when adjusting for *junk*, defined as volatile, low margin and highly leveraged stocks. Despite these findings, Alquist et al. (2018) fail to demonstrate equivalent results in markets outside the U.S.

Berk (1995a) introduces a different perspective on the size effect. He argues that because size, as measured by market capitalization, inherently includes a risk premium, there is a fundamental relationship between stock returns and size (Berk, 1995a). Therefore, according to Berk (1995a), to claim the presence of a size effect, stock returns must also be correlated with alternative measures of size, such as sales, book value of assets, and number of employees.

Inspired by Berk's perspective, Godvik and Ohma (2004) examine the size effect on the Oslo Stock exchange between 1992 and 2001. They find no significant relation between stock return and size, using Berk's measures (Godvik & Ohma, 2004). Although several studies explore the size effect when examining general market efficiency on the Oslo Stock Exchange (Borch, 2008; Isaksen, 2021), the literature lacks an in-depth study of the size effect in this market. This research gap lays the foundation for our thesis, in which we analyse the size effect on the Oslo Stock Exchange between 2000 and 2020.

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[4] Novy-Marx (2013) defines profitability as gross profit to assets.

3. Methodology

In this section we present the models and methodologies used to conduct our analysis. First, we discuss portfolio sorting, which constitutes the foundation for our analysis. This also includes double portfolio sorting, both conditional and unconditional. We proceed to explore the two asset pricing models utilized in the analysis, the CAPM and the Fama-French three-factor model.

3.1. Portfolio sorting

To establish whether the size effect is present on the Oslo Stock Exchange we use a portfolio sorting procedure, which is a widely used method in empirical finance to investigate stock returns (Cattaneo et al, 2016). The procedure entails dividing stocks into portfolios based on a characteristic, in our case size, to establish whether there are significant differences in average returns. Portfolio sorting is a non-parametric[5] method that offers an alternative to enforcing linearity in the relationship between size and return (Cattaneo et al, 2016). This method allows us to investigate the size effect while reducing idiosyncratic risk impacting our results.

Furthermore, this aligns with the findings of Dijk (2014) who summarizes studies on the size effect between 1936 and 2000. He highlights portfolio sorting as the commonly used method to establish the presence of the effect. However, we observe that the number of portfolios tends to vary across studies. Selecting the appropriate number of portfolios imposes a discussion on the bias-variance trade-off described by Cattaneo et al. (2016). Decreasing the number of portfolios will result in less bias, but more variation in the characteristics we are analyzing. In contrast, increasing the number of portfolios will impose more bias stemming from individual returns. Following the insight from Cattaneo et al. (2016) and studies of similar sample sizes (Dijk, 2014), we divide the data into decile portfolios. However, to ensure robustness in our findings we also conduct the procedure using octile and quintiles portfolios.

We employ this method by dividing the data into decile portfolios based on size. The portfolios are re-sorted in January each year, with an equal number of stocks in each portfolio. To evaluate the portfolio return we use value-weighted individual stock returns, as is most common in

[5] Non-parametric methods do not make assumptions (e.g. normal distribution) about the underlying distribution of the population from which the data is draw (Blum & Fattu, 1954).

previous studies (Dijk, 2014). To support robustness, we also repeat the procedure for equally weighted portfolios, see Appendix B1.

When examining the size effect, the portfolios of interest are the LargeCap and SmallCap decile portfolios. We construct a long-short equity portfolio, which is long the SmallCap and short the LargeCap decile portfolio. This composite portfolio is referred to as the decile spread portfolio. The average monthly return of the decile spread portfolio is used to establish whether the effect is present and significant in our data.

3.2. Double portfolio sorting

When exploring the connection between the size effect and liquidity, we employ a double portfolio sorting procedure. In this context, there are two main approaches to consider, conditional and unconditional sorting. Both methods are widely used in previous studies, relevant to this thesis (Amihud & Mendelson, 1986; Asness et al, 2018), but offer insight from different perspectives.

3.2.1. Conditional

In conditional portfolio sorting, stocks are first grouped by one factor, in this case size. Then, each size portfolio is subdivided into portfolios based on the second factor, in this case liquidity. As our data contains on average 209 stocks, and it is necessary to have a sufficient number of stocks in each subdivided portfolio, we start by forming five size portfolios and continue with subdividing each size portfolio into five liquidity portfolios, resulting in 25 portfolios, with an equal number of stocks in each portfolio. The method sort liquidity conditional on size, giving size the greatest weight in the formation of portfolios. The results yield an analysis of how returns vary based on liquidity within each size portfolio but do not consider the liquidity differences across size portfolios.

3.2.2. Unconditional

Unconditional portfolio sorting is initiated by sorting stocks independently on two characteristics. We form five portfolios based on size, and five based on liquidity. Following this independent sorting, 25 sub-portfolios are formed at the intersection between the size and liquidity portfolios. This method gives equal weight to the two characteristics and the number of stocks in each sub-portfolio will therefore vary conditionally on how many stocks are sorted in each intersection. Compared to conditional sorting, this method allows for a wider interpretation based on both size and liquidity. However, the method can yield portfolios containing zero or very few stocks, which can impact the ability to draw conclusions based on these portfolios.

3.3. The Capital Asset Pricing Model

Several studies attribute the size effect to smaller stocks having more exposure to systematic risk factors (Asness et al, 2018). To analyse the effect, we must therefore examine the returns from a systematic risk-adjusted perspective. We first employ the CAPM to retrieve the beta-adjusted effect, commonly referred to in the literature (Keim, 1983; Lamoureux & Sanger, 1989). This method exhibits whether the return can be explained merely by higher exposure to the market risk premium factor.

The CAPM aims to price assets by calculating their expected return. The model accounts for each asset's non-diversifiable market risk exposure (Sharpe, 1964; Lintner, 1965; Treynor, 1962; Mossin, 1966).

$$E(R_i) = R_f + [E(R_m) - r_f] \cdot \beta_i \tag{3.1}$$

$$\beta_i = \frac{cov(R_i, R_m)}{var(R_m)} \tag{3.2}$$

Equation 3.1 calculates the expected return as the risk-free rate plus the market risk premium, multiplied with an asset's sensitivity to the market risk, denoted as the beta, see equation 3.2. The CAPM offers an easy and intuitive approach to calculating expected return and remains the preferred choice for both finance professionals and academic (Graham & Harvey, 2001; Welch & Goyal, 2008). Despite being widely used in practice, the model has been criticized, as discussed in the literature review. Perhaps the most common critique is its neglection of other risk factors impacting expected return (Fama & French, 2004).

3.4. The Fama-French three-factor model

To extend our analysis of the risk-adjusted size effect we employ the Fama-French three-factor model. The model is an extension of the CAPM, where a size and value factor are included (Fama & French, 1992). Fama and French (1992) argue that size and value are systematic risk factors that must be accounted for when estimating expected return. This model allows us to evaluate the risk adjusted size effect after controlling for the size and value factors, in addition to the market risk premium. Furthermore, the model allows us to evaluate if the Fama-French size factor (SMB) can explain variation in the size effect, as measured by the decile spread portfolio.

$$r_i - R_f = \alpha_i + \left(R_m - R_f\right) \cdot \beta_i + SMB \cdot s_i + HML \cdot h_i + \epsilon_i \qquad (3.3)$$

$$R_{HML} = \tfrac{1}{2}\left(R_{B,H} + R_{S,H}\right) - \tfrac{1}{2}\left(R_{B,L} + R_{B,S}\right) \qquad (3.4)$$

$$R_{SMB} = \tfrac{1}{3}\left(R_{S,H} + R_{S,M} + R_{S,L}\right) - \tfrac{1}{3}\left(R_{B,H} + R_{B,M} + R_{B,L}\right) \qquad (3.5)$$

The Fama-French three-factor model is shown in equation 3.3. The market risk premium is identical to the factor in the CAPM. The size and value factors are constructed by long-short equity portfolios using portfolio sorting. The size-based sorting separates two portfolios by median market capitalization. The below median portfolio is labelled Small (S), and the above median portfolio is labelled Big (B). The value-based sorting uses the Book-to-market (B/M) ratio as the portfolio sorting criteria. The stocks are sorted into three portfolios, where the lowest 30 percent are labelled Low (L), the mid 40 percent are labelled Medium (M), and the highest 30 percent are labelled High (H).

The "HML – High Minus Low" is the value factor and seeks to capture difference in stock performance between value and growth stocks. The factor is based on the notion that value stocks tend to outperform growth stocks over time. The calculation of the HML factor is shown in equation 3.4.

The size factor is called the "SMB – Small Minus Big" and aims to capture the differences in stock performance between small and large market capitalization stocks. Like the size effect, the SMB factor is based on the notion that generally small capitalization stocks tend to outperform large capitalization stocks over time. The SMB equation is shown in equation 3.5.

Compared to the CAPM, the Fama-French three-factor model offers a more nuanced and accurate explanation for variation in stock return (Fama & French, 1992). However, there are continuous debates as to how the SMB and HML factors are persistent across time and markets (Daniel & Titman, 1997).

4. Data

The main dataset utilized is obtained from Titlon (2023) and contains daily data for all stocks listed on the Oslo Stock Exchange main list between January 2000 and November 2020. This does not include stocks listed on the other multilateral trading facilities including Euronext Growth (former Merkur Markets) and Euronext Expand (former Oslo Axess). In addition, data on the VIX is gathered from Yahoo Finance (2023). Given that our analysis is based on monthly observations, we have converted the relevant daily variables into a monthly format. The processes of screening and cleaning this data are detailed in Appendix A1, and an explanation of variables are provided in Appendix A2. We construct all systematic risk factors used in our analysis, including the MRP, the SMB and the HML, from our data in accordance with relevant literature.

4.1. Limitations

The first notable limitation is related to the number of stocks on the Oslo Stock Exchange. This issue is particularly pronounced in the double portfolio sorting. Due to the limited quantity of stocks, we had to reduce our initial sorting from ten to five portfolios. This prevents us from making direct comparisons between the results obtained from our portfolio sorting analyses.

Additionally, the availability of accounting data represents another limitation. Our data is confined to observations of equity, debt, and earnings. This prevents us from testing theories on quality, which has been shown to present interesting results in recent studies (Asness et al, 2018). Furthermore, we are unable to offer an analysis of Berk's (1995a) approach to measure size. This includes considering variables such as Property, Plant and Equipment, the number of employees, and sales, as alternative measures of size.

Furthermore, we only have accounting figures for approximately 80 percent of the stocks. However, in our analysis accounting figures are only used to calculate the HML factor. Therefore, this factor is limited to representing 80 percent of the stocks. This could potentially impact the representativeness and robustness of the analysis.

5. Re-examining the size effect

To initiate our analysis, we seek to establish whether a size effect is present on the Oslo Stock Exchange between 2000 and 2020. The basis of our analysis is a portfolio sorting procedure, as described in Section 3.1. Summary statistics for the decile portfolio sorting are reported in table 1.

Table 1 – Summary statistics for decile portfolios

This table reports summary statistics for the decile portfolios between January 2000 and November 2020. The table shows the average market capitalization for each portfolio in mNOK (column 2), average number of stocks in each portfolio (column 3), average monthly portfolio return (column 4) and monthly portfolio standard deviation (column 5). The range market capitalization calculates the difference between the highest and lowest observation of market capitalization for each portfolio in mNOK (column 6). Portfolio turnover calculates the number of unique stocks in each portfolio during the period, divided by the average number of stocks in the portfolio (column 7). Average market share exhibits the average market capitalization for the portfolio divided by the total market capitalization (column 8).

Portfolio	Avg. MCap (mNOK)	Avg. number of stocks	Avg. monthly return (%)	Monthly St.dev. (%)	Range MCap (mNOK)	Portfolio turnover	Avg. market share (%)
LargeCap	56,739	19.45	0.83	5.86	391,436	3.47	77.17
2	7,748	18.91	0.59	6.75	25,344	6.05	10.07
3	3,873	19.00	0.48	7.12	9,539	8.32	5.13
4	2,206	18.90	0.98	7.12	5,482	10.72	3.03
5	1,326	18.98	0.16	7.24	3,108	11.10	1.78
6	854	18.96	0.50	7.05	2,112	11.41	1.20
7	544	18.98	0.57	8.16	1,393	10.62	0.76
8	333	18.98	0.79	7.36	669	9.79	0.47
9	185	18.94	1.09	8.88	384	8.79	0.27
SmallCap	78	19.35	2.12	10.03	257	7.89	0.12

We observe a noticeable variation in market capitalization, especially in the LargeCap decile. Examining the data, we identify Equinor ASA which on average constitutes about 20 percent of the total market capitalization. We recognize the impact this has on the LargeCap decile portfolio, but do not see this as a problem for validity as we find similar results for equally weighted portfolios, see Appendix B1. Furthermore, when observing the portfolio turnover, we find the LargeCap decile to be considerably more stable compared to the SmallCap decile portfolio. Higher portfolio turnover implies higher transaction costs for rebalancing the position in the SmallCap decile portfolio, which impacts the ability to execute a trading strategy.

Table 1 shows no indication of a linear relationship between size and returns for the decile portfolios. This aligns with Banz's (1981) findings which highlight non-linearity as a key feature of the size effect. Furthermore, the non-linearity observed supports our choice to focus the analysis on the LargeCap and SmallCap decile portfolios.

The average number of shares listed on the Oslo Stock Exchange in our data is 209 per year. When adjusting for companies being delisted throughout the year, we are left with an average of approximately 19 stocks per portfolio. This could lead to a survivorship bias where the worst performers, who file for bankruptcy, are discarded from the analysis, causing portfolio returns to be artificially high. Because the nature of bankruptcy implies size moving towards zero, this can lead to a more prominent survivorship bias in the SmallCap decile portfolio. However, studies from the U.S. markets have found that less than half the delistings between 1995 and 2007 were involuntary, with most being due to M&A, or strategic considerations (Macey et al, 2008). Nevertheless, we consider the survivorship bias a limitation in our conclusion.

To establish the presence of a size effect, we primarily focus on the decile spread portfolio. Furthermore, to ensure a robust analysis we also include results for the octile and quintile portfolio spreads, in addition to the SMB described by Fama and French (1992). This provides nuanced insight into what extent the size effect is present, and significant. Because more stocks in each portfolio decrease the effect, we expect a potential effect to be most prominent in the more extreme spread portfolios. Furthermore, due to less diversification, we anticipate the more extreme portfolios to have noticeably higher volatility (standard deviation).

Table 2 – Re-examining the size effect on the Oslo Stock Exchange

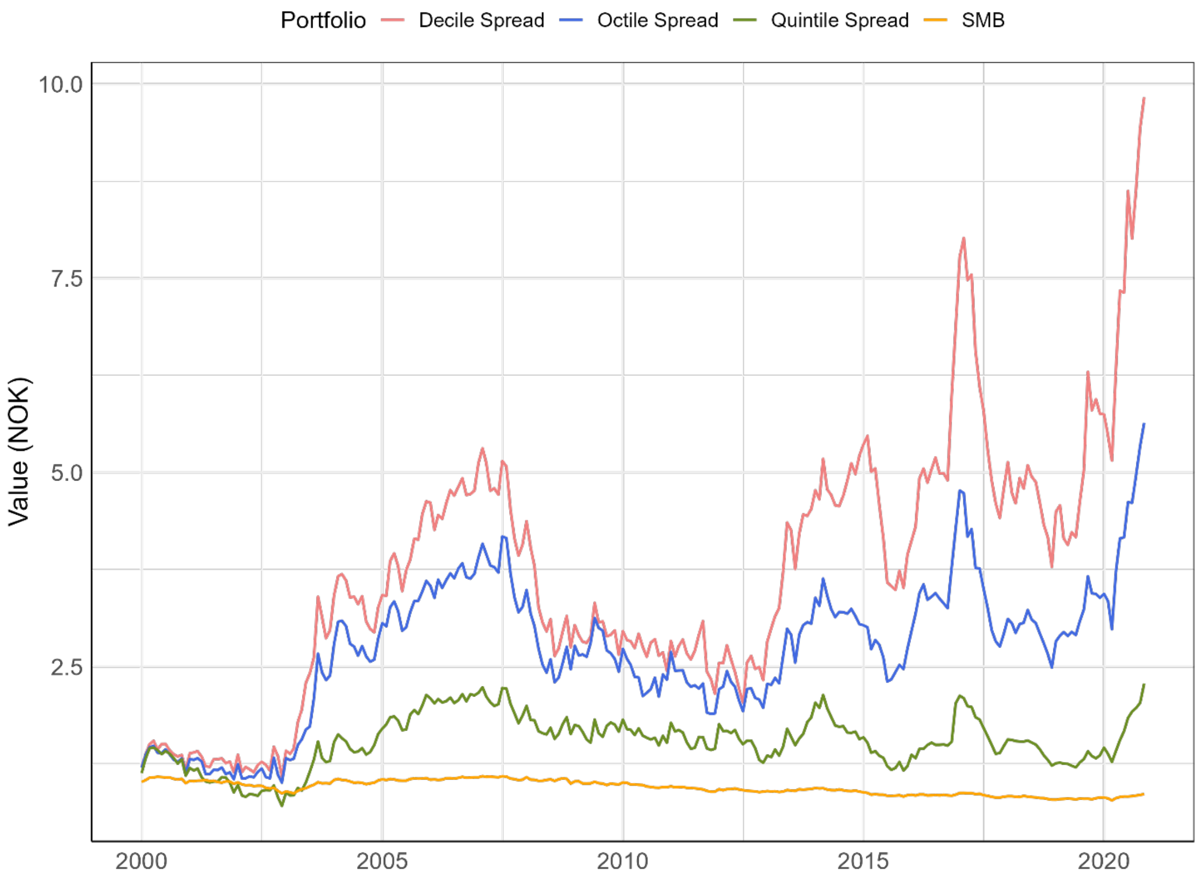
The table reports results of the size effect on the Oslo Stock Exchange between 2000-2020. We observe results for the decile spread portfolio, the octile spread portfolio, the quintile spread portfolio, and the SMB portfolio described by Fama and French (1992). The table evidence average monthly return (column 2), monthly standard deviation (column 3), and t-statistics (column 4).

Portfolio	Avg. monthly return (%)	Monthly St. dev. (%)	t- statistic
Decile spread	1.29	8.84	2.31
Octile spread	0.99	7.84	1.99
Quintile spread	0.57	7.00	1.28
SMB	-0.05	1.49	-0.51

We observe a 1.29 percent average monthly return for the decile spread portfolio, significantly different from zero. This indicates that the size effect is present on the Oslo Stock Exchange between 2000 and 2020. Results from the octile spread portfolio support the presence of the effect, with a monthly average return of 0.99 percent, significantly different from zero. This indicates an effect, also with more diversification. However, for the quintile spread and the SMB we find no significant return. As expected, we observe the volatility to be higher for the more extreme spread portfolios. Figure 1 exhibits the difference between the spread portfolios.

Figure 1 – Cumulative return of a 1 NOK investment in spread portfolios

The figure shows the cumulative return on a 1 NOK investment in the decile spread portfolio (red), octile spread portfolio (blue), quintile spread portfolio (green) and SMB portfolio (yellow). The figure shows monthly observations between 2000 and 2020.



While achieving an attractive return, the decile spread portfolio also exhibits significantly higher volatility. When sorted in fewer portfolios diversification leads to lower return and volatility, which mitigates the effect. This is especially evident in the SMB portfolio, which yields negative return over the time-period. This highlights that the size effect is present in the extremes of the data, which aligns with previous studies (Dijk, 2014).

6. Traditional risk-based theories

To further investigate the size effect, we proceed to adjust the effect for systematic risk factors. Because idiosyncratic risk can be eliminated with diversification, rational investors only seek compensation for taking on systematic risk. For the size effect to be relevant, we must therefore show the effect to be significant, also after adjusting for systematic risk factors.

In this section, we utilize two of the most recognized asset pricing models, the CAPM and the Fama-French three-factor model. When regressing the decile spread portfolio against the CAPM we control for return achieved merely from higher market risk exposure. When regressing the portfolio against the Fama-French three-factor model we control for both the HML and the SMB, in addition to the market risk premium factor.

6.1. The beta-risk adjusted size effect

To initiate the analysis, we use the CAPM, described in Section 3.3. The CAPM is a single-factor model adjusting for the exposure of an asset to the market risk premium (Sharpe, 1964; Lintner, 1965; Treynor, 1962; Mossin, 1966). Our motivation for using this model is to analyse if the size effect found in table 2 can be explained by exposure to the market risk premium factor. The regression results will provide us with a beta-adjusted size effect, as referred to in the literature (Keim, 1983; Lamoureux & Sanger, 1989).

For the size effect to be present when using the CAPM, we must find a significant alpha. To investigate the nuances of the effect, we present regression result for the SmallCap, and the LargeCap decile portfolio, in addition to the decile spread portfolio. To ensure valid results we also conduct the analysis for the octile and quintile portfolios. Results from these regressions are reported in Appendix B2.

For the LargeCap decile portfolio, we expect to find a beta near one and a high adjusted R^2 as we report the LargeCap decile portfolio to constitute on average 77.2 percent of the total market. Furthermore, given the market share, we do not expect to see a significant alpha for the LargeCap decile portfolio.

For the SmallCap decile portfolio, we anticipate a positive beta above one, based on the elevated standard deviation reported in table 1. Because the SmallCap decile reports a noticeably higher monthly return in table 1, we expect to find a positive and significant alpha. Furthermore, as the SmallCap decile accounts for merely 0.12 percent of the average total market, we expect the adjusted R^2 to be relatively low.

Table 3 - CAPM regression analysis

The table reports the CAPM regression results for the LargeCap decile, SmallCap decile, and decile spread portfolio. The market risk premium (MRP) factor captures the sensitivity to the excess return of the market. The constant reports the alpha for the portfolios.

	Dependent variable:		
	LargeCap decile	SmallCap decile	Decile spread
MRP	1.001***	0.926***	-0.075
	(0.008)	(0.092)	(0.096)
Constant	0.061	1.388**	1.327**
	(0.049)	(0.538)	(0.561)
Observations	251	251	251
R^2	0.983	0.290	0.002
Adjusted R^2	0.983	0.287	-0.002
Residual Std. Error (df = 249)	0.774	8.489	8.848
F Statistic (df = 1; 249)	14,310.050***	101.805***	0.613

Note: *p<0.1; **p<0.05; ***p<0.01

We observe the beta coefficient for the LargeCap decile portfolio to be close to one, with adjusted R^2 at 98.3 percent. Because the variation in return is largely captured in the market risk premium factor, we find a low and insignificant alpha. This indicates that the model largely captures the variation in return for the LargeCap decile portfolio. The SmallCap decile portfolio has a positive and significant beta-coefficient, slightly beneath one, suggesting that the market risk exposure cannot fully explain the elevated standard deviation in this portfolio. In line with the SmallCap decile portfolio constituting a minuscule amount of the total market, we observe a lower adjusted R^2 for the SmallCap decile portfolio. The alpha is significant and positive, which indicates that return cannot be explained merely by market risk exposure.

The regression of the decile spread portfolio reports the combination of the LargeCap, and the SmallCap decile portfolios. We observe no significant correlation with the market risk premium, corresponding to a low R^2. As a result, the alpha is similar to the monthly average return for the decile spread portfolio reported in table 2. We report a beta-adjusted size effect

of 1.33 percent per month. The regression for the octile and quintile portfolios aligns with these results, providing robustness to the analysis, see Appendix B2.
There are some concerns regarding the validity of the regression model, especially in the normality of the residuals. However, following the central limit theorem we conclude that the model assumptions are met, see discussion in Appendix C3.

6.2. The Fama-French adjusted size effect

To further analyse the effect, we utilize the Fama-French three-factor model, described in section 3.4. This model is an extension of the CAPM, adding both the SMB and the HML factors. This model is relevant for our analysis, particularly because the SMB factor aims to capture the same effect as the decile spread portfolio, that small stocks tend to outperform large stocks over time.

While aiming to capture the same effect, the decile spread portfolio and the SMB are structured differently. The SMB is a less extreme version of the decile spread portfolio, sorting all stocks into two portfolios. In table 2 we find the return of the SMB to be not significantly different from zero, contrary to the decile spread portfolio. However, this does not imply that the SMB factor cannot explain variation in return for the decile spread portfolio. With this analysis we wish to establish whether the decile spread portfolio return can be explained by high exposure to the SMB factor and thereby identify if the size effect can be attributed to systematic risk.

For the LargeCap decile portfolio, we expect to find significant negative exposure to the SMB factor because the SMB is short large stocks. For the HML factor we are uncertain about the results as we find no apparent reason for the return of the LargeCap decile portfolio to be correlated with neither value nor growth stocks. Following the expectations of the SMB being significant, we expect an insignificant alpha and an increased adjusted R^2 compared to table 3.

For the SmallCap decile portfolio, we expect the SMB coefficient to be positive and significant as the SMB is long small stocks. Because the SmallCap decile portfolio is more volatile (standard deviation) than the LargeCap decile portfolio, we expect the SMB coefficient to be higher, in absolute terms, compared to the LargeCap decile portfolio. The SMB coefficient will indicate how much of the variation in the decile spread portfolio can be attributed to SMB exposure. Furthermore, the SMB exposure will dictate the significance of the alpha. If the SMB explains the majority of variation in return for the SmallCap decile portfolio, we might find an insignificant alpha. This would indicate that return beyond SMB exposure is merely caused by higher volatility (standard deviation). As for the LargeCap decile portfolio, we have no specific expectation for exposure to the HML factor. Because we expect the SMB to be positive, we anticipate the adjusted R^2 to increase compared to table 3.

Table 4 – Fama-French Three-Factor Model regression analysis

The table reports the Fama-French three-factor model regression results for the LargeCap decile, SmallCap decile, and the decile spread portfolio. The market risk premium (MRP) captures the sensitivity to the excess market return. The Small-minus-Big (SMB) captures the exposure to small minus big stocks. The High-minus-Low (HML) captures the exposure to value minus growth stocks. The constant reports the alpha for the portfolios.

	Dependent variable:		
	LargeCap decile	SmallCap decile	Decile spread
MRP	0.994***	0.976***	-0.018
	(0.006)	(0.075)	(0.075)
SMB	-0.363***	3.233***	3.596***
	(0.024)	(0.303)	(0.305)
HML	-0.004	0.224	0.227
	(0.012)	(0.150)	(0.150)
Constant	0.047	1.510***	1.463***
	(0.035)	(0.434)	(0.437)
Observations	251	251	251
R^2	0.991	0.541	0.401
Adjusted R^2	0.991	0.536	0.394
Residual Std. Error (df = 247)	0.552	6.851	6.885
F Statistic (df = 3; 247)	9,458.841***	97.173***	55.069***
Note:			*p<0.1; **p<0.05; ***p<0.01

For the LargeCap decile portfolio, we observe significant negative exposure to the SMB and an increased adjusted R^2. Furthermore, we observe an insignificant alpha, aligned with our expectations. For the SmallCap decile portfolio, we find a significant positive exposure to the SMB. However, the alpha remains significant, as in table 3. This indicates that despite the SMB capturing a significant amount of the variation in return, the portfolio alpha cannot be attributed to merely higher volatility (standard deviation). The significant exposure to the SMB is also reflected in a higher adjusted R^2, compared to table 3. We find no significant exposure to the HML factor for neither the LargeCap, nor the SmallCap decile portfolio.

Because the SMB factor is negative for the LargeCap decile, and positive for the SmallCap decile, we find the decile spread portfolio to have a significant positive exposure to the SMB factor. Furthermore, including the SMB leads adjusted R^2 to increase, compared to table 3. The alpha is still significant and shows a size effect of 1.46 percent per month. This is slightly higher than in both table 2 and table 3, which is somewhat surprising considering the impact of the SMB factor. We also note that the significance of the alpha has increased, compared to table 3. This can be due to changes in the model specifications or omitted variable bias. Omitting relevant variables from factor models can lead to biases in both the estimation and interpretations of the model, which may explain the increased alpha (Cochrane, 2005). We recognize that this represents a significant weakness in the model. However, we note that the regression for the octile and quintile portfolios supports our results, providing robustness to the analysis, see Appendix B2.

As in the CAPM regression, there are some concerns regarding the normality of the residuals. However, following the same argument as presented for the CAPM regression, we conclude that the model assumptions are met, see Appendix C4.

7. Liquidity and liquidity risk

Another popular explanation for the size effect is liquidity and liquidity risk. For our analysis, we focus on two main theories. First, we consider the perspective of Amihud and Mendelson (1986) who argue that size matters merely because small stocks tend to be illiquid and large stocks tend to be liquid. Further, we investigate the perspective of Acharya and Pedersen (2005) who emphasize liquidity risk, arguing that investors require compensation for holding illiquid assets.

We initiate the analysis by discussing liquidity and assessing the relationship between size and liquidity for our decile portfolios. We proceed to test the results of Amihud and Mendelson (1986) by conducting a conditional portfolio sorting based on first size and then liquidity. We then test the results presented by Acharya and Pedersen (2005), by extending the Fama-French three-factor model to include a liquidity risk factor.

7.1. Size and liquidity

Liquidity is a broad and elusive concept, and there are many liquidity measures discussed in finance literature. Sarr and Lybek (2002) define stock liquidity as how easily an asset can be traded in relatively significant amounts without adversely affecting its price. In the analysis, we primarily focus on the percentage bid-ask spread as our liquidity measure.

7.1.1. Percentage bid-ask spread

The percentage bid-ask spread is a highly intuitive and recognized measure of stock liquidity (Chordia et al, 2001). The measure is calculated as the spread between the ask and the bid price, divided by the bid-ask midpoint, see equation 7.1. We calculate the measure on daily data before averaging it to monthly observations of liquidity. By using the percentage ratio, we are able to compare liquidity across different stocks.

$$Percentage\ bid\ ask\ spread = \frac{(Ask - Bid)}{\frac{Ask+Bid}{2}} \qquad (7.1)$$

The measure essentially reflects the transaction cost of executing trades. When liquidity is strong, market competition drives the spread to converge, which is reflected in a low percentage bid-ask spread. When liquidity is weak, the bid-ask spread diverges as there is less competition for the best prices, and the percentage bid-ask spread increases.

7.1.2. Examining liquidity

Liquidity based theories argue that the link between liquidity and size effect is that small stocks are generally illiquid and large stocks are generally liquid (Amihud & Mendelson, 1986; Acharya & Pedersen, 2005). Therefore, to further test these theories, we must first ascertain this relationship in our data.

Figure 2 – Boxplot of percentage bid-ask spread for decile portfolios

The figure shows the average monthly value-weighted percentage bid-ask spread for the decile portfolios. The horizontal black line within the boxes represents the median for the portfolios. The upper quantile (Q3) represents the value for the 75th percentile. The lower quantile (Q1) represents the value for the 25th percentile. The vertical lines from the boxes (whiskers) indicate the highest, and the lowest, values within a 1.5 times interquartile range (IQR) from the hinges.

We observe the median percentage bid-ask spread to be higher for the lower size decile portfolios. This indicates that the smaller stocks are generally less liquid than larger stocks. This is particularly evident in the SmallCap decile, and the LargeCap decile portfolios. We formally test this relation using a pairwise Wilcoxon test. This is a non-parametric test to identify differences in distribution between pairs of samples (Wilcoxon, 1945). In the test matrix, reported in Appendix C5, all but one pair is proven to have significantly different liquidity. These results support the relation between size and liquidity, observed in figure 2, which constitutes the foundation for liquidity theory in relation to the size effect.

7.2. Size and liquidity theory

In this analysis, we consider the impact of liquidity in explaining the size effect, inspired by the works of Amihud and Mendelson (1986). They argue that because small stocks are generally illiquid, and large stocks are generally liquid, the size effect is in fact a liquidity effect (Amihud & Mendelson, 1986).

To examine the specific impact of liquidity on the effect, we conduct a conditional portfolio sorting procedure, as described in Section 3.2.1. First, stocks are sorted based on size (market capitalization), and then based on liquidity (percentage bid-ask spread). If the size effect is in fact a liquidity effect, we would expect to see illiquid stocks, of similar size, outperforming liquid stocks over time. Furthermore, because of higher volatility (standard deviation) in small stocks, we expect the differences between illiquid and liquid stocks to be more pronounced in the lower size quintiles. For robustness, we replicate the analysis using another liquidity measure, see Appendix B3.

Table 5 – Conditional portfolio sorting analysis

The table reports average monthly return (Panel A) and t-statistics (Panel B) for 25 conditionally sorted portfolios. The portfolios are first sorted by size (market capitalization), and then liquidity (percentage bid-ask spread). Vertically, the portfolios span from the most liquid to the most illiquid. The last row considers the spread between the most illiquid and the most liquid portfolios. Horizontally, the portfolios span from the largest size portfolio to the smallest size portfolio. The last column shows the spread between the lowest and the highest size portfolios.

Panel A

Average monthly return (%)

	Large	2	3	4	Small	Large-Small
Liquid	0.97	1.35	1.46	3.60	4.01	3.04
2	1.16	1.29	0.63	0.22	1.38	0.22
3	0.35	0.69	0.39	-0.09	0.42	0.07
4	0.61	-0.33	-0.81	-0.35	1.21	0.60
Illiquid	-0.07	-0.31	-0.69	-0.35	0.31	0.38
Illiquid-Liquid	-1.04	-1.66	-2.15	-3.95	-3.70	

Panel B

T-statistics

	Large	2	3	4	Small	Large-Small
Liquid	1.45	1.21	1.22	2.20	2.21	1.57
2	1.43	1.39	0.59	1.20	0.88	0.12
3	0.40	0.81	0.40	-0.09	0.29	0.04
4	0.81	-0.33	-0.83	-0.34	0.69	0.32
Illiquid	-0.10	-0.35	-0.79	-0.31	0.16	0.19
Illiquid-Liquid	-1.03	-1.18	-1.45	-1.97	-1.41	

The results indicate that liquid stocks outperform illiquid stocks within each size quintiles, contrary to the findings of Mendelson and Amihud (1986). However, the results generally lack significance. Furthermore, the method itself has its limitations, being a non-parametric method. This prevents us from deriving explicit conclusions based on the results. We note that the replicating analysis conducted in Appendix B3 shows similar results, although with slightly more significant results.

Furthermore, we conduct an unconditional portfolio sorting procedure as described in Section 3.2.2. Results are reported in Appendix B4. The unconditional sorting gives equal weights to both size and liquidity as sorting characteristics. While conditional sorting merely permits analysing return within size quintiles, unconditional sorting allows us to analyse variation in return across both size and liquidity quintiles. However, unconditional portfolio sorting entails forming multiple sub-portfolios, each containing a varying number of stocks. This leads to several sub-portfolios with few stocks. As a result, our analysis is confined to the large liquid portfolio and the small illiquid portfolios.

The difference in returns between these two portfolios is not significantly different from zero. Despite the methodological differences between the conditional and the unconditional sorting, the findings from our unconditional sorting procedure align with the broader conclusions of this section, which is that we do not find conclusive evidence of illiquid stocks outperforming liquid stocks when controlling for size.

7.3. Size and liquidity risk theory

In our further analysis we consider the impact of liquidity risk on the size effect. Theories on liquidity risk emphasize the risk of holding illiquid assets as a systematic risk factor, for which investors demand a premium (Acharya & Pedersen, 2005). Therefore, they argue that variation in liquidity risk exposure can explain the size effect (Acharya & Pedersen, 2005).

There are several reasons why illiquidity risk may be worthy of a premium. First, illiquidity obstructs investors from efficiently restructuring their holdings. In this context, Adrian and Shin (2010) emphasize the equity effect of leverage, arguing that liquidity is particularly important in market downturns. In a declining market, investors using leverage may be forced to sell assets to cover losses or post margins. If the assets are illiquid, this might lead to significant slippage[6], or that investors are forced to sell their best holdings if these are liquid. We believe testing this effect may be particularly relevant on the Oslo Stock Exchange, as this market is relatively small and less liquid[7].

To test the impact of liquidity risk on the size effect we extend the Fama-French three-factor model by including the LIQ factor, described by Asness et al. (2018). This factor is constructed using portfolio sorting, with liquidity as the only characteristic. Based on the sorting, we form a long-short equity decile portfolio, buying the illiquid and selling the liquid decile, which

[6] Slippage refers to the difference between the expected price, and the executed price of a trade. In simple terms, slippage implies having to move the price to trade significant volume.

[7] The total market capitalization of the Oslo Stock Exchange is approximately NOK 3,960 bn. For reference this is 27.7% of the Nasdaq Stockholm, 7.7% of the London Stock Exchange, and 1.5% of the New York Stock Exchange. Data on market capitalization and exchange rates are gathered from Bloomberg, per 16.12.23.

constitutes the factor (Asness et al, 2018). The liquidity of each stock is measured by the percentage bid-ask spread, described in Section 7.1.1. To ensure robustness we also conduct the analysis using another liquidity risk factor, see Appendix B5.

If liquidity risk is in fact a key factor in explaining the size effect, we expect to find significant exposure of the decile spread portfolio to the LIQ factor. For the LargeCap decile portfolio this implies finding significant negative exposure to the LIQ factor, because large stocks are generally liquid, and the LIQ is short liquid stocks. Furthermore, if we assume the LIQ is significant, and based on the results from the Fama-French three-factor model regressions in Section 6.2, we anticipate the adjusted R^2 to be close to one with an insignificant alpha for the LargeCap decile portfolio.

For the SmallCap decile portfolios we anticipate significant positive exposure to the LIQ. This is because the LIQ is long illiquid stocks that tend to also be small. As a result, we expect the adjusted R^2 to increase, compared to the results in Section 6.2, and the alpha to decrease.

Table 6 – Extended Fama-French Three-Factor Model regression analysis

The table reports the extended Fama-French three-factor model regression results for the LargeCap decile, SmallCap decile, and the decile spread portfolio. The market risk premium (MRP) captures the sensitivity to the excess market return. The Small-minus-Big (SMB) captures the exposure to small minus big stocks. The High-minus-Low (HML) captures the exposure to value minus growth stocks. In addition to these three factors, we add the LIQ factor. This factor aims to capture liquidity risk, and is structured as a long illiquid, and short liquid, portfolio. Liquidity is measured using the percentage bid-ask spread. The constant reports the alpha for the portfolios.

	Dependent variable:		
	LargeCap decile	SmallCap decile	Decile spread
MRP	1.000***	1.081***	0.081
	(0.006)	(0.077)	(0.078)
SMB	-0.359***	2.729***	3.088***
	(0.025)	(0.319)	(0.320)
HML	-0.023*	0.301*	0.324**
	(0.012)	(0.154)	(0.155)
LIQ	0.011**	0.234***	0.223***
	(0.005)	(0.057)	(0.058)
Constant	0.068**	1.616***	1.549***
	(0.034)	(0.426)	(0.428)
Observations	245	245	245
R^2	0.992	0.577	0.438
Adjusted R^2	0.992	0.570	0.428
Residual Std. Error (df = 240)	0.524	6.609	6.644
F Statistic (df = 4; 240)	7,818.026***	81.889***	46.682***
Note:			*p<0.1; **p<0.05; ***p<0.01

We find the LIQ to be significantly positive for the LargeCap decile portfolio, contrary to our expectations. This indicates that the return of the LargeCap decile portfolio, is positively correlated with the liquidity risk factor. For the SmallCap decile portfolio we find positive significant exposure to the LIQ factor. This indicates that the portfolio is positivley exposed to the liquidity risk captured in the LIQ. Furthermore, the alpha of the SmallCap decile portfolio increases compared to previous results.

We find the alpha in the LargeCap decile to be significant, contrary to previous models. Furthermore, we observe that the HML factor becomes significant for all three portfolios, following the inclusion of the LIQ factor. This could be due to the LIQ changing the model's specification or omitted variable bias in the previous models. As a result, when including the LIQ, the true significanse of the HML might now be present. Another explanation is multicollinearity. However, VIF tests conducted rejects this explanation, see Appendix C5. Due to the complexity of financial markets, omitted variable bias is a general problem for factor models (Cochrane, 2005). Therefore, this weakness must be acknowledged when drawing conclusions from factor model regressions.

Because the coefficient for the SmallCap decile portfolio is higher than the LargeCap decile portfolio, the decile spread portfolio is significantly, positively exposed to the LIQ factor. However, the alpha remains significant. This indicates that despite the significant exposures to the factor, liquidity risk cannot explain the observed size effect.

When conducting the analysis using an alternative liquidity risk factor, we observe different results, which weakens the results of the analysis in table 6. In addition to the problem of omitted variable bias, this prevents us from drawing explicit conclusions from this analysis. Furthermore, the model validity test shows concerns regarding the normality of the residuals. However, following the central limit theorem, we conclude that the model assumptions are met, see discussion in Appendix C5.

8. The January effect

In this section, we examine the relationship between the January effect and the size effect. The January effect, first discovered by Sidney B. Wachtel (1942), and formally introduced by Rozeff and Kinney (1976), refers to the empirical observation of higher stock returns in January than in other months. Horowitz (2000a) even finds the size effect to be present in January and to reverse between February and December. Research, including Keim (1983), Reinganum (1981) and Roll (1983), shows a clear link between the January effect and the size effect, as the January effect is considerably more prominent for small stocks.

Finance literature suggests several explanations for the January effect. Haugen and Lakonishok (1988) argue the effect arises because asset managers reallocate their portfolios to hold stronger stocks for the year end reports. Theory from behavioural economics suggests that the effect can be explained by a general optimism going into a new year, leading to a "risk-on" mentality (Ciccone, 2011). Furthermore, another explanation is the tax-loss selling hypothesis. This hypothesis attributes the effect to investors selling losing stocks to reduce taxable income towards the end of the year (Wachtel, 1942). Despite several suggestions, research fails to show conclusive evidence of one particular explanation (Haug & Hirschey, 2006)[8].

8.1. The January effect and the size effect

In this section, we seek to analyse if the size effect remains after adjusting for the January effect. To conduct this analysis, we calculate the return for decile spread portfolios in January, and between February and December, separately. For robustness, we conduct the analysis for the octile and the quintile portfolios, see Appendix B6.

[8] The tax-selling hypothesis has perhaps been the most accepted explanation for the January effect. However, evidence of a January effect in countries with the fiscal years ending in other months contradicts this hypothesis (Haug & Hirschey, 2006).

Table 7 – Re-examining the size effect and the January effect

The table reports the size effect in January and February-December separately for the SmallCap decile, the LargeCap decile and the decile spread portfolio. The table considers average monthly return, monthly standard deviation, and t-statistics for the portfolios.

Portfolio	Avg. monthly return (%)	Monthly St. dev (%)	t-statistics
Decile spread – January	12.08	8.18	6.61
Decile spread – February-December	0.3	8.24	0.56
LargeCap decile – January	-0.36	6.43	-0.25
LargeCap decile – February-December	0.94	5.82	2.47
SmallCap decile – January	11.72	10.32	5.08
SmallCap decile – February-December	1.25	9.56	1.98

We observe that the average return for the decile spread portfolio is 12.08 percent in January and 0.30 percent between February and December. The latter is not significantly different from zero. Compared to the size effect of 1.29 percent found in table 2, the difference is remarkable. The results imply that the entire size effect found is concentrated in the month of January.

We observe the LargeCap decile portfolio to have negative average return in January, and positive between February and December. However, the return in January is not significantly different from zero. Despite this, we note that this is the first analysis where the short position in the LargeCap decile portfolio contributes positively to the return of the decile spread portfolio.

For the SmallCap decile portfolio, we observe a noticeable difference between average return in January and the rest of the year. Both results are significantly different from zero. We find the vast majority of the effect observed in the decile spread portfolio to come from the SmallCap decile. Results from the octile, quintile, and SMB portfolios supports the robustness of the analysis, see Appendix B6.

The analysis shows that the entire size effect established is concentrated in January. This raises the question of whether this is due to the size effect, or the January effect. Based on the analysis, we cannot draw conclusions without further analysing the explanations for the January effect. However, our results indicate that the size effect found on the Oslo Stocks Exchange between 2000 and 2020, might be the January effect in disguise. Reinganum (1981) described this as an anomaly within an anomaly.

9. The flight-to-safety effect

In this section we consider the flight-to-safety effect. This includes analysing the relation between the size effect, and market turmoil and uncertainty[9]. The flight-to-safety effect suggests that in times of uncertainty, investors become more risk averse, resulting in less risky investments outperforming riskier ones (Vayanos, 2004). The effect has reported consistent results across both market and different crises (Aslanidis et al, 2020). In relation to the size effect, the flight-to-safety effect would suggest a reversal of the effect in times of crisis, if small stocks are in fact subject to more systematic risk[10] and volatility.

9.1. Volatility risk

Perhaps the most common measure of market volatility is the VIX, also referred to as the fear index, which is a leading indicator measuring the 30-day expected volatility in U.S. stocks based on implicit volatility priced in put and call options on stocks in the S&P 500 Index (Kuepper, 2023). There is no equivalent index on the Oslo Stock Exchange. Therefore, we first examine whether the index is an appropriate measure of market volatility, also in this market.

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[9] We use the term "crisis" as a description of times with significant market turmoil and increased uncertainty.

[10] These theories are discussed previously. We refer to Section 6.1 (Keim, 1983; Lamoureux & Sanger, 1989), Section 6.2 (Asness, Frazzini, Israel, Moskowitz, & Pedersen, 2018), and Section 7.3 (Acharya & Pedersen, 2005).

Figure 3 – Three months moving average of OSEBX and S&P 500

The figure shows a three-months moving average of monthly return on the S&P 500 Index (blue line) and a three-months moving average of the OSEBX (red line), between 2000 and 2020.

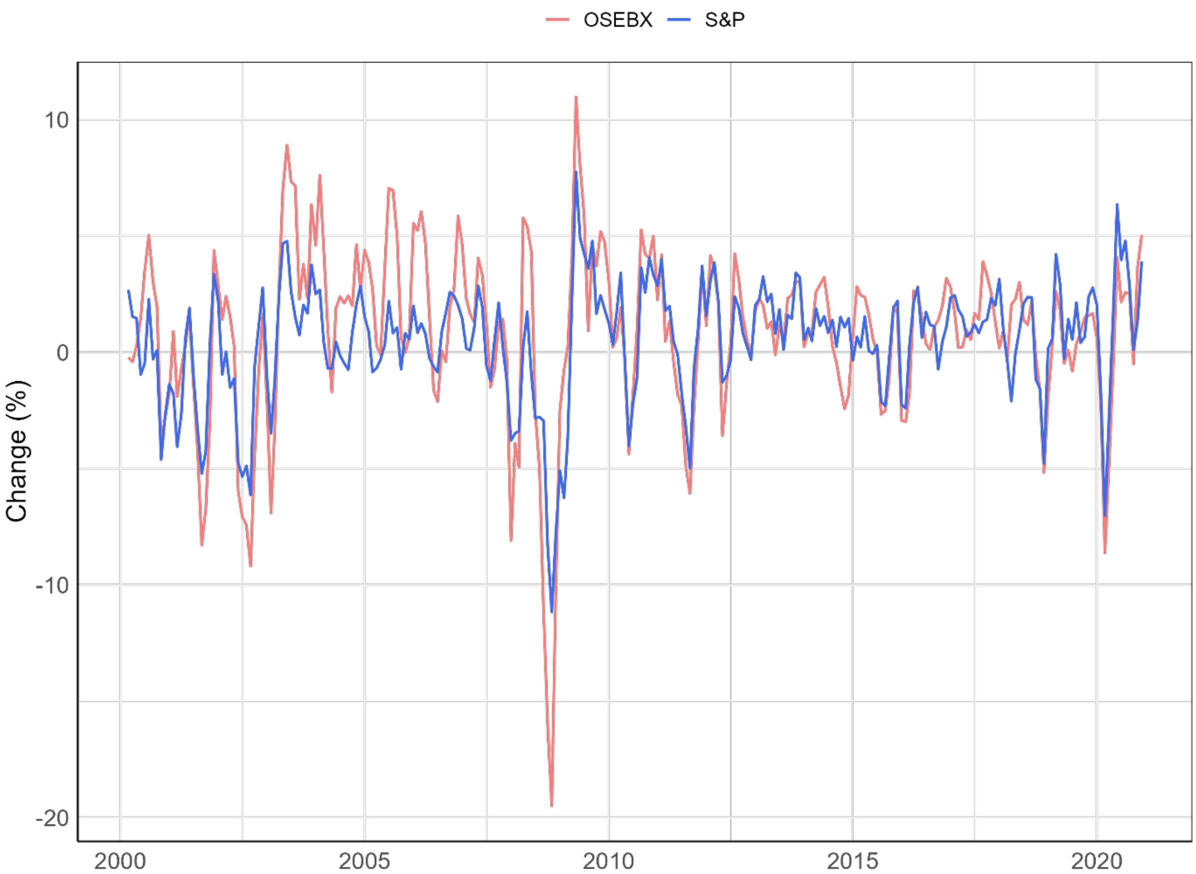
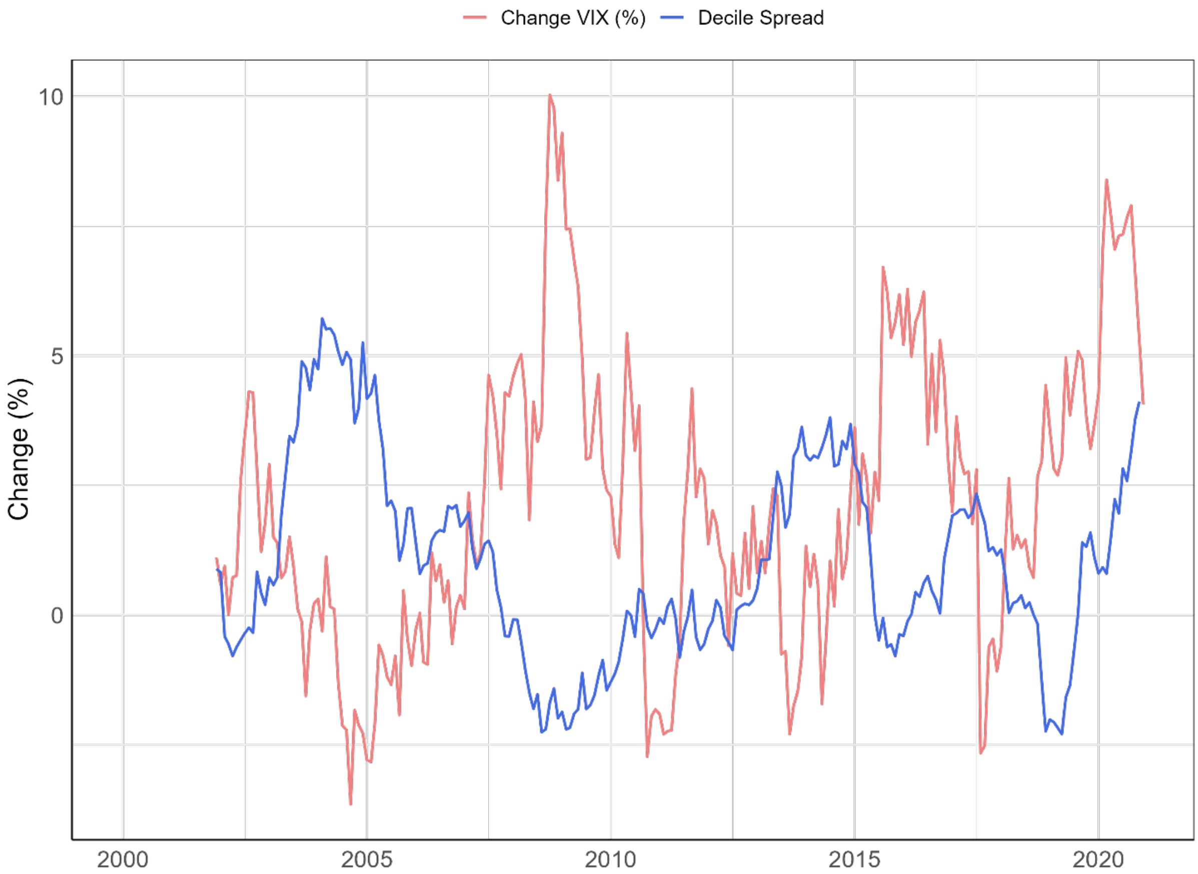


Figure 3 shows a strong correlation between the two indexes. We estimate the correlation between two moving averages to be 0.83. We conclude that this correlation is sufficient to argue that the VIX is also indicative of market volatility on the Oslo Stock Exchange. Furthermore, because both the VIX and the stock market are leading indicators, this strengthens the validity of the results. We do however recognize the challenges of using an indirect measure in our analysis.

Next, we consider the correlation between the monthly change in the VIX, and the decile spread portfolio. Assuming the flight-to-safety effect is present, we expect to find a negative correlation between the two variables, as increased volatility leads to risk aversion among investors. We expect this to be especially evident in months of abnormal spikes in the VIX.

Figure 4 – 24-months moving average of VIX and Decile Spread

The figure exhibits a 24-month moving average for the monthly percentage change in the VIX (red line) and a 24-month moving average for the monthly return of the decile spread portfolio (blue line), between 2000 and 2020.



We observe a negative correlation between the decile spread portfolio and the monthly change in the VIX. This suggests that when market volatility increases, large stocks generally outperform small stocks and vice versa. We estimate the correlation between two moving averages to be -0.46.

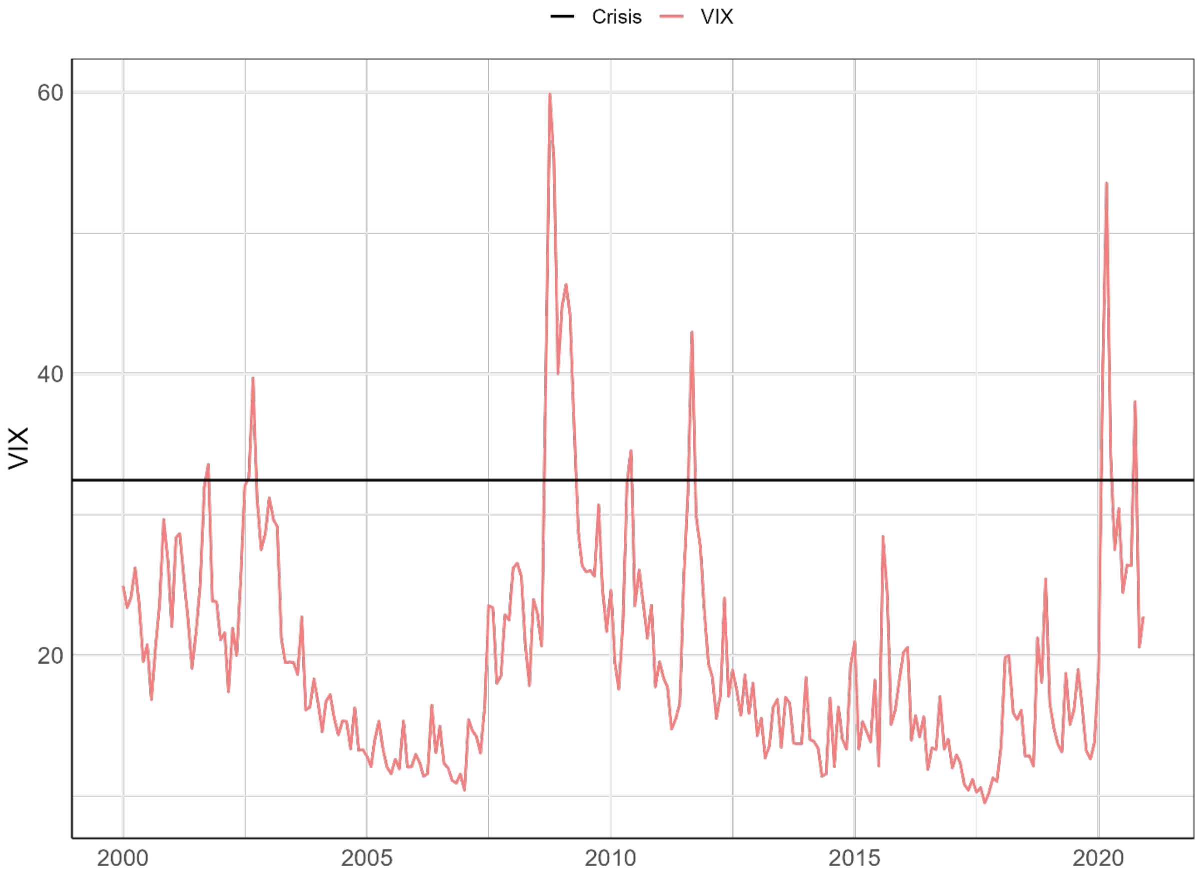
However, to draw conclusions about the relationship between the two variables, we must analyse raw monthly data. We estimate the monthly correlation between the two to be -0.04, not significantly different from zero. In months of negative development in the VIX, the correlation is slightly more prominent at -0.06 percent, but this is significantly different from zero.

$$Crisis_t = \begin{cases} 1 & if\ VIX \geq\ mean_{VIX} + std.dev_{VIX} \times 1.5 \\ 0 & if\ VIX <\ mean_{VIX} + std.dev_{VIX} \times 1.5 \end{cases} \quad (9.1)$$

In our further analysis, we examine the relation between the VIX and the size effect in times of crisis, meaning times of significant market turmoil and uncertainty. To conduct the analysis, we construct a dummy variable for times of crisis, defined as a multiple of 1.5 on the standard deviation above the mean value during our time-period, see equation 9.1. As a result, the crisis variable is 1 if the VIX is 32.44 or above, and 0 if not[11]. For robustness we also conduct the analysis using a 2.0x, and a 1.0x, multiple.

Figure 5 – VIX and crisis variable

The figure shows the average monthly VIX (red line) and the crisis variable (black line), defined as a in equation 9.1, between 2000 and 2020.



[11] This align with a common interpretation of the VIX being "high" when it exceeds 30 (Inman, 2018)

In figure 5, we consider average monthly observations of the VIX, and the crisis variable. We find 17 months of the VIX exceeding the crisis limit. In these months, the return of the decile spread portfolio is 0.96 percent, compared to 1.31 percent in the remaining months. However, the difference is not significantly different from zero. When conducting the analysis with the 1.0 multiple and the 2.0 multiple, we also find no significant results.

The results indicate no applicable correlation between the VIX, and the decile spread portfolio. This can be a result of the VIX not being an appropriate measure of volatility on the Oslo stocks Exchange, or that that volatility simply has no significant impact on the size effect. Furthermore, this can also be explained by different time horizons. While the VIX merely exhibits expected 30-day volatility, the stock market reflects several years of expectations.

9.2. Volatility and market liquidity

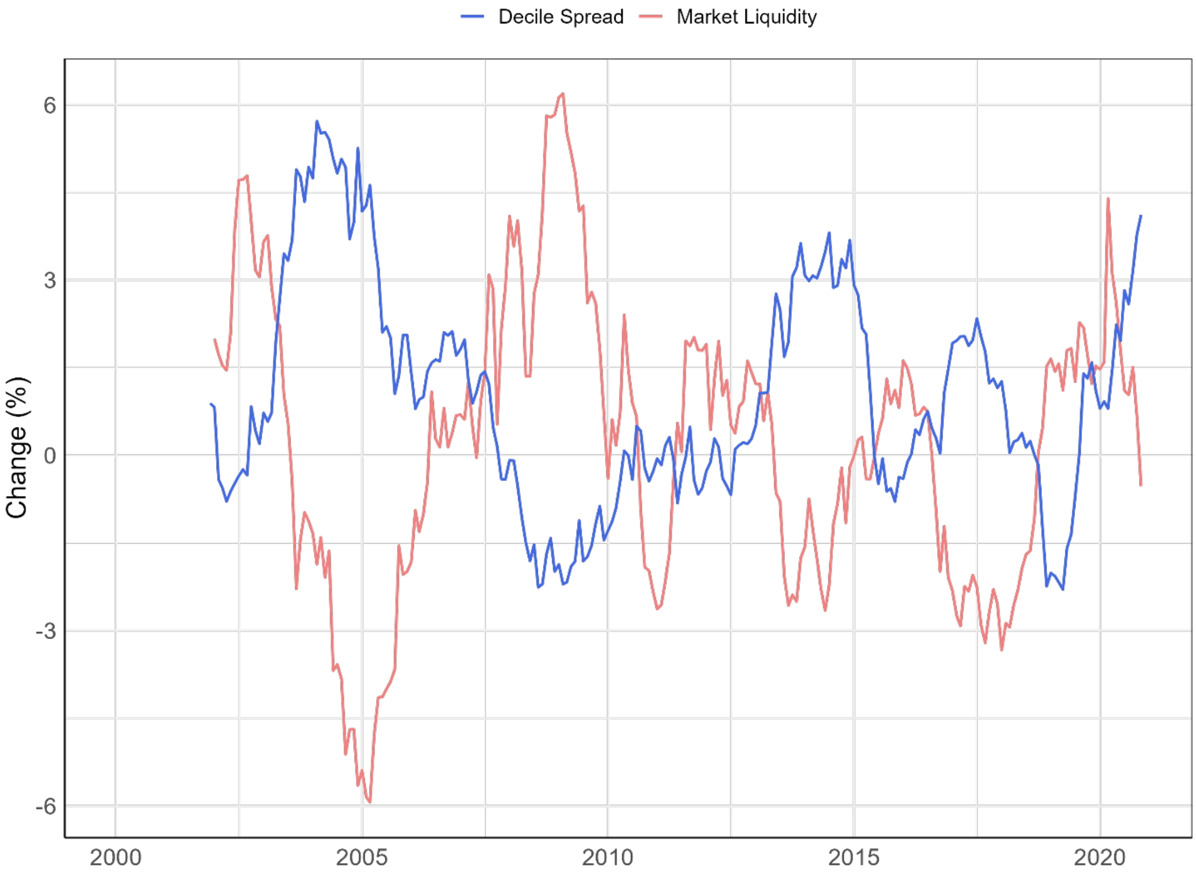
To ensure valid results we consider market liquidity, which has been shown to be highly correlated with market volatility (Pereira & Zhang, 2010). To calculate a measure for the market liquidity, we use the percentage bid-ask spread discussed in Section 7.1.1. We construct the factor using an equally weighted average for all stocks included on a monthly basis.

The relation between market liquidity and market volatility is relevant for several factors. First, the two are fundamentally combined, as wider bid-ask spreads and less volume, leads to traders having to move their prices more to trade, which directly impacts volatility (Pereira & Zhang, 2010). Second, both are historically sensitive to market turmoil and uncertainty (Pereira & Zhang, 2010). We estimate the correlation between the monthly change in the VIX and the monthly change in the market liquidity to be 0.4, significantly different from zero. This supports the VIX as a measure of market turmoil and uncertainty on the Oslo Stock Exchange, and subsequently the previous results of the analyses.

Furthermore, beyond being a test for the validity of the VIX, market liquidity offers insight to the ability of investors to execute a trading strategy. A strong correlation between the decile spread portfolio and the market liquidity measure, could potentially imply that when market liquidity decreases, there are not sufficient liquidity to trade significant volumes at the prices reported. This may have implications for the practical ability to profit from the size effect.

Figure 6 – 24-months moving average of Market Liquidity and Decile Spread

The figure considers a 24-month moving average of the change in the market liquidity measure (red line), and a 24-months moving average of the decile spread portfolio (blue line).



We estimate the correlation between the decile spread portfolio and market liquidity to be 0.18[12], significantly different from zero. This indicates that when liquidity is low, the decile portfolios tend to perform poorly, and vice versa. This can restrict investors from profiting from the effect if they cannot trade at the prices reported due to low liquidity. This issue could prove especially challenging as we find the size effect to stem primarily from the return in the SmallCap decile portfolio. Moreover, this could also explain the persistence of the size effect, as liquidity may have hindered investors from effectively capitalizing on the market inefficiency. We acknowledge the practical implications of this as a concern for executing a trading strategy based on the results the analysis.

[12] The correlation is positive because the bid-ask spread increase if liquidity decrease.

10. Conclusion

We establish the presence of an average size effect on the Oslo Stock Exchange between 2000 and 2020 of 1.29 percent per month. However, we acknowledge that the result may be skewed by a survivorship bias. When adjusting for several systematic risk factors, the effect remains significant. Further, we conclude that despite significant exposure to the SMB, this systematic risk factor cannot explain the observed effect. Examining liquidity, we find results indicating that liquid stocks outperform illiquid stocks of similar size, however these results are not conclusive. When analysing liquidity risk as a systematic risk factor, we find a significant exposure of the effect. However, neither these results are conclusive due to robustness results and potential biases in the model. Furthermore, we find an average size effect of 12.08 percent in January, and no significant return between February and December. This shows that the size effect established is only present in January. We find indications that market turmoil and uncertainty impact the effect negatively. However, these results are not conclusive. Lastly, we observe the market liquidity and the size effect to be negatively correlated. This could have implications for the ability to execute a trading strategy based on the effect and might explain the persistence of the size effect over time.

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Appendix 1: Data

A1 – Data screening and cleaning

From the Titlon dataset, we have collected a variety of data points, including SecurityId (which serves as a unique identifier for each firm), daily adjusted stock prices, and market capitalization values. Additionally, the dataset provides exchange and industry classification codes, along with daily best bid and ask prices. We also have access to the daily trading volume, expressed in NOK, and the daily rate for 3-month Norwegian Government Bills, sourced by Titlon from the Bondindex Table. Furthermore, Titlon also supplies us with balance sheet information, specifically data on equity and debt. All prices are denominated in NOK. To complement our data, we have sourced monthly data for the VIX index from Yahoo Finance (2023).

We calculate daily returns based on the percentage changes in daily adjusted prices. In line with the approach of Ince and Porter (2006), we exclude daily returns above 400 percent or below -85 percent to eliminate outliers and potential data errors. These returns, along with the risk-free rate, are then converted from daily to monthly values. We compute excess returns by subtracting the monthly risk-free rate from the monthly returns, and limit the data to common stocks.

The market capitalization as of January is used to determine the weight of each stock. Utilizing these weights, the total monthly market return is then computed. In the calculation of book to market we add equity and debt and divide by market capitalization as of January leaving us with yearly values for our HML sorting. The percentage bid-ask spread is determined by subtracting the best bid price from the best ask price and then dividing the result by the average of these two prices.

Our data underwent a thorough screening and cleaning processes. For future research using the Titlon dataset, we note that we encountered numerous duplicates across different firm identifiers. These duplications have been corrected in our finalized dataset.

A2 – Variable explanations

In our finalized dataset, before conducting additional calculations, we are left with one data frame containing monthly data, and one containing daily data including the following variables:

Monthly data frame:
SecurityId: Firm identifier
Date: Year/Month
return: Returns for individual stocks
excess_return: Returns for individual stocks excess of the market
total_market: Weighted return for all stocks included in the data
mktcap: Market Capitalization
b_m: Book to market ratio
VIX: Monthly Volatility Index

Daily data frame:
SecurityId: Firm identifier
Dailyreturns: Daily returns
Date: Year/Month/Day
bid_ask: Bid-Ask spread ratio
OfficialVolume: Daily volume in NOK

Appendix 2: Robustness analysis

B1 – Re-examining the size effect with equally weighted portfolios

In the analysis we re-examine the size effect using value-weighted portfolio sorting, following the standard procedure in the literature (Dijk, 2014). Using value-weighted portfolios may however lead issues as some stocks may skew the entire portfolios. We particularly refer to one observation, Equinor ASA which constitutes approximately 20 percent of the average total market size. We adjust for this potential issue by conducting the analysis using equally weighted portfolios.

Table 8 – Summary statistics for decile portfolios – Equally weighted

This table reports summary statistics for the decile portfolios with equally weighted decile portfolios between January 2000 and December 2020. The table shows the average market capitalization for each portfolio in mNOK (column 2), average number of stocks in each portfolio (column 3), average monthly portfolio return (column 4) and monthly portfolio standard deviation (column 5). The range market capitalization calculates the difference between the highest and lowest observation of market capitalization for each portfolio in mNOK (column 6). Portfolio turnover calculates the number of unique stocks in each portfolio during the period, divided by the average number of stocks in the portfolio (column 7). Average market share exhibits the average market capitalization for the portfolio divided by the total market capitalization (column 8).

Portfolio	Avg. MCap (mNOK)	Avg. number of stocks	Avg. monthly return (%)	Monthly St.dev. (%)	Range MCap (mNOK)	Portfolio turnover	Avg. market share (%)
High	56,739	19.45	0.66	6.71	391,436	3.47	77.17
2	7,748	18.91	0.68	6.66	25,344	6.05	10.07
3	3,873	19.00	0.46	7.01	9,539	8.32	5.13
4	2,206	18.90	0.87	7.06	5,482	10.72	3.03
5	1,326	18.98	0.17	7.18	3,108	11.10	1.78
6	854	18.96	0.48	7.08	2,112	11.41	1.20
7	544	18.98	0.61	8.19	1,393	10.62	0.76
8	333	18.98	0.82	7.36	669	9.79	0.47
9	185	18.94	0.99	8.75	384	8.79	0.27
Low	78	19.35	2.25	10.68	257	7.89	0.12

We observe similar results as in table 1. However, the average monthly returns are slightly higher for most of the decile portfolios. This is perhaps most noticeable in the SmallCap decile, with an average monthly return of 2.25 percent. We also find the volatility, as measured by standard deviation, to be slightly higher.

Table 9 – Re-examining the size effect on the Oslo Stock Exchange – Equally weighted

The table reports results of the size effect with equally weighted portfolios on the Oslo Stock Exchange between 2000-2020. The table evidence average monthly return (column 2), monthly standard deviation (column 3), and t-statistics (column 4). The first row reports results for the decile spread portfolio. The second row reports results for the octile spread portfolio. The third row reports results for the quintile spread portfolio. The fourth row report results from the SMB portfolio.

Portfolio	Avg. monthly return (%)	Monthly St. dev. (%)	t- statistic
Decile spread	1.65	9.02	2.14
Octile spread	0.96	7.89	1.98
Quintile spread	0.55	6.88	1.33
SMB	-0.02	1.64	-0.64

Considering the four spread portfolios, all show similar characteristics as in table 2. This strengthens the robustness of the analysis, as we conclude that the decision of using value weighted portfolios has minimal impact on the analysis.

B2 – CAPM and Fama-French regressions for octile and quintile portfolios

We base our choice of portfolios on the bias-variance trade-off presented by Cattabeo et al. (2016) and previous studies of similar sample sizes (Dijk, 2014). This might however have an important impact on the validity of the analysis. To ensure robust results we replicate the analyses from Section 6, using the octile and the quintile portfolios. If these analyses show noticeably different results, this might indicate that using decile portfolios is perhaps not the most appropriate method for analysing the effect. In table 10, we consider output from the CAPM regression, and in table 11, we consider the Fama-French three-factor model regression.

Table 10 – CAPM regression analysis - octile and quintile portfolios

The table reports CAPM regression results for the LargeCap, SmallCap, and the spread for both the octile and the quintile portfolios. The market risk premium (MRP) factor captures the sensitivity to the excess return of the market. The constant reports the alpha for the portfolios.

	Dependent variable:					
	LargeCap (Octile)	SmallCap (Octile)	Octile spread	LargeCap (Quintile)	SmallCap (Quintile)	Quintile spread
MRP	1.004***	0.935***	-0.069	1.003***	0.886***	-0.116
	(0.007)	(0.081)	(0.085)	(0.005)	(0.073)	(0.075)
Constant	0.032	1.055**	1.022**	0.031	0.658	0.627
	(0.042)	(0.474)	(0.497)	(0.028)	(0.427)	(0.443)
Observations	251	251	251	251	251	251
R^2	0.988	0.349	0.003	0.995	0.374	0.009
Adjusted R^2	0.988	0.347	-0.001	0.995	0.371	0.005
Residual Std. Error (df = 249)	0.655	7.481	7.845	0.434	6.731	6.983
F Statistic (df = 1; 249)	20,098.710***	133.604***	0.669	45,637.290***	148.460***	2.365
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 11 – Fama-French three-factor model regression analysis - octile and quintile portfolios

The table reports Fama-French three-factor model regression results for the LargeCap, SmallCap, and the spread for both the octile and the quintile portfolios. The market risk premium (MRP) captures the sensitivity to the excess market return. The Small-minus-Big (SMB) captures the exposure to small minus big stocks. The High-minus-Low (HML) captures the exposure to value minus growth stocks. The constant reports the alpha for the portfolios.

	Dependent variable:					
	LargeCap (Octile)	SmallCap (Octile)	Octile spread	LargeCap (Quintile)	SmallCap (Quintile)	Quintile spread
MRP	0.999***	0.997***	-0.002	0.999***	0.943***	-0.056
	(0.005)	(0.062)	(0.062)	(0.003)	(0.052)	(0.052)
SMB	-0.323***	3.319***	3.642***	-0.214***	3.192***	3.406***
	(0.019)	(0.250)	(0.252)	(0.013)	(0.209)	(0.210)
HML	-0.012	-0.005	0.007	-0.009	0.063	0.073
	(0.009)	(0.123)	(0.124)	(0.006)	(0.103)	(0.104)
Constant	0.020	1.182***	1.162***	0.023	0.780***	0.757**
	(0.028)	(0.357)	(0.360)	(0.018)	(0.299)	(0.301)
Observations	251	251	251	251	251	251
R^2	0.995	0.633	0.481	0.998	0.694	0.547
Adjusted R^2	0.995	0.629	0.475	0.998	0.690	0.541
Residual Std. Error (df = 247)	0.434	5.638	5.680	0.286	4.723	4.744
F Statistic (df = 3; 247)	15,362.300***	142.190***	76.381***	35,166.010***	186.794***	99.244***
Note:						*p<0.1; **p<0.05; ***p<0.01

Both the CAPM and Fama-French three-factor model show similar results as in section 6. The octile portfolios have similar properties as the decile portfolios, although with slightly lower values. The quintile portfolio shows no significant effect, which corresponds with the bias-variance trade-off previously discussed, and the results from Section 5.

B3 – Conditional portfolio sorting using the Amihud illiquidity measure

In Section 7 we examine the size effect using liquidity and liquidity risk theory. One key source of error in this analysis is the liquidity measure itself. For robustness, we therefore conduct the same analysis using another liquidity measure, the Amihud illiquidity measure.

$$Amihud\ illiquidity = \frac{|r_i|}{volume} \tag{B.1}$$

The Amihud illiquidity measure is calculated as the absolute return divided by the volume traded in million NOK, to capture the price impact per NOK traded (Amihud, 2002), see equation B.1. The higher the Amihud, the more illiquid the stock is. For our analysis we aggregate the measure to reflect average monthly illiquidity for each stock.

Table 12 - Conditional portfolio sorting analysis - Amihud illiquidity measure

The table reports average monthly return (Panel A) and t-statistics (Panel B) for 25 conditionally sorted portfolios. The portfolios are first sorted by size (market capitalization), and then liquidity (Amihud illiquidity measure). Vertically, the portfolios span from the most liquid to the most illiquid. The last row considers the spread between the most illiquid and the most liquid portfolios. Horizontally, the portfolios span from the largest size portfolio to the smallest size portfolio. The last column shows the spread between the lowest and the highest size portfolios.

Panel A

Average monthly return (%)

	Large	2	3	4	Small	Large-Small
Liquid	5.26	5.09	3.27	3.17	1.64	-3.62
2	2.33	0.91	0.28	1.28	1.30	-1.03
3	0.65	-0.32	-0.16	-0.01	0.63	-0.02
4	0.04	-0.72	-1.05	-0.59	0.16	0.12
Illiquid	-1.09	-0.83	-1.53	-0.91	-0.77	0.32
Illiquid-Liquid	-6.35	-5.92	-4.80	-4.08	-2.41	

Panel B

T-statistics

	Large	2	3	4	Small	Large-Small
Liquid	2.61	2.82	2.41	2.92	2.73	-1.70
2	1.48	0.72	0.42	1.10	1.65	-1.03
3	0.46	-0.25	0.12	-0.07	0.69	-0.02
4	-0.35	-0.60	-1.16	-0.80	0.19	0.12
Illiquid	-0.23	-1.95	-1.75	-1.21	-1.01	0.32
Illiquid-Liquid	-2.34	-2.94	-3.13	-2.98	-2.52	

In table 12 we replicate the unconditional portfolio sorting procedure, using the Amihud illiquidity measure. Like the analysis in table 5, we generally lack significant results. However, the results indicates that liquid stocks outperform the illiquid stocks in all size quintiles. These results are significantly different from zero, an corresponds to the findings in Section 7.2.

B4 – Unconditional portfolio sorting

In section 7.2 we conduct a conditional portfolio sorting to investigate variation in return across size quintiles. To further nuance the analysis, we conduct an unconditional portfolio sorting. This allows us to look at how returns are affected when sorting based on both size and liquidity independently.

We conduct the analysis by sorting the data in five quintiles based on size (market capitalization) and liquidity (percentage bid-ask spread). The 25 portfolios are then made from the intersection between the quintiles.

Table 13 – Unconditional portfolio sorting analysis

The table reports results from the unconditional portfolio sorting procedure. The stocks are sorted based on size (market capitalization) and liquidity (percentage bid-ask spread). In Panel A, we find the average number of stocks in each portfolio. In Panel B, we show output for the portfolios of interest, the Large Liquid portfolio and the Small Illiquid portfolio. Panel B reports average monthly return, average number of stocks in the portfolio, and t-statistics.

Panel A

Average number of stocks in each portfolio

	Large	2	3	4	Small
Liquid	30.00	9.62	2.80	1.29	1.50
2	7.67	17.19	9.81	5.76	2.38
3	2.89	8.00	12.38	12.29	6.48
4	2.00	2.76	9.71	12.57	14.57
Illiquid	1.29	3.53	7.62	10.71	19.52

Panel B

	Large Liquid	Small Illiquid
Average monthly return (%)	0.87	0.08
Average number of stocks	30	19.5
t-statistics	1.17	0.05

In table 13, Panel A, we show the number of stocks in each portfolio, when conducting the analysis. This shows that liquidity and size is connected as there are few small stocks categorized as liquid and few large stocks categorized as illiquid. The portfolios of interest are the Large Liquid portfolio and the Small illiquid portfolio which has the highest and second highest number of stocks respectively.

In table 13, Panel B we report the return of the Large Liquid portfolio and Small Illiquid portfolio. We observe that there are no significant differences in returns between the portfolios. This supports the results in section 7.2.

B5 – Fama French three-factor model regression using IML

We now seek to replicate the extended Fama-French analysis, using another liquidity risk factor. For this procedure we introduce a new factor, the IML (illiquid minus liquid) factor, as referred to by (Amihud, 2014). This factor is based on the return of a long-short equity portfolio which is long illiquid stocks and short liquid stocks, measured by the Amihud illiquidity measure (Amihud, 2014). In table 14 we consider the regression results.

Table 14 - Extended Fama-French Three-Factor Model regression analysis - IML

The table reports the extended Fama-French three-factor model regression results for the LargeCap decile, SmallCap decile, and the decile spread portfolio. The market risk premium (MRP) captures the sensitivity to the excess market return. The Small-minus-Big (SMB) captures the exposure to small minus big stocks. The High-minus-Low (HML) captures the exposure to value minus growth stocks. In addition to these three factors, we add the IML factor. This factor aims to capture liquidity risk, and is structured as a long illiquid, and short liquid, portfolio. Liquidity is measured using Amihud illiquidity measure. The constant reports the alpha for the portfolios.

	Dependent variable:		
	LargeCap decile	SmallCap decile	Decile spread
MRP	1.004***	0.982***	-0.022
	(0.007)	(0.089)	(0.089)
SMB	-0.354***	3.139***	3.493***
	(0.025)	(0.322)	(0.323)
HML	-0.016	0.323**	0.339**
	(0.012)	(0.162)	(0.163)
IML	0.021**	0.024	0.003
	(0.009)	(0.119)	(0.119)
Constant	0.055	1.469***	1.414***
	(0.034)	(0.440)	(0.441)
Observations	245	245	245
R^2	0.992	0.548	0.402
Adjusted R^2	0.992	0.540	0.393
Residual Std. Error (df = 240)	0.524	6.834	6.848
F Statistic (df = 4; 240)	7,804.417***	72.696***	40.413***
Note:			*p<0.1; **p<0.05; ***p<0.01

We observe some differences from the initial analysis. Most significant is that the IML is not statistically significant, like the LIQ factor is. This could indicate that the LIQ factor is a more appropriate factor to explain the size effect, meaning the decile spread portfolio, or that the liquidity risk is not significant in explaining the effect. We recognize this as a potential source of error, which impacts the conclusion of the analysis.

B6 – January effect for octile and quintile portfolios

In this section we re-examine the size effect, adjusting for the January effect, in the octile and quintile spread portfolios, and the SMB. This is to ensure the validity of the results found in the section 8.

Table 15 - Re-examining the size effect and the January effect – octile and quintile portfolios

The table reports the size effect in January and February-December separately for both the octile and the quintile portfolios. The table shows the average monthly return, monthly standard deviation, and t-statistics for the portfolios.

Portfolio	Avg. monthly return (%)	Monthly St. dev (%)	t-statistics
Octile spread – January	11.57	7.55	6.86
Octile spread – February-December	0.02	7.13	0.04
Quintile spread – January	9.20	6.82	6.03
Quintile spread – February-December	-0.22	6.48	-0.52
SMB – January	1.52	1.64	4.15
SMB – February-December	-0.19	1.40	-2.08

In table 15, we consider the results from the robustness analysis. We observe similar results as in section 9. Both the octile and quintile spread portfolios yield higher monthly returns in January, both significantly different from zero. For the months between February and December the results are close to zero, and not insignificant. For the SMB portfolio we also find an effect in January, and negative return between February and December, which is significantly different from zero. The supports the results in section 9, which indicate that when adjusting for January, the size effect is no longer significant.

Appendix 3: OLS assumptions and model diagnostics

C1 – Ordinary least square regression

We use Ordinary Least Squares (OLS) regression to examine the relationship between our portfolios and the systematic risk factors. OLS is a statistical method used to estimate the relationships between a dependent variable and one or more independent variables (Greene, 2003). This method allows us to measure the risk-adjusted excess return as the constant, referred to as the alpha. OLS models linear relationships and has some assumptions that need to be fulfilled for validity and inference purposes. These include linearity, no perfect multicollinearity, homoscedasticity, no autocorrelation, independence of errors, and normal distribution of error terms (Greene, 2003). Adherence to these assumptions ensures the validity and credibility of the model's inferences (Greene, 2003).

C2 – Validity tests

To validate the regression models reported, several diagnostic tests are conducted to evaluate whether the assumptions of OLS regression are met. The tests used are described below.

The Variance Inflation Factor (VIF) is a measure used to detect the presence of multicollinearity in regression models (Greene, 2003). VIF quantifies the extent of correlation between two explanatory variables in the model. A VIF value of one indicates no correlation, while a value greater than five or ten typically suggests significant multicollinearity (Greene, 2003).

The Shapiro-Wilk test is a statistical test for assessing the normality of the data (Shapiro & Wilk, 1965). It tests the null hypothesis that a sample comes from a normally distributed population. The test calculates a W-statistic where a value close to one suggests normal distribution (Shapiro & Wilk, 1965).

The Breusch-Pagan test is a statistical test that assesses the presence of heteroskedasticity in a regression model (Breusch & Pagan, 1979). The test checks the null hypothesis that the error variances are all equal, homoscedastic, against the alternative hypothesis that the error variances are a function of the independent variables in the model, heteroskedasticity. A significant test result indicates the presence of heteroscedasticity (Breusch & Pagan, 1979).

The Durbin-Watson test is a method for detecting the presence of autocorrelation in the residuals of a regression analysis (Durbin & Watson, 1950). The test statistic ranges from zero to four, where a value around two indicates no autocorrelation, values approaching zero suggest positive autocorrelation, and values close to four suggest negative autocorrelation (Durbin & Watson, 1950).

C3 – CAPM assessment

Table 16 – CAPM regression assumptions

The table reports results from diagnostics tests for LargeCap (Panel A), SmallCap (Panel B) and spread decile portfolio (Panel C) regressed against the CAPM. The table reports the type of test, statistic, test value and p-value in addition to a conclusion.

Panel A

Diagnostic Tests LargeCap decile portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.980	0.001	Non-normal residuals
Breusch-Pagan Test	BP statistic	0.978	0.323	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	1.944		No autocorrelation detected

Panel B

Diagnostic Tests SmallCap decile portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.989	0.3915	Residuals appear normal
Breusch-Pagan Test	BP statistic	2.303	0.1297	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	1.985		No autocorrelation detected

Panel C

Diagnostic Tests decile spread portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.980	0.001174	Non-normal residuals
Breusch-Pagan Test	BP statistic	0.978	0.3227	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	1.944		No autocorrelation detected

The diagnostic test results for the LargeCap decile and SmallCap decile portfolios show a potential concern regarding the normality of residuals. For both portfolios, the Shapiro-Wilk normality test indicates a low p-value suggesting the residuals do not follow a normal distribution. This can imply problems with inference and hypothesis testing. However, Greene (2003) discusses this issue and notes that following the central limit theorem this normality issue is accounted for with a sufficient number of observations. We therefore conclude that our model is valid from this perspective. Furthermore, none of the portfolios show evidence of heteroscedasticity or autocorrelation.

C4 – Fama-French three-factor model assessment

Table 17 – Fama-French three-factor regression assumptions

The table reports results from diagnostics tests for LargeCap (Panel A), SmallCap (Panel B) and spread decile portfolio (Panel C) regressed against the Fama-French three-factor model. The table reports the type of test, statistic, test value and p-value in addition to a conclusion.

Panel A

Diagnostic Tests LargeCap decile Portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.984	0.00543	Potential non-normal residuals
Breusch-Pagan Test	BP statistic	2.807	0.4224	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	1.927		No autocorrelation detected
Variance Inflation Factor (VIF)	MRP	1.019		No multicollinearity detected
Variance Inflation Factor (VIF)	SMB	1.091		No multicollinearity detected
Variance Inflation Factor (VIF)	HML	1.094		No multicollinearity detected

Panel B

Diagnostic Tests SmallCap Decile Portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.961	2.826e-06	Non-normal residuals
Breusch-Pagan Test	BP statistic	2.859	0.4139	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	1.844		Minimal autocorrelation
Variance Inflation Factor (VIF)	MRP	1.019		No multicollinearity detected
Variance Inflation Factor (VIF)	SMB	1.091		No multicollinearity detected
Variance Inflation Factor (VIF)	HML	1.094		No multicollinearity detected

Panel C

Diagnostic Tests Decile Spread Portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.964	6.294e-06	Non-normal residuals
Breusch-Pagan Test	BP statistic	3.091	0.3779	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	1.866		Minimal autocorrelation
Variance Inflation Factor (VIF)	MRP	1.019		No multicollinearity detected
Variance Inflation Factor (VIF)	SMB	1.091		No multicollinearity detected
Variance Inflation Factor (VIF)	HML	1.094		No multicollinearity detected

The diagnostic tests conducted on the LargeCap decile, SmallCap decile, and spread portfolios reveal issues with the normality of residuals. However, with the same argument used in the discussion of normality for the CAPM model, we conclude that the model is valid based on a sufficient number of observations. The results show no concerns regarding heteroskedasticity. The Durbin-Watson statistics for the SmallCap decile and decile spread portfolio show indications of minimal autocorrelation, but in a small regard that should not be a major problem for the model. The VIF results are close to one for the MRP, SMB, and HML variables across all portfolios, suggesting that multicollinearity is not an issue.

C5 – Extended Fama-French three-factor assessment

Table 18 – Extended Fama-French three-factor model assumptions

The table reports results from diagnostics tests for LargeCap (Panel A), SmallCap (Panel B) and spread decile portfolios (Panel C) regressed against the Fama-French three-factor model, plus the LIQ factor. The table reports the type of test, statistic, test value, and p-value in addition to a conclusion.

Panel A

Diagnostic Tests LargeCap Decile Portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.989	0.06374	Residuals are likely normal
Breusch-Pagan Test	BP statistic	4.581	0.333	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	2.064		No autocorrelation detected
Variance Inflation Factor (VIF)	MRP	1.163		No multicollinearity detected
Variance Inflation Factor (VIF)	SMB	1.249		No multicollinearity detected
Variance Inflation Factor (VIF)	HML	1.122		No multicollinearity detected
Variance Inflation Factor (VIF)	LIQ	1.298		No multicollinearity detected

Panel B

Diagnostic Tests SmallCap Decile Portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.965	1.15e-05	Non-normal residuals
Breusch-Pagan Test	BP statistic	6.633	0.1566	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	2.032		No autocorrelation detected
Variance Inflation Factor (VIF)	MRP	1.163		No multicollinearity detected
Variance Inflation Factor (VIF)	SMB	1.249		No multicollinearity detected
Variance Inflation Factor (VIF)	HML	1.122		No multicollinearity detected
Variance Inflation Factor (VIF)	LIQ	1.298		No multicollinearity detected

Panel C

Diagnostic Tests Decile Spread Portfolio

Test	Statistic	Value	P-Value	Conclusion
Shapiro-Wilk Normality Test	W-statistic	0.967	1.663e-05	Slight deviation from normality
Breusch-Pagan Test	BP statistic	6.587	0.1594	Homoscedasticity assumed
Durbin-Watson Test	DW statistic	2.037		No autocorrelation detected
Variance Inflation Factor (VIF)	MRP	1.163		No multicollinearity detected
Variance Inflation Factor (VIF)	SMB	1.249		No multicollinearity detected
Variance Inflation Factor (VIF)	HML	1.122		No multicollinearity detected
Variance Inflation Factor (VIF)	LIQ	1.298		No multicollinearity detected

The diagnostic test tables for the LargeCap decile, SmallCap decile, and decile spread portfolios reveal certain concerns regarding the normality of residuals for the SmallCap and decile spread portfolios. Based on previous argumentation we still conclude that the model is valid. For the LargeCap decile portfolio, the Shapiro-Wilk test suggests that residuals are normal at five percent significance level. The Breusch-Pagan test for all three portfolios yields p-values that do not suggest heteroskedasticity. Autocorrelation does not seem to be a concern, as indicated by the Durbin-Watson statistics being close to two for all portfolios. Moreover, VIF for the MRP, SMB, HML, and LIQ factors across all portfolios shows no evidence of multicollinearity.

C6 - Pairwise Wilcoxon test

We perform a pairwise Wilcoxon test to formally test if differences in liquidity between the decile portfolios are statistically significant. This is imperative for the further analyses testing liquidity-based theories, as they assume small stocks to be generally illiquid, and large stocks to be generally liquid (Amihud & Mendelson, 1986; Acharya & Pedersen, 2005).

Table 19 – Pairwise Wilcoxon test matrix

The matric show p-values for pairwise Wilcoxon tests for the decile portfolios.

	P1	P2	P3	P4	P5	P6	P7	P8	P9
P2	< 2e-16								
P3	< 2e-16	< 2e-16							
P4	< 2e-16	< 2e-16	< 2e-16						
P5	< 2e-16	< 2e-16	< 2e-16	< 2e-16					
P6	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16				
P7	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	6.8e-08			
P8	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	0.053		
P9	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	
P10	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16	< 2e-16

All but one pair show significantly different distributions. This supports the general assumption of small stocks being illiquid, and large stocks being liquid.